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應用 T-S 模糊模型之順滑模態可靠度控制研究
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計畫主持人：梁耀文

計畫參與人員：碩士班研究生-兼任助理人員：魏源廷
碩士班研究生-兼任助理人員：王世昕
碩士班研究生-兼任助理人員：丁立偉
碩士班研究生-兼任助理人員：吳家榮
碩士班研究生-兼任助理人員：鄭旭志
碩士班研究生-兼任助理人員：謝宜展
博士班研究生-兼任助理人員：徐勝均

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行政院國家科學委員會補助專題研究計畫成果報告

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(Study of reliable sliding mode control based on T-S fuzzy system model)

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共同主持人：
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一、中文摘要

隨著人類科技不斷的快速進步及推陳出新，使得大家的生活舒適程度及交通便捷性不斷的提昇，各種系統之設備規模、複雜程度、所投注的資金以及可能造成的災害威脅也因而大幅提高。因此，人們對於系統之安全性、可靠性及有效性的要求也變得格外殷切。由於系統的複雜程度不斷提昇，其所使用的控制策略、所需的計算時間以及其有效性便顯得格外重要，且這些因素往往決定了系統控制的品質與成敗。因此本計畫研究採用 T-S 模糊模型（T-S fuzzy model）結合順滑模態（Sliding mode control）理論來進行可靠度控制器的分析與設計工作。採用順滑模態控制的原因在於它所擁有的高度穩健特性（Robustness）及快速的反應能力；而採用 T-S 模糊模型來進行分析與設計的主要原因是它只需計算相關的線性系統模型資料，這對於具有高度非線性的複雜系統而言，可以有效的減少計算時間，達到有效控制的目的。因此，在本研究計畫裡我們進行 T-S 模糊模型及順滑模態控制策略結合之可靠度控制器的研發。

關鍵詞：非線性控制系統、順滑模態控制、TS 模糊模型、可靠度控制、穩定性分析、軌跡追蹤。

Abstract

Due to the growing demands for system reliability in a highly automated industrial system and in aerospace missions, where repair and maintenance often can not be achieved immediately, the study of reliable control has become paramount importance and has attracted considerable attention. On the other hand, since the modern control systems are constructed more and more complicated, the employed control strategy and the time for controller implementation have become extreme importance. In fact, the two mentioned-factors have a strong relation to the quality and the efficiency of the control mission. In this project, we combine the T-S fuzzy model approach and the Sliding Mode Control (SMC) scheme for alleviating the computational burden and promoting associated system reliability performances. The reason for adopting SMC scheme comes from its own advantages including responding rapidly and robustness to uncertainties and disturbances, while T-S fuzzy approach allows one to save lots of on-line computational burden, which is especially important for those systems with highly nonlinear and complicated dynamics. The combined scheme saves lots of on-line

computational burden while achieve efficiently control objective.

Keywords: Nonlinear control systems, sliding mode control, TS-fuzzy model, reliable control, stabilizability analysis, tracking performance.

二、緣由與目的

1、背景說明及計畫重要性

據美國軍用航空部門統計，美國在二次大戰期間飛機因為故障而損失的數量比起被擊落數量整整多了 1.5 倍，而運往遠東的設備經過運輸後有 60%不能使用，使用後的維修也成為重大問題，所以他們首先體認到可靠度控制的重要性與當系統缺乏可靠性的重大代價。有鑒於此，美國國防部在戰後投注大筆經費進行裝備可靠性的研究，開啟了錯誤偵測與診斷及可靠度控制的研究領域。近年來由於科技的發達及航太工業的突飛猛進，提供給人類舒適的生活環境及快捷之交通便利。各種系統之設備規模，複雜程度及所投注的資金也因而大幅提昇。因此，人們對於系統之安全性、可靠性及有效性的要求也越來越殷切。尤其是航空、太空、核電廠及化工廠等等具有高危險性的特殊機具，更可能由於系統的不穩定而導致重大災難。舉例來說：1979 年美國三里島核能電廠的意外事件、1986 年 1 月美國挑戰者號及 2003 年 2 月哥倫比亞號太空梭的不幸事件、1998 年 8 月至 1999 年 5 月的短短 10 個月之間，美國 3 種運載火箭“大力神”、“雅典娜”、“德爾他”共發生了 5 次發射失敗，造成 30 多億美元的直接經濟損失，美國的太空計劃也因此受到嚴重打擊。美國在 2004 年秋天正式啟用位於阿拉斯加葛瑞利堡的美國飛彈防禦系統基地，根據調查報導，這項斥資超過 1000 億美元研發的飛彈防禦系統效用受到嚴重地懷疑，在關鍵系統研發屢次的延遲下，只有進行了基本的試射測試，由於相關的可靠度控制和錯誤偵測與診斷的效能也尚未完全，所以目前實際上的效用據評估可能僅有 20%的防禦能力，而美國在未來 5 年內，每年還將繼續投入 90 到 100 億美元在此飛彈防禦系統上。這些相關的事件與分析都充分說明了錯誤偵測與診斷和可靠度控制這研究主題的重要性。

由於這股研究熱潮方興未艾，在學術界方面也持續的吸引了許多世界各地學者專家高度的重視。目前已有許多關於此方面的研究成果和理論不斷的被發表和提出，而諸多的重要國際學術會議如美國控制研討會(ACC)、IEEE 控制與決策會議(CDC)及國際控制聯合大會(IFAC)也都將此項研究主題列為重要的討論議題。美國電機電子工程師學會(IEEE)亦成立可靠度學會(IEEE Reliability Society)，出版關於可靠度方面的期刊：*IEEE Transactions on Reliability*，而在其他電機、電子、控制和資訊等相關的期刊，也有許多專家學者專門研究探討關於可靠度控制的問題，足見此方面研究主題的重要及迫切性。

2、研究目的

誠如華衛二號衛星(ROCSAT-2)在其設計中運用了四個旋轉輪(reaction wheel)來做姿態控制，其目的就是希望當有一具旋轉輪發生故障時，衛星姿態仍能獲得有效的控制，也就是說系統被要求能具有容錯的能力。而這容錯的能力也正是可靠度控制研究的主要目的：希望系統不論在正常或部分故障時仍能維持特定的性能表現。因為對於遠在太空的衛星系統而言，一旦系統發生故障，其回收維修相當不容易而且回收所需耗費的成本極高。而對於一般系統而言，當系統發生故障時，所需的零件及維修通常也無法及時有效的供

應。因此，對於控制系統可靠度的要求益發引起人們的關注。一般而言，“故障”會使系統表現出不希望的特性，而當動態系統中出現部分功能喪失時，將會導致整個系統性能的惡化，甚至引發系統的不穩定。為了有效的提升及確保系統在正常操作及部分故障時的性能，我們除了可以經由一些製造技術來提高產品的品質之外，另一個重要而且可行的方式，則是透過不斷監測系統運行的狀態並預測發展之趨勢，盡可能把可能發生的故障消除在剛開始發生的階段。而為了有效獲知系統的運作狀況，“故障偵測與診斷”(fault detection and diagnosis)的學問乃因應而生。此故障偵測與診斷機制的主要目的就是希望當系統發生故障或異常現象時能及時的發出警告訊號，並分離出異常狀況的原因，來源及嚴重程度，提供給決策機制採取最合理、最正確的處置以及採取最適當有效的控制策略，以避免設備的損壞及造成可能的不幸。由於任何一個控制系統均無法避免故障或異常現象的發生，因此如何有效的避免故障的發生，或者是如何能適度地容忍輕微的故障發生，是一個相當實際且重要的研究課題。因此。本計劃之研究目的是希望能研發新一代的可靠度控制技術使系統具有容錯能力及高可靠性，並希望能將此研發技術應用至實際系統上使其具有安全及容錯的目標。同時，經由此研發的過程為國家培養相關之電機控制研究人才。相信藉由研究訓練、技術累積和研發成果，對於國內學術的研究發展，國防科技的技術提升和工業的實務應用都必定能有所助益。

三、結果與討論

考慮如下非線性二階系統：

$$\dot{\mathbf{x}}_1 = \mathbf{x}_2 \quad 與\ \dot{\mathbf{x}}_2 = \mathbf{f}(\mathbf{x}) + B\mathbf{u} + \mathbf{d} \quad (1)$$

其中

$\mathbf{x}_1 = (x_1, \cdots x_n)^T \in \mathbf{R}^n, \mathbf{x}_2 = (x_{n+1}, \cdots x_{2n})^T \in \mathbf{R}^n$，$\mathbf{x} = (\mathbf{x}_1^T, \mathbf{x}_2^T)^T$ 為系統狀態，$\mathbf{u} = (u_1, \cdots, u_{n+m})^T \in \mathbf{R}^{n+m}$ 為控制輸入，$\mathbf{d} = (d_1, \cdots, d_n)^T \in \mathbf{R}^n$ 為可能之模型不確定性及外界干擾，$\mathbf{f}(\mathbf{x}) \in \mathbf{R}^n$ 及 $G(\mathbf{x}) \in \mathbf{R}^n$ 是平滑函數，$(\cdot)^T$ 表示一向量或矩陣之轉置。在本研究，我們假設 $\mathbf{f}(\mathbf{0}) = \mathbf{0}$。在系統(1)中的描述，我們已經假設系統有冗餘的控制輸入。將制動器分為兩群 H 與 F，代表的是健康的制動器與允許損壞的制動器。系統(1)可以重寫成：

$$\dot{\mathbf{x}}_1 = \mathbf{x}_2\ 與\ \dot{\mathbf{x}}_2 = \mathbf{f}(\mathbf{x}) + B_H\mathbf{u}_H + B_F\mathbf{u}_F + \mathbf{d}. \quad (2)$$

