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使用 AHP 爲基礎的群體決策法去評估與選擇行銷策略

Fuzzy AHP-based GDM Method for Assessing and Selecting Marketing Strategies

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摘要：傳統的統計分析技術，由於缺乏一套客觀與透明的群體決策共識達成機制，去處理成員間的衝突與看法上的不一致，以致行銷部門最後決選出來的行銷方案，往往在充滿被質疑、無法團結一致與同心協力的環境下執行。也因此，本文提出一個模糊層級分析群體決策法，可讓每一位參與決策的人員都感受到被尊重、而且有一套公平與透明的機制，逐漸地縮小決策者間的衝突與看法上的不一致。據此，最後所選出來的方案，將可在內部最小阻力與最小分裂的情況下得以被執行。

關鍵詞：行銷策略、層級分析法、群體共識

Abstract：The traditional statistics techniques, the final group option is frequently debatable and stuck in a harassing working environment owing to lack of a objective and transparent group consensus reaching process to deal with contradictory and conflicting judgments among marketing members. Accordingly, the proposed fuzzy AHP-based group decision making (GDM) method aims to which

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enable each group member is made to feel respected, judgments are articulated fairly, and the interactive solution process is completely transparent. Therefore, the final chosen solution would be executed smoothly and with a minimum of internal disruption.

***Keywords* :** Marketing Strategy, Analytic Hierarchy Process, Group Consensus

1. Introduction

At the start of every year, marketing management focuses mainly not on effectively allocating a limited budget among various marketing events, but instead on how to assess the effectiveness of various marketing strategies and choose the most effective one. Although every marketing strategy would be effective to a certain but limited extent by the end of the marketing period, their effectiveness remains uncertain and is subject to market trends and the strategies of competitors during strategy planning, evaluating and selecting. Consequently, marketing departments are constantly challenged to assess the marketing programs to ascertain which deserve corporate support, which are worthwhile to be executed, and which can achieve marketing goals. However, the collection and aggregation of opinions of all members within marketing departments is a tedious, laborious and costly task. Moreover, developing methods of convincing the majority to accept the final option with high consensus is another tough and difficult task.

In contrast to the prevailing marketing strategy assessment and selection, which is generally treated using statistical techniques, this investigation proposes a fuzzy AHP-based group decision making (GDM) method for collaboratively assessing and choosing marketing strategies. The main drawbacks of using statistical techniques alone are the difficulty of incorporating the experience and knowledge expressed by marketing members into the framework of strategy analysis and heavily reliance on the historical data. Incorporating the knowledge and experience into a well-formulated analytical model can be taken advantage of treating the complexity of the marketing phenomena and the multiplicity of marketing strategies (Dyer and Forman, 1991; Dyer et al., 1992). Besides, unless the strategy being assessed resembles previous strategies or target markets, historical data collected from previous surveys are of limited relevance (Bult and Foekens, 1993, Davies, 1994; Dyer and Forman, 1991). Since marketing decisions generally simultaneously involve quantitative and qualitative decision-making domains, as well as integrating opinions from multiple decision makers (DMs) (Keane, 1969), many marketing managers have become frustrated with using conventional stand-alone statistical techniques to developing a marketing-strategy comprehensive effectiveness measurement

(Dyer and Forman, 1991).

Generally, quantitative decision methods are much more efficient than qualitative ones for treating structured and certain decision problems. Meanwhile, qualitative decision-making methods are significantly better than quantitative ones when treating ill-structured and uncertain decision issues. The analytic hierarchy process (AHP), a participation-oriented and multi-attribute decision analysis methodology proposed by Saaty in 1971 (Saaty, 1988), has been broadly employed in modeling unstructured economic, social, political and management sciences problems during recent decades (Meade and Presley, 2002). One of the primary advantages of AHP is enabling the transformation of intangible qualitative judgments into tangible quantitative values (Badri, 2001). Through pair-by-pair comparison values for a set of objects, AHP elicits a corresponding priority vector that interprets the preferred information from the DMs. Additionally, in practice DMs sometimes or very often cannot give all or part of the preferred information with exact numerical values, and naturally express a rough value or vague knowledge about the preference. In particular, when encountering a changing marketing environment, a degree of uncertainty is associated with some or all pairwise comparison values in an AHP structure.

Based on the above discussion, this work proposes an applicable fuzzy AHP-based GDM model for measuring the consensus and minimizing the differences among the opinions of DMs via an objective, persuasive, and interactive decision process. Since the proposed method can generate a group preferred priority after a goal programming (GP) computation, sensitivity analysis can be straightly performed to review the solution robustness, and the developed method can be easily implemented into a computerized decision support system. This study presents a case example using data from marketing strategy selection at one of top four dairy companies in Taiwan. An acceptable solution with a stable majority consensus within the company is achieved by employing the proposed method.

2. Building A Fuzzy AHP-Based GDM Model

Among numerous current AHP methods for deriving the priority vector by comparing all pairs of criteria and decision alternatives, the eigenvector method (EM), least squares method (LSM), and logarithmic least squares method (LLSM) are the three most common and popular approaches (Saaty, 1988). Although EM, LSM, and LLSM have been demonstrated to satisfy “correctness in the consistent cases”, “comparison order variance”, and “smoothness”, only LLSM further satisfies “power invariance” (Fichtner, 1986). Based on the LLSM property, an AHP problem is formulated as follows (Saaty, 1988):

Model 1

$$\text{Minimize} \quad \sum_{1\le i<j\le k} (\ln a_{ij} - \ln(\frac{v_i}{v_j}))^2 \tag{1}$$

$$\text{Subject to:} \quad a_{ij} \ge 0, \tag{2}$$

where a_{ij} denotes the evaluator's preference between objects i and j, k is the number of objects, the ratio v_i / v_j represents the comparison between each pair of objects i and j, and $a_{ij} = 1 / a_{ji}$ in a positive reciprocal matrix.

To generate the priority vector from AHP problems by using GP techniques is not a new idea. The advantages of using GP are not only fulfilling the four axioms discussed in the previous paragraph (Fichtner, 1986), but also fulfills another axiom called single outlier neutralization (Bryson, 1995; Bryson et al, 1995; Li et al., 2001) and can be combined with other decision tools to deal with complex real-world problems (Chen and Hwang, 1992). Consequently, by combining the LLSM and GP properties, Model 1 is reprogrammed as the following model:

Model 2

$$\text{Minimize} \quad \sum_{1\le i<j\le k} (\delta_{ij}^{+} + \delta_{ij}^{-}) \tag{3}$$

$$\text{Subject to:} \quad \ln a_{ij} - (\ln v_i - \ln v_j) - \delta_{ij}^{+} + \delta_{ij}^{-} = 0, \text{ and } a_{ij} \ge 0, \tag{4}$$

where $(v_1, v_2, ..., v_k)$ is an un-normalized vector and is used to produce the normalized vector $(\omega_1, \omega_2, \ldots, \omega_k)$ with $\frac{v_i}{v_j} = \frac{\omega_i}{\omega_j}$ and $\sum_{i=1}^{k} \omega_i = 1$.

Due to the inherent subjectivity, imprecision and vagueness of human beings in expressing opinions, especially when encountering fluctuating evaluation scenarios and insufficient or incomplete information available for making judgments, DMs may feel that giving uncertain ratings is more comfortable and natural than giving precise ratings. Consequently, the following corollary is presented for treating a fuzzy assessment.

