IEEE TRANSACTIONS ON RELIABILITY, VOL. 45, NO. 3, 1996 SEPTEMBER 413

A Cut-Based Method for Terminal-Pair Reliability

Yu G. Chen
National Chiao Tung University, Hsinchu
Maria C. Yuang, Member IEEE
National Chiao Tung University, Hsinchu

Key Words — **Terminal-pair reliability, Factoring, Partitioning, Network reduction**

Summary & Conclusions — **This paper assesses two categories of partition techniques for computing terminal-pair reliability (path-based and cut-based algorithms) by experimenting on published benchmarks; the criteria are the number of subproblems and the computation time. The cut-based algorithm is superior to the path-based algorithm with respect to the computation time for most benchmarks. A refinement of the cut-based algorithm (using network reduction) profoundly outperforms the path-based algorithm (with reduction) for all benchmarks.**

1. INTRODUCTION

Acronyms[1]

D&G	Dotson & Gobien algorithm [4]
CB, PB	[cut, path] based
CBR, PBR	[CB, PB] algorithm with reduction
R&P	Deo & Medidi algorithm [3]
RKP	Rai, Kumar, Prasad — CB partition algorithm [10]
R-CB	refined cut-based algorithm (presented in this paper).

The analysis of network reliability has been given considerable attention. In particular, terminal-pair reliability [1 - 6, 8 - 14] deals with the determination of the reliability between two nodes (source and sink) of a network, given failure probabilities of all links. Existing terminal-pair reliability algorithms, based on the partition technique, fall into two categories: PB or CB. In the PB technique, D&G, the most efficient PB algorithm [14], used a partition algorithm based on the shortest path from the source to the sink. R&P further combined D&G with network-reduction [8] and improved performance even more. On the other hand, RKP performed the partition by a source-cut — separating the source from the remaining network nodes.

This paper assesses the PB and CB partition algorithms in terms of the number of subproblems and computation time, by experimenting on published benchmarks. In both D&G and R&P, the numbers of subproblems generated by partitioning are locally minimized at the expense of executing the path-searching algorithm for finding the partition basis in each subproblem [3]. On the other hand, RKP makes no attempt to minimize locally the number of subproblems, but greatly reduces the computation time for the partitioning of each subproblem. Our experimental results show the superiority of the CB algorithm over the PB algorithm with respect to the computation time for most benchmarks. This paper also refines the CB algorithm using network reduction. R-CB profoundly outperforms the PB algorithm (with reduction) for all benchmarks.

Section 2 gives an overview of the D&G, R&P, and RKP, and presents R-CB. Section 3 compares the performance between the PB and CB algorithms.

2. OVERVIEW OF PB & CB ALGORITHMS

Four algorithms are summarized:

- D&G — PB algorithm,
- R&P — PBR algorithm,
- RKP — CB algorithm,
- R-CB — CBR algorithm.

Notation

Rel(G)	terminal-pair reliability of network G
s, t	source, sink
e_i	set of links, $i=1,\dots,l$
p_i, q_i	[success, failure] probability of e_i; $p_i + q_i \equiv 1$
$*$, $-$	[contracting, deleting] operation of links.

Assumptions (for these algorithms)

1. The network is modeled as directed graph.
2a. The p_i, q_i, $i=1,\dots,l$ are known for the links.
2b. Nodes are perfect (do not fail).
3. All failure events are mutually statistically independent.

2.1 PB Algorithm — D&G

D&G computes Rel(G) from s to t in G by Boolean algebra. Given a set of links $\{e_1,e_2,\dots,e_l\}$ constituting an s-t path (a path from s to t), and based on the factoring theorem [8]:

$$\mathrm{Rel}(G) = q_1\cdot\mathrm{Rel}(G-e_1) + p_1\cdot q_2\cdot\mathrm{Rel}(G^*e_1-e_2) + \dots + \left[\prod_{i=1}^{l-1} p_i\right]\cdot q_l\cdot\mathrm{Rel}(G^*e_1{}^*e_2{}^*\dots{}^*e_{l-1}-e_l)$$

$$+ \left[\prod_{i=1}^{l} p_i\right]\cdot\mathrm{Rel}(G^*e_1{}^*e_2{}^*\dots{}^*e_l).$$

To minimize the number of subproblems generated by partitioning, the shortest s-t path is chosen as the basis for the partition. The subproblems are recursively processed until the source & sink are contracted or disconnected.