我們假設預先選取之健康的制動器滿足 $\mathbf{u}_H \in \mathbf{R}^n$ 與 $B_H \in \mathbf{R}^{n\times n}$ 為非奇異矩陣。

本計劃的目標是組織一個適當的控制律 $\mathbf{u}_H$ 與 $\mathbf{u}_F$ 使得即使當所有的或部分的制動器在 F 集合中發生損壞的狀況時，閉迴路系統的原點仍然漸近穩定。

A. T-S 模糊系統模型

T-S 模糊模型是藉由多個線性系統之組合而成的系統來近似原有的非線性系統(1)。假設有第 i 個($i = 1, 2, \cdots, p$)T-S 模糊模型對系統(2)的規則描述如下：

如果 ς_1 為 $M_{1_i}, \cdots, \varsigma_q$ 為 $M_{q_i}, i = 1, \cdots, p,$ 則

$$\dot{\mathbf{x}}_1 = \mathbf{x}_2\ 及\ \dot{\mathbf{x}}_2 = \mathbf{f}(\mathbf{x}) + B_H\mathbf{u}_H + B_F\mathbf{u}_F \quad (3)$$

其中 $\varsigma_1, \cdots, \varsigma_q$ 為假定變數，$M_{1_i}, \cdots, M_{q_i}$ 為對假定變數之隸屬函數，p 和 q 標示為規則與假定變數的個數，及 $A_i \in \mathbf{R}^{n\times n}$。T-S 模糊則根據各別線性系統(4)系統狀態的權重而被建立，如下所示：

$$\dot{\mathbf{x}}_1 = \mathbf{x}_2\ 及\ \dot{\mathbf{x}}_2 = \sum_{i=1}^{p} \alpha_i(\mathbf{x}) A_i \mathbf{x} + B_H\mathbf{u}_H + B_F\mathbf{u}_F \quad (4)$$

B. SMC 可靠度控制器設計

藉著與 T-S 模糊模型的關聯，系統(2)可以改寫為

$$\dot{\mathbf{x}}_1 = \mathbf{x}_2 \quad (5)$$

$$及\ \dot{\mathbf{x}}_2 = \sum_{i=1}^{p} \alpha_i(\mathbf{x}) A_i \mathbf{x} + \Delta\mathbf{f} + (B_H\mathbf{u}_H + B_F\mathbf{u}_F) + \mathbf{d} \quad (6)$$

其中 $\Delta\mathbf{f} := \mathbf{f}(\mathbf{x}) - \sum_{i=1}^{p} \alpha_i A_i(\mathbf{x})$。

因為(1)為一個二階系統的集合，我們可以假設順滑面為

$$\mathbf{s} := (s_1, s_2, \cdots, s_n)^T = \mathbf{x}_2(t) + M\mathbf{x}_1(t) \quad (7)$$

其中 $M \in \mathbf{R}^{n\times n}$ 為一個正定矩陣。清楚的，假設系統狀態仍然在順滑平面上時，則想要之 $\mathbf{x}(t) \to \mathbf{0}$ 之穩定化表現可以指數性的逼近。為了補償干擾或不確定的效果，我們引入假設

假設 1：存在非負函數 $\rho_j(\mathbf{x},t), j = 1,\cdots,n,$ 使得 $|(B_F\mathbf{u}_F^*)_j| + |(\Delta\mathbf{f})_j| + |d_j| \le \rho_j(\mathbf{x},t)$，其中 $\mathbf{u}_F^*$ 為 $\mathbf{u}_F$ 可能數值及 $(\cdot)_j$ 標示為第 j 列的向量。

遵循 SMC 設計的流程[2]，我們選擇

$$\mathbf{u}_H = -B_H^{-1}\left(\sum_{i=1}^{p}\alpha_i(\mathbf{x})A_i\mathbf{x} + M\mathbf{x}_2 + \Lambda_H \cdot \mathrm{sgn}(\mathbf{s})\right) \quad (8)$$

其中 $\Lambda_H = \mathrm{diag}(\rho_1(\mathbf{x},\mathrm{t}) + \eta_1, \cdots, \rho_\mathrm{n}(\mathbf{x},\mathrm{t}) + \eta_\mathrm{n})$，對 $j = 1,\cdots,n$，$\eta_j > 0$，$\mathrm{sgn}(\cdot)$ 標示為訊號函數且 $\mathrm{sgn}(\mathbf{s}) := (\mathrm{sgn}(s_1),\cdots,\mathrm{sgn}(s_n))^T$，即在控制器 $\mathbf{u}_H$ 下，它遵循(5)-(7)及假設 1 使得 $\mathbf{s}^T\dot{\mathbf{s}} \le \sum_{j=1}^{n}\eta_j \cdot |s_j|$。這個不等式使得系統狀態無論當在 F 內的控制器健康或者損壞時都可以在有限時間內接觸到順滑平面 [2]。

除了 $\mathbf{u}_H$ 的設計外，當部分或者全部在 F 中的制動器為健康時，我們也將設計 $\mathbf{u}_F$ 來提升整個系統的表現。從(5)-(6) 及(8)，則我們有 $\mathbf{s}^T\dot{\mathbf{s}} \le \mathbf{s}^T B_F \mathbf{u}_F - \sum_{j=1}^{n}\eta_j \cdot |s_j|$. 很清楚的，使得系統狀態接近順滑平面比 $\mathbf{u}_F = \mathbf{0}$ 來的快的一個 $\mathbf{u}_F$ 的選擇是

$$\mathbf{u}_F = -\Lambda_F \cdot \mathrm{sgn}(B_F^T\mathbf{s}) \quad (9)$$

其中 $\Lambda_F = diag(\eta_{n+1},\cdots,\eta_{n+m})$ 及 $\eta_{n+\iota} \ge 0$ 當所有 $\iota = 1,\cdots,m.$。這些推導證明了控制增益 $\eta_{n+\iota}$，$\iota = 1,\cdots,m$，的大小對在 F 集合中的制動器保證可穩定化的表現，可以從零到可允許的最大控制輸入的變化範圍。這允許了在 F 中的制動器可以完全損壞，部分損壞，吸收或放大在任何階及任何組合的狀況下，因此有了以下定理。

定理 1：如果假設 1 滿足，在(8)與(9)的控制律之下，即使當部分或全部的制動器為不正常操作，系統(2)的原點為局部漸近穩定。

C. 應用於衛星系統

一個在圓形軌道上的衛星姿態控制可以描述成(1)的形式，其中 n=3 [11]。六個狀態標示為三個尤拉角 (ϕ,θ,φ) 與它們的微分。為了簡化，我們假設在本研究中，火箭推進器只能用控制力及有一個冗餘的制動器來表現可靠度任務。藉著使 $\mathbf{x} = (\phi,\theta,\varphi,\dot{\phi},\dot{\theta},\dot{\varphi})^T$ 與 $\mathbf{f}(\mathbf{x}) = (f_1(x), f_2(x), f_3(x))^T$，整個系統動態可以描述如[2]

$$\text{其中 } B = \begin{pmatrix} 0.67 & 0.67 & 0.67 & 0.67 \\ 0.69 & -0.69 & -0.69 & 0.69 \\ 0.28 & 0.28 & -0.28 & -0.28 \end{pmatrix}.$$