Lemma 1. Figure 1 illustrates a triangular membership function $\mu(a_d)$ which can be represented as

$$\mu(a_d) = s_{d,R} \times a_d - (s_{d,R} - s_{d,L})a_{d2} + (s_{d,R} - s_{d,L})\delta - s_{d,L} \times a_{d1}, \tag{5}$$

$$a_d - a_{d2} + \delta \ge 0, \delta \ge 0, \tag{6}$$

where $s_{d,L}$ and $s_{d,R}$ are left-hand and right-hand side slopes, respectively.

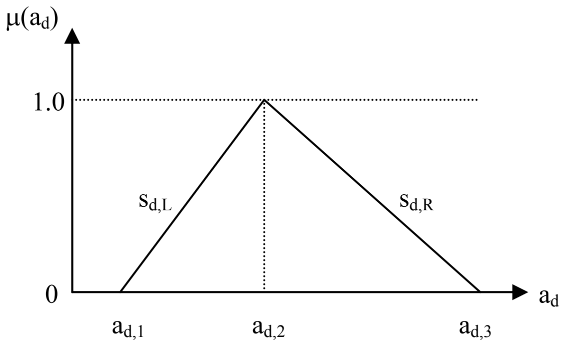

Fig. 1 A triangle membership function

Proof. Please see Appendix 1.

Taking Fig. 2 as an example, the problem of maximizing $\mu(a_1)$ can be programmed as follows:

Example 1

$$\text{Maximize } \mu(a_1) \tag{7}$$

$$\text{Subject to } \mu(a_1) = -1\times a_1+1.5\times 4-1.5\times\delta-0.5\times 2= -a_1-1.5\times\delta+5,\ a_1-4+\delta \geq 0,\ \delta \geq 0. \tag{8}$$

Inserting the constraint $a_1 \leq 3.7$ into Example 1 for illustrative purposes, after executing this program in LINGO or CPLEX the generated solution set is $\mu(a_1) = 0.85$, $a_1 = 3.7$, and $\delta = 0.3$. Accordingly, the fuzzy version of Model 3 can be formulated as below:

Model 3

$$\text{Minimize } \sum_{1\leq i<j\leq k}(\delta_{ij}^{+}+\delta_{ij}^{-}) \tag{9}$$

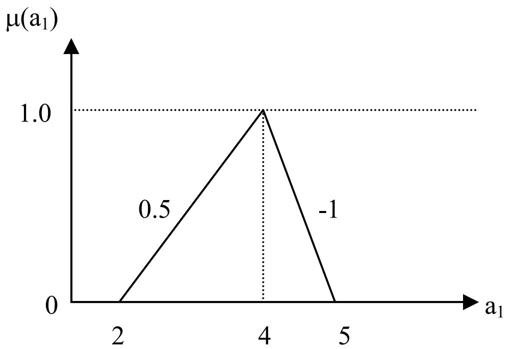

Fig. 2 A fuzzy value a_1

Subject to: $\ln a_{ij} - (\ln v_i - \ln v_j) - \delta_{ij}^{+} + \delta_{ij}^{-} = 0,$ (10)

$$\mu(a_{ij}) = s_{ij,R} \times a_{ij} - (s_{ij,R} - s_{ij,L}) a_{ij2} + (s_{ijR} - s_{ij,L}) \delta_{ij} - s_{ij,L} \times a_{ij1} \quad (11)$$

$$a_{ij} - a_{ij2} + \delta_{ij} \geq 0,\ \delta_{ij} \geq 0, \text{ and } a_{ij} \geq 0, \quad (12)$$

where the defuzzified solution is generated through a GP computation directly, which implies that tedious calculation (i.e., fuzzy mathematical operations) or iterative procedures (i.e., α-cut method) to defuzzify the fuzzy priority required by prevailing fuzzy GDM methods are not necessary. In practice DMs intuitionally and frequently input vague judgments, but prefer a crisp solution (Meade and Presley, 2002).

3. Aggregating Individual Opinions

Suppose that a marketing group containing nine DMs wishes to assess the importance of the consumer-oriented and the marketing-channel-oriented strategies. For example, Dell sells products straightly to end-buyers, while HP-Compaq sells products to end-buyers by collaborating with diverse channels. Since different products have different characteristics and different geographic markets also have different features, every strategy has its own advantages and disadvantages. The fuzzy ratings judged by the nine DMs are (2, 4, 5), (2, 3, 4), (1, 2.5, 3.5), (2, 3, 5), (2, 3, 4), (2, 3, 4), (1, 2, 4), (2, 3, 4), and (5, 6, 7). Figure 3 displays the preferred fuzzy values of each DM, and the defuzzified collective group position is described later on.

Referring to Lemma 1, uncertain judgments of each DM can be interpreted as follows:

$$\mu(a_{112}) = -a_{112} - 1.5 \times \delta_{112} + 5,\ a_{112} - 4 + \delta_{112} \geq 0,\ \delta_{112} \geq 0, \quad (13)$$

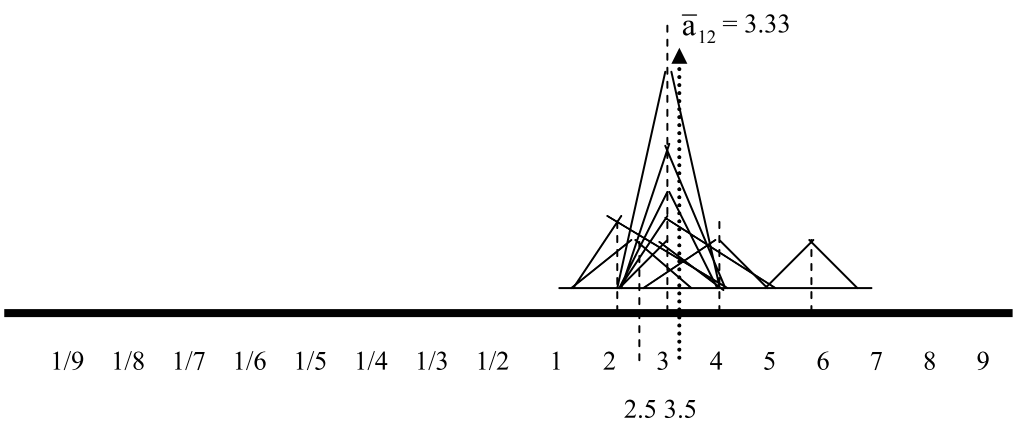

Fig. 3 Nine DMs' preferences and the collective group solution.

$$\mu(a_{212}) = -a_{212} - 2\times\delta_{212} + 4,\ a_{212} - 3 + \delta_{212} \geq 0,\ \delta_{212} \geq 0, \quad (14)$$

$$\mu(a_{312}) = -a_{312} - 1.6667\times\delta_{312} + 2.50005,\ a_{312} - 2.5 + \delta_{312} \geq 0,\ \delta_{312} \geq 0, \quad (15)$$

$$\mu(a_{412}) = -0.5a_{412} - 1.5\times\delta_{412} + 2.5,\ a_{412} - 3 + \delta_{412} \geq 0,\ \delta_{412} \geq 0, \quad (16)$$

$$\mu(a_{512}) = -a_{512} - 2\times\delta_{512} + 4,\ a_{512} - 3 + \delta_{512} \geq 0,\ \delta_{512} \geq 0, \quad (17)$$

$$\mu(a_{612}) = -a_{612} - 2\times\delta_{612} + 4,\ a_{612} - 3 + \delta_{612} \geq 0,\ \delta_{612} \geq 0, \quad (18)$$

$$\mu(a_{712}) = -0.5a_{712} - 1.5\times\delta_{712} + 2,\ a_{712} - 2 + \delta_{712} \geq 0,\ \delta_{712} \geq 0, \quad (19)$$

$$\mu(a_{812}) = -a_{812} - 2\times\delta_{812} + 4,\ a_{812} - 3 + \delta_{812} \geq 0,\ \delta_{812} \geq 0, \quad (20)$$

$$\mu(a_{912}) = -a_{912} - 2\times\delta_{912} + 7,\ a_{912} - 6 + \delta_{912} \geq 0,\ \delta_{912} \geq 0. \quad (21)$$

