行政院國家科學委員會專題研究計畫 成果報告

可逆馬可夫過程的 L2 切割時間
研究成果報告(精簡版)

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研究成果報告內容

Since Markov introduced such a stochastic process in 1906, many people have been engaged in developing its theory and applications. The classical theory of Markov chains focused on the qualitative analysis of fixed chains such as the rate of convergence to the stationarity as time tends to infinity. In the past two decades, as interest in chains with large state spaces has increased, a different asymptotic analysis has emerged. Some target distance to the stationarity is specified and the number of steps required to reach this target is called the mixing time. As a result, the modern theory of Markov chains falls on the study of mixing times and one goal is to understand how the mixing time grows as the size of the state space increases.

In the quantitative analysis of Markov chains, a remarkable phenomenon known as the cutoff phenomenon was introduced by Aldous and Diaconis in early 1980s to capture the fact that some Markov chains converge abruptly to their stationarity. This brilliant idea reveals the observation that the distribution of the chain is a good approximation to the limiting distribution after the so called ``cutoff time" but never fits well before. Such a phenomenon has a close relation to the mixing time and is sensitive to measurement mechanics.

Diverse techniques have been introduced to study the total variation mixing of Markov processes since 1980s. Coupling and strong uniform time are discussed by Aldous, Diaconis and Fill. Jerrum and Sinclair use conductance to bound the spectral gap and the mixing time. Applications of representation theory are introduced by Diaconis and Shahshahani. Diaconis and Saloff-Coste used comparison techniques in bounding the spectral gap and eigenvalues.

We have successfully used the above developed techniques to determine the mixing time of interesting models in [1]. For a generalization on the total variation distance, we consider the L^p-cutoff of families of Markov processes and derive its necessary and sufficient conditions. This has been summarized in [2]. Recently, a spectral representation of the L^2-distance makes it possible to determine the L^2-cutoff using the spectral decomposition.

The goal of this yearly project is twofold. One side is to make up the theory on the existence of L^2-cutoffs and on the formula of L^2-mixing times using the spectral information. The other side is to implement the theoretical results on a couple of interesting and practical examples. As it is well-known that both the cutoff and the mixing time are closely related, we (Laurent Saloff-Coste and Guan-Yu Chen) address both problems (the existence of L^2-cutoffs and the computation of the L^2-mixing time) at the same time using the spectral theory. This is the first time that the cutoff phenomenon is determined theoretically. Such a method should be regarded as solving the L^2-mixing-time problem using functional analysis even though it depends strongly on the prerequisite of complete spectral formation.

For an illustration, we supply a series of classical examples to clarify the intuition on the L^2-convergence of Markov chains, which can be very different from the total variation convergence. The result is not too surprising because it fits one's intuition but this is the first time that the L^2-convergence is studied in a very precise way.

This work has been composed in [3] and accepted by Journal of Functional Analysis. The authors of this article were invited by Persi Diaconis to present their results in the conference entitled "Analysis and Probability in Nice, 2nd session" which is held in Nice, France during June 15-19, 2009.

Reference

[1] Guan-Yu Chen and Laurent Saloff-Coste *The cutoff phenomenon for randomized riffle shuffle.* Random Structures and Algorithms 32 (2008), no. 3, 346--374.

[2] Guan-Yu Chen and Laurent Saloff-Coste. *The cutoff phenomenon for ergodic Markov processes.* Electronic Journal of Probability, 13 (2008), 26--78.

[3] Guan-Yu Chen and Laurent Saloff-Coste. *The L^2-cutoff for reversible Markov processes*. Accepted by Journal of Functional Analysis. (59 pages)