這裡，I_x, I_y 與 I_z 為對應三個體座標軸的慣性矩，ω_0 標示為常數軌道率，c 與 s 標示為 cos 與 sin 函數。

為了獲得適當的 T-S 模型來近似原始非線性動態，我們首先令 $\mathbf{f}(\mathbf{x}) = A(\mathbf{x})\mathbf{x}$。一個 $A(\mathbf{x})$ 項的集合有下列的形式：

$$(A(x))_{1,1} = \frac{I_y - I_z}{I_x}\left[\omega_0^2 c^2 x_3 \frac{s(2x_1)}{2x_1} - \omega_0^2 s^2 x_2 s^2 x_3 \frac{s(2x_1)}{2x_1} - 3\omega_0^2 c^2 x_2 \frac{s(2x_1)}{2x_1}\right],$$

$$(A(x))_{1,2} = \frac{I_y - I_z}{I_x}\left[\frac{1}{4}\omega_0^2 s(2x_3) c^2 x_1 \frac{sx_2}{x_2} - \frac{1}{4}\omega_0^2 s(2x_3)\frac{sx_2}{x_2} s^2 x_1\right],$$

$$(A(x))_{1,3} = \frac{I_y - I_z}{I_x}\left[\frac{1}{2}\omega_0^2 \frac{s(2x_3)}{x_3} c^2 x_1 s x_2 - \frac{1}{2}\omega_0^2 \frac{s(2x_3)}{2x_3} s x_2 s^2 x_1\right],$$

$$(A(x))_{1,4} = 0,$$

$$(A(x))_{1,5} = -\omega_0 s x_3 s x_2 + \frac{I_y - I_z}{I_x} \cdot \left[-\frac{1}{2}x_6 + \omega_0 c x_1 s x_3 s x_2 + \omega_0 c x_3 s x_1\right],$$

$$(A(x))_{1,6} = \omega_0 c x_3 c x_2 + \frac{I_y - I_z}{I_x}\left[\frac{1}{2}x_5 + \omega_0 c x_3 c x_1 - \omega_0 s x_3 s x_2 s x_1\right],$$

$$(A(x))_{2,1} = \frac{I_z - I_x}{I_y}\left[-\frac{1}{4}\omega_0^2 c x_2 \frac{s x_1}{x_1} s(2x_3)\right],$$

$$(A(x))_{2,2} = \frac{I_z - I_x}{I_y}\left[-\omega_0^2 \frac{s(2x_2)}{2x_2} s^2 x_3 c x_1\right.$$

$$-3\omega_0^2 \frac{s(2x_2)}{2x_2} cx_1 \Bigg],$$

$$(A(x))_{2,3} = \frac{I_z - I_x}{I_y} \left[-\frac{1}{2}\omega_0^2 cx_2 sx_1 \frac{s(2x_3)}{2x_3} \right],$$

$$(A(x))_{2,4} = \omega_0 cx_3 cx_1 + \omega_0 sx_3 sx_2 sx_1 + \frac{I_z - I_x}{I_y}$$

$$\cdot \left[-\frac{1}{2} x_6 + \omega_0 cx_1 sx_3 sx_2 + \omega_0 cx_3 sx_1 \right],$$

$$(A(x))_{2,5} = \omega_0 sx_3 cx_2 sx_1,$$

$$(A(x))_{2,6} = \omega_0 sx_3 cx_1 + \omega_0 cx_3 sx_2 sx_1$$

$$+ \frac{I_z - I_x}{I_y} \left[\frac{1}{2} x_4 - \omega_0 sx_3 cx_2 \right],$$

$$(A(x))_{3,1} = \frac{I_x - I_y}{I_z} \left[-\frac{3}{4}\omega_0^2 s(2x_2) \frac{sx_1}{x_1} \right],$$

$$(A(x))_{3,2} = \frac{I_x - I_y}{I_z} \Bigg[\omega_0^2 s^2 x_3 sx_1 \frac{s(2x_2)}{2x_2}$$

$$-\frac{3}{2}\omega_0^2 \frac{s(2x_2)}{2x_2} sx_1 \Bigg],$$

$$(A(x))_{3,3} = \frac{I_x - I_y}{I_z} \left[-\omega_0^2 \frac{s(2x_3)}{2x_3} cx_2 cx_1 \right],$$

$$(A(x))_{3,4} = \omega_0 sx_3 sx_2 sx_1 - \omega_0 cx_3 cx_1 + \frac{I_x - I_y}{I_z}$$

$$\cdot \left[\frac{1}{2} x_5 + \omega_0 cx_3 cx_1 - \omega_0 sx_3 sx_2 sx_1 \right],$$

$$(A(x))_{3,5} = -\omega_0 sx_3 cx_2 cx_1,$$

$$+ \frac{I_z - I_x}{I_y} \left[\frac{1}{2} x_4 - \omega_0 sx_3 cx_2 \right],$$

$$(A(x))_{3,6} = -\omega_0 sx_2 cx_1 cx_{13} + \omega_0 sx_3 sx_1$$

其中 $(A(\mathbf{x}))_{i,j}$ 標示為矩陣 $A(\mathbf{x})$ 之 (i,j)-項。

為了建構相對應的 T-S 模型。我們將從可能的工作區間來選擇適當的操作點，使得衛星的動態可以被此 T-S 模型很好的近似。在這個例子中，我們假設 $I_x = I_z = 2000 N \cdot m \cdot s^2,\ I_y = 400 N \cdot m \cdot s^2$，$\omega_0 = 1.0312 \times 10^{-3} rad/s$，同時我們假設 $x_1 \in [-\pi/2, \pi/2]$，$x_2 \in [-\pi, \pi]$ 與 $x_3 \in [-\pi/2, \pi/2]$。為了觀察前鑑部變數(premise variable)的個數的效果，我們考慮下列兩種方案。

案例 A：（考慮三個角度為前鑑部變數）

在本案例，操作點選擇為 $\{x_{i,j,k} = (x_{1,i}, x_{2,j}, x_{3,k}, 0,0,0)^T \mid i = 1,\cdots,n_1, j = 1,\cdots,n_2, k = 1,\cdots,n_3\}$ 其中 $\{x_{1,1},\cdots,x_{1,n_1}\}$，$\{x_{2,1},\cdots,x_{2,n_2}\}$ 與 $\{x_{3,1},\cdots,x_{3,n_3}\}$ 為 $[-\pi/2, \pi/2]$，$[-\pi, \pi]$ 與 $[-\pi/2, \pi/2]$，為相對應三個所選擇的切割點。在本案例，我們選擇 $n_1 = n_2 = n_3 = 5$ 與引入三角隸屬函數，如同圖 1 所描述。

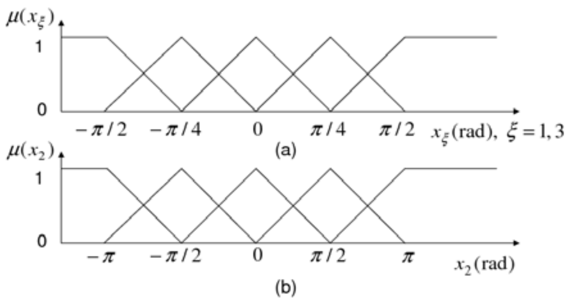

圖 1 案例 A 之三角隸屬函數

案例 B：（考慮六個前鑑部變數）

在本案例，操作點選成 $\{x_{i1,i2,i3,i4,i5,i6} = (x_{1,i1}, x_{2,i2}, x_{3,i3}, x_{4,i4}, x_{5,i5}, x_{6,i6})^T \mid 1 \le i_j \le n_j$ 及 n_j 為可能的整數，其中 $j = 1,\cdots,6\}$。在本例中，我們選擇 $n_j = 2$，$j = 1,\cdots,6$，並且也使用了三角隸屬函數，如圖 2 所示

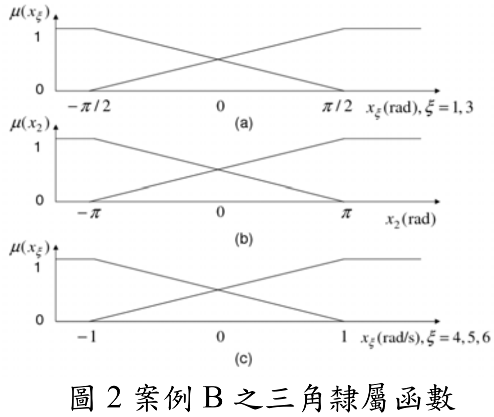

圖 2 案例 B 之三角隸屬函數

模擬結果總結在圖 3-5。我們使用了下列三種控制方式：SMC 可靠度設計[2](標示為 SMC)，T-S 模型為基礎之可靠度方法對應於不同的前鑑部變數個數(標示為 Case A 與 Case B)。SMC 可靠度設計之參數為 $M = 2I_3$，$\eta_j = 0.5$ 對所有 η_j 在 Λ_H 與 Λ_F，$\mathbf{d} = (0.1\sin(t), 0.1\cos(t), 0.1\cos(5t))^T$，$\mathbf{x}(0) = (-0.7, -0.07, 1.5, 0.3, 1.3, -0.2)^T$，$|u_j| \le 1$ 對所有的 j，訊號函數替換為飽和函數，飽和函數的邊界寬度為 0.05 來減輕訊號函數所產生

的切跳現象。此外，我們選擇$\mathbf{u}_2$當做可能損壞的控制器，也就是$H=\{u_1,u_3,u_4\}$ 與 $F=\{u_2\}$且$\mathbf{u}_2$在$t=2$時損壞。可以從圖 3 觀察在上述的三種控制方式中，穩定化表現都能成功的被達成。然而，因為 T-S 模型在案例 B 中非常接近原始非線性模型狀態軌跡，順滑變數與案例 B 的控制曲線與 SMC 可靠度設計非常接近，這可以從圖 3-5 了解。透過直接計算消耗能量與平方性能，有如下之關係：

$$(\int \mathbf{u}^T\mathbf{u})_{SMC} \approx 3.917 \le (\int \mathbf{u}^T\mathbf{u})_{CaseB} \approx 3.930 \le (\int \mathbf{u}^T\mathbf{u})_{CaseA} \approx 8.314$$

與

$$(\int \mathbf{x}^T\mathbf{x})_{CaseB} \approx 5.254 \le (\int x^T x)_{SMC} \approx 5.256$$
$$\le (\int x^T x)_{CaseA} \approx 5.602$$

