行政院國家科學委員會專題研究計畫 成果報告

常數彈性變異數過程下考量破產程序的資本結構模型與實證分析

研究成果報告(精簡版)

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中文摘要： 本文延伸 Francois and Morellec (2004)與 Broadie and Kaya (2007)的研究，使用二元樹模型，評價具到期日與第十一章破產法規架構下之公司債。為了使模型更有彈性，並更貼近實際，我們允許標的資產價格波動度變動，亦即發展一個常數彈性變異數(CEV)過程下，考量破產程序的資本結構模型。本研究數值分析結果指出，當重整的期限越長，或是 CEV 過程的彈性係數越小時，公司債價值越低。

英文摘要：

附件一

行政院國家科學委員會補助專題研究計畫 ■ 成 果 報 告
□期中進度報告

常數彈性變異數過程下考量破產程序的資本結構模型與實證分析

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成果報告類型(依經費核定清單規定繳交)：■精簡報告　□完整報告

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執行單位： 國立交通大學財務金融研究所

中　華　民　國　100 年　10　月　31　日

1. Introduction

The structural credit risk modeling or defaultable claim modeling is pioneered by the seminal paper by Black and Scholes (1973) in which corporate liabilities can be viewed as a covered call — own the asset but short a call option. Later on, Merton (1974) rigorously elaborates the pricing of corporate debt. This Black-Scholes-Merton approach of modeling default claims is named structural approach since the model explicitly ties default risk to the firm value process and its capital structure. In the capital structure models, the most notable developments are the works by Leland (1994) and Leland and Toft (1996). These two studies extend the endogenous default approach by Black and Cox (1976) to include the tax shield of debt and bankruptcy costs, and explicitly analyze the static tradeoff theory of capital structure. Various important issues including the endogenous default boundary, leverage ratio, debt value, debt capacity, yield spread, and even potential agency costs are discussed in detail. Leland (1994) and Leland and Toft (1996) provide closed-form expressions of corporate securities as well as comparative statics. Their methodology and stationary debt structures are widely adopted in the literature. In addition, these two models have been served as benchmark models in the optimal capital structure literature.

According to the Leland (1994) and Leland and Toft (1996) models, bankruptcy is triggered when shareholders find that running the company is no longer profitable, even when the cash flows produced by the assets may still capable of servicing the debt. Therefore, bankruptcy is determined endogenously rather than by a certain level of net asset or cash flow constraint While the Leland (1994) and the Leland and Toft (1996) models provide some insights of the capital structure issues, their predicted optimal leverage and yield spreads seem not to be consistent with historical average. Consequently, the more recent studies introduce additional realistic features in order to meliorate the model-predicted leverage and yield spreads. A line of research has been advanced by Fan and Sundaresan (2000) who introduce a bargaining game between shareholders and debtholder to determine the optimal bankruptcy boundary. Later on, Francois and Morellec (2004) further incorporate the Chapter 11 bankruptcy proceedings into the capital structure and bankruptcy decisions.

.The U.S. bankruptcy code, which includes a liquidation process (Chapter 7) and reorganization process (Chapter 11), is used to deal with the issues of a firm facing financial distress. Contractual agreements and bankruptcy laws may cause different outcome when the firm fails to make debt payments and declares bankruptcy. For example, bankruptcy may lead to liquidation under Chapter 7, reorganization under Chapter 11, or the debt renegotiation between debt holders and equity holders. Due to the reorganization period granted by the bankruptcy code of Chapter 11, one can therefore regard the corporate equity as a consecutive down-and-out Parisian option, which permits a period of time the underlying asset can stay below the bankruptcy boundary before liquidation.

On the other hand, in order to introduce realism into the possible movement of the firm value, Hilberink and Rogers (2002) and Chen and Kou (2009) add jumps into the asset value process, which implies that the asset value of the firm can suddenly drop drastically and causes a default. Hence, default is not a predictable event any more, and the credit spread increases in the short-term. However, stochastic volatility feature, another important and commonly adopted assumption in option pricing, is still rare in structural modeling literature. It is also reasonable to conjecture that the volatility of firm value process is not a constant through time. The firms value volatility could be higher while the firm value is low (The well-known leverage effect in option pricing).

However, in order to obtain closed-form solutions of corporate securities, researchers need to impose some unrealistic assumptions to avoid time and path dependence, for example, the stationary debt structure, as assumed by Leland (1994) and the Leland and Toft (1996). These models usually price infinite maturity bonds or continuously rollover bonds although these bonds are rarely used in practice, especially when firms are in financial distress. By contrast, for finite maturity bonds, it is difficult to obtain analytical solutions in models of bankruptcy proceedings that include grace periods since it introduces path dependency. To overcome the difficulties, Broadie and Kaya (2007) are the first to introduce binomial lattice approach, widely adopted in option pricing literature, into structural credit risk modeling. Therefore, in this study, we extend the work by Broadie and Kaya (2007) to develop a capital structure model with the feature of Chapter 11 bankruptcy proceedings under the CEV process.

The CEV model proposed by Cox (1975) and Cox and Ross (1976) is complex enough to allow for changing volatility and simple enough to provide a closed form solution for options with only two parameters. The CEV process has the advantage that the volatility of the underlying asset is linked to the level of underlying asset, which is consistent with the empirical observation that the underlying asset volatility tend to change as the underlying asset moves up and down. Although the CEV model is not as general and flexible as the stochastic volatility models, its simplicity may still be worth exploring since those generalized models are expensive to implement, especially when one applies to a capital structural model with a more realistic and complex setting of bankruptcy proceedings. When applied to structural models, especially in the case of Chapter 11 modeling with path-dependency, high-dimensional lattice models are very expensive. One distinguish feature of the CEV model as opposed to other stochastic volatility models is that it requires only a two-dimensional lattice (Nelson and Ramaswamy (1990) and Boyle and Tian (1999)) as the models under geometric Brownian motion assumption. Its parsimonious setting accommodates the well-known leverage effect, the inverse relationship between the level of the underlying primitive variable and its variance of return.

This study contributes to existing literature in applying the CEV process to structural credit risk modeling, which has not yet been explored in literature. We extend the binomial lattice method by Broadie and Kaya (2007) to develop a capital structure model, which incorporates finite maturity as well as the feature of Chapter 11 bankruptcy proceedings. Our numerical results also show that the value of risky debt can be substantially affected by these features we considered – When the reorganization period is longer or the elasticity constant β is smaller, the value of corporate risky debt will be lower. Moreover, as has been mentioned, this model equipped with changing volatility feature can be implemented without too much extra computational effort and can be used in practical applications.

The remainder of this paper is organized as follows. Section 2 reviews literature. Section 3 describes our model and how to incorporate the CEV process, Chapter 11, and finite maturity features by using the binomial lattice method. Section 4 presents the numerical results in detail and illustrates the model implications. Section 5 summarizes and concludes our paper.

2. Literature Review

2.1 Modeling Bankruptcy Proceedings in Capital Structure Models

We summarize related literature in two of the most relevant fields, namely, modeling bankruptcy proceedings in capital structure models, and the CEV process and option pricing.

(I) Modeling bargaining between holders and equity holders – The Fan and Sundaresan Model (2000)

Fan and Sundaresan (2000) propose a game-theoretic setting which incorporates equity's ability to force concessions and varying bargaining powers to the debt holders and equity holders. Two cases are discussed in their study when the firm's asset value falls below a certain boundary: Debt-equity swap and strategic debt service. In the case of debt-equity swap, the debtholders exchange their claims for equity. In strategic debt service, borrowers stop making the contractual coupon and start servicing debt strategically until the firm's asst goes back above the boundary again.

Some key assumptions under their framework include: (1) During the default period, the tax benefits are lost. (2) Asset sales for dividend payments are prohibited. (3) The firm can be liquidated only at a cost. The proportional cost is $\alpha (0 \le \alpha \le 1)$ and the fixed cost is $K (K \ge 0)$. Note that debt holders have strict absolute priority upon liquidation.

(4) The asset value of the firm follows the lognormal diffusion process

$$dV = (\mu - q)Vdt + \sigma V dB_t \qquad \textbf{(1)}$$

where μ is the instantaneous expected rate of return on the firm gross of all payout, and q is the firm's cash payout ratio.

We present only the results of strategic debt service which will be further explored in our study. If the equity and debt holders can reach an agreement of temporary coupon reduction when the firm is under financial distress, the firm will not lose its potential future tax benefits. At the trigger point for the strategic service $\tilde{V}_S$, both parties will bargain the total value of the firm $v(V)$.

The driving force behind strategic behavior is the presence of proportional and fixed costs of liquidation. The bargaining power of equity holder clearly depends on the liquidation cost $\alpha V_S + K$. Fan and Sundaresan (2000) endogenize both the reorganization boundary and the optimal sharing rule between equity and debt holders upon default.

The total value of the firm is

$$v(V) = \begin{cases} V + \frac{T_C C}{r} - \frac{\lambda_+}{\lambda_+ - \lambda_-} \frac{T_C C}{r} \left(\frac{V}{\tilde{V}_S} \right)^{\lambda_-}, & when\ V > \tilde{V}_S \\ V + \frac{-\lambda_-}{\lambda_+ - \lambda_-} \frac{T_C C}{r} \left(\frac{V}{\tilde{V}_S} \right)^{\lambda_+}, & when\ V \le \tilde{V}_S \end{cases} \quad \textbf{(2)}$$

where $\lambda_\pm = 0.5 - \frac{r-\beta}{\sigma^2} \pm \sqrt{\left[0.5 - \frac{r-\beta}{\sigma^2}\right]^2 + \frac{2r}{\sigma^2}}$

For any $V \le \tilde{V}_S$, Fan and Sundaresan set the optimal sharing rule between shareholders and debtholders as

$$\tilde{E}(V) = \tilde{\theta} v(V) \text{ and } \tilde{D}(V) = (1-\tilde{\theta}) v(V). \quad \textbf{(3)}$$

The Nash solution to the bargaining game can be characterized as

$$\tilde{\theta}^* = \arg\ \max \{\tilde{\theta} v(V) - 0\}^{\eta} \{(1-\tilde{\theta}) v(V) - \max[(1-\alpha)V - K, 0]\}^{1-\eta}$$
$$= \min\left[\eta - \eta \frac{(1-\alpha)V - K}{v(V)}, \eta\right] \quad \textbf{(4)}$$

where η denotes bargaining power of shareholders.