---

[1] The singular & plural of an acronym are always spelled the same.

0018-9529/96/$5.00 ©1996 IEEE

2.2 PBR Algorithm — R&P

To gain better performance, R&P combined D&G with network reduction. The network is simplified by using network reduction. R&P used 6 reduction rules [8], including removing valueless links (such as entering the source) and *series-parallel* link reduction. The PB partition is in turn performed based on a shortest *s-t* path. Each generated subproblem is recursively processed by reduction & partition until the source & sink are contracted or disconnected.

2.3 CB Algorithm — RKP

Instead of partitioning based on the shortest *s-t* path, the CB partition uses the source-cut consisting of all links emanating from the source. Given source-cut $\{e_1, e_2, \ldots, e_l\}$, then Rai *et al*, recursively factor Rel(G) analogous to D&G until the source & sink are contracted or disconnected.

2.4 CBR Algorithm — R-CB

A CB algorithm can incur more generated subproblems owing to the consideration of valueless links during the partition. This difficulty can be eliminated by using network reduction. In CBR, network reduction is always performed prior to the partition of each subproblem.

3. PERFORMANCE COMPARISONS

The CB algorithm results in lower complexity of determining the partition basis than the PB algorithm. Given an $n \times n$ adjacency matrix representing the connections in an input network with n nodes, the worst-case computation time to find a shortest *s-t* path by a breadth-first search [7] is $O(n^2)$. By contrast, the computation time to determine the source-cut is only $O(n)$. To justify this, we implemented the 4 algorithms (see section 2) in the C language and executed them in IBM RISC System/6000 using a collection of input-network benchmarks [3, 4, 8, 11 - 14], as shown in figure 1. Each input network with n nodes is represented by an $n \times n$ adjacency matrix denoting the connections in the network.

The performance of PB & CB are compared in terms of the number of subproblems and computation time. Figure 2 shows that the number of subproblems generated by PB is unsurprisingly less than that generated by CB for most of the benchmarks. As for the computation time, however, CB outperforms PB in most of the benchmarks, as shown in Figure 3. Figures 2 & 3 also compare performance of PBR & CBR for published benchmarks. Figure 2 shows that the numbers of subproblems generated by CBR have been greatly reduced, and become comparable to those of PBR. As for the computation time shown in Figure 3, CBR outperforms PBR for all benchmarks.