清楚的，SMC 與案例 B 在如下之兩種性能指標$\int \mathbf{u}^T\mathbf{u}$ 與$\int \mathbf{x}^T\mathbf{x}$都非常接近，而案例 A 消耗較更多能量與較大的$\int \mathbf{x}^T\mathbf{x}$。值得一提的是從圖 5(b)可看出$\mathbf{u}_2$在兩秒後損壞且$\mathbf{u}_2$在 0.75 秒附近改變正負號，$\mathbf{u}_2$符號的改變可以從$B_2\mathbf{s}$的正負號改變來確認，這與(9)相符合。此外，由於在模擬中使用飽和函數，案例 A 之$\mathbf{u}_2$的值在$0.75 \le t \le 2$時是 0.5，這並不與(9)相違背因為此時$|B_F^T\mathbf{s}|>1$。最後，當重複計算控制器5×10^4次後，T-S 型設計(包含隸屬權重決定)，CPU 運算時間較傳統 SMC 為少，各控制方法所耗費的計算時間有著以下關係：

$$(\text{CPU})_{CaseA} \approx 5.087 \le (\text{CPU})_{CaseB} \approx 7.453$$
$$\le (\text{CPU})_{SMC} \approx 10.313$$

。從這些模擬，可以發現 SMC 與案例 B 的性能彼此非常接近，且比案例 A 好。然而，案例 A 消耗較少的時間在控制器實現上，因為它只有使用三個角度當作前提變數。此外，所提出的 T-S 型方法不只減輕線上運算負擔，也可以有效的完成穩定化任務，如同 SMC 設計一般，且以 T-S 模型為基礎的方法在前鑑部變數之分割變得更細的時候不會產生額外線上運算負擔,。

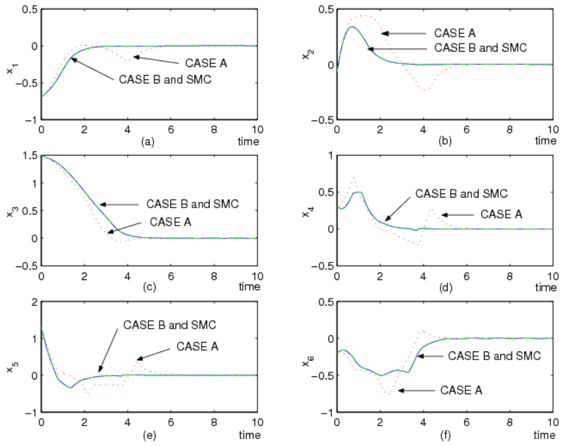

圖 3 系統之六個狀態

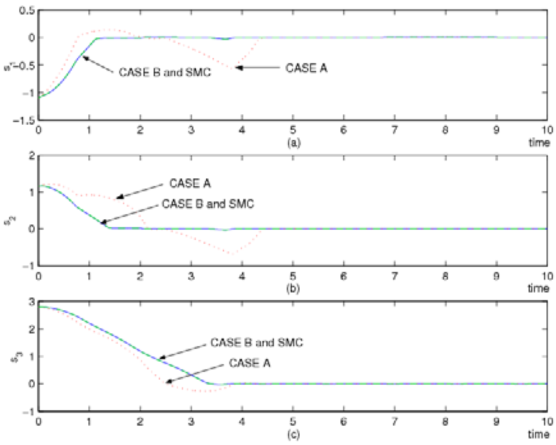

圖 4 系統之順滑變數

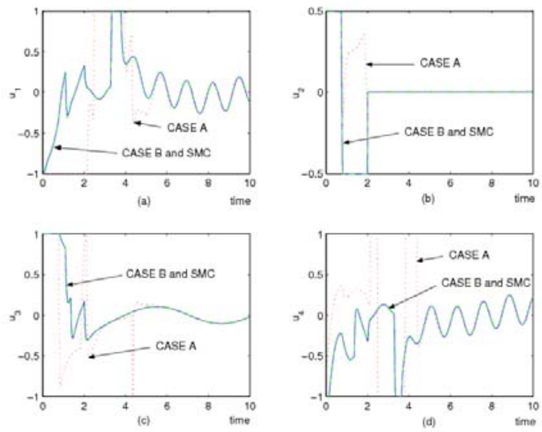

圖 5 控制器輸入

本計畫所獲得的研究成果已整理並在期刊及研討會發表（詳計畫成果自評及附件）。

四、計畫成果自評

本計畫的主要目的在於探討如何結合TS-模糊模型及變結構控制策略進行穩定性及軌跡追蹤的可靠度控制任務。針對本計畫之研究主題，我們在這一年內已完成下列工作項目：

1. 提出 T-S 模糊模型為基礎的可靠度設計，即使當可能損壞的制動器發生部分損壞或者全部損壞時依然可以達到穩定化之性能表現。
2. 提供的方法可以大量減輕線上運算負擔，因為所使用來近似原始非線性系統的T-S模糊系統模型之系統參數大部分可以離線取得。
3. 所提供的方法有著快速響應以及穩健的特性，因為採納了順滑模控制技術補償了額外干擾及在非線性與 T-S 模糊系統之間模型的不確定性。
4. 即使增加模糊法則的數量也不會產生額外的線上運算負擔，衛星系統的模擬例子清楚的展示了所提方案之效率與好處。

就計畫而言，我們已經達到了預期的成果。這些研究成果有些已發表於國外著名期刊及研討會，包括

[1] Y.-W. Liang, S.-D. Xu, D.-C. Liaw, and C.-C. Chen, “A Study of T-S Model-Based SMC Scheme With Application to Robot Control,” *IEEE Transactions on Industrial Electronics,* Vol. 55, No. 11, pp. 3964-3971, 2008.

[2] Y.-W. Liang, S.-D. Xu, and L.-W. Ting, “T-S Model-Based SMC Reliable Design for a Class of Nonlinear Systems,” *IEEE Transactions on Industrial Electronics,* accepted for publication, 2009.

[3] Y.-W. Liang, S.-D. Xu, and C.-C. Chen, “Study of a combination of the T-S fuzzy and the SMC approaches,” *CACS Automatic Control Conference (中華民國自動控制研討會)*, Tainan, Taiwan, Nov. 21-23, 2008.

五、參考文獻

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出席國際學術會議心得報告

計畫編號	NSC 97-2221-E-009-087
計畫名稱	應用 TS 模糊模型之順滑模態可靠度控制研究
出國人員姓名 服務機關及職稱	梁耀文，國立交通大學電機與控制系，副教授
會議時間地點	August 18-21, 2009，日本，福岡(Fukuoka)
會議名稱	ICCAS-SICE 2009
發表論文題目	SMC Reliable Design for T-S Model-Based Systems

一、參加會議經過

此次國際學術研討會議名稱為 ICROS-SICE International Joint Conference 2009 (ICCAS-SICE 2009)，是由 The Society of Instrument and Control Engineers (SICE)及 The Institute of Control, Robotics and Systems (ICROS)主辦，協辦單位包括有 IEEE Industrial Electronics Society、IEEE Robotics and Automation Society、IEEE Control Systems Society、The Instrumentation, Systems, and Automation Society (ISA) 、 Asian Control Association (ACA) 、China Instrument and Control Society (CIS)、Chinese Association of Automation (CAA)、Chinese Automatic Control Society (CACS) 、 International Measurement Confederation (IMEKO)、IEEE Japan Council、IFAC NMO-Japan、The Institute of Electrical Engineers of Japan。會議期間為西元 2009 年八月十八日至八月二十一日，地點為日本福岡之 Fukuoka International Congress Center。我們在八月十八日由新竹出發至中正機場，搭機飛往日本福岡機場。本人之論文報告日期被安排在八月十九日週三上午九點第一位上台報告，會場有多位學者專家對本人之研究成果甚感興趣，除了聽取這些學者專家的意見與建議外，本人也積極的與他們分享研究成果，自覺收穫豐碩。除此之外，會議期間我們也聆聽了諸多與自己研究領域相關之最新的研究報告，對於專家學者之認真與專注及求甚解之精神印象深刻，自覺收穫良多。會議後我們也遊覽福岡及其附近之旅遊景點，包括有將台灣割讓給日本之馬關條約簽約現場，感觸良多。並於八月二十二日返回台灣。

二、與會心得

由於參加此次研討會的學者專家甚多，因此這次的學術研討會議包含的研究主

題也相當廣泛。除了有許多的學術理論研究成果外，也包含有許多工業應用的應用成果展現。其中有量測、控制、系統資訊、系統判別、計算機及工業電力電子等相關應用之議題。這次的會議主要是以兩種形式呈現，一種是以海報的方式展現研究成果，另一種則是以口頭報告的形式呈現研究心得。我是以口頭報告的形式參與發表研究成果。會議期間觀摩各地學者專家呈現研究成果的方式，並互相交換研究心得。在控制領域方面，研究成果包括有控制理論、非線性系統控制、智慧型控制、模糊理論、類神經網路、基因演算法、電力系統、電力電子、工業資訊學、電腦與控制技術、感應器與致動器等等。此次會議之主要目的在於探討目前控制理論之趨勢及其應用的最新發展，與會人士除了地主國日本外，還有許多來自世界各地之學者專家。大家齊聚一堂共同研討及分享彼此之研究成果與研究心得。參加此次會議不但能親自目睹及體會著名學者的風範及研究執著精神，萌生見賢思齊的成長動力。同時能增進國際會議發表論文的膽識及獲致國際學術界最新的研究動態。在會議期間許多控制界的前輩所提出的問題及意見不但具體而深入，且往往能點出問題之所在及提供進一步研究方向與可能解答之輪廓，個人自覺收穫良多。也希望自己能有多些機會參與類似的國際會議。在這次的會議後，我們攜帶回來了大會的光碟資料，使我們可以盡覽此次會議的所有成果。在此願再一次的感謝國科會的經費補助，使我能順利的參與這次的研究成果發表饗宴。

