訊息分配與市場資訊效率性之研究
Information Distribution and the Informative Efficiency

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摘要：本研究提供建構股價模型之理論基礎。研究中特別將私人訊息分配之情形納入模型中，並假設投資人之絕對風險厭惡係數為常數，各自追求預期效用極大，得出市場結清均衡時，均衡價格為市場訊息按照其精確度與觀察頻率之乘積加權平均。主要發現有：第一，市場訊息分配均等將導致均衡價格取代私人訊息而成為市場中各種訊息之充分統計量，此時證券市場具有資訊效率，投資人失去蒐集私人訊息之動機。第二，市場訊息分配與資訊觀察值成正比為市場具有資訊效率之充要條件。然實際上述條件難以成就，投資人仍具有蒐集私人資訊之動機。

關鍵詞：訊息分配、資訊效率、充分統計量

Abstract：This paper investigates the determinants of the informative efficiency of the stock market. In different to previous studies, the paper considers the factor of "information distribution" and shows that the equilibrium price aggregates the market information according to their information frequency and precision.　It is shown that if the distribution of market information is equal, then the equilibrium price serves as a sufficient statistic for the market information.　When all the market information frequencies are proportional to the information observed, the equilibrium price is a sufficient statistic for market information.　However, the market information is not equally allocated; it is impossibility for price informativeness.　The traders still have an incentive to collect and to search for valuable

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information to improve their estimate of the true value of the risky asset.　　This will be helpful to build a stable model of stock market.

Notation

The following notation is used throughout:

There are N traders in the stock market.

$\tilde{x}_i = \tilde{v} + \tilde{\varepsilon}_i$: The ith private information. Where $\tilde{v}$ denotes the true value of the risky asset; and $\tilde{\varepsilon}_i \sim N(0, \Lambda_i^{-1})$ denotes the error term of the information $\tilde{x}_i$, $i = 1,2,...,I$.

$\Lambda_i \equiv Var^{-1}(\tilde{\varepsilon}_i)$: The precision of the information $\tilde{x}_i$.

$\{f_1, f_2, ..., f_I\}$: Distribution of the market information $\tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_I$.

r : The absolute risk aversion of investor.

$\tilde{w}_i$: The wealth gained from trading of the information $\tilde{x}_i$

$\tilde{p}$: The equilibrium price of risky asset

q_i : The informative trading quantity depending on the information $\tilde{x}_i$.

1. Introduction

The role of prices in aggregating and conveying information is central to the study of resource allocation in a competitive economy.　　This paper address the question of "What determines the informativeness of the stock price?", and "How informationally efficient is the market price?"

The literature on the determinants of stock prices is extensive.　　Traditional economic theories of the stock market hold that the investors invest and revise their beliefs until a market-clearing price is established.　　Hence, the information observed by investors would affect the market equilibrium. They model the price formation process with each piece of information is observed by the same amount of investors[1].　　This leads to the assumption that the market information is equally allocated (See Baigent, 2003; Bray, 1981; Grossman, 1976; Grossman, 1978; Grossman, 1981; Grossman and Stiglitz, 1980; Hellwig, 1980; Jonathan, 1993; Lintner, 1969).

Grossman (1976) analyzes an economy under assumptions of homogeneity in allocation of market

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[1] Usually assuming each piece of information is observed by one investor.

information, and claims that the equilibrium price is a simple average over the market information collected by traders and it serves as a sufficient statistic for the market information, namely the market equilibrium price can transmit all the market information.　However, they present a "self-destruction" model.　A major limitation of this result is that when traders take the price as given, they have no incentive to acquire any information when the market is free of noise (such as a supply shock: see Diamond and Verrecchia, 1981).　In this situation, the private information is a redundancy to investors, and both the number of informed traders and the informative efficiency would be reduced.

Despite their dominance in economic theories, the models cited above have limitations because the information frequency is omitted in determining stock prices.

To remedy this lack in prior theory, this paper introduces the concept of "Information allocation" in determining stock prices to a competitive stock market and builds a model in which each source of information could be observed by different amount of traders.

To be an illustration, there are six informed investors A, B, C, D, E, and F.　There are three sources of information $\tilde{x}_1$, $\tilde{x}_2$, and $\tilde{x}_3$. Assume that the information $\tilde{x}_1$ is observed by investor A; the information $\tilde{x}_2$ is observed by investors B and C; and the information $\tilde{x}_3$ is observed by investors D, E, and F. The distribution frequencies of the information $\tilde{x}_1$, $\tilde{x}_2$, and $\tilde{x}_3$ are {1,2,3}.