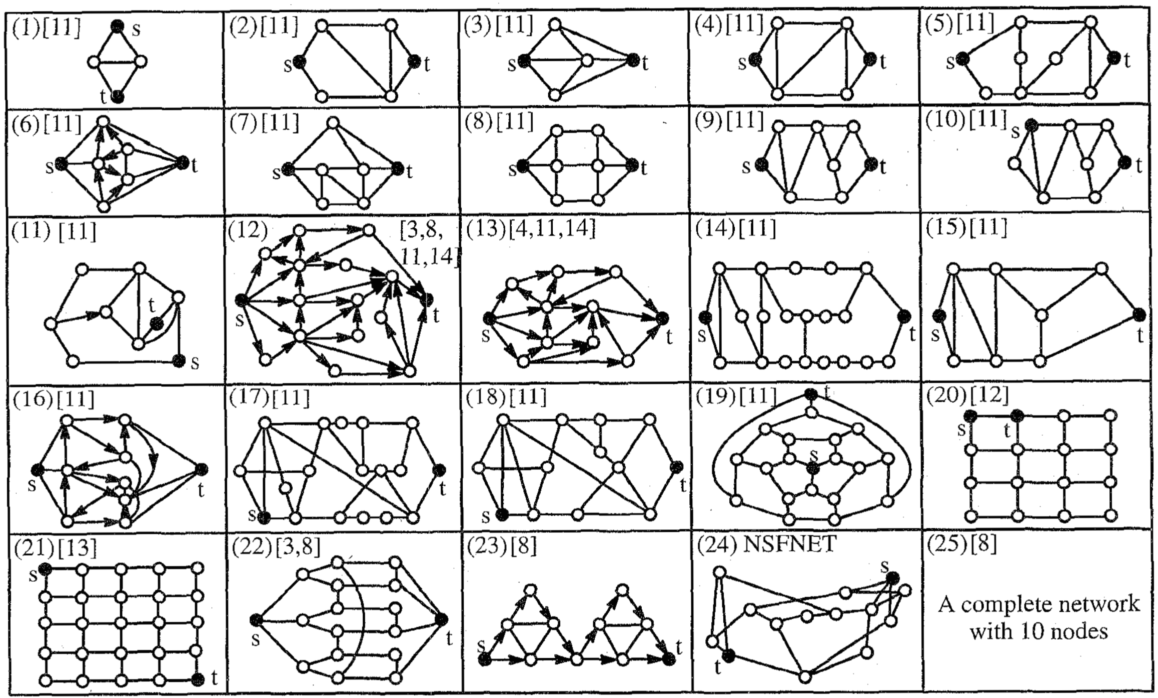

Figure 1. Benchmarks

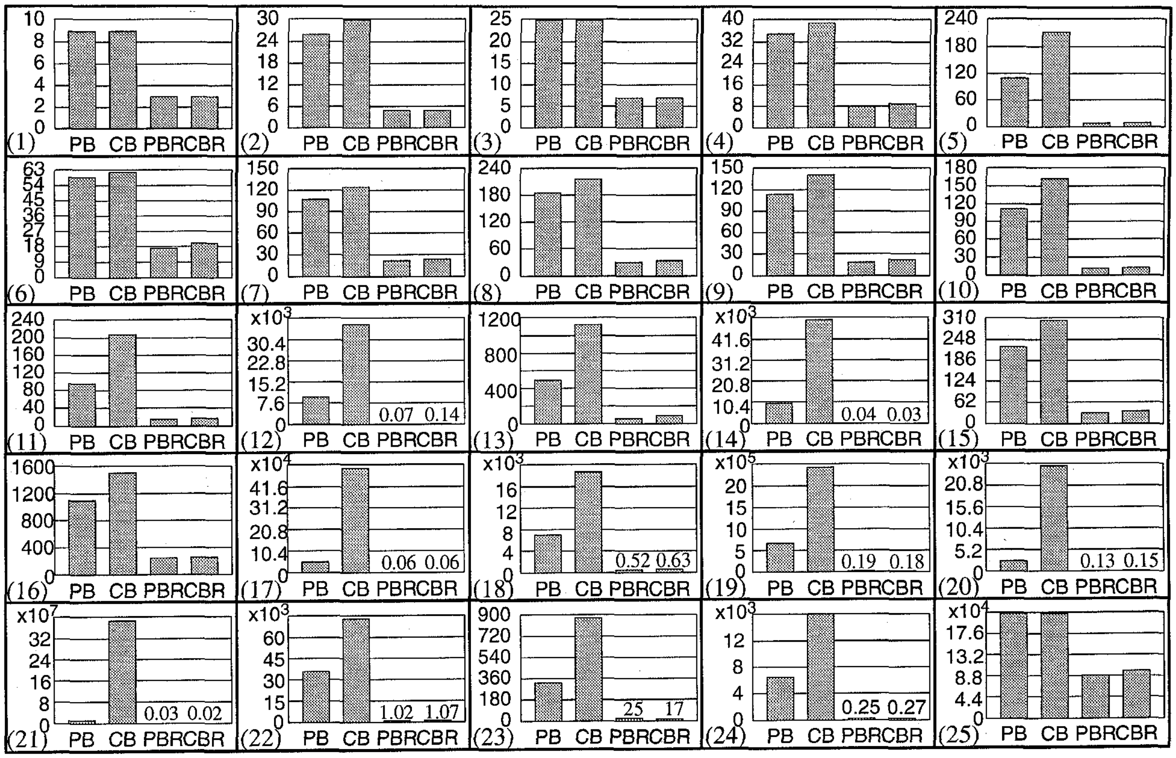

Figure 2. Number of Subproblems for the Benchmarks

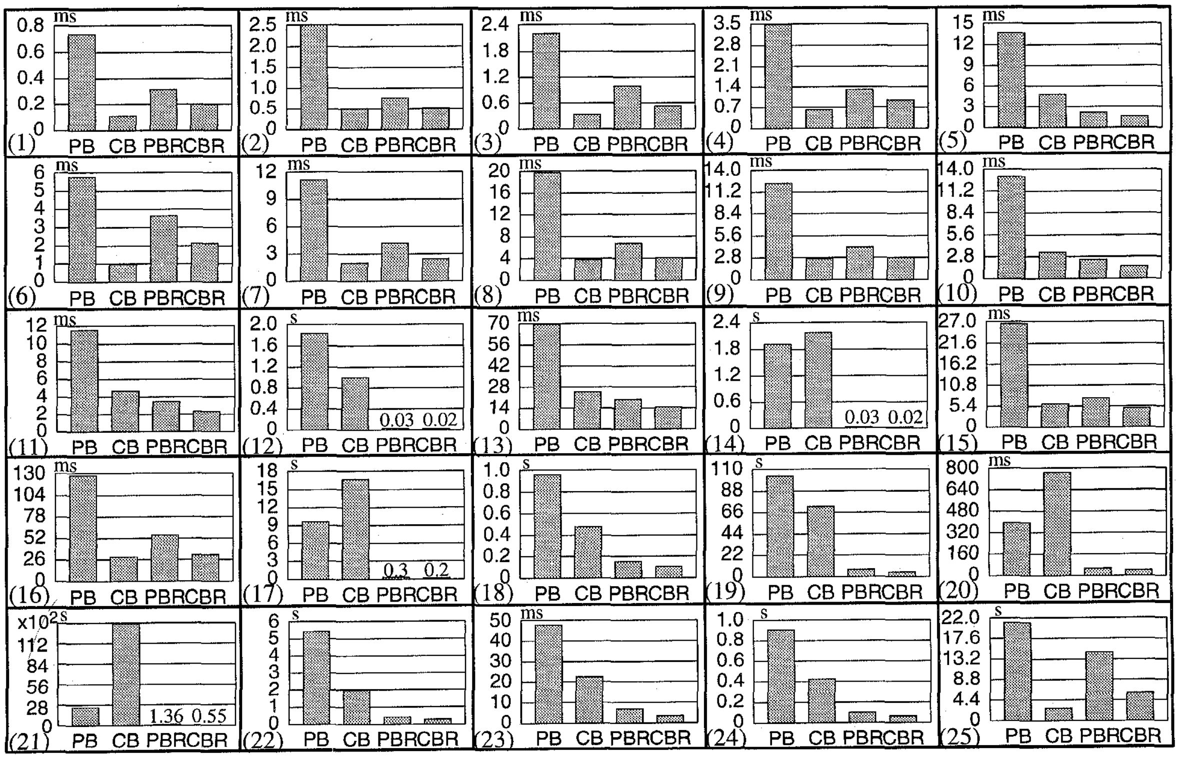

Figure 3. Computation Time for the Benchmarks

REFERENCES

[1] J.A. Abraham, "An improved algorithm for network reliability", *IEEE Trans. Reliability*, vol R-28, 1979 Apr, pp 58-61.

[2] M.O. Ball, "Computational complexity of network reliability analysis: an overview", *IEEE Trans. Reliability*, vol R-35, 1986 Aug, pp 230-239.