SMC Reliable Design for T-S Model-Based Systems

Yew-Wen Liang, Sheng-Dong Xu, Der-Cherng Liaw, Cheng-Chang Chen, and Li-Wei Ting

Department of Electrical and Control Engineering, National Chiao Tung University, Taiwan
(Tel: +886-3-5712121 Ext: 31669; E-mail: ywliang@cn.nctu.edu.tw)

Abstract: This paper studies the robust reliable control issues based on the Takagi-Sugeno (T-S) fuzzy system modeling method and the sliding mode control (SMC) technique. The combined scheme is shown to have the merits of both approaches. It not only alleviates the on-line computational burden by using the T-S fuzzy system model to approximate the original nonlinear one (since most of the system parameters of the T-S model can be computed off-line) but it also preserves the advantages of rapid response and robustness of the SMC schemes. Moreover, the combined scheme does not require on-line computation of any nonlinear term of the original dynamics and the increase in the partition number of the region of premise variables does not create extra on-line computational burdens for the scheme. Under the design, the control mission can continue safely without prompt external support even when the susceptible actuators fail to operate. The proposed analytical results are also applied to the attitude control of a spacecraft. Simulation results demonstrate the benefits of the proposed scheme.

Keywords: Sliding mode control, reliable control, nonlinear control systems, T-S fuzzy system model.

1. INTRODUCTION

Recently, the study of reliable (or fault-tolerance) control has attracted considerable attention (see, e.g., [4], [9], [10], [11], [12], [17], [19]). The objective of reliable control is to design an appropriate controller such that the closed-loop system can tolerate the abnormal operations of specific control components and retain the overall system stability with acceptable system performance. Among the existing reliable control studies, several approaches have been presented. These approaches include the linear matrix inequality (LMI)-based approach [12], the algebraic Riccati equation (ARE)-based approach [17], the coprime factorization approach [18], the Hamilton-Jacobi (HJ)-based approach [9], [19], and the sliding mode control (SMC)-based approach [4], [10], [11]. Among the above-mentioned reliable control studies, only the HJ-based and the SMC-based approaches deal with reliability issues for nonlinear systems. However, the reliable controller of the HJ-based approach explicitly depends on the solution of an associated Hamilton-Jacobi equation which is in general difficult to solve. Though a power series method [8] may alleviate the difficulty through computer calculation, the obtained solution is only approximate and the computation load grows quickly when the system is complicated. In contrast, the SMC reliable controllers do not require the solution of any HJ equation, while retain the advantages of conventional SMC designs [11]. Those advantages include rapid response, robustness, and ease of implementation [11], [13], [14], [15].

On the other hand, because of the conceptual simplicity and the fact that most of the system parameters can be computed off-line, the Takagi-Sugeno (T-S) modeling scheme has become a popular and powerful fuzzy system modeling approach (see, e.g., [1], [13], [16]). The basic idea of the T-S approach is first to decompose a nonlinear system into several linear models according to different cases where the associated linear models best fit the nonlinear one, and then to aggregate each individual linear model into a single nonlinear one in terms of each model's membership functions. Though the concept is simple, the T-S fuzzy system model has been theoretically justified as a universal approximator which makes the T-S fuzzy system model become particularly useful, especially when the nonlinear model is complicated. In order to compensate for the additional uncertainties resulting from the difference between the original and the T-S models, a combined scheme incorporated with the SMC technique was recently proposed (see, e.g., [13]). The combined scheme not only alleviates the on-line computational burden since the T-S fuzzy system model is utilized to approximate the original nonlinear one, but it also preserves the advantages of rapid response and robustness of the SMC schemes. In light of those remarkable benefits, this paper will investigate the reliability issues from the combined scheme viewpoint.

2. PROBLEM STATEMENT

Consider a class of 2nd-order nonlinear control systems

$$\dot{\mathbf{x}}_1 = \mathbf{x}_2 \;\text{ and }\; \dot{\mathbf{x}}_2 = \mathbf{f}(\mathbf{x}) + B\mathbf{u} + \mathbf{d} \tag{1}$$

where $\mathbf{x}_1 = (x_1, \cdots, x_n)^T \in \mathbb{R}^n$, $\mathbf{x}_2 = (x_{n+1}, \cdots, x_{2n})^T \in \mathbb{R}^n$ and $\mathbf{x} = (\mathbf{x}_1^T, \mathbf{x}_2^T)^T$ is the system states, $\mathbf{u} = (u_1, \cdots, u_{n+m})^T \in \mathbb{R}^{n+m}$ is the control inputs, $\mathbf{d} = (d_1, \cdots, d_n)^T \in \mathbb{R}^n$ denote possible model uncertainties and/or external disturbances, $\mathbf{f}(\mathbf{x}) \in \mathbb{R}^n$ is a smooth function, and $(\cdot)^T$ denotes the transpose of a vector or a matrix. In this study, we assume that $\mathbf{f}(\mathbf{0}) = \mathbf{0}$. It is important to note that in the description of the system given by Eq. (1) we have assumed that the system has control input redundancy. We divide the actuators into two groups $\mathcal{H}$ and $\mathcal{F}$, within which we assume that all of the actuators in $\mathcal{H}$ are healthy while those in $\mathcal{F}$ are allowed to fail during the operation. System (1) can be rewritten as

$$\dot{\mathbf{x}}_1 = \mathbf{x}_2 \text{ and } \dot{\mathbf{x}}_2 = \mathbf{f}(\mathbf{x}) + B_{\mathcal{H}}\mathbf{u}_{\mathcal{H}} + B_{\mathcal{F}}\mathbf{u}_{\mathcal{F}} + \mathbf{d}. \quad (2)$$

Since the nonsingularity assumption of $B_{\mathcal{H}}$ is necessary for the existence of equivalent control in SMC design when all the actuators in $\mathcal{F}$ fail to operate [6], we assume that the pre-selected healthy actuators satisfy $\mathbf{u}_{\mathcal{H}} \in \mathbb{R}^n$, and $B_{\mathcal{H}} \in \mathbb{R}^{n\times n}$ is a nonsingular matrix.

The objective of this study is to organize an appropriate $\mathbf{u}_{\mathcal{H}}$ and $\mathbf{u}_{\mathcal{F}}$ so that the origin of the closed-loop system is asymptotically stable even when all or some of the actuators in the set $\mathcal{F}$ fail to operate.

3. T-S MODEL-BASED SMC RELIABLE DESIGN

In light of the advantages of the T-S modeling and SMC approaches as stated above, this study will combine the two schemes for the design of reliable controllers.

3.1 T-S Fuzzy Model Description

It is known that a nonlinear system can be approximated by a T-S fuzzy model ([13], [16]), which is described by a combination of several linear models with suitable weighting. The ith ($i = 1, 2, \cdots, p$) rule of the T-S fuzzy model for System (2) has the following form:
If ζ_1 is $M_{1_i}, \cdots, \zeta_q$ is M_{q_i}, $i = 1, \cdots, p$, then

$$\dot{\mathbf{x}}_1 = \mathbf{x}_2 \text{ and } \dot{\mathbf{x}}_2 = A_i\mathbf{x} + B_{\mathcal{F}}\mathbf{u}_{\mathcal{F}} + B_{\mathcal{H}}\mathbf{u}_{\mathcal{H}} \quad (3)$$

where $\zeta_1, \cdots, \zeta_q$ are premise variables, $M_{1_i}, \cdots, M_{q_i}$ are membership functions for premise variables, p and q denote the number of rules and premise variables, respectively, and $A_i \in \mathbb{R}^{n\times n}$. The T-S fuzzy model is then constructed according to the weight of the system state on each linear model as (4) below:

$$\dot{\mathbf{x}}_1 = \mathbf{x}_2 \text{ and } \dot{\mathbf{x}}_2 = \sum_{i=1}^{p} \alpha_i(\mathbf{x})A_i\mathbf{x} + B_{\mathcal{F}}\mathbf{u}_{\mathcal{F}} + B_{\mathcal{H}}\mathbf{u}_{\mathcal{H}} \quad (4)$$

where the weightings $\alpha_i(\mathbf{x}) \geq 0$ for all i and $\sum_{i=1}^{p} \alpha_i(\mathbf{x}) = 1$.