This paper extends the existing competitive models and considers that the market information could not be equally allocated.　It shows that the equilibrium price aggregates the market information according to their frequency and precision. Second, following prior literatures, the term "informative efficiency" refers to statistical efficiency.　(See Admati and Pfleiderer, 1988; Foster and Viswanathan, 1990; Grossman and Stiglitz, 1980; Jonathan, 1993; Kyle, 1984, Kyle, 1985, Kyle ,1989; Spiegel and Subrahmanyam, 1992).　I focus on the relationships between the information allocation and the informative efficiency of the stock market.　It find that the price informativeness of the stock market are affected by the allocation of market information.

The remainder of the paper is structured as follows: Section 2 sets out the basic economic environment to be analyzed.　Section 3 solves for the market equilibrium of the model. In particular, this paper considers the impact of the allocation of market information.　The major propositions on conditions for the market efficiency are in section 4 and also the nature of the parameter which determines whether the stock market is informatively efficient is discussed. Conclusions are provided in Section 5.

2. The Model

2.1 Information Structure Assumptions

This is a two-period model, in which it is assumed that the true value of the risky asset is distributed normally with zero mean[2] and its variance is Λ_v^{-1}, i.e. $\tilde{v} \sim N(0, \Lambda_v^{-1})$. Following Titman and Trueman (1986), and it is reasonable to assume that the prior distribution of $\tilde{v}$ is diffuse (i.e. Λ_v is trivial)[3].

There are I kinds of information, $\tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_I$ available in the stock market. Each kind of information is related to the true value of the risky asset and is an unbiased estimate of that true value. Specifically,

$$\tilde{x}_i = \tilde{v} + \tilde{\varepsilon}_i , \quad i = 1,2,...,I \tag{1}$$

There is a noise term, $\tilde{\varepsilon}_i \sim N(0, \Lambda_i^{-1})$, which prevents traders from learning the true value of $\tilde{v}$. And the $Var^{-1}(\tilde{\varepsilon}_i) \equiv \Lambda_i$ can be seen as the precision of the information $\tilde{x}_i$[4]. It is assumed that the noise terms $\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, ..., \tilde{\varepsilon}_I$ are jointly normally distributed and their covariance are zero, i.e. $Cov(\tilde{\varepsilon}_i, \tilde{\varepsilon}_j) = 0 \quad \forall i \leq I, j \leq I, \quad i \neq j$.

This paper investigate a model of short horizons, which means that investors cannot leave their money in the market for an indefinite period of time but instead must close out their position quickly (Froot *et al.*, 1992). In the current period (before trading starts), each investor searches for the information $\tilde{x}_i$. After a trader observes the information $\tilde{x}_i$, he has the basis to estimate the true value of the risky asset and to make a decision as to hold how many quantity in the firm's security, q_i, to maximize his own utility in uncertain circumstances. Hence the trading size chosen by an investor is determined endogenously.

2.2 The Trading Assumption

Consider an economy where there are N traders with constant values of the Arrow-Pratt measure of absolute risk aversion (CARA). $-\dfrac{U''(w)}{U'(w)} = r$, and $U'(w) > 0 > U''(w)$.

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[2]   This assumption for zero mean of intrinsic value of a risky asset is just for simplicity and will not influence the result of our analysis.

[3]   It is reasonable that the variance of $\tilde{v}$ is relatively larger than the variance of private information $\tilde{x}_i$.

[4]   Grossman (1976) assumed the precisions of market information are homogeneous, this paper generalize this assumption so that the precisions of market information are heterogeneous.

For simplicity, we consider a representative trader with homogeneous expectations. Thus the expectation of the true value of the risky asset of any trader who has observed the same information is assumed to be equal (on average).

Let random variable $\tilde{w}_i$ be the gain of financial wealth of any investor who has observed the information $\tilde{x}_i$. Let $g(\tilde{v} \mid x_i)$ be the probability density of $\tilde{v}$, given $\tilde{x}_i = x_i$.

The objective of any trader who has observed the information $\tilde{x}_i$ is to maximize his own expected utility derived from financial wealth, i.e. $Max \quad EU(\tilde{w}_i) \equiv \int_{-\infty}^{+\infty} U(\tilde{w}_i) g(\tilde{v} \mid x_i) dv$. The objective function can be written:

$$\underset{q_i}{Max} \quad EU(\tilde{w}_i) = EU[(\tilde{v} - p)q_i \mid x_i] \tag{2}$$

Where E is the expectations operator conditioned on the investor's information. Let $\tilde{p}$ be the share price of the risky asset. Suppose also that shares of the stock are perfectly divisible and are traded at no cost in a competitive stock market.

3. Market Clear Equilibrium

Let us now consider the equilibrium of the model defined in the preceding section.

The informed investors observe and utilize some kind of the market information to form a more precise estimate for the expected value of the risky asset and to decide how many shares to invest to maximize their own utility. This generates informative trading and price movement in the stock market. An individual will eventually observe that the probability distribution of returns conditional on the observable market information (Grossman and Stiglitz, 1976). The equilibrium price and the trading volume can be derived by solving the equilibrium of the economy.