Among various methods of aggregating individual opinions to determine the group solution, the geometric mean method and weighted arithmetic mean (WAM) method are two widely accepted methods. When equal importance is assigned to all DMs, the geometric mean method is a more appropriate way of synthesizing judgments (Aczel and Saaty, 1983), while the WAM method casters to situations in which some members are expected to exert a strong influence compared to others (Saaty, 1988). Since a subset of members frequently exists whose positions make them likely to be effective agents for consensus-building in real-life cases (Bryson, 1996), this study employs the WAM method to gather individual opinions and thus reach a group opinion. Using the WAM method, the aggregated group positions between strategies 1 and 2 can be obtained by calculating $\bar{a}_{12} = (\sum_{d=1}^{n} w_d a_{d12})$ where w_d denotes the influence weight from the d'th DM.

From the above discussion, the maximum majority consensus is generated by the following model.

Model 4

Minimize $$\sum_{d=1}^{9} (\delta\delta_{d12}^{+} + \delta\delta_{d12}^{-}) \quad (22)$$

Subject to: $$a_{d12} - \bar{a}_{12} - \delta\delta_{d12}^{+} + \delta\delta_{d12}^{-} = 0, \text{ for each d, d=1, 2, …, 9,} \quad (23)$$

$$\bar{a}_{12} = (\sum_{d=1}^{9} w_d a_{d12}),\ (14) - (21), \quad (24)$$

where $$\sum_{d=1}^{n} w_d = 1.$$

Where other constraints are ignored, and according to the seniority, experience, and job titles of nine DMs, the adequate influence weights are presented to reflect the real-world GDM situation, where $w_1 = w_2 = 0.1875$, $w_3 = w_4 = w_5 = 0.125$, and $w_6 = w_7 = w_8 = w_9 = 0.0625$. Running this program in LINGO or CPLEX computes total disagreement deviation of 2.5, implying that the average dissimilarity for each DM to the group position is 0.2778 obtained by 2.5 / 9. The other defuzzified solutions are $\overline{a}_{12}$ = 3.33, $a_{112} = a_{212} = a_{412} = a_{512} = a_{612} = a_{712} = a_{812} = 3.33$, $a_{312} = 2.5$, and $a_{912} = 5$ in which $\mu(a_{112}) = 0.665$, $\mu(a_{212}) = \mu(a_{512}) = \mu(a_{612}) = \mu(a_{812}) = 0.67$, $\mu(a_{312}) = 1$, $\mu(a_{412}) = 0.835$, $\mu(a_{712}) = 0.335$ and $\mu(a_{912}) = 0$. Notably, the assessments of all of the DMs approach the aggregated majority assessments with decreasing deviations, as illustrated in Fig. 3 above. Hence, the smaller the deviations are, the closer the opinions of DMs are.

4. Treating A Trade-Off "Consensus" Preferred Option

Suppose we have k strategies (alternatives, objectives, candidates, issues, …), s = 1, 2, …, k, and n DMs, d = 1, 2, …, n. These DMs evaluate the strategies based on each criterion c, c = 1, 2, …, m, and also judge the importance of the criteria pertaining to a certain overall objective under the AHP-based GDM environment. Based on Models 3 and 4, an integrated AHP-based GDM model can be constructed as follows:

Model 5

Minimize $[\sum_{d=1}^{n}\sum_{c'>c}^{m}(\delta_{dcc'}^{+}+\delta_{dcc'}^{-})+\sum_{d=1}^{n}\sum_{c=1}^{m}\sum_{s'>s}^{k}(\delta_{dcss'}^{+}+\delta_{dcss'}^{-})]+[\sum_{d=}^{n}\sum_{ss=1}^{k}(\delta_{ds}^{+}+\delta_{ds}^{-})]$

Subject to:

$$\ln a_{dcc'} - (\ln v_{dc} - \ln v_{dc'}) - \delta_{dcc'}^{+} + \delta_{dcc'}^{-} = 0, \tag{25}$$

$$\ln a_{dcss'} - (\ln v_{dcs} - \ln v_{dcs'}) - \delta_{dcss'}^{+} + \delta_{dcss'}^{-} = 0, \tag{26}$$

$$\overline{v}_{s} - v_{ds} - \delta_{ds}^{+} + \delta_{ds}^{-} = 0, \text{ for each s under each d,} \tag{27}$$

$$v_{ds} = \sum_{c=1}^{m} v_{dc} v_{dcs}, \text{ for each s under each d,} \tag{28}$$

$$\mu(a_{dcc'}) = s_{dcc'R} \times a_{dcc'} - (s_{dcc'R} - s_{dcc'L}) a_{dcc'2} + (s_{dcc'R} - s_{dcc'L}) \delta_{dcc'} - s_{dcc'L} a_{dcc'1} \tag{29}$$

$$a_{dcc'} + a_{dcc'2} + \delta_{dcc'} \geq 0, \delta_{dcc'} \geq 0, \tag{30}$$

$$\mu(a_{dcss'}) = s_{dcss'R} \times a_{dcss'} - (s_{dcss'R} - s_{dcss'L}) a_{dcc'2} + (s_{dcss'R} - s_{dcss'L}) \delta_{dcss'} - s_{dcss'L} a_{dcss'1} \tag{31}$$

$$a_{dcss'} + a_{dcss'2} + \delta_{dcss'} \geq 0, \delta_{dcss'} \geq 0, \tag{32}$$

$$d \in \{1, 2, …, n\}, (c, c') \in \{1 \leq c < c' \leq m\}, (s, s') \in \{1 \leq s < s' \leq k\}, \tag{33}$$

where $a_{dcc'}$ denotes the d'th DM's fuzzy preference between criterion c and c', $a_{dcss'}$ represents the d'th DM's fuzzy comparison value to which strategy s is preferable to strategy s' under the c'th criterion, d = 1, 2, …, n, c = 1, 2, …, m, s = 1, 2, …, k, $\overline{v}_{s}$ denotes the group consensus opinion for the s'th strategy, v_{ds} represents the d'th DM's opinion for the s'th strategy, $s_{dcc'L}$ and $s_{dcc'R}$ are separately left-hand and right-hand side slopes of fuzzy value $a_{dcc'}$, and $s_{dcss'L}$ and $s_{dcss'R}$ are separately left-hand and right-hand side slopes of fuzzy value $a_{dcss'}$.

Notably, $(v_1, v_2, ..., v_k)$ in (27) is the un-normalized and aggregated group strategy priority vector. Apparently, the first term in the objective function is the derived consistent deviation across the whole AHP model, while the second term is the generated consensus deviation between the collective group judgment and individual DMs judgments regarding strategies. Noticeably, while the first term of the objective function in Model 5 is to maximize the overall evaluation consistency for entire DMs, the second term is concerned not with every pairwise comparison of each level within the AHP structure but rather than with final alternative assessment and priority. The advantage of this assumption is that if DMs are drawn from different areas of expertise with different/multiple disciplines, each DM is allowed to build separate AHP criteria structure under the same predetermined ultimate goal and marketing strategies. Since this assumption fits the real-world situation reasonably closely, the proposed model can treat GDM problems effectively by focusing on the final scoring and ranking of strategies.

