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Theoretical and Experimental study on
two-dimensionally periodic structures

計畫主持人：彭松村 教授
共同主持人：黃瑞彬 研究副教授
交通大學電子與資訊研究中心
計畫編號：NSC 89-2213-E009-074

Guiding Characteristics of Periodic Impedance Surface

R. B. Hwang and S. T. Peng

Microelectronics and Information Systems Research Center
And Department of Communication Engineering
National Chiao Tung University
Hsinchu, Taiwan, ROC

Abstract

In this paper, we present a systematic analysis of the dispersion characteristics of two-dimensionally periodic impedance surfaces. The method of mode matching is employed analyze the boundary-value problem rigorously in terms of the complete set of both TE- and TM-polarized plane waves in the uniform medium above the impedance surface. Numerical results are obtained extensively and are presented graphically in the form of phase diagram, to explain various physical effects that are peculiar to the class of structures under consideration. More importantly, the unique characteristics of slanted stopband of 2D periodic structures are investigated to develop useful criteria for practical design and to identify potential applications.

1. Introduction

In recent years, considerable attention has been focused on the study of wave phenomena associated with two-dimensional (2D) periodic structures, and many applications have been demonstrated, particularly in the aspects of surface wave suppression by using the Bragg Reflection in bound-wave region [1,2]. Because of their mathematical complexity, the research works so far have been limited mostly to experimental studies or have to rely on numerical simulations. Recently, we have presented a rigorous analysis of the scattering and guiding of waves by 2D periodic impedance surfaces, in order to understand better the wave phenomena involved in the 2D periodic structures. The concept of 1D reactive surface has been employed in the analysis of surface wave along corrugated metal surface by Hessel and Oliner [3] for explaining the Wood's anomaly for the light scattering by 1D periodic structures. Here, we extend the 1D model to the 2D case and present a simple method of analysis that provides physical insights into the wave processes associated with 2D periodic structures. Thus, a clear physical picture can be well established and design criteria can be developed for practical applications.

2. Statement of Problem and method of analysis

Fig. 1 depicts a 2D periodic structure that is horizontally infinite in extent. As shown, the periodic layer is covered by the air half-space and may be supported by a stack of uniform layers, and it is periodic along the x-axis with the periodic a and along the y-axis with the period b. Here, we assume that the multilayer structure has a vertical distribution of the dielectric constants, such that it acts as a waveguide even in the absence of the periodic layer. It is well known that such a type of multilayer periodic structure is amenable to a rigorous analysis as a boundary-value problem by the method of mode matching. While the fields in the uniform regions can be simply expressed as a superposition of plane waves, those for the periodic layer have to be represented in terms of the Floquet mode functions. It is noted that in the present case, we have a 3D boundary-value problem that requires the simultaneous presence of both TE and TM constituent waves with respect to the vertical z-axis, in order to satisfy the continuity conditions on the tangential components of the electromagnetic fields at periodic interfaces. In particular, to the fields in the air region, the structure below the top surface of the periodic layer may be approximated by a 2D periodic surface impedance that can be generally represented by an infinite double Fourier series. For simplicity, we consider here only the first harmonic in both *x* and *y* directions, such that the surface impedance is written as:

$$Z(x,y) = Z_s\left[1 + 2\delta_x \cos\frac{2\pi x}{a} + 2\delta_y \cos\frac{2\pi y}{b} + 4\delta_{xy} \cos\frac{2\pi x}{a}\cos\frac{2\pi y}{b}\right] \quad (1)$$

In general, the surface impedance is a complex quantity:

$$Z_s = R_s + jX \quad (2)$$

where R_s is the surface resistance representing the ohmic loss of the structure, and X_s is the surface reactance, and the δ's are the modulation indices. If needed, more harmonics may be added to the representation, and the ensuing theory remains valid. The approximation of a periodic structure by an impedance surface had been successfully employed for the explanation of Wood's anomaly for the scattering of light in 1D periodic structures [3], and this work can be considered as an extension of the earlier work to the 2D case. Consequently, the problem is reduced to the determination of the fields in the air region to satisfy the boundary condition:

$$\underline{z}_o \times \underline{E}_t(x,y,0) = Z(x,y)\underline{H}_t(x,y,0) \quad (3)$$

where $\underline{E}_t$ and $\underline{H}_t$ are the tangential vectors of the electromagnetic fields. With the plane wave representation for the fields in the air region, the last boundary condition yields, in the absence of the incident wave, a set of linear recurrence relations in the vector form will be constructed. The condition for the existence of non-trivial solutions requires the vanishing of the coefficient matrix and this defines the dispersion relation of the waveguiding structures.