Since the informative trading for the risky asset depends on the information observed, it is natural to think of the market clearing equilibrium (price and trading quantity) and the optimal expected utility as functions of the market information $\tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_I$. This implies that different market information about the true value of the risky asset leads to different market trading behavior of the risky asset and gives different market equilibrium regimes.

3.1 Determinants of Optimal Trading

Traders act competitively after they receive the private information about the stock's true value in

the stock market. This informational trading gives rise to demand and supply for the risky asset.

Given $\tilde{x}_i = x_i$, we can prove that the conditional expectation and the inverse of the variance of the true value of the risky asset are equal to x_i and Λ_i respectively. This can be written as:

$$E(\tilde{v} \mid x_i) = x_i \tag{3}$$

$$Var^{-1}(\tilde{v} \mid x_i) = \Lambda_i \tag{4}$$

See Appendix A for proof

By the equations (3) and (4) , the posterior distribution of $\tilde{v}$ conditioned on x_i is normally distributed with mean x_i and variance Λ_i^{-1}, where Λ_i means the conditional accuracy given the information x_i.

Lemma 1:

Under the economy defined in the section 2, the optimal shareholding for the risky asset conditioned on the private information $\tilde{x}_i$ depends on the risk attitude of investors r, the expected price change (capital gain per share, $E[\tilde{v} - p \mid x_i]$) and the conditional precision of the information observed, $\mathrm{var}^{-1}(\tilde{v} \mid x_i)$. Specifically, I could derive the informative trading function of the risky asset depending on the information $\tilde{x}_i = x_i$ as:

$$q_i = \frac{1}{r}\mathrm{var}^{-1}(\tilde{v} \mid x_i)E[\tilde{v} - p \mid x_i] \tag{5}$$

See Appendix B for proof.

As in the Grossman and Stiglitz (1976; 1980) model, the informed traders in this model know how to form rational expectations, given the information available to them.   By the lemma 1, the optimal trading volume is negatively related to the risk attitude of investors and the conditional variance given information available.   Intuitionally, it is positively related to the capital gain from the informative stock trading.

Insert the equations (3) and (4) into the equation (5) and gives the demand for stock:

$$q_i = \frac{\Lambda_i(x_i - p)}{r} \tag{6}$$

The demand is a function of the asset price and of trader's predictions of the return.

3.2 The market clearing condition

Traditional economic theory (e.g. Lintner, 1969) of the stock market holds that the investors invest

and revise their beliefs until a market-clearing price is established.　　The equilibrium price aggregates the market information.　　Hence, the information observed by investors would affect the market equilibrium.　　The model presented here is almost a generalization of that of Grossman and Stiglitz (1976)

There are I kinds of information in the market. Let $0 \leq f_i \leq N$, means the frequency of the information $\tilde{x}_i$ observed by investors and $\sum_{i=1}^{I} f_i = N$.　　The allocation of market information, $\tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_I$, is denoted by $F \equiv [f_1, f_2, ..., f_I]$.　　The informative trading is denoted by $Q \equiv [q_1, q_2, ..., q_I]'$.　　Where q_1 , q_2 ,…, and q_I are the trading quantity function given by the equation (6).　　An equilibrium price must satisfy the market clearing condition[5], that is,

$$FQ = 0 \qquad (7)$$

The equation (7) states that the aggregate trading quantity over market information set $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_I)$ for the risky asset must equal zero[6].

In the following proposition 1, I shall prove that the equilibrium price of risky asset is a linear function of all the market information.　　It reflects the allocation of the market information.

Proposition 1:

Under the Walrasian market defined in the section 2, the equilibrium price $\tilde{p}$ aggregates all the market information $\tilde{x}_1, \tilde{x}_2, ...\tilde{x}_I$ about a risky asset according to their corresponding frequency, $f_1, f_2, ..., f_I$ and the precision, $\Lambda_1, \Lambda_2, ..., \Lambda_I$. Specifically, $\tilde{p}$ is determined by

$$\tilde{p} = \frac{\sum_{i=1}^{I} f_i \Lambda_i \tilde{x}_i}{\sum_{i=1}^{I} f_i \Lambda_i} \qquad (8a)$$

$$\text{Or,} \quad \tilde{p} = \tilde{v} + \frac{\sum_{i=1}^{I} f_i \Lambda_i \tilde{\varepsilon}_i}{\sum_{i=1}^{I} f_i \Lambda_i} \qquad (8b)$$

See Appendix C for proof.

The equation (8a) indicates that the market equilibrium price is a weighted average of all the

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[5] Since the utility of investors is identical, the property of initial endowment irrelevance holds.