Model 5 clearly aims to concurrently optimize the overall preference consistency of all DMs as well as the degrees of agreements of DMs with the group consensus. However, a truly optimization solution must be based on a single objective and cannot be obtained by a multiple objective model (Dyer et al., 1992). Hence, a single solution capable of optimizing all the goals generally does not exit. When solving problems with multiple and non-commensurable goals, the simultaneous optimization of two terms in the objective function is typically considered a trade-off problem. Nevertheless, Lemma 2 is presented to resolve this problem.

Lemma 2. Model 5 can be treated as the following model:

Model 6

Minimize
$$[\sum_{d=1}^{n}\sum_{c'>c}^{m}(\delta^{+}_{dcc'}+\delta^{-}_{dcc'})+\sum_{d=1}^{n}\sum_{c=1}^{m}\sum_{s'>s}^{k}(\delta^{+}_{dcss'}+\delta^{-}_{dcss'})](1/(\mathrm{M}^2-\mathrm{M}^1)) +$$
$$[\sum_{d=}^{n}\sum_{s=1}^{k}(\delta^{+}_{ds}+\delta^{-}_{ds})] \quad (34)$$

Subject to: (25) – (33).

Proof. In Model 5, the smaller deviation within the first term indicates that a higher consistent priority vector is generated. This implies that higher DMs' desirability is derived. The smaller deviation within the second term displays that higher concordance with the group solution is derived. This implies that higher group consensus is achieved. Consequently, initially, without considering the first term in Model 5, after calculating Model 5 to minimize the second term, M^1 is used to denote the calculated minimal deviation within the second term. Likewise, using Model 5 to maximize the second term without considering the first term, the generated solution representing the maximal deviation within the second term is denoted by M^2. Based on the concepts of the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) originally developed by Yoon et al. (Hwang and Yoon, 1981) and LINnear programming techniques for Multidimensional Analysis of Preference (LINMAP) originally presented by Srinivasan et al. (Hwang and Yoon, 1981), the trade-off weighting value can be determined as $1/(M^2 - M^1)$. In this way, Model 5 thus can be reformulated as Model 6.

5. Determining the Group Consensus

Suppose that a marketing group contains three DMs, who give comparison values between objects A and B of 3.7, 3 and 5, respectively. The average group assessment value for comparing objects A and B is then calculated as (3.7 + 3 + 5) / 3 = 3.9. Accordingly, the degree of agreement of the opinion of the first DM compared to the average group rating is 0.9487, calculated by 1 - | (3.7 – 3.9) / 3.9 |. Consequently, the agreement of the first DM to the group average assessment value is 94.87%. Similarly, the agreements of the second and third DMs to the average assessment value of the group are found to be 76.92% and 71.76%, respectively. Therefore, the group consensus is 81.18%, computed by (94.87% + 76.92% + 71.76%) / 3, which implies the average agreement degree from all DMs to the aggregated group opinion. In an extreme case, when all three DMs give identical comparison values when comparing objects A and B, then the group consensus becomes 100% which means all DMs totally agree with the group decision.

From the above discussion, the group consensuses for each pair of criteria and overall criteria assessments can be obtained by $(\sum_{d=1}^{n}(1-|\frac{a_{dcc'}-\bar{a}_{cc'}}{\bar{a}_{cc'}}|))/n$ and $[\sum_{d=1}^{n}[(\sum_{1\le c<c'\le m}(1-|\frac{a_{dcc'}-\bar{a}_{cc'}}{\bar{a}_{cc'}}|))/(m(m-1)\div 2)]]/n$, respectively. Likewise, the group consensuses for each pair of strategies and overall strategies evaluation is obtained by $(\sum_{d=1}^{n}(1-|\frac{a_{dcss'}-\bar{a}_{css'}}{\bar{a}_{css'}}|))/n$ and $[\sum_{d=1}^{n}[(\sum_{1\le s<s'\le k}(1-|\frac{a_{dcss'}-\bar{a}_{css'}}{\bar{a}_{css'}}|))/(k(k-1))\div 2)]]/n$, respectively. The $\bar{a}_{cc'}=(\sum_{d=1}^{n}w_d a_{dcc'})$ and $\bar{a}_{css'}=(\sum_{d=1}^{n}w_d a_{dcss'})$ are the aggregated group opinions separately for criteria and strategies. The calculation and defuzzifing process of $\bar{a}_{cc'}$ and $\bar{a}_{css'}$ are introduced in preceding Model 4 and Section 3. Accordingly, Model 6 can be reformulated as below.

Model 7

Minimize $[\sum_{d=1}^{n}\sum_{c'>c}^{m}(\delta_{dcc'}^{+}+\delta_{dcc'}^{-})+\sum_{d=1}^{n}\sum_{c=1}^{m}\sum_{s'>s}^{k}(\delta_{dcss'}^{+}+\delta_{dcss'}^{-})](1/(M^2–M^1))$ +

$$[\sum_{d=}^{n}\sum_{s=1}^{k}(\delta_{ds}^{+}+\delta_{ds}^{-})] \tag{35}$$

Subject to: (25) – (33), $\overline{a}_{cc'}=(\sum_{d=1}^{n}w_d a_{dcc'})$ and $\overline{a}_{css'}=(\sum_{d=1}^{n}w_d a_{dcss'})$, (36)

where the group consensuses for criteria and strategies can be computed after generated the values of $\overline{a}_{cc'}$ and $\overline{a}_{css'}$.

6. Setting A Rigid Disagreement Threshold

To avoid the final choice to be executed in a harassing and rickety working environment, GDM problems not only desire to maximize the degree of agreement to the group consensus and the desirability of each DM but also desire to reduce the number of strong objection members. Traditionally, all individuals within a group completely agreeing with all issues are called completely agreement (absolute consensus). However, this situation is counterintuitive and unlikely to exist in the real world (Kacprzyk et al., 1992). In practice, GDM aims to derive a solution that can be accepted by at least half of the DMs, where the solution can sufficiently represent the thoughts of the majority. However, in many cases, a stable and rigid majority may require the consent of over 75% of DMs. That is, a firm majority requires opposition of less than 25% among DMs.

Accordingly, a GDM method solely considering single agreement or disagreement usually results in the generated solution falling on the functions of instability and manipulability (Kacprzyk et al., 1992; Numi, 1982, 1983, 1986). Restated, the key question here is: are there any or some of the criteria, strategies, alternatives, or objectives are seriously controversial, disputed, or even conflicting? Moreover, if these issues were treated using a strong disagreement threshold, would the final group choice be more stable and practical? Notably, this work considers two assumptions. The first assumption is how many members oppose the best alternative (rank one) when only the first position among alternatives is selected. The second assumption is to consider how many members have priorities (ranking) differing from the group priorities when either some alternatives are selected or an appropriate proportion of the budget is allocated to these predetermined alternatives.

Given a marketing department containing four DMs, the computed final group rank for a three-strategy evaluation case is s2 > s1 > s3 where the private opinions of four DMs are s3>s1>s2, s2>s1>s3, s2>s3>s1, and s1>s3>s2, respectively. The first assumption is that only the alternative of the first position will be selected and executed. It means that the marketing department is only concerned with preventing rank 1 of the generated priority receiving objection from over 25% of DMs. In this situation, the second and third DMs agree with the group choice, while the first and fourth DMs disagree that choice. Accordingly, the total group object degree is 2 / 4 = 50%, exceeding the 25% threshold of a rigid disagreement.