3.2 SMC Reliable Design

By incorporating with the T-S fuzzy model, System (2) can be rewritten as

$$\dot{\mathbf{x}}_1 = \mathbf{x}_2, \quad (5)$$

$$\text{and } \dot{\mathbf{x}}_2 = \sum_{i=1}^{p} \alpha_i(\mathbf{x})A_i\mathbf{x} + \Delta\mathbf{f} + (B_{\mathcal{F}}\mathbf{u}_{\mathcal{F}} + B_{\mathcal{H}}\mathbf{u}_{\mathcal{H}}) + \mathbf{d} \quad (6)$$

where $\Delta\mathbf{f} := \mathbf{f}(\mathbf{x}) - \sum_{i=1}^{p} \alpha_i(\mathbf{x})A_i\mathbf{x}$. Since System (1) contains a set of 2nd-order systems, we may assume the sliding surface to be

$$\mathbf{s}(t) := (s_1, s_2, \cdots, s_n)^T = \mathbf{x_2}(t) + M\mathbf{x_1}(t) \quad (7)$$

where $M \in \mathbb{R}^{n\times n}$ is a positive definite matrix. Clearly, if the system state remains on the sliding surface, then the desired stabilization performance of $\mathbf{x}(t) \to \mathbf{0}$ can be exponentially achieved. To compensate for the effects of disturbances and/or uncertainties, we impose the next assumption:

Assumption 1: There exist nonnegative scalar functions $\rho_j(\mathbf{x}, t)$, $j = 1, \cdots, n$, such that $|(B_{\mathcal{F}}\mathbf{u}^*_{\mathcal{F}})_j| + |(\Delta\mathbf{f})_j| + |d_j| \leq \rho_j(\mathbf{x}, t)$, where $\mathbf{u}^*_{\mathcal{F}}$ describes the possible values of $\mathbf{u}_{\mathcal{F}}$ and $(\cdot)_j$ denotes the jth entry of a vector.

Following the SMC design procedure [11], we select

$$\mathbf{u}_{\mathcal{H}} = -B_{\mathcal{H}}^{-1}\left(\sum_{i=1}^{p} \alpha_i(\mathbf{x})A_i\mathbf{x} + M\mathbf{x}_2 + \Lambda_{\mathcal{H}} \cdot \text{sgn}(\mathbf{s})\right) \quad (8)$$

where $\Lambda_{\mathcal{H}} = \text{diag}(\rho_1(\mathbf{x}, t) + \eta_1, \cdots, \rho_n(\mathbf{x}, t) + \eta_n)$ with $\eta_j > 0$ for $j = 1, \cdots, n$, sgn$(\cdot)$ denotes the sign function, and sgn$(\mathbf{s}) := (\text{sgn}(s_1), \cdots, \text{sgn}(s_n))^T$. Under the control $\mathbf{u}_{\mathcal{H}}$, it follows from (5)-(7) and Assumption 1 that $\mathbf{s}^T\dot{\mathbf{s}} \leq \sum_{j=1}^{n} \eta_j \cdot |s_j|$. This inequality implies that the system states will reach the sliding surface in a finite amount of time [11] no matter whether the actuators in $\mathcal{F}$ are healthy or not.

In addition to the design of $\mathbf{u}_{\mathcal{H}}$ as discussed above, we now investigate the design of $\mathbf{u}_{\mathcal{F}}$ to promote the overall system performance when some or all of the actuators in $\mathcal{F}$ are healthy. From (5)-(6) and (8), we have $\mathbf{s}^T\dot{\mathbf{s}} \leq \mathbf{s}^T B_{\mathcal{F}}\mathbf{u}_{\mathcal{F}} - \sum_{j=1}^{n} \eta_j \cdot |s_j|$. Clearly, one of the choices of $\mathbf{u}_{\mathcal{F}}$ to make system states approach the sliding surface faster than in the case of $\mathbf{u}_{\mathcal{F}} = \mathbf{0}$ is

$$\mathbf{u}_{\mathcal{F}} = -\Lambda_{\mathcal{F}} \cdot \text{sgn}(B_{\mathcal{F}}^T\mathbf{s}) \quad (9)$$

where $\Lambda_{\mathcal{F}} = \text{diag}(\eta_{n+1}, \cdots, \eta_{n+m})$ and $\eta_{n+\iota} \geq 0$ for all $\iota = 1, \cdots, m$. These derivations show that the magnitude of control gains $\eta_{n+\iota}, \iota = 1, \cdots, m$, for the actuators in the set $\mathcal{F}$ that guarantee stabilization performance may vary from 0 to the allowable maximum control input magnitude. That is, it allows the situation of actuators in $\mathcal{F}$ to be total failure, partial failure, attenuation or amplification in any order and in any combination. From the derivations above, we then have the following result.

Theorem 1: Suppose that Assumption 1 holds. Then the origin of System (2) is locally asymptotically stable under the control law given by (8) and (9) even when some or all of the actuators in $\mathcal{F}$ experience abnormal operation.

4. APPLICATION TO SPACECRAFT ATTITUDE STABILIZATION

An attitude model for a spacecraft in a circular orbit can be described in the form of (1) with $n = 3$ [11]. The six state variables denote the three Euler's angles (ϕ, θ, ψ) and their derivatives. For simplicity, we assume in this study that the thruster is the only applied control force and there is an actuator redundancy to perform the reliable task. By letting $\mathbf{x} = (\phi, \theta, \psi, \dot{\phi}, \dot{\theta}, \dot{\psi})^T$ and $\mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), f_3(\mathbf{x}))^T$, the overall system dynamics are described as follows [11]:

$$f_1(\mathbf{x}) = \omega_0 x_6 cx_3 cx_2 - \omega_0 x_5 sx_3 sx_2 + \frac{I_y - I_z}{I_x}\Big[x_5 x_6 + \omega_0 x_5 cx_1 sx_3 sx_2 + \omega_0 x_5 cx_3 sx_1 + \omega_0 x_6 cx_3 cx_1$$

$$
\begin{aligned}
&+\frac{1}{2}\omega_0^2 s(2x_3)c^2x_1sx_2+\frac{1}{2}w_0^2c^2x_3s(2x_1)\\
&-\omega_0x_6sx_3sx_2sx_1-\frac{1}{2}\omega_0^2s^2x_2s^2x_3s(2x_1)\\
&-\frac{1}{2}\omega_0^2s(2x_3)sx_2s^2x_1-\frac{3}{2}\omega_0^2c^2x_2s(2x_1)\Bigg],\\
f_2(\mathbf{x}) \;=\;& \omega_0x_6sx_3cx_1+\omega_0x_4cx_3sx_1+\omega_0x_6cx_3sx_2sx_1\\
&+\omega_0x_5sx_3cx_2sx_1+\omega_0x_4sx_3sx_2cx_1\\
&+\frac{I_z-I_x}{I_y}\Bigg[x_4x_6+\omega_0x_4cx_1sx_3sx_2\\
&+\omega_0x_4cx_3sx_1-\omega_0x_6sx_3cx_2\\
&-\frac{1}{2}\omega_0^2s(2x_2)s^2x_3cx_1-\frac{1}{2}w_0^2cx_2sx_1s(2x_3)\\
&+\frac{3}{2}\omega_0^2s(2x_2)cx_1\Bigg],\\
f_3(\mathbf{x}) \;=\;& \omega_0x_4sx_1sx_3sx_2-\omega_0x_6cx_1cx_3sx_2\\
&-\omega_0x_5cx_1sx_3cx_2+\omega_0x_6sx_3sx_1-\omega_0x_4cx_3cx_1\\
&+\frac{I_x-I_y}{I_z}\Bigg[x_4x_5+\omega_0x_4cx_3cx_1\\
&-\omega_0x_4sx_3sx_2sx_1-\omega_0x_5sx_3cx_2\\
&-\frac{1}{2}\omega_0^2s(2x_3)cx_2cx_1+\frac{1}{2}w_0^2s^2x_3sx_1s(2x_2)\\
&-\frac{3}{2}\omega_0^2s(2x_2)sx_1\Bigg],\\
B \;=\;& \begin{pmatrix} 0.67 & 0.67 & 0.67 & 0.67\\ 0.69 & -0.69 & -0.69 & 0.69\\ 0.28 & 0.28 & -0.28 & -0.28\end{pmatrix}.
\end{aligned}
$$

Here, I_x, I_y, and I_z are the inertia with respect to the three body coordinate axes, ω_0 denotes the constant orbital rate, and c and s denote the $\cos$ and $\sin$ functions, respectively.