[6] This paper investigate a model of short horizons, which means that investors cannot leave their money in the market for an indefinite period of time but instead must close out their position quickly (see Froot *et al.* 1992)

market information with weights of the corresponding frequency and the precision of the market information $(f_1 \Lambda_1, f_2 \Lambda_2, ..., f_I \Lambda_I)$ and it shows that the equilibrium price of a risky asset is determined more by the information with higher frequency. Also, all the traders' expectations of the future price of the risky asset enter the price function.   This implies that the informationally efficient price system aggregates diverse information ($\tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_I$) by an auction mechanism with traders, and completely characterizes the price movement around a trade under a competitive market.

By ignoring the effect of the market information frequencies, the Grossman (1976) and related works that follows a similar line argue that the market equilibrium price is a simple average of the market information collected by traders.   However, the equation (8a) shows that the Grossman's result is only a special case in which the aggregate precision of each kind of information is uniform. i.e. $f_1 \Lambda_1 = f_2 \Lambda_2 = ... = f_I \Lambda_I$.

4. Informative efficiency

The issues "Is the equilibrium price a sufficient statistic for the information about the true value of firms?" and "Is the price informativeness of stock market optimal" are central to the study about the informative efficiency of stock markets..

Grossman (1976) analyzes an economy under a strong assumption of homogeneity in allocation of market information, and claims that the equilibrium price serves as a sufficient statistic for the market information, namely the market equilibrium price can transmit all the market information.   A major limitation of this result is that when traders take the price as given, they have no incentive to acquire any information when the market is free of noise (such as a supply shock: see Diamond and Verrecchia, 1981).   In this situation, the private information is a redundancy to investors, and both the number of informed traders and the informative efficiency would be reduced. Hence, it presents a "self-destruction" model.

The stability of informative trading of a simple asset market model is studied in a situation where there is different frequency distribution among market information.   This section will proves that the equal allocation of market information is a sufficient condition for the equilibrium price to be a sufficient statistic for the market information about the true value of the risky asset.   Hence, the previous conclusions that the market equilibrium price is a sufficient statistic for the information about the true value of the risky asset might be just an inevitable outcome for ignoring the heterogeneity of the allocation of market information.

Following prior literatures (See Admati and Pfleiderer, 1988; Foster and Viswanathan, 1990; Grossman and Stiglitz, 1980; Kyle, 1984; Kyle, 1985; Kyle, 1989; Spiegel and Subrahmanyam, 1992), the term "informative efficiency" refers to statistical efficiency.　It gets the following definition.

Definition: informative efficiency condition

If and only if $q(\tilde{x}_k, \tilde{p}) = q(\tilde{p})$, then $\tilde{p}$ is a sufficient statistic for the information $\tilde{x}_k$, $k = 1,2,...,I$.

It means that when $\tilde{p}$ is a sufficient statistic for the information $\tilde{x}_k$, the trading decisions of investor using the information set { $\tilde{x}_k, \tilde{p}$ } and the information { $\tilde{p}$ } are indifference.

Recall the equation (5), the informative trading decisions of investors are completely characterized by the conditional mean and variance of the information about the intrinsic value of the asset. Hence it gets the following Lemma. (Also see Bray, 1981)[7]

Lemma 2:

If and only if $\dfrac{E(\tilde{v} \mid x_k, p)}{Var(\tilde{v} \mid x_k, p)} = \dfrac{E(\tilde{v} \mid p)}{Var(\tilde{v} \mid p)}$, then $\tilde{p}$ is a sufficient statistic for the information $\tilde{x}_k$,

$k = 1,2,...,I$.

Proposition 2:

If and only if

$$x_k = \frac{f_k[\sum f_i \Lambda_i x_i]}{\sum f_i^2 \Lambda_i} , \forall k = 1,2,...,I \tag{9}$$

then the random variable $\tilde{p}$ is a sufficient statistic for the information $\tilde{x}_k$, $k = 1,2,...,I$

See the Appendix D for proof.

Corollary 1:

The sufficient and necessary condition for informative efficiency is

$$\frac{f_i}{x_i} = \frac{f_k}{x_k}, \forall i,k \tag{10}$$

It is obvious by the equation (10).　When all the market information frequencies are proportional to the information observed the equilibrium price is a sufficient statistic for market information. Under this impracticable and strong hypothesis, when traders take the price as given, they have no incentive to acquire any information when the market is free of other noise.　However, the observational value of market information depends upon investors' subjective beliefs about the

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[7] This necessary and sufficient condition is different and more precise than that proposed by Bray (1981).

distribution; and also the information frequency is determined exogenously. It is impossibility for informative efficiency of stock market for lack of an automatic mechanism to attain the condition. The stock market will not break down as in the case described by Grossman (1976). This will be very helpful to build a capital asset pricing model.

Proposition 3: sufficient condition for informative efficiency

If $Var(\tilde{p}) = Cov(\tilde{p}, \tilde{x}_k)$, then the equilibrium price is a sufficient statistic for market information.