At equilibrium, Fan and Sundaresan (2000) show that debt holders accept less than the contractual coupon and still permit shareholders to run the company. This results in deviations from APR, in which the shareholders will get a bigger share and the debtholders will get a less proportion of the firm. But both parties will be better off. Therefore, the concession of debtholders is well explained under the Fan and Sundaresan's framework. They then solve the trigger point for strategic debt service (K=0 case) and the strategic service amount paid to debt holders $S(V) = (1-\eta\alpha)qV$.

The Fan and Sundaresan model shows that debt renegotiation encourages early default and increases credit spreads on corporate debt, given that shareholders can renegotiate in distress to avoid inefficient and costly liquidation. It might be the interest of debt holders to forgive part of the debt service payments if it can avoid the wasteful liquidations, which can be shared by the two claimants. If shareholders have no bargaining power, no strategic debt service takes place and the model converges to the Leland model. Furthermore, by introducing the possibility of renegotiating the debt contract, the default can occur at positive equity value. This is in contrast to the Leland's (1994) model in that the default occurs when the equity value reaches zero as a consequence of that issuing new equity is costless and the APR is respected.

(II) Modeling Chapter 11 Bankruptcy Proceedings – Francois and Morellec (2004)

Francois and Morellec (2004) extend the Fan and Sundaresan (2000) model to incorporate the possibility of Chapter 11 filings. Under their setup, shareholders hold a Parisian down-and-out option on the firm's asset, i.e., shareholders have a residual claim on the cash flows generated by the firm unless the value of these assets reaches the default threshold and remains below that threshold for the exclusive period. It is generally acknowledged that there are two types of defaulting firms: First, firms that are economically sound, but default only due to temporary financial distress, and recover under Chapter 11. Second, firms that are economically unsound, keep on losing value under Chapter 11, and eventually liquidated under Chapter 7. The modeling philosophy comes from the empirical studies which show that most firms emerge from Chapter 11. Only a few are eventually liquidated under Chapter 7 after filing Chapter 11.

Following the Nash bargaining game of Fan and Sundaresan's approach[1], Francois and Morellec presume the firm renegotiates its debt obligations whenever the asset value falls below a constant threshold V_B. However, a major difference from Fan and Sundaresan's model is that Francois and Morellec assume that a proportional costs φ are borne by the company during the renegotiation process. The costs of financial distress incurred by the firm filing Chapter 11 are ignored in the prior research. Even though the costs implied by private workouts generally are low, Chapter 11 filings are associated with large costs of financial distress that may affect shareholder's default decision. Note that Leland (1994) only allows liquidation while Fan and Sundaresan (2000) only permit private workouts.

Francois and Morellec solve the endogenous default barrier by maximizing equity value, and provide closed-form solutions for corporate debt and equity values. They find that the possibility to renegotiate the debt contract has ambiguous impact on leverage choices and increases credit spreads on corporate debt. The sharing rule of cash flows during bankruptcy has a large impact on optimal leverage, while credit spread on corporate debt shows little sensitivity to the varying bargaining power.

(III) Binomial Lattice Method for Modeling Chapter 11 Proceedings – Broadie and Kaya (2007)

Broadie and Kaya (2007) develop a binomial lattice method that can be used handle more real but complex structural models such as the finite maturity case of Leland (1994) or the models of Brodie, Chernov, and Sundaresan (2007). Broadie and Kaya (2007) assume that the firm's asset value V_t of is independent of the capital structural choices and its evolution under the risk-neutral measure Q as follows:

$$\frac{dV_t}{V_t} = (r-q)dt + \sigma dW_t \qquad \textbf{(5)}$$

[1] Note that the specification for the bargaining game within Francois and Morellec's framework is slightly different from that of Fan and Sundaresan (2000). Francois and Morellec focus on Chapter 11 filings— court-supervised debt renegotiation in contrast to the private workouts by Fan and Sundaresan. Therefore, the automatic stay of assets prevents shareholders from liquidating the firm's assets. Nevertheless, renegotiation plan requires each participant to receive a payoff that exceeds the liquidating value of its claims. As a result, bondholders' payoff has to exceed $(1-\alpha)V_B$ and this results in the difference of the specification for the bargaining game.

where W_t is a standard Brownian motion under *Q*, *r* is the constant risk-free rate, *q* is the payout ratio (or cash flow) of the firm, σ is the volatility of asset returns.

The instantaneous cash generated by the firm is denoted as δ_t and $\delta_t = qV_t$. They assume that the firm issues a bond that promises to pay coupons at a continuous constant rate C, until a default event occurs. The coupon is paid from the cash flow δ_t generated by the firm at time *t*. Equity holders receive the remainder $\delta_t - C$ in the form of dividends. In the case that cash flows are not enough to make coupon payments, i.e., $C > \delta_t$, $(\delta_t - C)$ is the negative cash flow for equity holders. Broadie and Kaya (2007) show in their Proposition 1 that this is equivalent to dilution of equity by the firm. More importantly, this treatment does not violate the limited liability requirement.

They next apply the standard binomial branching process and compute the claim of equity holders after debt issuance, *E*. The claim of the debt holders, *D*, and the total firm value, *F*, at each node are given as follows:

$$E = e^{-r\Delta t}(pE_u + (1-p)E_d) \qquad \textbf{(6)}$$

$$D = e^{-r\Delta t}(pD_u + (1-p)D_d) \qquad \textbf{(7)}$$

$$F = e^{-r\Delta t}(pF_u + (1-p)F_d) \qquad \textbf{(8)}$$

The present value of equity ignoring the current coupon payment and the cash flow is denoted as $\tilde{E}$. If the difference between the coupon payment and the current cash flow is denoted by $\overline{C} = C - \delta$, Broadie and Kaya (2007) show that

$$E = \begin{cases} 0 & if\ \tilde{E} \le \overline{C} \\ \tilde{E} > \overline{C} & if\ \tilde{E} > \overline{C} \end{cases} \qquad \textbf{(9)}$$

This result indicates that one needs not go through tedious calculations of equity dilution at each step even if the firm's cash flow is not enough to cover the coupon payment.

2.2 Parisian Options

Under the setup of Francois and Morellec (2004), shareholders hold a Parisian down-and-out option on the firm's asset in the presence of Chapter 11 filings. Parisian option is a variant of barrier option. A barrier option is activated or deactivated as a threshold has been reached while the activation of a Parisian option depends on the time spend above or below the barrier. For example, a down-and-out option is void when the underlying spends more than a specified time strictly below the barrier.

Parisian option can be valued by various methods. Chesney et al. (1997) use Laplace inversion to compute Parisian option value. Avellaneda and Wu (1999) obtain a lattice scheme for calculating the price and sensitivities of such options. Costabile (2002) provides a discrete time algorithm to evaluate European Parisian options. Bernard et al. (2005) develop a new inverse Laplace transform method that is quick and appropriate to the pricing problem. Costabile (2002) provides a discrete time algorithm to evaluate European Parisian options with flat or exponential barriers. His approach is based on a combinatorial tool for counting the number of paths that remains below a barrier for a period strictly smaller than a pre-specified time interval.

In this paper, we use a variant of the lattice-based method, called the forward shooting grid (FSG), which has been successfully applied to price path-dependent options. The FSG approach was developed by Hull and White (1993) and Ritchken, Sankarasubramanian and Vijh (1993) for the pricing of American- and European-style Asian and lookback options. The FSG approach uses auxiliary state vector at each node on the lattice. The state vector can be used to capture the path-dependent feature of the option contract, like grace period of the asset price. This feature is closely related to the Chapter 11 bankruptcy code in the context of capital structure modeling.

2.3 The CEV Process and Option Pricing

An important issue in option pricing is to find a stock return distribution that allows returns to stock and its volatility to be correlated with each other. There is considerable empirical evidence that the returns to stocks are heteroscedastic and the volatility of stock returns changes with stock price. A great deal of empirical evidence indicates that stock volatility is negatively related to stock price, and it is so-called leverage effect first discussed by Black (1976). To accommodate this leverage effect, the Constant Elasticity of Variance (CEV) model by Cox (1975) and Cox and Ross (1976) relaxes the constant volatility assumptions of the Black-Scholes model and treats volatility as a deterministic function – as a power function of the price of the underlying asset. The rationale for an inverse relationship between the stock price and its variance of return can be explained by some simple economic arguments. Researchers use both financial and operating leverage arguments. A decline in a leveraged firm's stock price may lead to an increase in its debt-equity ratio, hence the riskiness of the stock increases. Even if a firm has no debt, the decline of the stock price can make it more difficult for the firm to meet its fixed costs and thus has effect to increase volatility.

The CEV model assumes the diffusion process for the stock is

$$dS = \mu S dt + \delta S^{\beta/2} dz, \qquad \textbf{(10)}$$

and the instantaneous variance of the percentage price change or return, σ^2, follows deterministic relationship:

$$\sigma^2(S,t) = \delta^2 S^{(\beta-2)} \qquad \textbf{(11)}$$

where the elasticity of this variance with respect to the stock price equals β.

If β =2, prices are lognormally distributed and the variance of returns is constant, which is the same as the well-known Black-Scholes model. If β <2, the stock price is inversely related to the volatility. Cox originally restricted $0 \le \beta < 2$. Emanuel and MacBeth (1982) extended his analysis to the case $\beta > 2$ and discuss its properties. However, Jackwerth and Rubinstein (2001) find that typical values of the β can fit market option prices well for post-crash period only when $\beta < 0$, and they called the model with $\beta < 0$ unrestricted CEV model[2]. In their empirical study, the difference of pricing performance of restricted CEV model ($\beta \ge 0$) and BS model is not significant.