In the second assumption concerning the whole alternative rank, which implies that the marketing department may attempt to allocate a certain portion of budget to these predetermined strategies or select some among them. In this respect, the management will be concerned that the entire priority/ranking will not be objected to by over 25% of DMs. In this case, comparing the group rank s2 > s1 > s3 and the first DM's rank s3>s1>s2, the object degree is 2 /3 for the first DM. Similarly, the individual object degree of the 2nd, 3rd, and 4th DMs can be obtained as 0, 2/3, and 2/3, respectively. Accordingly, the entire group object level is (2/3 + 0 + 2/3+ 3/3) / 4 = 58.33%, also surpassing the rigid disagreement threshold of 25%.

Suppose $\bar{v}$ [s] denotes the group judgment for the s'th strategy, v_d [s] represents the judgment of the d'th DM for the s'th strategy, and D[d] is the object degree of the d'th DM to the majority priority for strategies. Taking the above example as an illustration, algorithms for calculating the rigid disagreement threshold are presented as follows:

Algorithm 1.

While DMs are only concerned with the rank 1 among strategies

nn = 0, n = 9, $\bar{v}$ [ ] and v_d [ ] are array variables,

for d = 1 to n,

if $\bar{v}$ [1] $\neq$ v_d [1] then nn = nn +1,

next d,

the rigid disagreement threshold is nn / n,

End

Algorithm 2.

While DMs are concerned with the entire strategy priority

n = 9, kk = 0, $\bar{v}$ [ ], v_d [ ], and D[d] are array variables,

for d = 1 to n,

for s = 1 to k,

if $\bar{v}$ [s] $\neq$ v_d [s] then kk = kk +1,

next s,

the d'th DM's object degree is kk / k,

D[d] = kk / k,

kk = 0,

next d,

the rigid disagreement threshold is $\sum_{d=1}^{n} D[d]$ / n,

End

Unless an acceptable majority consensus exits, it is premature to use mathematical techniques to determine the final choice. Accordingly, the next section conducts a process for reaching group consensus before applying mathematical operations to derive a "consensus" preferred decision.

7. Developing A Group Consensus Reaching Process

GDM problems relate to various research fields and have been the subject of numerous investigations and meta-analysis over the past decades. Since no matter which group choice procedure is employed, it would satisfy some set of plausible conditions but not another set of equally plausible ones. Besides, in practice, DMs prefer using intuition way to make/adjust their decisions speedily rather than using complex mathematical models for decision optimization. Consequently, attempting to design new, more sophisticated solution procedures do not appear very promising. Accordingly, the direction with greatest potential real-life application is to develop an interactive process that allows each DM to rapidly rethink and adjust his/her opinion.

Consequently, an interactive GDM procedure is presented below to coordinate a dynamic and iterative group discussion process and help DMs make their decisions more closely resemble the majority position.

Interactive GDM Solution Process

Step 1. Construct an AHP structure and Input the pairwise assessments of each DM.

Step 2. Assign suitable weights to each DM, specify the rigid disagreement threshold (the default value is 25%), and model the problem as Model 7.

Step 3. Calculate the M^1 and M^2.

Step 4. Generate the group solution by GP computation.

Step 5. Compare the generated disagreement degree with the rigid disagreement threshold, and if the against level is below the threshold then go to Step 8.

Step 6. Display the distance maps among the group position and DMs positions across the AHP structure using cluster analysis.

Step 7. Update DMs opinions and formulate the problem again as Model 7. Proceed to Step 3.

Step 8. Since the most promising group-consensus solution is reached with the rigid and stable majority consent, sensitivity analysis can be executed if necessary to examine the robustness of the final group choice.

The first advantage of using the proposed interactive solution process instead of a mediator is that the influence of human subjectivity is reduced, the entire process is transparent, and the laborious interactive process can be handed by a computerized GDM system. Owing to Model 7 being convenient to compute using many popular linear programming packages, like LINGO, and Algorithms 1 and 2 are simple and clear-cut, the proposed method is easily coded using many common programming languages combined with the execution of LINGO. The second advantage of the proposed process is that incorporating cluster analysis into the interactive framework can increase the possibility of encouraging revision by comparing how well a set of individual judgments over the AHP structure compare with the group judgment via the generated distance maps. Moreover, the third advantage is capable of using sensitive analysis directly after a rigid and stable majority option is reached. In this way, the solution derived following the presented interactive GDM solution process in some sense is a robust and virtually equivalent optimal solution.

8. An Application

At the beginning of each year, the marketing management of Company T faces of not only effectively allocating a limited budget among numerous marketing events, but also (more importantly) of how to assess the effectiveness of various marketing strategies and choose an effective one with maximum group consensus. Because of future market uncertainty, a successful marketing strategy is much better evaluated by multiple criteria in a subjective/objective and quantitative/qualitative assessment framework rather than a stand-alone technique. Moreover, at the time of evaluation, the contribution of a given marketing strategy is uncertain, doubtful and subject to the dynamic market situation and the responses of competitors. Hence, such a marketing strategy assessment and selection problem is usually ambiguous, vague and full of conflicting judgments among DMs. In past years, strong disagreement or contradictory opinions have frequently occurred in the Company T, and how to treat the group consensus has become a crucial matter, when planning and selecting annual marketing strategies. However, although each marketing member has different perspectives and interests, they are all belong to same company and responsible for finding the most promising and stable “consensus” solution and share responsibility for the final decision. Accordingly, in contrast with conventional marketing strategy assessment, which is generally conducted with statistical methods alone, this investigation applies the proposed AHP-based GDM method to assist Company T in assessing marketing strategies and selecting the most effective one with a minimum of internal hassle.

8.1 Background Information

According to a market survey by Company T and research by Taiwan’s Food Research and Development Institute (Author, 2001), 71.3% of Taiwan’s populations drink fresh milk and 40.1% of this group is frequent drinkers. The major products in the fresh milk market are generally categorized into full-cream fresh milk, calcium enriched fresh milk, low fat milk, and vitamin added fresh Milk. Full-cream fresh milk, calcium enriched fresh milk, low fat milk, and vitamin added fresh milk hold market shares of 60%, 10%, 23% and 7%, respectively. Regarding packaging and size, large plastic bottles (over 946cc), large paper bottles (over 946cc), medium cardboard cartons (437~500cc), small cardboard cartons (200~236cc), small plastic bottles (below 250cc), small aluminum cans, small glass bottles, and other small packages have market shares of 59.64%, 4.49%, 0.85%, 14.36%, 4.28%, 6.74%, 3.32% and 6.3%, respectively. Regarding sales channels, department stores and supermarkets, chain convenience stores, chain shopping malls, breakfast stores, and traditional grocers have market shares of 21.06%, 26.32%, 26.32%, 15.78% and 10.52%, respectively. From a geographical perspective, 37.5%, 43.3%, 43.2%, and 32.3% of the population in northern, middle, southern and eastern of Taiwan are frequent drinkers, respectively. Meanwhile, in terms of age, 35%, 35%, 43.5%, 50%, 44.1%, 35%, 35%, 35% and 25% of the population in the age groups of under 20, 20~25, 25~29, 30~35, 35~39, 40~45, 45~49, 50~55 and over 55, respectively, are frequent drinkers. Interestingly, 43.4% of the female populations are frequent drinkers, compared to only 36.9% of the male population. Finally, a positive association exists between frequent fresh milk drinking and educational level.

The primary obstacles facing the marketing planners of Company T are summarized as follows:

(1) Although the market share of Brand T is extremely close to the leading Brand Y and leads third placed Brand P by 9%, Brand T remains markedly lower than Brand Y in terms of brand image and is nearly the same as Brand Z in the minds of consumers;

(2) Brand T leads in the department store and supermarket markets; however, Brand Z owns over 3000 chain convenience stores. Consequently, although Brand W holds a larger market share than Brand Z, Brand Z poses a large threat to Brand T in channel control, particularly given that 26.32% of fresh milk is sold through chain convenience stores [2];

(3) Owing to the rapid diversification of Taiwan's social structure (i.e. consumer preferences) in recent years, the image of Brand T is rapidly aging, particularly since Brand T is a long established brand. Meanwhile, the Brands Y and X have waged extremely successful advertising campaigns, resulting in the average age of the customers of company T being higher than that of companies Y and X; and

(4) Although Brand X only has an 8% market share, half of Brand X's sales are through chain shopping malls, and Brand X success in this area has created a significant challenge to Brand T.