See appendix E for proof.

It makes sense that the coefficient $\dfrac{Cov(\tilde{p}, \tilde{x}_k)}{Var(\tilde{p})} = 1$ is analogous to the coefficient of $\tilde{p}$, when one regresses $\tilde{x}_k$ on $\tilde{p}$ using OLS.

Proposition 4 :

If the observational frequency of market information is equal, i.e. $f_1 = f_2 = ... = f_I$, then the equilibrium price is a sufficient statistic for market information.

See appendix F for proof.

It proves that the equal allocation of market information is a sufficient condition for the equilibrium price to be informatively efficient. Hence, the previous conclusions (e.g. Grossman, 1976) that the market equilibrium price is a sufficient statistic for the information about the true value of the risky asset might be just an inevitable outcome for ignoring the heterogeneity of the allocation of market information.

5. Conclusions and suggestions

The issues "Is the equilibrium price a sufficient statistic for the information about the true value of firms?" and "Is the price informativeness of stock market optimal" are central to the study about the informative efficiency.

Grossman (1976) characterizes a special case in which the information allocation is uniform. i.e. $f_i = f_j, \forall i \neq j$ and proposes an informationally efficient price system which aggregates diverse information perfectly, but in doing this the price system also eliminates the private incentive for collecting the information.

This paper considers the factor of the information allocation in a competitive stock market assuming that each source of market information could be observed by different number of investors. Also, I explore the conditions under which the equilibrium price could be a sufficient statistic for the market information. Several main findings are documented as follows.

First, the equal allocation of market information leads the market to be informatively efficient. Hence, the previous conclusions that the market equilibrium price is a sufficient statistic for the information about the true value of the risky asset might be just an inevitable outcome for ignoring the heterogeneity of the allocation of market information.

Second, when all the market information frequencies are proportional to the information observed the equilibrium price is a sufficient statistic for market information.　The equilibrium price's ability to predict the intrinsic value of risky asset or "the price informativeness" will be maximized.　However, the market information is not equally allocated; it is impossibility for price informativeness and the informative efficiency of stock market. The market will not break down as in the case described by Grossman (1976).　The traders still have an incentive to collect and to search for valuable information to improve their estimate of the true value of the risky asset. This will be helpful to build a stable model of stock market.

Finally, this paper suggests that the equilibrium price aggregates the market information according to their information frequency and precision.　Our analysis has valuable empirical applications.　For example, this result proposes that the magnitude of the stock price response to some information is also determined by the observational frequency and precision of that information.　This provides the setting sufficient for a further test of the relation between the magnitude of market response and the observational frequency and precision of market information.

Appendix A

Given that $\tilde{x}_i = \tilde{v} + \tilde{\varepsilon}_i$, $\tilde{\varepsilon}_i \sim N(0, \Lambda_i^{-1})$, $\tilde{v} \sim N(u_v, \Lambda_v^{-1})$,

Let $\sum = Var\begin{bmatrix} \tilde{v} \\ \tilde{x}_i \end{bmatrix} = Cov\begin{bmatrix} \tilde{v}\tilde{v} & \tilde{v}\tilde{x}_i \\ \tilde{x}_i\tilde{v} & \tilde{x}_i\tilde{x}_i \end{bmatrix} = \begin{bmatrix} \Lambda_v^{-1} & \Lambda_v^{-1} \\ \Lambda_v^{-1} & \Lambda_v^{-1} + \Lambda_i^{-1} \end{bmatrix}$

Using moment generating function, we get

$\tilde{v} \mid x_i \sim N[u_v + \sum_{vi} \sum_{ii}^{-1}(x_i - u_i), \sum_{vv} - \sum_{vi} \sum_{ii}^{-1} \sum_{iv}]$, where $u_v \equiv E(\tilde{v}) = 0$, $u_i \equiv E(\tilde{x}_i) = 0$, $\Sigma_{vv} = \Lambda_v^{-1}$, $\Sigma_{vi} = \Sigma_{iv} = \Lambda_v^{-1}$, and $\Sigma_{ii} = \Lambda_v^{-1} + \Lambda_i^{-1}$

$Var(\tilde{v} \mid x_i) = \sum_{vv} - \sum_{vi} \sum_{ii}^{-1} \sum_{iv} = \Lambda_v^{-1} - \Lambda_v^{-1} \dfrac{1}{\Lambda_v^{-1} + \Lambda_i^{-1}} \Lambda_v^{-1} = \dfrac{1}{\Lambda_v + \Lambda_i}$

$E(\tilde{v} \mid x_i) = u_v + \sum_{vi} \sum_{ii}^{-1}(x_i - u_i) = 0 + \dfrac{\Lambda_i}{\Lambda_v + \Lambda_i}(x_i - 0)$

Following Titman and Trueman (1986), it is reasonable to assume that the Λ_v is trivial, and we

have

$$Var^{-1}(\widetilde{v} \mid x_i) = \Lambda_i \tag{A1}$$

$$E(\widetilde{v} \mid x_i) = x_i \tag{A2}$$

Q.E.D.