When β <2, the nondividend-paying CEV call pricing formula is as follows:

$$C = S\left[\sum_{n=0}^{\infty} g(S' \mid n+1) G(K' \mid n+1+\frac{1}{2-\beta})\right] - Ke^{-r\tau}\left[\sum_{n=0}^{\infty} g(S' \mid n+1+\frac{1}{2-\beta}) G(K' \mid n+1)\right] \quad \textbf{(12)}$$

When β >2, the CEV call pricing formula is as follows:

[2] The unrestricted CEV model is mathematically legitimate. However, there are some economic arguments supporting on a restriction on the parameter β. For example, it is inconceivable for the stock index to have a significant probability of bankruptcy while this is likely with sufficiently negative β. See the detail in Jackwerth and Rubinstein (Page 12; 2001).

(Eq 2.1.4)

$$C = S\left[1-\sum_{n=0}^{\infty} g(S' \mid n+1+\frac{1}{2-\beta})G(K' \mid n+1)\right] - Ke^{-r\tau}\left[1-\sum_{n=0}^{\infty} g(S' \mid n+1)G(K' \mid n+1+\frac{1}{2-\beta})\right] \quad \textbf{(13)}$$

where $S' = \left[\dfrac{2re^{r\tau(2-\beta)}}{\delta^2(2-\beta)(e^{r\tau(2-\beta)}-1)}\right]S^{2-\beta}$

$$K' = \left[\frac{2r}{\delta^2(2-\beta)(e^{r\tau(2-\beta)}-1)}\right]K^{2-\beta}$$

$g(x \mid m) = \dfrac{e^{-x}x^{m-1}}{\Gamma(m)}$ is the gamma density function

$$G(x \mid m) = \int_x^{\infty} g(y \mid m)dy$$

C is the call price; *S*, the stock price; τ, the time to maturity; *r*, the risk-free rate of interest; *K*, the strike price; and β and δ, the parameters of the formula.

Schroder (1989) expressed the CEV call option pricing formula in terms of the noncentral chi-square distribution:

When β<2,

$$C = S_t Q(2y;2+2/(2-\beta),2x) - e^{-rt}K(1-Q(2x;2+2/(2-\beta),2y)) \quad \textbf{(14)}$$

When β>2,

$$C = S_t Q(2x;2+2/(2-\beta),2y) - e^{-rt}K(1-Q(2y;2+2/(2-\beta),2x)) \quad \textbf{(15)}$$

$Q(z;v,k)$ is a complementary noncentral chi-square distribution function with z, v, and k being the evaluation point of the integral, degree of freedom, and noncentrality, respectively, where

$$k = \frac{2r}{\delta^2(2-\beta)(e^{r(2-\beta)\tau}-1)}$$

$$x = kS_t^{2-\beta}e^{r(2-\beta)\tau}$$

$$y = kK^{2-\beta}$$

3. The model

This section presents the details of our capital structure model under the CEV process. We first denote the asset value of the firm V_t and use it as the primitive variable. We assume that the asset value V_t is independent of the capital structure and other financial decisions. The diffusion process follows the constant elasticity of variance (CEV) of Cox and Ross (1976) under the risk-neutral measure Q and it is given by

$$dV_t = (r-q)V_t dt + \sigma V_t^{\frac{\beta}{2}} dW_t \quad \textbf{(16)}$$

where W_t is a standard Brownian motion under Q, q is the payout ratio of the firm, and σ is the volatility of asset returns. β is a constant, know as the elasticity factor, and $0 \le \beta < 2$. In the case when $\beta = 2$, equation (1) reduces to geometric Brownian motion, and this implies that geometric Brownian motion is a special case of the CEV process.

3.1 A binomial lattice under the CEV process

We construct a discrete approximation for the CEV process using the binomial method. Nelson and Ramaswamy (1990) derived a binomial approximation of the stochastic process described in (1) in which a "computationally simple binomial tree" is proposed in order to let the number of nodes in the tree structure grows linearly with number of time intervals. We let

$$y = y(t,V).$$

Applying Ito's Lemma, the stochastic differential equation for y is

$$dy_t = \left(\frac{\partial y}{\partial t} + (r-q)V_t \frac{\partial y}{\partial V} + \frac{1}{2}\sigma^2 V_t^{\beta} \frac{\partial^2 y}{\partial V^2} \right) dt + \frac{\partial y}{\partial V} \sigma V_t^{\frac{\beta}{2}} dW_t \quad \textbf{(17)}$$

In order to have a constant diffusion coefficient for the Y-process, we let

$$\frac{\partial y}{\partial V} \sigma V_t^{\frac{\beta}{2}} = \upsilon, \quad \textbf{(18)}$$

for some positive constant υ. Equation (3) is equal to

$$\frac{\partial y}{\partial V} = \frac{\upsilon}{\sigma} V_t^{-\frac{\beta}{2}},$$

For $\beta \ne 2$, the transformation can be given by

$$y_t = \frac{\upsilon}{\sigma\left(1-\frac{\beta}{2}\right)} V_t^{1-\frac{\beta}{2}} \qquad \textbf{(19)}$$

and for $\beta = 2$, the transformation is given by

$$y_t = \frac{\upsilon}{\sigma}\ln\left(V_t\right)$$

Thus we can build up a computationally simple binomial tree to approximate the y-process. The two-dimensional grid in the (t, y)-space can be built as follows. The value y_t of the process at time *t*, after one period at time $t+1$, can rise to $y_t + \upsilon\sqrt{\Delta t}$ or decrease to $y_t - \upsilon\sqrt{\Delta t}$. In this way, we can build up the value of the y-process as

$$y_{t+i\Delta t}^{j} = y_t + \left(2j - i\right)\upsilon\sqrt{\Delta t}\,, \qquad i = 0,\ldots,n\,, \qquad j = 0,\ldots,i$$

where $y_{t+i\Delta t}^{j}$ represents the value on the binomial tree under y-process at time $t+i\Delta t$ after j up steps and $i-j$ down steps. Next, to build up a binomial tree with V-process on the two-dimensional grid in the (t,V) space, we can use the inverse transformation of (19)

$$V_t = \begin{cases} \left[\dfrac{\sigma\left(1-\frac{\beta}{2}\right)}{\upsilon} y_t\right]^{\frac{1}{1-\frac{\beta}{2}}} & , \text{ if } y_t > 0 \\ 0, & \text{otherwise.} \end{cases} \qquad \textbf{(20)}$$

Once we have constructed the binomial tree with V-process, we can define the probability of each up step. First we define $V_{t+i\Delta t}^{\underline{j}}$ to be the greatest $V_{t+i\Delta t}^{j}$, $j = 0,\ldots,i$, making $e^{r\Delta t}V_{t+(i-1)\Delta t}^{j} - V_{t+i\Delta t}^{j} \geq 0$, and $V_{t+i\Delta t}^{\overline{j}}$ to be the smallest $V_{t+i\Delta t}^{j}$, $j = 0,\ldots,i$, making $V_{t+i\Delta t}^{j} - e^{r\Delta t}V_{t+(i-1)\Delta t}^{j} \geq 0$. The probability with (t,V) space makes an up steps is

$$p_{t+(i-1)\Delta t}^{j} = \begin{cases} \dfrac{e^{r\Delta t}V_{t+(i-1)\Delta t}^{j} - V_{t+i\Delta t}^{\underline{j}}}{V_{t+i\Delta t}^{\overline{j}} - V_{t+i\Delta t}^{\underline{j}}} & \text{if } V_{t+i\Delta t}^{\overline{j}} > 0 \\ 0 & \text{otherwise} \end{cases} \qquad \textbf{(21)}$$

The probability of down steps is thus $q_{t+(i-1)\Delta t}^{j} = 1 - p_{t+(i-1)\Delta t}^{j}$. With the definition given above, $p_{t+(i-1)\Delta t}^{j}$ represents a probability for the evolution of the value of underlying asset in the binomial tree.

Next, we would like to compute the value of equity, debt and firm on the lattice. We denote equity value as E, the value of the debt holder as D, and the total firm value by F. For simplicity, we drop the indexes of the probability $p^{j}_{t+(i-1)\Delta t}$ hereafter. At current node, the present value of the equity is given by

$$E = e^{-r\Delta t}\left(pE_u + (1-p)E_d\right) \quad \textbf{(22)}$$

The values of D and F also can be calculated in the same way:

$$D = e^{-r\Delta t}\left(pD_u + (1-p)D_d\right) \quad \textbf{(23)}$$

$$F = e^{-r\Delta t}\left(pF_u + (1-p)F_d\right) \quad \textbf{(24)}$$

Assume that at the current node, the firm has to pay coupon payment C and firm's cash flow is δ_t. The present value of equity which do not consider current coupon payment and current firm cash flow is given by

$$\tilde{E} = e^{-r\Delta t}\left(pE_u + (1-p)E_d\right) \quad \textbf{(25)}$$

We denote the difference between the coupon payment and firm cash flow $\bar{C} = C - \delta_t$. When the coupon payment is less than the firm cash flow, $\bar{C}$ is negative and it means that excess firm cash flow over the coupon payment can be receive by equity holders. If $\bar{C}$ is positive, it means equity holders should raise money by equity dilution. Broadie and Kaya (2007) have shown that equity value at the current node is as follows:

$$E = \begin{cases} 0 & \text{if } \tilde{E} \le \bar{C} \\ \tilde{E} - \bar{C} & \text{if } \tilde{E} > \bar{C} \end{cases} \quad \textbf{(26)}$$

This result indicates that one needs not go through tedious calculations of equity dilution at each step even if the firm's cash flow is not enough to cover the coupon payment.