To derive an appropriate T-S model to approximate the original nonlinear dynamics, we first express $\mathbf{f}(\mathbf{x}) = A(\mathbf{x})\mathbf{x}$. A set of entries of $A(\mathbf{x})$ have the following forms:

$$
\begin{aligned}
(A(\mathbf{x}))_{1,1} &= \frac{I_y-I_z}{I_x}\Bigg[w_0^2c^2x_3\frac{s(2x_1)}{2x_1}\\
&\quad -\omega_0^2s^2x_2s^2x_3\frac{s(2x_1)}{2x_1}-3\omega_0^2c^2x_2\frac{s(2x_1)}{2x_1}\Bigg],\\
(A(\mathbf{x}))_{1,2} &= \frac{I_y-I_z}{I_x}\Bigg[\frac{1}{4}\omega_0^2s(2x_3)c^2x_1\frac{sx_2}{x_2}\\
&\quad -\frac{1}{4}\omega_0^2s(2x_3)\frac{sx_2}{x_2}s^2x_1\Bigg],\\
(A(\mathbf{x}))_{1,3} &= \frac{I_y-I_z}{I_x}\Bigg[\frac{1}{2}\omega_0^2\frac{s(2x_3)}{2x_3}c^2x_1sx_2\\
&\quad -\frac{1}{2}\omega_0^2\frac{s(2x_3)}{2x_3}sx_2s^2x_1\Bigg],\\
(A(\mathbf{x}))_{1,4} &= 0,\\
(A(\mathbf{x}))_{1,5} &= -\omega_0sx_3sx_2+\frac{I_y-I_z}{I_x}\\
&\quad \cdot\Bigg[\frac{1}{2}x_6+\omega_0cx_1sx_3sx_2+\omega_0cx_3sx_1\Bigg],\\
(A(\mathbf{x}))_{1,6} &= \omega_0cx_3cx_2+\frac{I_y-I_z}{I_x}\Bigg[\frac{1}{2}x_5+\omega_0cx_3cx_1\\
&\quad -\omega_0sx_3sx_2sx_1\Bigg],\\
(A(\mathbf{x}))_{2,1} &= \frac{I_z-I_x}{I_y}\Bigg[-\frac{1}{4}w_0^2cx_2\frac{sx_1}{x_1}s(2x_3)\Bigg],\\
(A(\mathbf{x}))_{2,2} &= \frac{I_z-I_x}{I_y}\Bigg[-\omega_0^2\frac{s(2x_2)}{2x_2}s^2x_3cx_1\\
&\quad +3\omega_0^2\frac{s(2x_2)}{2x_2}cx_1\Bigg],\\
(A(\mathbf{x}))_{2,3} &= \frac{I_z-I_x}{I_y}\Bigg[-\frac{1}{2}w_0^2cx_2sx_1\frac{s(2x_3)}{2x_3}\Bigg],\\
(A(\mathbf{x}))_{2,4} &= \omega_0cx_3sx_1+\omega_0sx_3sx_2cx_1+\frac{I_z-I_x}{I_y}\\
&\quad \cdot\Bigg[\frac{1}{2}x_6+\omega_0cx_1sx_3sx_2+\omega_0cx_3sx_1\Bigg],\\
(A(\mathbf{x}))_{2,5} &= \omega_0sx_3cx_2sx_1\ ,\\
(A(\mathbf{x}))_{2,6} &= \omega_0sx_3cx_1+\omega_0cx_3sx_2sx_1\\
&\quad +\frac{I_z-I_x}{I_y}\Bigg[\frac{1}{2}x_4-\omega_0sx_3cx_2\Bigg],\\
(A(\mathbf{x}))_{3,1} &= \frac{I_x-I_y}{I_z}\Bigg[-\frac{3}{4}\omega_0^2s(2x_2)\frac{sx_1}{x_1}\Bigg],\\
(A(\mathbf{x}))_{3,2} &= \frac{I_x-I_y}{I_z}\Bigg[w_0^2s^2x_3sx_1\frac{s(2x_2)}{2x_2}\\
&\quad -\frac{3}{2}\omega_0^2\frac{s(2x_2)}{2x_2}sx_1\Bigg],\\
(A(\mathbf{x}))_{3,3} &= \frac{I_x-I_y}{I_z}\Bigg[-\omega_0^2\frac{s(2x_3)}{2x_3}cx_2cx_1\Bigg],\\
(A(\mathbf{x}))_{3,4} &= \omega_0sx_1sx_3sx_2-\omega_0cx_3cx_1+\frac{I_x-I_y}{I_z}\\
&\quad \cdot\Bigg[\frac{1}{2}x_5+\omega_0cx_3cx_1-\omega_0sx_3sx_2sx_1\Bigg],\\
(A(\mathbf{x}))_{3,5} &= -\omega_0cx_1sx_3cx_2\\
&\quad +\frac{I_x-I_y}{I_z}\Bigg[\frac{1}{2}x_4-\omega_0sx_3cx_2\Bigg],\\
(A(\mathbf{x}))_{3,6} &= -\omega_0cx_1cx_3sx_2+\omega_0sx_3sx_1
\end{aligned}
$$

where $(A(\mathbf{x}))_{i,j}$ denotes the (i,j)-entry of the matrix $A(\mathbf{x})$. Next, a set of operating points will be selected for the construction of the associated linear models. These operating points are selected from the possible workspace, so that the motion of the spacecraft can be well approximated by using a convex combination of the associated linear models. For demonstration, we assume that $I_x = I_z = 2000\ N\cdot m\cdot s^2$, $I_y = 400\ N\cdot m\cdot s^2$, $\omega_0 = 1.0312\times 10^{-3}\ rad/s$, and the angular positions are constrained to be $x_1\in[-\pi/2,\pi/2]$, $x_2\in[-\pi,\pi]$, and $x_3\in[-\pi/2,\pi/2]$. To investigate the effects of the number of premise variables, we consider the following two cases: The first in which the three angles are chosen as the premise variables, while in the second case all six states are included.

4.1 Case for Three Premise Variables

In this case, the operating points are chosen in the form of $\{\mathbf{x}_{i,j,k} = (x_{1,i}, x_{2,j}, x_{3,k}, 0, 0, 0)^T | \; i = 1, \cdots, n_1, \quad j = 1 \cdots, n_2, \quad k = 1, \cdots, n_3\}$, where $\{x_{1,1}, \cdots, x_{1,n_1}\}$, $\{x_{2,1}, \cdots, x_{2,n_2}\}$ and $\{x_{3,1}, \cdots, x_{3,n_3}\}$ are three selected partitions of $[-\pi/2, \pi/2]$, $[-\pi, \pi]$, and $[-\pi/2, \pi/2]$, respectively. In this case, we select $n_1 = n_2 = n_3 = 5$ and employ the triangular membership functions. Under these settings, we have $5^3 = 125$ operating points. The associated 125 linear models can then be easily obtained. Two of them are listed below:

$$A_{1,1,1} = 10^{-6} \cdot \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -824.96 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -206.24 & 0 & 1031.2 \end{pmatrix},$$

$$A_{2,2,4} = 10^{-6} \cdot \begin{pmatrix} 0 & 0 & 0 & 0 & -729.17 & 583.33 \\ 0 & 0 & 0 & 0 & -729.17 & 0 \\ 0.41 & 0.27 & 0 & 145.83 & -583.33 & -1031.2 \end{pmatrix}$$

where $A_{i,j,k} = A(\mathbf{x}_{i,j,k})$. After determining the 125 linear models, the T-S fuzzy system model can be easily determined when the angular positions of the spacecraft are available. Define the region $D_{i,j,k} := \{\mathbf{x} | x_{1,i} \leq x_1 \leq x_{1,i+1}, \; x_{2,j} \leq x_2 \leq x_{2,j+1}, \; x_{3,k} \leq x_3 \leq x_{3,k+1}, \; -1 \leq x_l \leq 1, \; l = 4, 5, 6\}$. The upper bounds of $||\Delta \mathbf{f}||_{\infty, D_{i,j,k}} := \sup_{\mathbf{x} \in D_{i,j,k}} ||\Delta \mathbf{f}(\mathbf{x})||$ over the region $D_{i,j,k}$ can be computed off-line, and it is found that the maximum value of $||\Delta \mathbf{f}||_{\infty, D_{i,j,k}}$ among all of the regions $D_{i,j,k}$ is $\max_{i,j,k}(||\Delta \mathbf{f}||_{\infty, D_{i,j,k}}) \approx 1.14$. Since the T-S type controller only uses three premise variables with triangular membership functions, it therefore triggers at most eight rules (i.e., at most 2^3 linear models) at each time instant. Thus, it does not create an extra on-line computational burden if the partition for the regions of x_1, x_2 and x_3 are made finer. However, since the maximum value of a function over a smaller sub-region is smaller than or equal to that of the same function over the whole region, it follows that a finer partition for the region of x_1, x_2 and x_3 will result in a smaller magnitude of $\rho_j(\mathbf{x}, t)$ as stated in Assumption 1. Thus, the control magnitude will be smaller so that the physical control magnitude constraint is easier to fulfill for practical applications if the partition of x_1, x_2 and x_3 are made finer.