Appendix B

Assuming that traders are price takers, we describe the optimization problem below.

The trading volume caused when the information $\widetilde{x}_i$ is observed by a trader is denoted by q_i, and that trade can produce a random financial wealth $\widetilde{w}_i$ and the expected utility $EU(\widetilde{w}_i)$. The expected utility is concave in w_i, i.e. $U' > 0$ and $U'' < 0$

The objective of a potential trader who has observed the information $\widetilde{x}_i$ is to choose trading a quantity, q_i, to maximize his own expected utility of financial wealth. That is:

$$\underset{q_i}{Max} \quad EU(\widetilde{w}_i) = EU[(\widetilde{v} - p)q_i],$$

The utility-maximizing value of q_i is given by the following first-order condition.

F.O.C. $E[U'(\widetilde{w}_i) \cdot (\widetilde{v} - p) \mid x_i] = 0$

$$E[U'(\widetilde{w}_i) \cdot (\widetilde{v} - p) \mid x_i] = E[U'(\widetilde{w}_i) \mid x_i] \cdot E[(\widetilde{v} - p) \mid x_i] + Cov[U'(\widetilde{w}_i), (\widetilde{v} - p) \mid x_i] = 0 \tag{B1}$$

By Stein's Lemma:

$$Cov[U'(\widetilde{w}_i), (\widetilde{v} - p) \mid x_i] = Cov[\widetilde{v}, (\widetilde{v} - p) \mid x_i] \cdot E[U''(\widetilde{w}_i) \mid x_i] q_i = Var(\widetilde{v} \mid x_i) \cdot E[U''(\widetilde{w}_i) \mid x_i] q_i \tag{B2}$$

Substituting equation (B.2) back into equation (B.1) gives

$$q_i = \frac{E[\widetilde{v} - p \mid x_i]}{-\dfrac{U''}{U'} Var[\widetilde{v} \mid x_i]} = \frac{1}{r} Var[\widetilde{v} \mid x_i]^{-1} E[\widetilde{v} - p \mid x_i]$$

Q.E.D.

Appendix C: The Equilibrium

Recall equation (6), and $\widetilde{p}$ denotes the equilibrium price which solves the market clearing equation. I have the market clearing equation as:

$$\sum_{i=1}^{I} \frac{f_i \Lambda_i}{r} (x_i - p) = 0 \tag{C1}$$

Solving equation (C1) for the equilibrium price $\tilde{p}$, according to the assumption that $\tilde{x}_1, \tilde{x}_2, ... \tilde{x}_I$ are independent of each other, $\tilde{p}$ is a linear function of a vector of joint normal distributed variables and is also normally distributed, we can rewrite price as a random form as

$$\tilde{p} = \frac{\sum_{i=1}^{I} f_i \Lambda_i \tilde{x}_i}{\sum_{i=1}^{I} f_i \Lambda_i} \tag{C2}$$

or equivalently (recall that $\tilde{x}_i = \tilde{v} + \tilde{\varepsilon}_i$)

$$\tilde{p} = \tilde{v} + \frac{\sum_{i=1}^{I} f_i \Lambda_i \tilde{\varepsilon}_i}{\sum_{i=1}^{I} f_i \Lambda_i} \tag{C3}$$

Q.E.D.

Appendix D: Informative Efficiency Condition

Recall the assuming that random variables are joint normal and $Cov(\tilde{v}, \tilde{x}_k) = Cov(\tilde{v}, \tilde{p}) = Var(\tilde{v}) \equiv \Lambda_v^{-1}$, and traders have constant absolute risk aversion,

Step1:

Let $Var\begin{bmatrix} \tilde{v} \\ \tilde{p} \end{bmatrix} \equiv Cov\begin{bmatrix} \tilde{v}\tilde{v} & \tilde{v}\tilde{p} \\ \tilde{p}\tilde{v} & \tilde{p}\tilde{p} \end{bmatrix} = \begin{bmatrix} \Lambda_v^{-1} & \Lambda_v^{-1} \\ \Lambda_v^{-1} & \Lambda_p^{-1} \end{bmatrix}$, $\Sigma_{vv} = \Lambda_v^{-1} \equiv Var(\tilde{v})$, $\Sigma_{vp} \equiv \Lambda_v^{-1}$, $\Sigma_{pp} = \Lambda_p^{-1} \equiv Var(\tilde{p})$