3.2 Setup for the bankruptcy with grace period and bargaining

In the real world, the equity holders can liquidate the firm under Chapter 7 of the U.S. bankruptcy code or renegotiate debt payments under Chapter 11. Under Chapter 11, the bankruptcy court allows the firm to restructure its debt during a certain grace period. Chapter 11 also prevents debt holders from liquidating the firm's asset. Therefore, a firm may declare bankruptcy under Chapter 11 when it is in financial distress, and it spends some time as a bankruptcy firm which does not make full coupon payment, and then recover to become a healthy firm.

Following Francois and Morellec (2004), we assume that equity holders decide to declare bankruptcy at a certain level of the firm asset value V_B, and a grace period G is granted by bankruptcy court. If the firm does not come out from bankruptcy at the end of the grace period, the firm is liquidated. When the firm is in bankruptcy, distress cost ω reduces the net firm cash flow. When the firm asset value is under the default boundary V_B, debt is serviced strategically. At the time bankruptcy is declared, the debt service is determined by the bargaining game between debt holders and equity holders.

We follow the setting of Fan and Sundaresan (2000) to determine the debt service using a Nash bargaining game. Denote the proportional liquidation cost as α. If the firm is liquidated at the bankruptcy point, the debt holders receive $(1-\alpha)V_B$ and equity holders receive nothing. If the firm is not liquidated, firm asset value will be F_{V_B}, and will be share between debt holders and equity holders.

We assume the bargaining power of the equity holders is η and the bargaining power of debt holders is $1-\eta$. If we denote the sharing rule at the bankruptcy point as θ, the incremental value gained by equity holders is θF_{V_B} and the incremental value gained by debt holders is $(1-\theta)F_{V_B}-(1-\alpha)V_B$.

The optimal sharing rule is

$$\theta^* = \arg\max\left\{\left[\theta F_{V_B}\right]^{\eta}\left[(1-\theta)F_{V_B}-(1-\alpha)F_{V_B}\right]^{1-\eta}\right\} \qquad \textbf{(27)}$$

and its solution is

$$\theta^* = \eta\left(1-\frac{(1-\alpha)V_B}{F_{V_B}}\right) \qquad \textbf{(28)}$$

As a result, at the bankruptcy point, the value of the claim of the equity holders is

$$\theta^* F_{V_B} = \eta\left(F_{V_B}-(1-\alpha)V_B\right) \qquad \textbf{(29)}$$

and the value of the claim of the debt holders is

$$\left(1-\theta^*\right)F_{V_B} = \left(1-\eta\right)\left(F_{V_B} - \left(1-\alpha\right)V_B\right) + \left(1-\alpha\right)V_B \qquad \textbf{(30)}$$

The bargaining game determines the value of equity and debt at the bankruptcy point through equation (29) and (30). Thus we do not need to know how the debt is serviced when firm is in bankruptcy, just need to know the total firm value F_B.

3.3 Binomial lattice computations

We use the binomial lattice under the CEV process as described in section 3.1. If the bond is a consol bond, the bankruptcy boundary will be constant and time independent. However, if the bond has finite maturity, the bankruptcy boundary will be time dependent. First we price infinite maturity debt. The bankruptcy boundary V_B is assumed to be known at each time step in the lattice. The optimal level of V_B can be solve numerically later. We assume that the default boundary V_B fits with the level of nodes on the lattice. If it is not on the lattice, we use the first node level that is higher than V_B to approximate V_B. We assume firm has issued a consol bond with coupon payment C. The effective tax rate is τ and all interest payments are tax deductible. In the binomial lattice, due to discrete time steps, the total firm cash flow at a certain node with asset value is given by:

$$\delta_t = V_t e^{q\Delta t} - V_t \qquad \textbf{(31)}$$

Following Broadie and Kaya (2007), according to the position of nodes on lattice, we next perform the binomial lattice computation among the following three conditions.

***<u>Condition (i): The firm is in a healthy condition</u>* (Nodes with $V > V_B$)**

The firm is in a healthy state in these nodes, the coupon payments are paid either by firm cash flow or equity dilution if firm cash flow is not enough to pay the coupon payments. The effective tax rate is τ. Equity, debt, and firm value can update as follows:

$$\text{If } \tilde{E} + \delta_t \geq \left(1-\tau\right)C\Delta t: \quad \begin{aligned} E &= \tilde{E} + \delta_t - \left(1-\tau\right)C\Delta t, \\ D &= C\Delta t + e^{-r\Delta t}\left(pD_u + \left(1-p\right)D_d\right), \\ F &= \delta_t + e^{-r\Delta t}\left(pF_u + \left(1-p\right)F_d\right) + \tau C\Delta t. \end{aligned} \qquad \textbf{(32)}$$

$$\text{If } \tilde{E}+\delta_t<(1-\tau)C\Delta t: \quad \begin{aligned} E &= 0, \\ D &= (1-\alpha)(V_t+\delta_t), \\ F &= (1-\alpha)(V_t+\delta_t). \end{aligned}$$

where $\tilde{E}$ is given in (10) and δ_t is given in (16).

***<u>Condition (ii): The firm is in bankruptcy</u>* (Nodes with $V<V_B$)**

The firm is in bankruptcy. The debt is serviced strategically and we do not know how the firm cash flow is shared between debt holders and equity holders. We can use equation (29) and (30) to determine the value of debt and equity at the bankruptcy point, so we only need to track of the firm value F when the firm is in bankruptcy. In addition, there are no tax benefits while the firm is in bankruptcy.

The total time spent below the default boundary V_B needs to be recorded. Let g record the length of time the firm spends in bankruptcy. Because we are working on the binomial lattice, g can only take discrete values. Let $\bar{g}$ denote the maximum number of time steps that the firm can spend in bankruptcy. We have $\bar{g}=G/\Delta t$, where G is the grace period. Assume $\bar{g}$ is an integer, and then g will be the value in $[0,1,\ldots,\bar{g}-1,\bar{g}]$. For a given node and a given g, there exists three possibilities in the next time steps. First, the firm comes out of bankruptcy next time step. Second, if $g=\bar{g}-1$ in the current node, and $V<V_B$ in the next time step, then the grace period will be in expiration and the firm will be liquidated. Finally, the firm can still be in bankruptcy without expired grace period in the next time step. Thus the value of g will be one higher than the current node. For each node, we need to keep track of the firm value in every possible state of g. Thus, $F[i]$ will represent the firm value at the current node when $g=i$. We can update the firm value as follows:

$$F[i]=\begin{cases} \bar{\delta}_t+e^{-r\Delta t}\left(pF_u[i+1]+(1-p)F_d[i+1]\right) & \text{for } i=1,\ldots,\bar{g}-1 \\ (1-\alpha)(V_t+\bar{\delta}_t) & \text{for } i=\bar{g} \end{cases} \quad \textbf{(33)}$$

where

$$\bar{\delta}_t=V_te^{(q-\omega)\Delta t}-V_t \quad \textbf{(34)}$$

and $\overline{\delta}_t$ represents the distress cost adjusted cash flow of the firm.

***<u>Condition (iii): The last healthy state before the firm into bankruptcy or the first state after the firm out of bankruptcy</u>* — (Nodes with $V = V_B$)**

This node is the last healthy state before firm goes into bankruptcy or the first healthy state the firm just comes out from bankruptcy. The equity and debt values can be calculated using equation (29) and (30) after firm value is computed. We update equity, debt, and firm values as follows:

$$\begin{aligned}
F[0] &= \delta_t + e^{-r\Delta t}\left(pF_u + (1-p)F_d[1]\right), \\
F[i] &= \overline{\delta}_t + e^{-r\Delta t}\left(pF_u + (1-p)F_d[1]\right) \text{ for } i = 1,\ldots,\overline{g}, \\
E &= \eta\left(F[0] - (1-\alpha)V_B\right), \\
D &= (1-\eta)\left(F[0] - (1-\alpha)V_B\right) + (1-\alpha)V_B.
\end{aligned} \quad \textbf{(35)}$$

$F[0]$ represents the value of the firm at the bankruptcy boundary V_B that has never been in bankruptcy, and it is the value for the node reaching V_B from above. The $F[i]$ is the value of the firm at the bankruptcy boundary V_B just coming out from bankruptcy. As the result, $F[i]$ takes into account the distress cost, while $F[0]$ does not.

3.4 Pricing finite maturity debt

We can use the procedure described in Section 3.3 for pricing finite maturity bond with coupon C, face value P and maturity T.At maturity, the face value and the coupon payment should be paid; otherwise the firm will be liquidated. If the firm is still under the bankruptcy boundary V_B when the bond matures, the firm will be liquidated. The terminal values will be calculated as follows:

1. **Nodes with $V > V_B$**

$$\begin{aligned}
&\text{If } V_T + \delta_T \geq (1-\tau)C\Delta t + P: && E = V_T + \delta_T - (1-\tau)C\Delta t - P \\
& && D = C\Delta t + P \\
& && F = V_T + \delta_T + \tau C\Delta t \\
&\text{If } V_T + \delta_T < (1-\tau)C\Delta t + P: && E = 0 \\
& && D = (1-\alpha)(V_T + \delta_T) \\
& && F = (1-\alpha)(V_T + \delta_T)
\end{aligned} \quad \textbf{(36)}$$

2. Nodes with $V = V_B$

$$F[i] = (1-\alpha)\left(V_T + \overline{\delta}_T\right) \text{ for } i = 1,\ldots,\overline{g} \quad \textbf{(37)}$$

3. Nodes with $V < V_B$

$$\begin{aligned}
&\text{If } V_T + \delta_T \geq (1-\tau)C\Delta t + P: && E = V_T + \delta_T - (1-\tau)C\Delta t - P \\
& && D = C\Delta t + P \\
& && F[0] = V_T + \delta_T + \tau C\Delta t \\
& && F[i] = V_T + \overline{\delta}_T + \tau C\Delta t \text{ for } i = 1,\ldots,\overline{g} \\
&\text{If } V_T + \delta_T < (1-\tau)C\Delta t + P: && E = 0 \\
& && D = (1-\alpha)(V_T + \delta_T) \\
& && F[0] = (1-\alpha)(V_T + \delta_T) \\
& && F[i] = (1-\alpha)\left(V_T + \overline{\delta}_T\right) \text{ for } i = 1,\ldots,\overline{g}
\end{aligned} \quad \textbf{(38)}$$

<u>Optimal bankruptcy boundary</u>

We assume that V_B is a vector that contains the bankruptcy boundary for each time steps. The optimal V_B in the finite maturity setting will not be constant, but will be time dependent since the remaining value of the bond is changing over time. Therefore, we assume a functional form for the bankruptcy boundary and let the equity holders choose a parameter of that function to maximize the equity value. Following Broadie and Kaya (2007), we use a linear function of the riskless bond price.

$$V_B^t = \phi P_t \quad \textbf{(39)}$$

V_B^t is the bankruptcy boundary at an intermediate time t, P_t is the riskless bond price at time t, and ϕ is a positive number that is time independent. Also, if V_B^t is not on the lattice, we use the first node level that is higher than V_B^t to approximate V_B^t.