3. Numerical Results and Discussions

Based on the numerical analysis in the previous section, we are now in a position to carry out the qualitative and quantitative analysis for the guiding characteristics of 2D periodic impedance surface. First, before embarking on elaborate computations, we present a simple perturbation procedure to obtain approximate results with ease. Moreover, a graphical method is developed to identify and explain various physical phenomena associated with the structure at hand and this will be particularly useful for practical design considerations. Second, for a numerical simulation, the infinite dimensions of system equations should be truncated into a finite order for numerical analysis. Extensive numerical data are carried out systematically in order to identify all possible physical phenomena associated with the structure under investigation. Finally, the numerical data are displayed in the form of the phase diagram for physical interpretations.

According to the theory of mode coupling, when the perturbations are small the dispersion root of the periodically perturbed structure should differ only slightly from that of the unperturbed one, but the propagation characteristics could change qualitatively from a propagating wave to a decaying wave in a stopband region. For the low-frequency operation, the normalized radius of each circle ($k_{sw}a/2\pi$) is intentionally set to be smaller than $1/\sqrt{2}$, so that the intersections occur only between the circles centered along either k_x-axis or k_y-axis, but not between those along the diagonal directions. The implication of these diagrams is that the 2D periodic structure behaves like a 1D one in either *x*- or *y*-direction.

On the other hand, at higher frequencies, k_{sw} becomes larger, and additional interactions may take place between the circles centered along the diagonal directions, such as the interactions between (0, 0) and (-1, -1) harmonics and also between (-1, 0) and (0, -1) harmonics. The four extra stopbands are slanted at an angle with respect to the k_x-axis, as previously reported by us [5]; they are mainly due to the cross modulation term containing δ_{xy}. Inside these slanted stopbands, k_y is complex for a given real k_x, and so is k_x for a given real k_y. These extra stopbands provide additional incident conditions for the suppression of surface waves.

To further understand the characteristics of the slanted stopbands, Fig. 2 shows the change in the range of incident angle to be within the stopband for the two normalized frequencies. Fig. 2(a) and (b) shows the real part of k_x and imaginary part as a function of k_y. To achieve the surface wave suppression for a large angle of incidence, we have designed a special case with the normalized impedance surface $Z_s = j3$, the periods $a = b$, and the modulation indices are $\delta_x = \delta_y = \delta_{xy} = 0.1$. In this case, the slanted stopbands merge into a large one, as will become clearer latter on. Since the curves behave generally in the same form around the frequency range from $a/\lambda = 0.215$ to 0.235, we plot them for the two values of a/λ, for succinctness.

As shown in Fig. 2(a), the dashed line is the unperturbed phase diagram for the case of $a/\lambda = 0.235$. The curves in heavy solid line are for the perturbed phase diagram of $a/\lambda = 0.235$, while the light ones are for that of $a/\lambda = 0.215$. Due to the reflection symmetry, the phase diagram is drawn for the angle ϕ from 0 to 90 degrees, where ϕ is measured from the k_x-axis toward the k_y-axis. As we trace the phase diagram along the fundamental harmonic of surface wave ($m = 0$, $n = 0$), the pass-bands are diminishing and the two slanted stopbands merge into a large one to form a large angle of surface wave stopband. From the values of k_x and k_y at the band edges for the case of $a/\lambda = 0.235$, it is estimated that the guided wave stays within the stopband for the incident angle ranging from 21^o to 71^o, as measured from x-axis, as shown in Fig. 2(a). Moreover, inside the slanted stopband, there is an extra vertical stopband at $Re[k_x a/2\pi] = 0.5$. It is due to the contribution of modulation index δ_x; that is the periodic variation along x-direction. Fig. 2(b) shows the imaginary part of $k_x a/2\pi$, also exhibiting the merge of the two slanted stopbands. Again, the effect of vertical stopband contributed from the x-direction periodicity can also be observed from the extra stopband appearing in the center portion of the slanted stopband. Conceivably, it will provide a stronger surface-wave suppression around such a region. The general behavior of stopbands for the case of $a/\lambda = 0.215$ are similar to that of $a/\lambda = 0.235$ are not repeated here, for succinctness.

A surface wave can be excited in any lateral direction along the structures; therefore, the range of incident angle within a stopband should be investigated in detail for the practical applications. Here, we plot the variation of angular stopband versus operation frequency, for the same parameters as in Fig. 2. As depicted in Fig. 3, the two dashed lines are for the upper- and lower-bound of a stopband. In the frequency range investigated, the angle spectrum for surface-wave suppression covers, at least, from 35 to 55 degrees. In other words, the surface wave can be totally eliminated over the range of 20^o of the incident angle for the frequency range from $a/\lambda = 0.215$ to 0.235. This permits us to control the propagation of surface waves in certain frequency range by a proper design of 2D periodic structures.