$E[\tilde{v} \mid p] = u_v + \Sigma_{vp} \Sigma_{pp}^{-1} (p - u_p)$, $Var[\tilde{v} \mid p] = \Sigma_{vv} - \Sigma_{vp} \Sigma_{pp}^{-1} \Sigma_{pv}$ where $u_v \equiv E(\tilde{v})$, $u_p \equiv E(\tilde{p})$

$$E[\tilde{v} \mid p] = \frac{Var(\tilde{v})}{Var(\tilde{p})} p \tag{D1}$$

$$Var[\tilde{v} \mid p] = Var(\tilde{v}) - \frac{[Var(\tilde{v})]^2}{Var(\tilde{p})} \tag{D2}$$

Step 2:

$$Var\begin{bmatrix} \tilde{v} \\ \tilde{p} \\ \tilde{x}_k \end{bmatrix} = Cov\begin{bmatrix} \tilde{v}\tilde{v} & \tilde{v}\tilde{p} & \tilde{v}\tilde{x}_k \\ \tilde{p}\tilde{v} & \tilde{p}\tilde{p} & \tilde{p}\tilde{x}_k \\ \tilde{x}_k\tilde{v} & \tilde{x}_k\tilde{p} & \tilde{x}_k\tilde{x}_k \end{bmatrix} = \begin{bmatrix} \Lambda_v^{-1} & \Lambda_v^{-1} & \Lambda_v^{-1} \\ \Lambda_v^{-1} & \Lambda_p^{-1} & \sigma_{pk} \\ \Lambda_v^{-1} & \sigma_{pk} & Var(\tilde{x}_k) \end{bmatrix}, \sigma_{pk} \equiv Cov(\tilde{p}, \tilde{x}_k)$$

Let $\theta = \begin{bmatrix} \tilde{p} \\ \tilde{x}_k \end{bmatrix}$

$$\sum_{vv} = \Lambda_v^{-1}, \sum_{v\theta} = [\Lambda_v^{-1} \quad \Lambda_v^{-1}], \sum_{\theta v} = \begin{bmatrix} \Lambda_v^{-1} \\ \Lambda_v^{-1} \end{bmatrix}, \quad \sum_{\theta\theta} = \begin{bmatrix} \Lambda_p^{-1} & \sigma_{pk} \\ \sigma_{pk} & Var(\tilde{x}_k) \end{bmatrix}$$

$$\sum_{\theta\theta}^{-1} = \frac{\begin{bmatrix} Var(\tilde{x}_k) & -\sigma_{pk} \\ -\sigma_{pk} & \Lambda_p^{-1} \end{bmatrix}'}{\begin{vmatrix} \Lambda_p^{-1} & \sigma_{pk} \\ \sigma_{pk} & Var(\tilde{x}_k) \end{vmatrix}}$$

$E[\tilde{v} \mid p, x_k] = u_v + \sum_{v\theta} \sum_{\theta\theta}^{-1}(\theta - u_\theta)$; $Var[\tilde{v} \mid p, x_k] = \sum_{vv} - \sum_{v\theta} \sum_{\theta\theta}^{-1} \sum_{\theta v}$;

$$E[\tilde{v} \mid p, x_k] = Var(\tilde{v})\frac{[Var(\tilde{x}_k) - Cov(\tilde{p}, \tilde{x}_k)]p + [Var(\tilde{p}) - Cov(\tilde{p}, \tilde{x}_k)]x_k}{Var(\tilde{p})Var(\tilde{x}_k) - Cov^2(\tilde{p}, \tilde{x}_k)} \tag{D3}$$

$$Var[\tilde{v} \mid p, x_k] = Var(\tilde{v}) - \frac{[Var(\tilde{v})]^2}{Var(\tilde{p})Var(\tilde{x}_k) - \sigma_{pk}^2}[Var(\tilde{x}_k) + Var(\tilde{p}) - 2\sigma_{pk}] \tag{D4}$$

Step3:

(D3)-(D1)= $E[\tilde{v} \mid p, x_k] - E[\tilde{v} \mid p]$

$$= Var(\tilde{v})\frac{Cov(\tilde{p}, \tilde{x}_k)[Cov(\tilde{p}, \tilde{x}_k) - Var(\tilde{p})]p + Var(\tilde{p})[Var(\tilde{p}) - Cov(\tilde{p}, \tilde{x}_k)]x_k}{Var(\tilde{p})[Var(\tilde{p})Var(\tilde{x}_k) - Cov^2(\tilde{p}, \tilde{x}_k)]}$$

$$= \frac{Var(\tilde{v})[Var(\tilde{p}) - Cov(\tilde{p}, \tilde{x}_k)]}{Var(\tilde{p})[Var(\tilde{p})Var(\tilde{x}_k) - Cov^2(\tilde{p}, \tilde{x}_k)]}[Var(\tilde{p})x_k - Cov(\tilde{p}, \tilde{x}_k)p] \tag{D5}$$