4. Numerical Result

4.1 Numerical method

4.1.1 The FSG (Forward Shooting Grid) approach

We have already described the binomial lattice computation under the CEV process in section 3. The most crucial and time consuming part is to deal with the path-dependent feature of the grace period. In this paper, we adopt the Forward Shooting Grid (FSG) approach developed by Hull and White (1993) to cope with nodes which are under the bankruptcy boundary. Let $g(k, j)$ denote the grid function. The binomial tree of the FSG algorithm can be represented by

$$V[m-1, j;k] = \left\{ p_u V[m, j+1; g(k, j+1)] + p_u V[m, j+1; g(k, j+1)] \right\} e^{-r\Delta t} \qquad \textbf{(40)}$$

$$g(k, j) = (k+1)\mathbf{1}_{\{x_j \le V_B\}} \qquad \textbf{(41)}$$

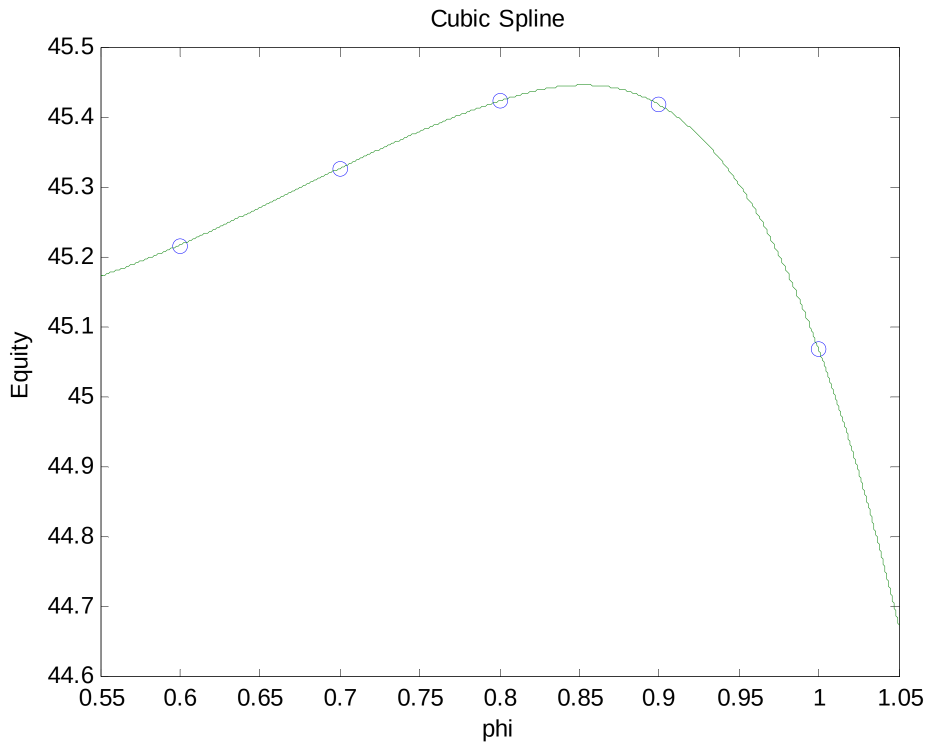
Note that before we move to next time step, it is necessary to compute $V[m, j;k]$ for all index k for $k = \overline{g}-1, \overline{g}-2, \cdots, 0$. In order to shorten the computation time and enhance efficiency, we do not compute k in all nodes. We only need to compute the nodes which are under the bankruptcy boundary. In addition, it is also not necessary to compute V of various k for the nodes which are below the bankruptcy boundary for more than $\overline{g}$ levels. This is because the asset value of the firm can by no means go back to the level of bankruptcy boundary within grace period.

4.1.2 Determination of optimal bankruptcy boundary

The optimal bankruptcy boundary must be chosen to maximize the equity value numerically. In the case of infinite maturity debt, we choose an arbitrary bankruptcy boundary that is lower than optimal boundary. And next we start to increase the boundary on the lattice and reprice the equity value. The equity value first increases and then starts to decrease after it reaches the maximum value when we move the boundary up on the boundary. Therefore, we stop moving the boundary when the equity value starts to decrease. As a result, we can obtain the discrete observation points and fit a cubic spline interpolation to approximate the exact functional form and use it to find the maximum value of equity and the optimal bankruptcy boundary. In the finite maturity debt case, under the assumption of $V_B^t = \phi P_t$, equity holders will choose a ϕ to maximize the equity value. Similar to the case of infinite maturity debt, we choose arbitrary ϕs and reprice the equity values. We then need to search for the appropriate ϕ and fit a cubic spline interpolation for the maximum equity value. Figure 1 illustrates the cubic spline interpolation to find the equity maximizing boundary, $V_B^t = \phi P_t$, of finite maturity debt case.

Figure 1 Illustration of Cubic Spline Interpolation for Equity

The model parameters are $V_0 = 100$, $\sigma = 20\%$, $C = 3$, $P = 60$, $r = 5\%$, $q = 3\%$, $\alpha = 50\%$, ω=1%, $\tau = 25\%$, $\eta = 50\%$, $G = 1$, $\beta = 1$, $T = 5$ $\Delta t = 0.005$ *years* equity is 45.4476

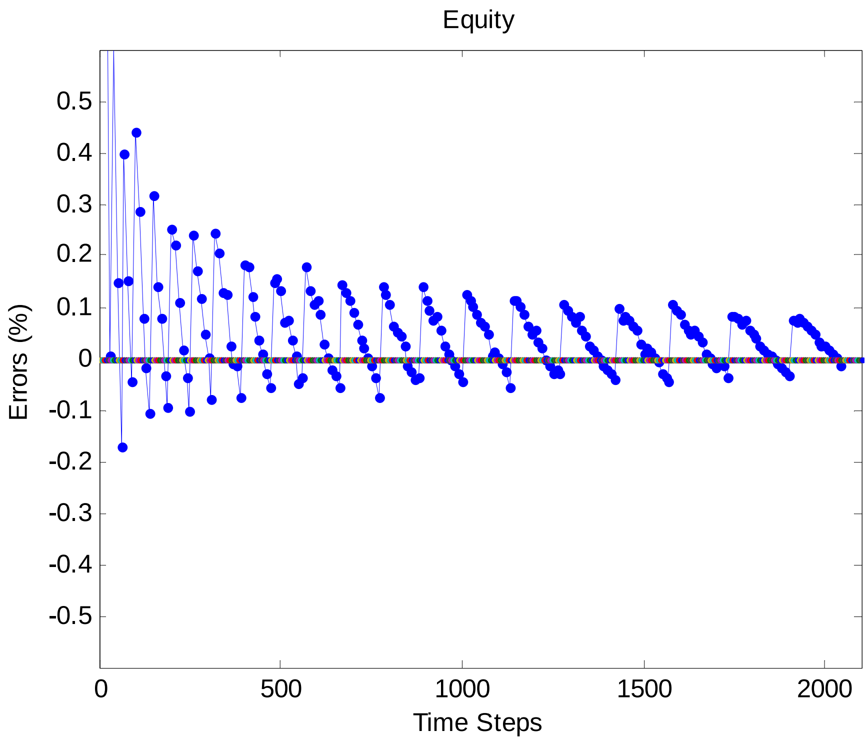


4.1.3 Convergence of the binomial lattice method

To decide the appropriate number of time steps in our numerical study, the convergence of lattice approach must be analyzed. Therefore, we compute the equity and debt values with 5000 time steps and set them as the true values. Then we analyze our pricing results by comparing the values obtained from the lattice approach with the "true value" describe above. Figure 2 and Figure 3 show, under $\beta = 1$ and $\beta = 0.5$, respectively, the convergence of equity and debt pricing errors as the number of time steps increases. The oscillation of pricing errors is due to the relative positioning of the lattice nodes and the bankruptcy boundary. Since the bankruptcy boundary is determined endogenously by maximizing equity value, there is no way to force the bankruptcy boundary to be laid on the lattice node to solve the oscillating behavior of the errors. Nonetheless, one can observe that the size of pricing errors is relatively small when we use more than 1000 time steps. The largest error is less than 0.2% of the true value, and thus we will use Δt =0.005 (=5/1000) to perform the numerical study in the later section.

Figure 2. Convergence of Equity and Debt Errors (β =1)

The model parameters are $V_0 = 100$, $\sigma = 20\%$, $C = 3$, $P = 60$, $r = 5\%$, $q = 3\%$, $\alpha = 50\%$, ω=1%, $\tau = 25\%$, $\eta = 50\%$, $G = 1$, $\beta = 1$, $T = 5$. The true value of equity is 45.4671, and the true value of debt is 55.0929.