8.2 Marketing Strategies for Company T

Building on the above market situation, Company T's ultimate marketing goal is not only maintain profits, but also to become the leading Brand in terms of the market share (that is, to gain a market share exceeding that of Brand Y). Following extended discussion among senior marketing managers, five marketing strategies are launched, as described below:

(1) Strategy A: Rejuvenating brand image

Strategy A involves redesigning product packaging, reforming corporate image, and blending in with young people's current fashion through the mass media. This strategy can hopefully be achieved by instigating innovative changes to create new consuming motives and attract the attention of a new generation. However, Strategy A could potentially lead to the loss of some current customers. Furthermore, packaging and image renew do not guarantee expanded market share. Particularly, Brand T is a long established brand, making its old image difficult to quickly transform and rejuvenate. Building a new and young brand image acquires considerable time and resources. Marketing planners also recognize that Brand T involves in a group of products and so must satisfy different market segments. Thus, effectively integrating its entire product line and nurturing a new and young brand image is a big challenge.

(2) Strategy B: Launching a new product mix

Strategy B involves launching a series of totally new products and varies product promotion according to market segments and consumer clusters. Moreover, strategy B also involves identifying and creating new markets for all products in the Brand W family. Launching a new product mix will not only help Company T to attract new customers, but will also help it to develop a new, young and positive image with its current (original) customers. However, implementing strategy B will involve some side effects. First, developing and launching a new product mix is quite expensive. Additionally, the timing and pace control of launching a series of new products is a relatively difficult task, particularly coordinating research, production, distribution, and marketing. Furthermore, a new product mix will not only invade the markets of other brands, but will also damage Brand T's existing markets. Comparing Strategy B with other strategies reveals that it potentially has the highest profits among five strategies, but that the costs of failure are also the highest.

(3) Strategy C: Designing intensive promotion activities by continuously responding to the latest consumer trends

Strategy C strives to expand current customer networks by capturing the latest consumer trends. Promotions, sales activities, distribution channel policies, product pricing and discounts, and other responses to changes in consumer behavior can be adjusted over time subject to continuous market surveys. Strategy C is highly attractive in terms of the relative ease of identifying the drawbacks of competitors, compensating for the weaknesses of current sales policy, effectively responding to changes in consumer tastes, and quickly adjusting key promotional events. Nevertheless, Strategy C has several limitations, as follows: (a) A large marketing staff is required to execute intensive promotion activities and market surveys. (b) Although sales may increase rapidly in the short-term, long-term sales may suffer. (c) Excessive changes such as promotions, sales activities, channel policies, and responses to customers may gradually create a negative and unstable image and damage the existing stability and maturity of Brand T.

(4) Strategy D: Focusing on the promotion and development of healthy milk products

Strategy D is to launch a series of health products aimed at health conscious customers. This strategy also heavily emphasizes nurturing a positive and new health image for Brand W and offering diverse health milk products to customers. However, most people consider even ordinary fresh milk to already be a health product, and furthermore, the existing image ranking of the top four brands in terms of healthiness in the Taiwan milk market is Brand Y>X>T>Z. Therefore, promotion of healthy fresh milk products may have unintended effect of boosting sales of Brands Y and X. Comparing Strategy D with other strategies reveals it to have the lowest projected budget, a middle projected profit, and the medium risk of possible failure among five strategies.

(5) Strategy E: Doubling sales networks

Increasing the opportunities of consumers to access the fresh milk products of Brand W is the crux of Strategy E. Obviously, buying milk is completely different from buying a car, and most consumers are reluctant to shop around before deciding which brand of milk to buy. Consequently, increasing consumer access to a given brand should increase the sales of that brand. Take Brand Z as an example. Although Brand Z has the least healthy image and lowest product diversity among the four brands, it retains a healthy third placed market share and has enjoyed the highest growth rate of any brand during the past five years. This success is attributed primarily to the following two factors: (a)

Brand Z owns over 3000 chain convenience stores and (b) only Brand Z's fresh milk products are permitted be sold in these convenience stores. Thus, if Brand T could expand its influence in the more accessible distribution channels (i.e., chain convenience stores and chain shopping malls), its sales should increase significantly. However, annual sales in the milk market have an upper limit, and doubling sales networks would not lead to a doubling of fresh milk sales. Additionally, Strategy E cannot solve the major concern of the marketing planners of Company T, namely the problem of image aging.

9. Solution Illustration

According to the presented interactive GDM solution process, Step 1 first aims to construct an AHP evaluation structure. As is well known, customer responses vary with customer cluster, geographical area, packaging and sizes, sales channels, and product mix. In lieu of complex and uncertain market information, a good marketing strategy cannot merely consider one dimension since such a strategy would provide Company T with partial market information and favor partly marketing members. Thus, having thoroughly reviewed five strategies and conduced an interview survey of expert opinion, this investigation summarizes ten evaluation criteria for assessing five marketing strategies considered by nine marketing DMs. Figure 4 illustrates the evaluating structure among the ultimate marketing goal, dimensions, criteria, and strategies.

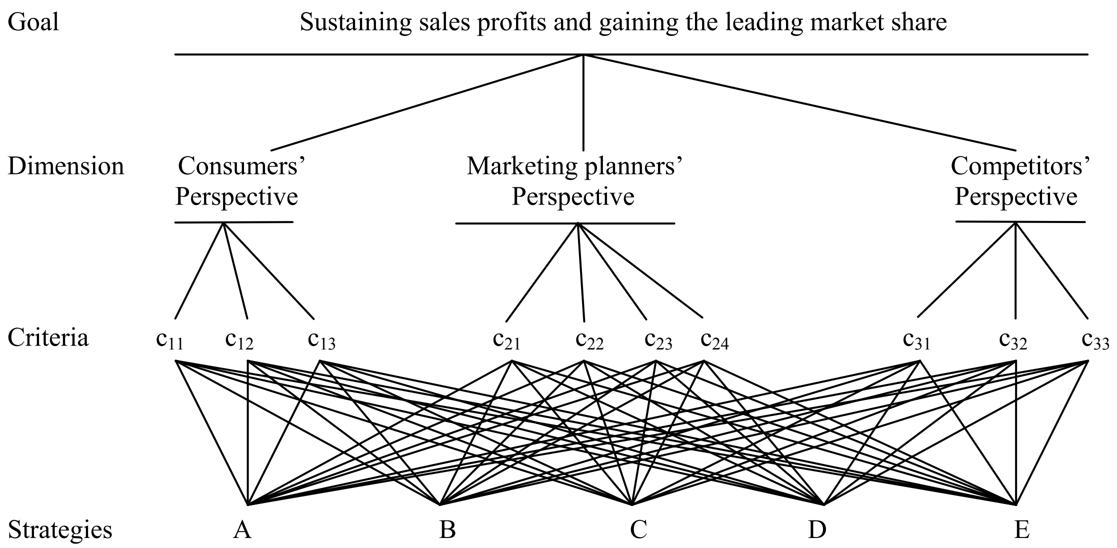

Fig. 4 The structure for evaluating five marketing strategies

In Fig. 4, q_1 represents consumers' perspective, q_2 denotes marketing channels' perspective, q_3 stands for competitors' Perspective, c_{11} denotes the new and young image of the product, c_{12} represents purchase convenience and availability, c_{13} stands for the attractiveness of promotions, c_{21} interprets better understanding of changes in consumers and sales channels, c_{22} expresses better understanding of the competitive strengths of other products, c_{23} denotes maximizing product market share, c_{24} stands for promoting the product's quality and healthy image, c_{31} represents the preference of major customers, c_{32} expresses product profit, and c_{33} interprets product circulation rate. The comparison judgments elicited from nine DMs are partially summarized in Tables 1 – 4, while other lengthy data are available on request.