(D4)-(D2)= $Var[\tilde{v} \mid p, x_k] - Var[\tilde{v} \mid p]$

$$= -[Var(\tilde{v})]^2 \frac{[Var(\tilde{p}) - Cov(\tilde{p}, \tilde{x}_k)]^2}{Var(\tilde{p})[Var(\tilde{p})Var(\tilde{x}_k) - Cov^2(\tilde{p}, \tilde{x}_k)]} \leq 0 \tag{D6}$$

Step4:

Recall the lemma 2, if and only if $\tilde{p}$ is a sufficient statistic for $\tilde{x}_k$ about $\tilde{v}$, then

$$\frac{E[\tilde{v} \mid p, x_k]}{Var[\tilde{v} \mid p, x_k]} = \frac{E[\tilde{v} \mid p]}{Var[\tilde{v} \mid p]}$$

$$\Leftrightarrow \frac{E[\tilde{v} \mid p, x_k] - E[\tilde{v} \mid p]}{Var[\tilde{v} \mid p, x_k] - Var[\tilde{v} \mid p]} = \frac{E[\tilde{v} \mid p]}{Var[\tilde{v} \mid p]}$$

$$\Leftrightarrow \frac{(D3) - (D1)}{(D4) - (D2)} = \frac{(D1)}{(D2)}$$

$$\Leftrightarrow \frac{Var(\tilde{p})x_k - Cov(\tilde{p}, \tilde{x}_k)p}{-Var(\tilde{v})[Var(\tilde{p}) - Cov(\tilde{p}, \tilde{x}_k)]} = \frac{p}{Var(\tilde{p}) - Var(\tilde{v})}$$

$$\Leftrightarrow \frac{Cov(\tilde{p}, \tilde{x}_k) - Var(\tilde{v})}{Var(\tilde{p}) - Var(\tilde{v})} = \frac{x_k}{p} \tag{D7}$$

Further recalling that $p = \dfrac{\sum f_i \Lambda_i x_i}{\sum f_i \Lambda_i}$, $Cov(\tilde{p}, \tilde{x}_k) - Var(\tilde{v}) = \dfrac{f_k}{\sum f_i \Lambda_i}$,

$Var(\tilde{p}) - Var(\tilde{v}) = \dfrac{\sum f_i^2 \Lambda_i}{(\sum f_i \Lambda_i)^2}$, it follows that the sufficient and necessary condition for informative

efficiency is that:

$$x_k = \frac{f_k [\sum f_i \Lambda_i x_i]}{\sum f_i^2 \Lambda_i}, \forall k = 1, 2, ..., I \tag{D8}$$

Q.E.D.

Appendix E: Sufficient Condition

By (D5), $Var(\tilde{p}) = Cov(\tilde{p}, \tilde{x}_k) \Longrightarrow E[\tilde{v} \mid \tilde{p}, \tilde{x}_k] = E[\tilde{v} \mid \tilde{p}]$ and By (D6), $Var(\tilde{p}) = Cov(\tilde{p}, \tilde{x}_k) \Longrightarrow$ $Var[\tilde{v} \mid \tilde{p}, \tilde{x}_k] = Var[\tilde{v} \mid \tilde{p}]$

Hence, if $Var(\tilde{p}) = Cov(\tilde{p}, \tilde{x}_k)$ then $\dfrac{E[\tilde{v} \mid p, x_k]}{Var[\tilde{v} \mid p, x_k]} = \dfrac{E[\tilde{v} \mid p]}{Var[\tilde{v} \mid p]}$, i.e. price is sufficient statistic

for market information.

Q.E.D.

Appendix F

It is easy to show that if market information is equally allocated then the equilibrium price is a sufficient statistic for the information.

First prove that if $f_1 = f_2 = ... = f_I$, then $Var(\tilde{p}) = Cov(\tilde{x}_k, \tilde{p})$

$$Cov(\tilde{x}_k, \tilde{p}) - Var(\tilde{p}) = \frac{f_k}{\sum_{i=1}^{I} f_i \Lambda_i} - \frac{\sum_{i=1}^{I} f_i^2 \Lambda_i}{(\sum_{i=1}^{I} f_i \Lambda_i)^2} = \frac{\sum_{i=1}^{I} (f_k - f_i) f_i \Lambda_i}{(\sum_{i=1}^{I} f_i \Lambda_i)^2}$$

Hence, if $f_1 = f_2 = ... = f_I$, then $Var(\tilde{p}) = Cov(\tilde{x}_k, \tilde{p}) \Rightarrow \dfrac{E[\tilde{v} \mid p, x_k]}{Var[\tilde{v} \mid p, x_k]} = \dfrac{E[\tilde{v} \mid p]}{Var[\tilde{v} \mid p]}$.

Q.E.D.

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