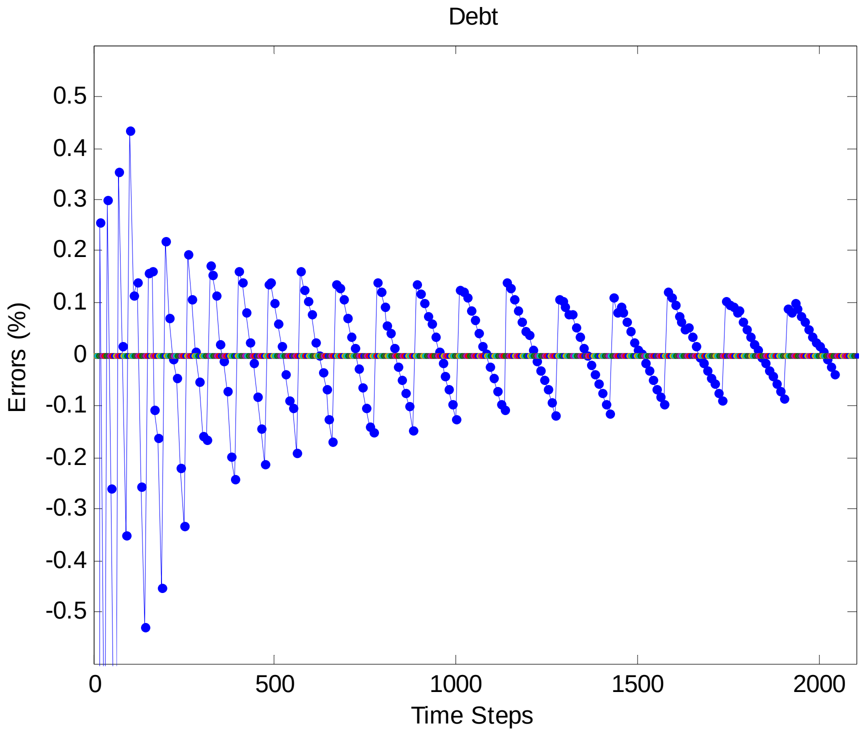
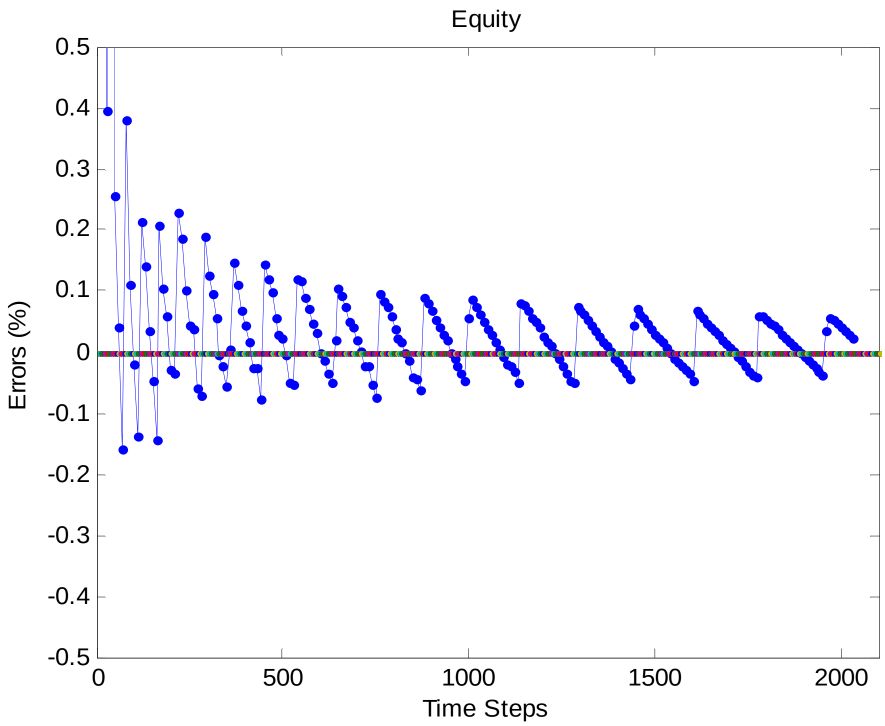
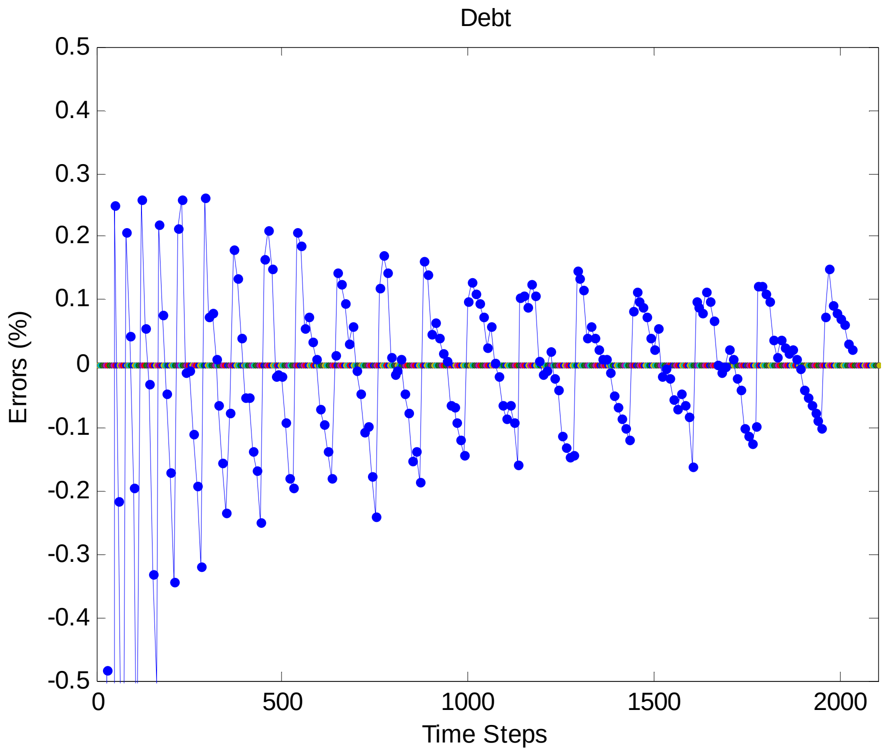


Figure 3. Convergence of Equity and Debt Errors (β =0.5)

The model parameters are $V_0 = 100$, $\sigma = 20\%$, $C = 3$, $P = 60$, $r = 5\%$, $q = 3\%$, $\alpha = 50\%$, ω=1%, $\tau = 25\%$, $\eta = 50\%$, $G = 1$, $\beta = 0.5$, $T = 5$. The true value of equity is 45.8437, and the true value of debt is 54.9405.





4.2 Application and Analysis: Numerical Example

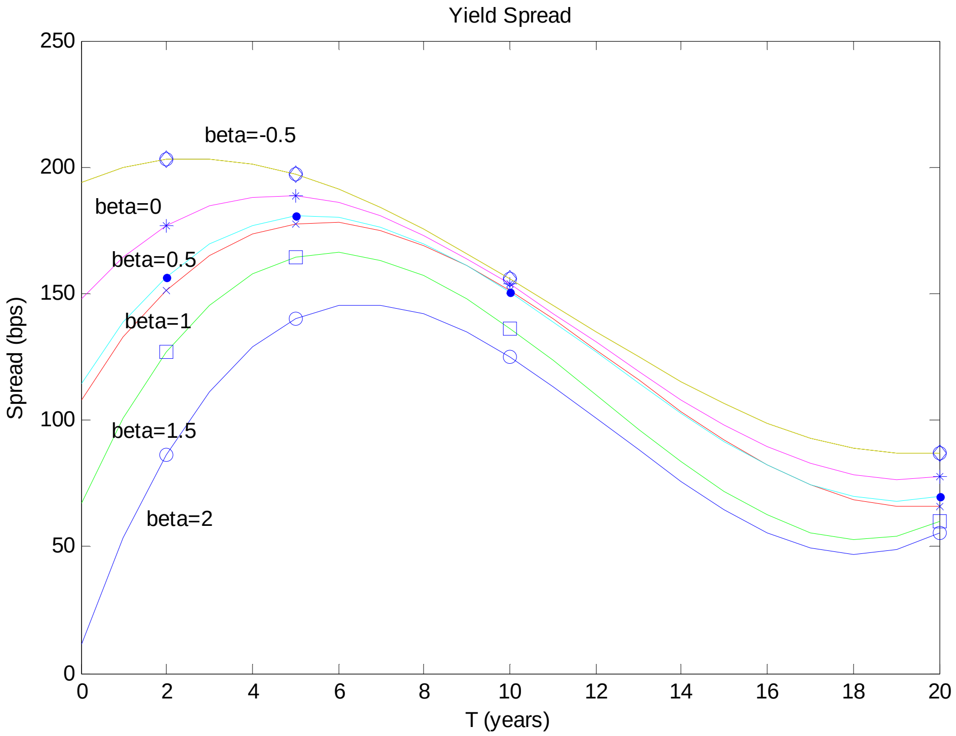
In this section, we first investigate the debt prices and the yield spreads of coupon bond with finite maturity under various elasticity parameter β of the CEV process, and next extend the analysis to the effect of different grace periods. We are aware of that the length of the granted grace period in the Chapter 11 setting may have different impact on bonds with different maturity. Therefore, we first set G=0 to examine the effect of maturity on debt value and yield spreads in Figure 4. It is apparent that as the debt maturity increases, the price of the finite maturity coupon bond converged to the price of the consol bond.

Figure 4 presents relationship between debt maturities and yield spread, equity value, and debt value. The yield spread is defined as *Credit Spreads*=*C/D-r* , where r is a risk-free rate. Consistent with the results of Leland and Toft (1996), under moderate leverage (V=100 and P=60), the yield spread first rises with the increase of debt maturity until it reaches the peak for around 5 to 6 years to maturity. The yield spreads then decrease at a decreasing rate as maturities prolonged. Since a company under financial distress may not easily issue new debts, it is crucial that term structure of yield spread in our study agrees with prior literature without having to have the restrictive assumption about the continuously rollover debt.