4.2 Case for Six Premise Variables

The operating points in this case are chosen in the form of $\{\mathbf{x}_{i_1,i_2,i_3,i_4,i_5,i_6} = (x_{1,i_1}, x_{2,i_2}, x_{3,i_3}, x_{4,i_4}, x_{5,i_5}, x_{6,i_6})^T | \; 1 \leq i_j \leq n_j$ and n_j are positive integers for $j = 1, \cdots, 6\}$. In this example, we select $n_j = 2$ for $j = 1, \cdots, 6$ and also employ the triangular membership functions. Under these settings, we have $2^6 = 64$ operating points and linear models which are determined from the relation $A_{i_1,i_2,i_3,i_4,i_5,i_6} = A(\mathbf{x}_{i_1,i_2,i_3,i_4,i_5,i_6})$. Details are omitted. The T-S model can then be easily determined when all of the angular positions and velocities of the spacecraft are available. Since the T-S type controller uses six premise variables for this case, it triggers 64 rules (i.e., 2^6 linear models) at each time instant. Furthermore, it does not create an extra on-line computational burden if the partition for the regions of the system states is made finer, as seen in the previous case. Moreover, it is found that $||\Delta \mathbf{f}||_\infty \approx 0.005$, which can be computed off-line. This implies that the difference between the T-S model and the original dynamics for this case is much smaller than that of Case A, though this case consumes more time (since it triggers 64 rules at every time instant) to evaluate the T-S model than that of Case A (only triggers 8 rules at each time instant).

Numerical results are summarized in Figs. 1-3. Among these, we use the following three control schemes: One is the SMC reliable design [11] (labeled by SMC), and the other two are the T-S model-based SMC reliable scheme with a different number of premise variables as stated in Cases 4.1 and 4.2 above (Labeled by Case A and Case B, respectively). The parameters of these SMC reliable designs are set to be $M = 2I_3$, $\eta_j = 0.5$ for all η_j in $\Lambda_\mathcal{H}$ and $\Lambda_\mathcal{F}$, $\mathbf{d} = (0.1\sin(t), 0.1\cos(t), 0.1\cos(5t))^T$, $\mathbf{x}(0) = (-0.7, -0.07, 1.5, 0.3, 1.3, -0.2)^T$, $|u_j| \leq 1$ for all j, and the sign function is replaced by the saturation function with a boundary layer width of 0.05 to alleviate the chattering produced by the sign function. In addition, we select u_2 as the susceptible actuator, that is, $\mathcal{H} = \{u_1, u_3, u_4\}$ and $\mathcal{F} = \{u_2\}$, and assume that u_2 fails at $t = 2$. It is observed from Fig. 1 that the stabilization performance is, as expected, achieved for all of the three control schemes. However, since the T-S model for Case B is very close to the original nonlinear model, the state curves, the sliding variables and the control curves for Case B and the SMC reliable design are also very close to each other, which can be recognized from Figs. 1-3. By direct calculation, the consumed energy and the quadratic performance have the following relations: $(\int \mathbf{u}^T\mathbf{u})_{\mathrm{SMC}} \approx 3.917 \leq (\int \mathbf{u}^T\mathbf{u})_{\mathrm{CaseB}} \approx 3.930 \leq (\int \mathbf{u}^T\mathbf{u})_{\mathrm{CaseA}} \approx 8.143$ and $(\int \mathbf{x}^T\mathbf{x})_{\mathrm{CaseB}} \approx 5.254 \leq (\int \mathbf{x}^T\mathbf{x})_{\mathrm{SMC}} \approx 5.256 \leq (\int \mathbf{x}^T\mathbf{x})_{\mathrm{CaseA}} \approx 5.602$. Clearly, the two performance indices $\int \mathbf{u}^T\mathbf{u}$ and $\int \mathbf{x}^T\mathbf{x}$ of the two control schemes SMC and Case B are found to be close to each other, while Case A consumes more energy and experiences a larger value of $\int \mathbf{x}^T\mathbf{x}$ than the other two schemes. It is worth noting from Fig. 3(b) that u_2 fails after $t = 2$ and changes sign around $t = 0.75$ for all of the three schemes. The sign change of u_2 is verified by the sign change of $B_2\mathbf{s}$, which agrees with Eq. (9). Furthermore, owing to the use of the saturation function for simulation, the magnitude of u_2 for Case A is seen to be less than 0.5 rather than equal to 0.5 during the time period $0.75 \leq t \leq 2$, which does not contradict Eq. (9) when $|B_\mathcal{F}^T\mathbf{s}| < 1$. Finally, when repeatedly computing the controllers 5×10^4 times, the T-S type design (including the determination of membership weightings) consumes less CPU time than the classic SMC design in the relation of $(\mathrm{CPU})_{\mathrm{CaseA}} \approx 5.087 \leq (\mathrm{CPU})_{\mathrm{CaseB}} \approx 7.453 \leq (\mathrm{CPU})_{\mathrm{SMC}} \approx 10.313$. Based upon these simulations, it is observed that the performances of the two schemes SMC and Case B are very close to each other and is better than those of Case A; however, Case A consumes less time for controller implementation because it only uses three angles as premise variables. Besides, the proposed T-S type approach not only alleviates the on-line computational burden, but it is also able to efficiently perform the stabilization mission as that of the SMC design. The T-S model-based SMC schemes do not create an extra on-line computational burden when the partition of the premise variables is made finer.

5. CONCLUSIONS

A T-S model-based SMC reliable design has been presented for a set of 2nd-order nonlinear control systems. The proposed reliable scheme is shown to be able to continue the control mission safely without prompt maintenance and achieve the stabilization performance even when the susceptible actuators experience outage. Besides, the presented scheme retains both the benefits of the T-S and the SMC approaches. It not only alleviates the on-line computational burden since it uses the T-S model to approximate the original nonlinear one and most of the system parameters of the T-S model can be computed off-line. It also preserves the advantages of rapid response and robustness of the SMC schemes. Moreover, the increase in the partition number of the region of premise variables in this T-S scheme does not create extra on-line computational burdens for the scheme. Simulation results have demonstrated the benefits of the proposed scheme.

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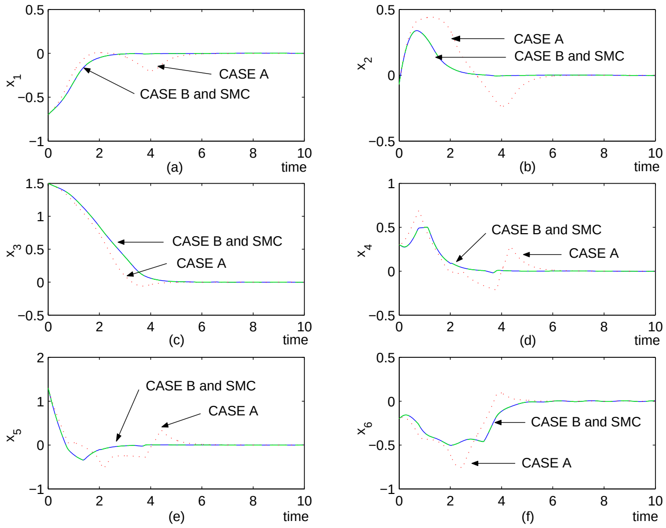

Fig. 1 Time history of the six system states.

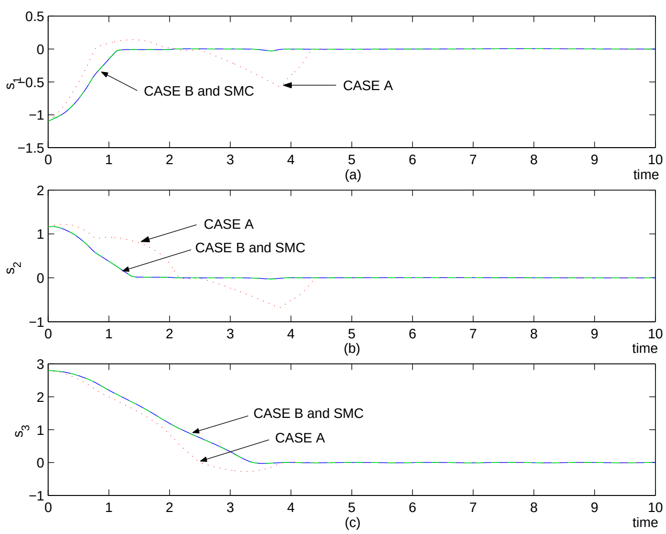

Fig. 2 Time history of the sliding variables.

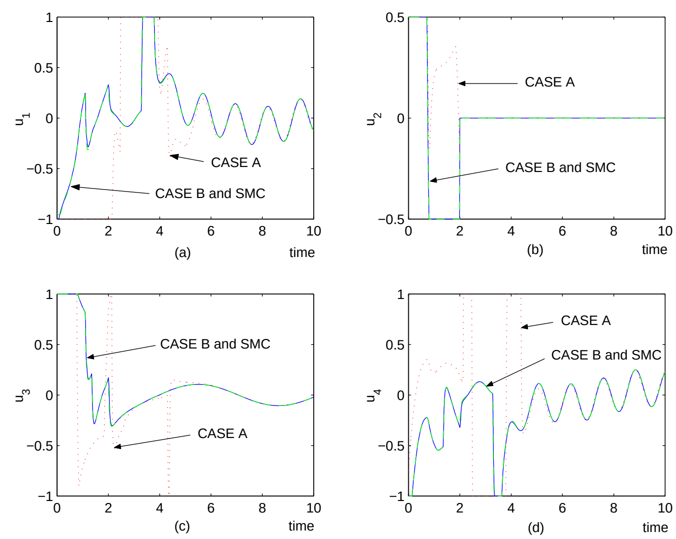

Fig. 3 Time history of the control inputs.