[3] N. Deo, M. Medidi, "Parallel algorithm for terminal-pair reliability", *IEEE Trans. Reliability*, vol 41, 1992 Jun, pp 201-209.

[4] W.P. Dotson, J.O. Gobien, "A new analysis technique for probabilistic graphs", *IEEE Trans. Circuits & Systems*, vol GAS-26, 1979 Oct, pp 855-865.

[5] S. Hariri, C.S. Raghavendra, "SYREL: A symbolic reliability algorithm based on path and outset methods", *IEEE Trans. Computers*, vol C-36, 1987 Oct, pp 1224-1232.

[6] M. Macgregor, W.D. Grover, U.M. Maydell, "Connectability: A performance metric for reconfigurable transport networks", *IEEE J. Selected Areas in Communications*, vol 11, 1993 Dec, pp 1461-1469.

[7] U. Manber, *Introduction To Algorithms*, 1989; Addison-Wesley.

[8] L.B. Page, J.E. Perry, "Reliability of directed networks using the factoring theorem", *IEEE Trans. Reliability*, vol 38, 1989 Dec, pp 556-562.

[9] S. Rai, A. Kumar, "Recursive technique for computing system reliability", *IEEE Trans. Reliability*, vol R-36, 1987 Apr, pp 38-44.

[10] S. Rai, A. Kumar, E.V. Prasad, "Computer terminal reliability of computer network", *Reliability Engineering*, vol 16, 1986, pp 109-119.

[11] S. Soh, S. Rai, "CAREL: Computer aided reliability evaluator for distributed computing networks", *IEEE Trans. Parallel & Distributed Systems*, vol 2, 1991 Apr, pp 199-213.

[12] D. Torrieri, "An efficient algorithm for the calculation of node-pair reliability", *Proc. IEEE MILCOM* '91, 1991 Nov, pp 0187-0192.

[13] D. Torrieri, "Calculation of node-pair reliability in large networks with unreliable nodes", *IEEE Trans. Reliability*, vol 43, 1994 Sep, pp 375-377.

[14] Y.B. Yoo, N. Deo, "A comparison of algorithms for terminal-pair reliability", *IEEE Trans. Reliability*, vol 37, 1988 Jun, pp 210-215.

AUTHORS

Yu G. Chen; Dept. Computer Science and Information Eng'g; National Chiao Tung Univ; 1001 Ta Hsueh Rd; 30050 Hsinchu, TAIWAN - R.O.C.
Internet (e-mail): ygchen@csie.nctu.edu.tw

Yu G. Chen (born in Taiwan, 1970) received the BS (1992) in Computer Science and the MS (1994) in Information Engineering from the National Chiao Tung University, Hsinchu. He is a PhD candidate in the same department. His current research interests include network reliability analysis, ATM network management, high speed networking, and multimedia communications.

Dr. Maria C. Yuang; Dept. Computer Science and Information Eng'g; National Chiao Tung Univ; 1001 Ta Hsueh Rd; 30050 Hsinchu, TAIWAN - R.O.C.
Internet (e-mail): mcyuang@csie.nctu.edu.tw

Maria C. Yuang received the BS (1978) in Applied Mathematics from the National Chiao Tung University, Hsinchu; the MS (1981) in Computer Science from the University of Maryland, College Park, and the PhD (1989) in Electrical Engineering and Computer Science from the Polytechnic University, Brooklyn. From 1981 to 1990, she was with AT&T Bell Laboratories and Bell Communications Research (Bellcore), where she was a member of technical staff working on high-speed networking and protocol engineering. She has been an Associate Professor in Computer Science and Information Engineering at the National Chiao Tung University since 1990. Her current research interests include high speed networking, ATM network management, performance modeling & analysis, multimedia communications, and protocol engineering.

Manuscript received 1996 June 25

Publisher Item Identifier S 0018-9529(96)08123-7

◄TR►