In addition, the yield spreads are negatively related to the elasticity parameter β of the CEV process. This is in line with the fact that the value of equity increases and the value of the debt decreases as the β decreases. The results are reasonable because the decrease in β is associated with the rise of asset volatility. Therefore, equity, which can be viewed as a call option on asset value[3], increases its value. In contrast, debt holders who expect the firm to have steady cash flow may suffer from the higher volatility caused by the lower β.

Figure 4. Effect of Maturity on Yield Spreads, Equity, and Debt for a Coupon Bond for Various β

The model parameters are $V_0 = 100$, $\sigma = 20\%$, $C = 3$, $P = 60$, $r = 5\%$, $q = 3\%$, $\alpha = 50\%$, ω=1%, $\tau = 25\%$, $\eta = 50\%$, G=0. The time increment in the lattice is $\Delta t = 0.005$ *years*.



[3] Note that Leland and Toft (1996) indicated that equity in the capital structure model is not precisely analogous to an ordinary call option for several reasons: First, default may occur at any time but not only at debt maturity. Second, the bankruptcy boundary may vary with the risk of the firm's activities. Last and most importantly, the existence of tax benefits and their potential loss in bankruptcy imply that debt and equity holders do not split a claim whose value depends only on the underlying asset value.

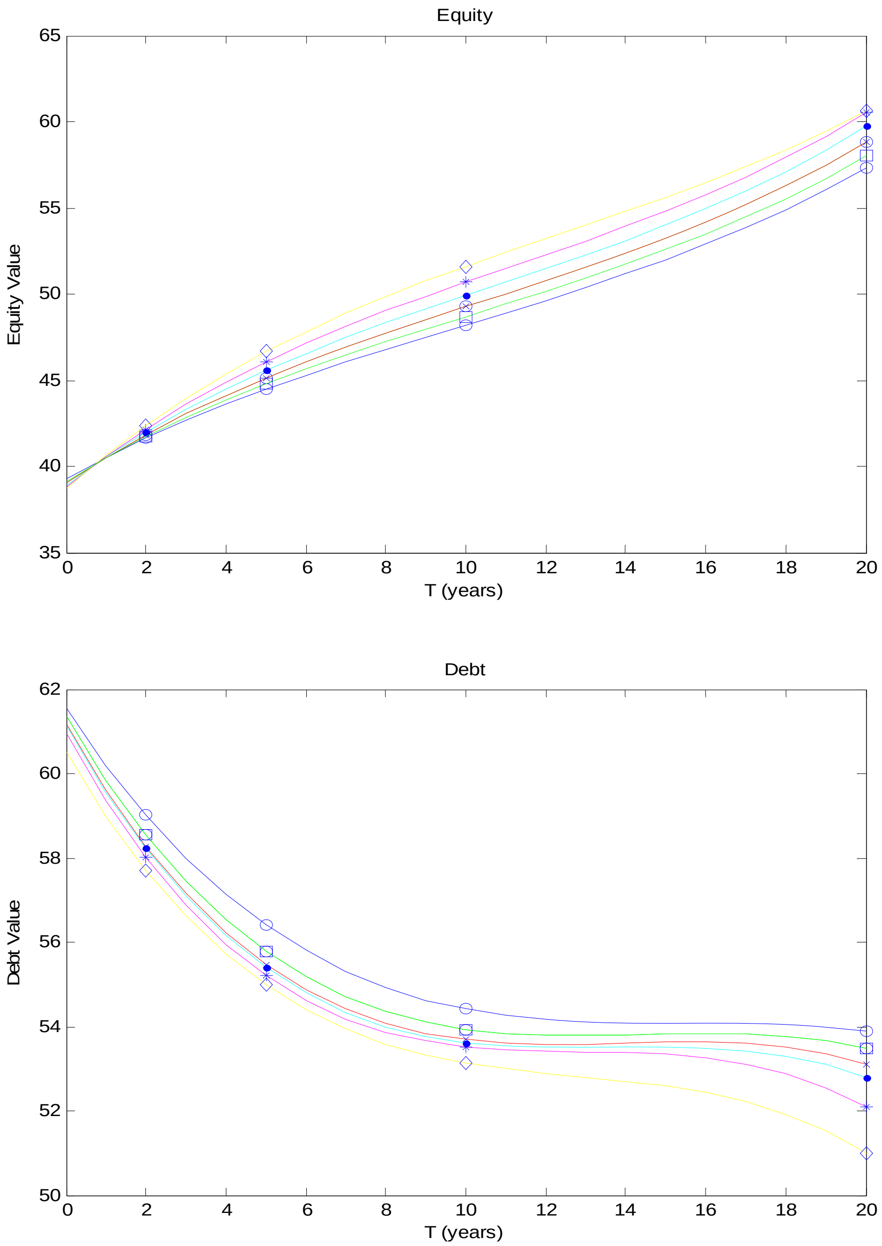
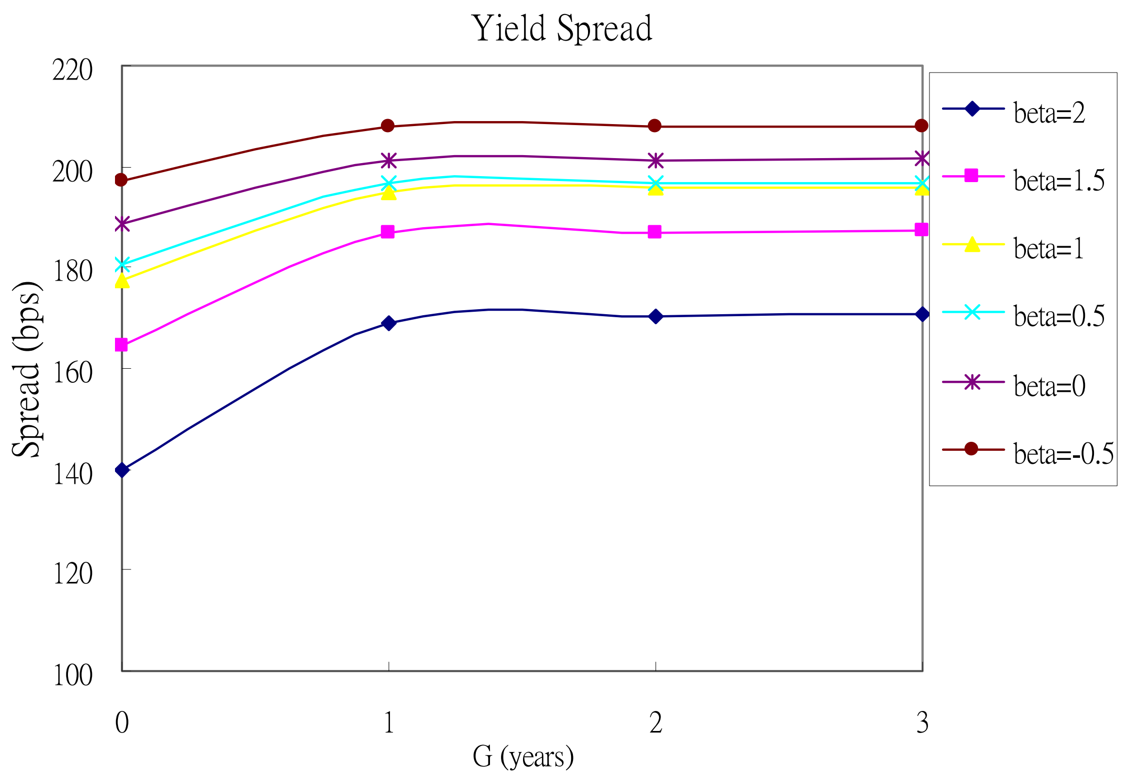
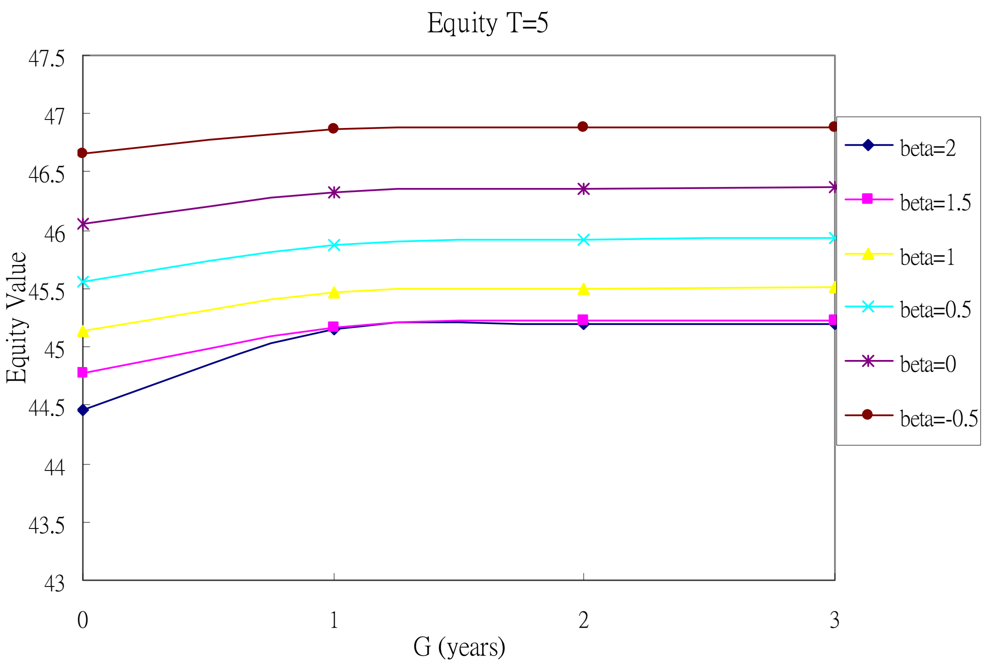