Table 1 Comparison values among three dimensions

	Consumers' Perspective (q_1)	Marketing Channels' Perspective (q_2)	Competitors' Perspective (q_3)
Consumers' Perspective (q_1)	1	a_{112}=(2, 4, 5), a_{212}=(2, 3, 4), a_{312}=(1, 2.5, 3.5), a_{412}=(2, 3, 5), a_{512}=(2, 3, 4), a_{612}=(2, 3, 4), a_{712}=(1, 2, 4), a_{812}=(2, 3, 4), a_{912}=(5, 6, 7)	a_{113}=(1, 3, 4), a_{213}=(3, 4, 5), a_{313}=(3, 3.5, 5), a_{413}=(5, 5.5, 7), a_{513}=(4, 5, 6), a_{613}=(1, 2, 3), a_{713}=(1, 3, 5), a_{813}=(1, 3, 4), a_{913}=(5, 6, 7)
Marketing Channels' Perspective (q_2)		1	a_{123}=(1/4, 1, 2), a_{223}=(1/3, 1/2, 1), a_{323}=(1, 2, 3), a_{423}=(1, 2, 2.5), a_{523}= (1/4, 1/3, 1), a_{623}=(1/3, 1/2, 1), a_{723}= (1/3, 1, 2), a_{823}=(1/2, 1, 1.5), a_{923}= (1/3, 1, 2)
Competitors' Perspective (q_3)			1

Table 2 Comparison values among three criteria under the q_1 dimension

	c_1	c_2	c_3
c_1	1	a_{1112}=(1/6, 1/5, 1/4), a_{2112}=(2, 3, 4), a_{3112}= (1/6, 1/5, 1/3.5), a_{4112}=(1/1.5, 1/2, 1/2.5), a_{5112}=(1/2, 1, 2), a_{6112}=(1.5, 2, 3.5), a_{7112}= (2, 2.5, 3), a_{8112}=(1/3, 1/2.5, 1/2), a_{9112}= (1/1.5, 1, 1.5)	a_{1113}=(1/4, 1/3, 1/2), a_{2113}=(1/1.5, 1, 1.5), a_{3113}=(1/1.5, 1, 1/2), a_{4113}=(1/4, 1/3, 1/2), a_{5113}=(1, 1.5, 2), a_{6113}=(2, 2.5, 3), a_{7113}= (1/3, 1/2, 1/1.5), a_{8113}=(1/1.5, 1, 2), a_{9113}= (1/1.5, 1, 1.5)
c_2		1	a_{1123}=(1, 1.5, 2), a_{2123}=(1/2.5, 1/2, 1/1.5), a_{3123}=(3, 4, 5), a_{4123}=(1/2, 1/1.5, 1/1.2.5), a_{5123}=(1/1.5, 1, 2), a_{6123}=(1.5, 2, 2.5), a_{7123}=(1/1.5, 1, 1.5), a_{8123}=(1/1.5, 1, 1.5), a_{9123}=(1/4, 1/3, 1/2)
c_3			1

Table 3 Comparison values among four criteria under the q_2 dimension

	c_{21}	c_{22}	c_{23}	c_{24}
C_{21}	1	a_{1212}=(1/1.5, 1, 1.5), a_{2212}=(1, 2, 3), a_{3212}=(1/2, 1/1.5, 1), a_{4212}=(1/1.5, 1/1.2, 1), a_{5212}= (1, 1.2, 1.5), a_{6212}=(1/2, 1, 1.5), a_{7212}= (1, 1.5, 2), a_{8212}= (1/1.5, 1, 2), a_{9212}=(1, 2, 2.5)	a_{1213}=(1, 1.25, 1.5), a_{2213}=(1/1.5, 1, 2), a_{3213}= (1/2, 1, 2), a_{4213}=(1/1.5, 1, 2), a_{5213}= (1, 1.2, 1.5), a_{6213}= (1/2, 1, 1.5), a_{7213}= (1, 1.5, 2), a_{8213}= (1/3, 1/2, 1), a_{9213}= (1, 1.5, 2)	a_{1214}=(1/1.5, 1/1.25, 1), a_{2214} =(1, 1.5, 2), a_{3214}= (1/1.5, 1, 2), a_{4214}=(1/1.5, 1, 1.5), a_{5214} =(1, 1.2, 1.5), a_{6214}= (1/2, 1, 1.5), a_{7214}= (1, 1.5, 2), a_{8214}=(1/3, 1/2, 1), a_{9214}= (1/2, 1, 2)
c_{22}		1	a_{1223}=(1/1.5, 1, 2), a_{2223}=(1/2, 1, 1.5),a_{3223}= (1/2, 1, 2), a_{4223} =(1.25, 1.5, 2), a_{5223}=(1/1.5, 1, 1.5), a_{6223}=(1/2, 1, 2), a_{7223} =(1/1.5, 1, 1.5), a_{8223}= (1/3, 1/2, 1), a_{9223}=(1/2, 1/1.5, 1)	a_{1224}=(1/2, 1/1.5, 1), a_{2224}=(1/3, 1/2, 1/1.5), a_{3224}=(1, 2, 3), a_{4224}=(1, 1.5, 2), a_{5224} =(1/1.5, 1, 1.5), a_{6224}=(1/2, 1, 2), a_{7224}= (1/1.5, 1/1.2, 1), a_{8224}= (1/3, 1/2, 1), a_{9224}= (1/2.5, 1/2, 1/1.5)
c_{23}			1	a_{1234}=(1/2, 1/1.5, 1), a_{2234}=(1, 1.5, 2), a_{3234}= (1, 1.5, 2), a_{4234} =(1/1.5, 1, 1.5), a_{5234} =(1/1.5, 1, 1.5), a_{6234}=(1/1.5, 1, 1.5), a_{7234}= (1/1.5, 1/1.2, 1), a_{8234}= (1/1.5, 1, 1.3), a_{9234}= (1/1.5, 1, 1.5)
c_{24}				1

Table 4 Comparison values among three criteria under the q_3 dimension

	c_{31}	c_{32}	c_{33}
c_{31}	1	a_{1312}=(1, 1.5, 2), a_{2312}=(1, 2, 3), a_{3312}= (1/1.2, 1, 2), a_{4312}=(1, 1.5, 2), a_{5312}=(1, 2, 3), a_{6312}=(1/2, 1, 1.5), a_{7312}= (1, 2, 3), a_{8312} =(1/1.5, 1, 1.5), a_{9312}= (1, 2, 2.5)	a_{1313}=(1/2, 1/1.5, 1), a_{2313}=(1, 1.5, 2), a_{3313}=(1/1.5, 1, 1/1.5), a_{4313}=(2, 4, 6), a_{5313}=(1, 1.5, 2), a_{6313}=(1/5, 1/4, 1/2), a_{7313}= (1/4, 1/3, 1/2), a_{8313}=(1, 1.5, 2), a_{9313}= (2, 3, 4)
c_{32}		1	a_{1323}=(1/3, 1/2, 1/1.5), a_{2323}=(1/2.5, 1/2, 1/1.5), a_{3323}=(1/2, 1/1.5, 1), a_{4323}=(2, 3, 4), a_{5323}=(1/2, 1/1.5, 1), a_{6323}=(1/3, 1/2, 1), a_{7323}=(1/6, 1/5, 1/4), a_{8323}=(1, 1.5, 2), a_{9323}=(1, 2, 3)
c_{33}			1