4. CONCLUSIONS

In this paper, we have modeled a periodic structure by using a periodic impedance surface. The guiding characteristics of surface wave on the planar periodic impedance surface have been rigorously treated. Numerical results are systematically carried out to show the stop-bands structure of the dispersion curves, with particular attention paid to the slanted stopbands. In the case of a large modulation index, we have achieved the stopband operation for a wide angle of incidence by the merge of the slanted stopbands. The existence of this type of stop-bands provides more degrees of freedom for the design of microwave and millimeter wave circuits and antennas, such as the suppression of leaky surface waves. More data will be given in the presentation to explain the underlying concepts and potential applications.

Acknowledges

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References

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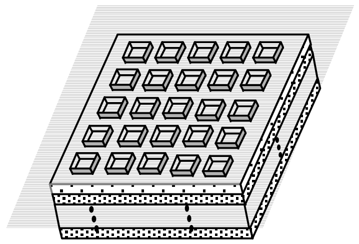

Fig. 1Structural configuration of a 2D periodic structures.

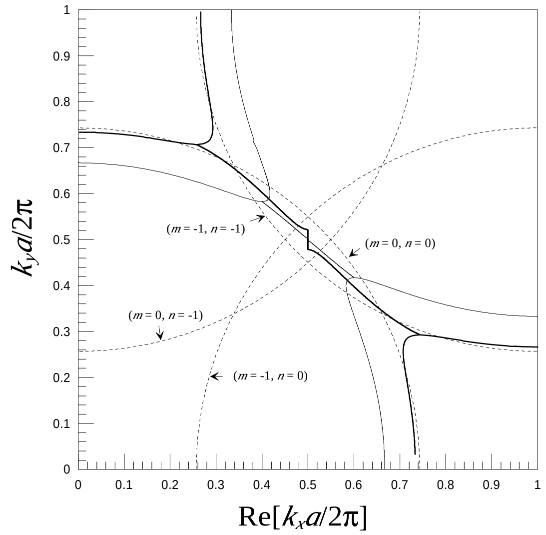

Fig. 2(a): Phase diagram for the case of $Z_s = j3$; $a = b$; $\delta_x = \delta_y = \delta_{xy} = 0.1$; computation of k_x for a given k_y

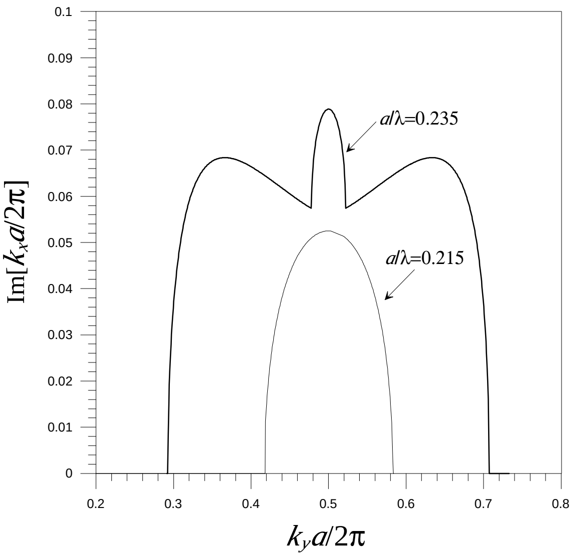

Fig. 2(b): Stopband behavior with the nromalized frequency as a parameter for the case as same in Fig. 2(a)

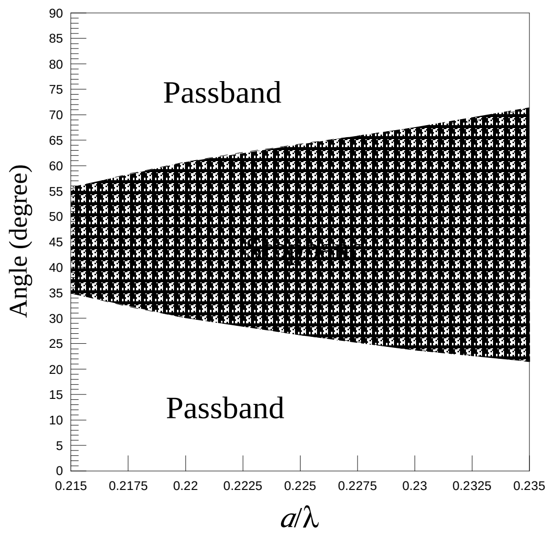

Fig. 3: Variation of angular stopband for a certian frequency range for the case as the same for fig. 2.