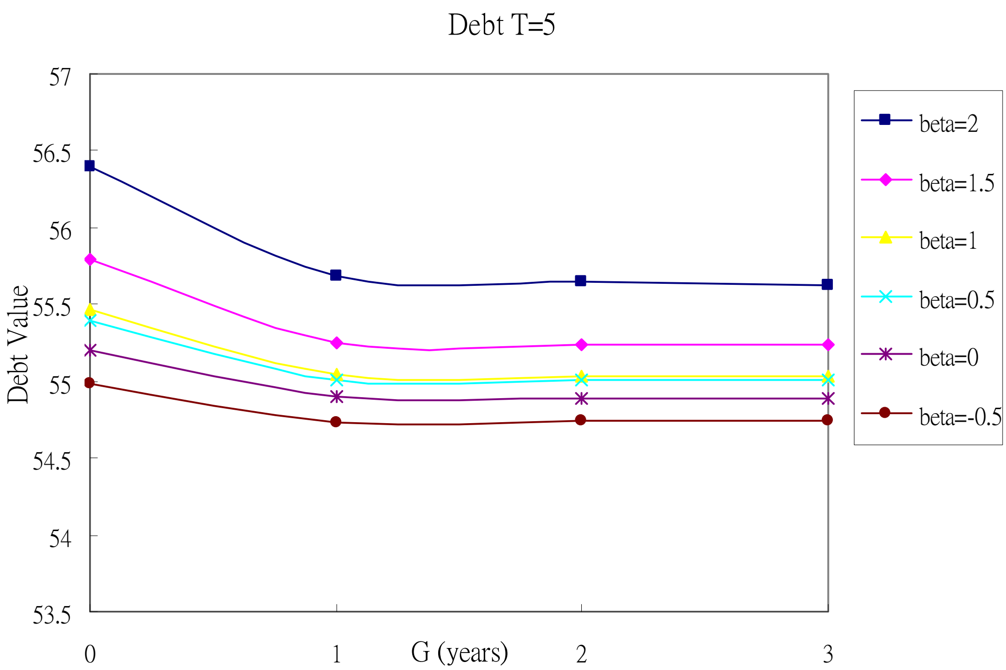
Figure 5 extends the analysis to the cases in the presence of grace period. Similar to the previous results without grace period, when the β of the CEV process decreases, the value of the equity increases and the value of the debt decreases. The reason is the same as before since the volatility comes into play. Furthermore, it appears that as the grace period increases, the equity value and yield spread increase at a decreasing rate and eventually converge to a certain level. By contrast, debt value goes down as grace period prolonged. This is because the existence of the grace period can benefit equity holders via reorganization while debt holders could not benefit from it. Intuitively, from the viewpoint of equity holders, if there is no grace period in the case of bankruptcy, the firm is liquidated immediately and equity holders may receive nothing. But if there is grace period for equity holders, equity holders can wait for firm recovery from financial distress, not just only residual value after liquidation. More importantly, as indicated by Francois and Morellec (2004), Chapter 11 filings involves potential debt service reduction and lead to higher leverage ratio and hence higher yield spread.

Figure 5. Effect of Grace Period on Equity, Debt and Yield Spread for a Coupon Bond with Various β

The model parameters are $V_0 = 100$, $\sigma = 20\%$, $C = 3$, $P = 60$, $r = 5\%$, $q = 3\%$, $\alpha = 50\%$, ω=1%, $\tau = 25\%$, $\eta = 50\%$, $T = 5$ The time increment in the lattice is $\Delta t = 0.005$ *years*







5. Conclusion

In this paper, we introduce constant elasticity of variance (CEV) process into the popular structural modeling framework of capital structure analysis. The CEV process allows the asset volatility to change with the current level of asset value, and thus give the risky corporate debt pricing model a more realistic touch. We extend the work by Francois and Morellec (2004) and Broadie and Kaya (2007) to develop a capital structure model, which incorporates finite maturity as well as the feature of Chapter 11 bankruptcy proceedings. While evaluating a single corporate debt with finite maturity or complex bankruptcy proceedings, no analytical solution is available and one needs to resort to numerical methods. Therefore, we adopt a lattice approach to price risky corporate debt and generate results that are consistent with the limited liability of equity principle. Furthermore, our study can be beneficial to corporate debt pricing model as well. While many existing models use infinite maturity bonds to obtain a close-form solution, our method can be used to price finite maturity debt.

In general, our numerical results for finite maturity debt are consistent with those of previous literature. We first analyze the term structure of yield spreads for finite maturity debt and leave out the Chapter 11 bankruptcy code. We find that under moderate leverage yield spreads first rise up and then drop with the increase of debt maturity at a decreasing rate to a certain stable level. Under different grace periods in Chapter 11 bankruptcy proceedings, the equity value increases and the debt value decreases when the grace period prolonged. The results are as expected because increasing in grace period is the extra benefit for equity holders while it could potentially hurt debtholders. Furthermore, we also examine the effect of parameter β of the CEV process. It is apparent that the decrease of β under the CEV process, implying higher volatility, leads to a lower debt value, a higher yield spreads, and a higher equity value.

The future work of our study could be adding empirical investigation to examine the model implications: Whether or not the low-asset-value firm indeed has a lower β (high volatility)? Whether the grace period of Chapter 11 bankruptcy proceeds is positively associated with yield spread and equity value? In addition to empirical aspects, improving the efficiency of the numerical approach to shorten the computing time should be crucial for applying our model in pricing risky bonds.

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國科會補助計畫衍生研發成果推廣資料表

日期:2011/10/29

<table>
<tr><td rowspan="3">國科會補助計畫</td><td>計畫名稱: 常數彈性變異數過程下考量破產程序的資本結構模型與實證分析</td></tr>
<tr><td>計畫主持人: 李漢星</td></tr>
<tr><td>計畫編號: 99-2410-H-009-025-　　　　　學門領域: 財務</td></tr>
<tr><td colspan="2">無研發成果推廣資料</td></tr>
</table>

99 年度專題研究計畫研究成果彙整表

計畫主持人：李漢星	計畫編號：99-2410-H-009-025-
計畫名稱：常數彈性變異數過程下考量破產程序的資本結構模型與實證分析	

成果項目			量化			單位	備註（質化說明：如數個計畫共同成果、成果列為該期刊之封面故事...等）
			實際已達成數（被接受或已發表）	預期總達成數(含實際已達成數)	本計畫實際貢獻百分比		
國內	論文著作	期刊論文	0	1	100%	篇	
		研究報告/技術報告	0	0	100%		
		研討會論文	0	0	100%		
		專書	0	0	100%		
	專利	申請中件數	0	0	100%	件	
		已獲得件數	0	0	100%		
	技術移轉	件數	0	0	100%	件	
		權利金	0	0	100%	千元	
	參與計畫人力（本國籍）	碩士生	3	3	100%	人次	
		博士生	0	0	100%		
		博士後研究員	0	0	100%		
		專任助理	0	0	100%		
國外	論文著作	期刊論文	0	0	100%	篇	
		研究報告/技術報告	0	0	100%		
		研討會論文	0	0	100%		
		專書	0	0	100%	章/本	
	專利	申請中件數	0	0	100%	件	
		已獲得件數	0	0	100%		
	技術移轉	件數	0	0	100%	件	
		權利金	0	0	100%	千元	
	參與計畫人力（外國籍）	碩士生	0	0	100%	人次	
		博士生	0	0	100%		
		博士後研究員	0	0	100%		
		專任助理	0	0	100%		

其他成果 (無法以量化表達之成果如辦理學術活動、獲得獎項、重要國際合作、研究成果國際影響力及其他協助產業技術發展之具體效益事項等，請以文字敘述填列。)	無

	成果項目	量化	名稱或內容性質簡述
科教處計畫加填項目	測驗工具(含質性與量性)	0	
	課程/模組	0	
	電腦及網路系統或工具	0	
	教材	0	
	舉辦之活動/競賽	0	
	研討會/工作坊	0	
	電子報、網站	0	
	計畫成果推廣之參與（閱聽）人數	0	

國科會補助專題研究計畫成果報告自評表

請就研究內容與原計畫相符程度、達成預期目標情況、研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）、是否適合在學術期刊發表或申請專利、主要發現或其他有關價值等，作一綜合評估。

1. 請就研究內容與原計畫相符程度、達成預期目標情況作一綜合評估

■達成目標

□未達成目標（請說明，以 100 字為限）

□實驗失敗

□因故實驗中斷

□其他原因

說明：

2. 研究成果在學術期刊發表或申請專利等情形：

論文：□已發表 □未發表之文稿 ■撰寫中 □無

專利：□已獲得 □申請中 ■無

技轉：□已技轉 □洽談中 ■無

其他：（以 100 字為限）

3. 請依學術成就、技術創新、社會影響等方面，評估研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）（以 500 字為限）

本研究首先整理現有討論破產程序資本結構模型的相關研究成果，以及巴黎路選擇權的訂價文獻。模型上我們延伸了 Francois and Morellec (2004) 的研究，並導入常數彈性變異數隨機過程(CEV)，以探討在波動度可隨資產價值變動下的收益率差以及破產程序對公司債價值的影響。數值方法上，本研究延伸 Broadie and Kaya (2007)的研究，使用二元樹模型，此方法不須過去學者為排除公司證券價值與時間的相關性，所加入的一些不符合實際之假設，因此可評價具到期日與第十一章破產法規架構下之公司債。本研究數值分析結果指出，當破產保護寬的期限越長，或是 CEV 過程彈性係數越小時，公司債價值越低。此研究在學術上將常數彈性變異數過程應用至結構信用風險模型，增加了不同公司價值下，公司價值波動度差異的可能性，此為過去研究中所未見的，亦並未增加模型估計上過度的複雜度，也因此可避免因模型估計困難而產生無法實際應用的狀況。另外，本研究亦將常數彈性變異數過程下的結構風險模型之概似函數估計加以處理，應在學術上與風險管理應用上皆有所貢獻。此外，此研究模型實證部份，目前正在進行進一步的實證分析，應可於近期投稿國際學術研討會。