Subsequently, Step 2 assigns the adequate influence weights to nine marketing members based on their seniority, experiences, and job titles, to reflect the real marketing GDM scenario of Company T, where $w_1 = w_2 = 0.1875$, $w_3 = w_4 = w_5 = 0.125$, and $w_6 = w_7 = w_8 = w_9 = 0.0625$. The rigid disagreement threshold is set as 25%. Assuming that $a_{dqq'}$ denotes the fuzzy preference of the d'th

DM' between dimension q and q', $a_{dqcc'}$ represents the fuzzy preference of d'th DM between criterion c and c' under the q'th dimension, $a_{dqcss'}$ is the comparison value of the d'th DM regarding which strategy s is preferable to strategy s' under the c'th criterion and q'th dimension, $\bar{v}_s$ denotes the group consensus opinion for the s'th strategy, and v_{ds} represents the opinion of the d'th DM regarding the s'th strategy, then this marketing problem is modeled as follows:

Problem Model

$$\text{Minimize} \quad [\sum_{d=1}^{9}\sum_{q'>q}^{3}(\delta^{+}_{dqq'}+\delta^{-}_{dqq'})+\sum_{d=1}^{9}\sum_{q=1}^{3}\sum_{c'>c}^{c_q}(\delta^{+}_{dqcc'}+\delta^{-}_{dqcc'})+\sum_{d=1}^{9}\sum_{q=1}^{3}\sum_{c=1}^{c_q}\sum_{s'>s}^{5}(\delta^{+}_{dqcss'}+\delta^{-}_{dqcss'})](1/(M^2-M^1))+$$

$$[\sum_{d=}^{n}\sum_{s=1}^{k}(\delta^{+}_{ds}+\delta^{-}_{ds})] \qquad (37)$$

Subject to:

$$\ln a_{dqq'}-(\ln v_{dq}-\ln v_{dq'})-\delta^{+}_{dqq'}+\delta^{-}_{dqq'}=0, \qquad (38)$$

$$\ln a_{dqcc'}-(\ln v_{dqc}-\ln v_{dqc'})-\delta^{+}_{dqcc'}+\delta^{-}_{dqcc'}=0, \qquad (39)$$

$$\ln a_{dqcss'}-(\ln v_{dqcs}-\ln v_{dqcs'})-\delta^{+}_{dqcss'}+\delta^{-}_{dqcss'}=0, \qquad (40)$$

$$\bar{v}_s - v_{ds} - \delta^{+}_{ds} + \delta^{-}_{ds} = 0, \text{ for each s under each d}, \qquad (41)$$

$$v_{ds} = \sum_{q=1}^{3}\sum_{c=1}^{c_q} v_{dq}v_{dqc}v_{dqcs}, \text{ for each s under each d}, \qquad (42)$$

$$\mu(a_{dqq'})= s_{dqq'R}\times a_{dqq'}-(s_{dqq'R}-s_{dqq'L})a_{dqq'2}+(s_{dqq'R}-s_{dqq'L})\delta_{dqq'}-s_{dqq'L}a_{dqq'1} \qquad (43)$$

$$a_{dqq'}+a_{dqq'2}+\delta_{dqq'}\geq 0,\ \delta_{dqq'}\geq 0, \qquad (44)$$

$$\mu(a_{dqcc'})=s_{dqcc'R}\times a_{dqcc'}-(s_{dqcc'R}-s_{dqcc'L})a_{dqcc'2}+(s_{dqcc'R}-s_{dqcc'L})\delta_{dqcc'}-s_{dqcc'L}a_{dqcc'1} \qquad (45)$$

$$a_{dqcc'}+a_{dqcc'2}+\delta_{dqcc'}\geq 0,\ \delta_{dqcc'}\geq 0, \qquad (46)$$

$$\mu(a_{dqcss'})= s_{dqcss'R}\times a_{dqcss'}-(s_{dqcss'R}-s_{dqcss'L})a_{dqcc'2}+(s_{dqcss'R}-s_{dqcss'L})\delta_{dqcss'}-s_{dqcss'L}a_{dqcss'1} \qquad (47)$$

$$a_{dqcss'}+a_{dqcss'2}+\delta_{dqcss'}\geq 0,\ \delta_{dqcss'}\geq 0, \qquad (48)$$

$$\bar{a}_{qq'}=(\sum_{d=1}^{9}w_d a_{dqq'}),\ \bar{a}_{qcc'}=(\sum_{d=1}^{9}w_d a_{dqcc'}),\ \bar{a}_{qcss'}=(\sum_{d=1}^{9}w_d a_{dqcss'}), \qquad (49)$$

$$d = 1, 2, \ldots, 9,\ q = 1, 2, 3,\ c = 1, 2, \ldots, c_q,\ c_1=c_3=3,\ c_2=4,\ s = 1, 2, \ldots, 5,$$

where all fuzzy assessments are formulated as Lemma 1, $s_{dqq'L}$ and $s_{dqq'R}$ are separately left-hand and right-hand side slopes of fuzzy value $a_{dqq'}$, $s_{dqcc'L}$ and $s_{dqcc'R}$ are separately left-hand and right-hand side slopes of fuzzy value $a_{dqcc'}$, and $s_{dqcss'L}$ and $s_{dqcss'R}$ are separately left-hand and right-hand side slopes of fuzzy value $a_{dqcss'}$.

Referring to Step 3, the calculated M^1 = 0.85157 and M^2 = 3.7956. The Problem Model then is executed by LINGO in Step 4. Following normalization, the derived each DM and majority consensus ranks for five strategies are summarized in Tale 5.

Table 5 Generated defuzzified scores and ranks of the majority and each DM

Strategy	Aggregated Group Judgments	Individual DM's Judgment (Rank)								
		1	2	3	4	5	6	7	8	9
A	0.1796 (4)	0.165 (4)	0.128 (5)	0.235 (2)	0.161 (4)	0.226 (2)	0.115 (5)	0.184 (3)	0.209 (3)	0.243 (1)
B	0.2316 (1)	0.165 (4)	0.242 (2)	0.299 (1)	0.301 (1)	0.275 (1)	0.222 (2)	0.183 (4)	0.124 (5)	0.205 (3)
C	0.1778 (5)	0.209 (3)	0.132 (4)	0.183 (3)	0.214 (2)	0.154 (4)	0.140 (4)	0.142 (5)	0.212 (2)	0.226 (2)
D	0.2101 (2)	0.250 (1)	0.296 (1)	0.137 (5)	0.171 (3)	0.139 (5)	0.318 (1)	0.219 (2)	0.166 (4)	0.127 (5)
E	0.2009 (3)	0.211 (2)	0.202 (3)	0.146 (4)	0.153 (5)	0.206 (3)	0.205 (3)	0.272 (1)	0.289 (1)	0.199 (4)

To simplify the illustration, Company T is assumed to only seek the best execution strategy. Although the aggregated majority ranks strategies B > D > E > A > C, and Strategy B is ranked first supported by three DMs where these three DMs hold influential weights of 0.375, Table 5 reveals that Strategy D is also ranked first by three DMs, and two of these three are the most influential members in this GDM process, while Strategy E is ranked first by two DMs, and Strategy A is ranked first by one DM. Following the step 5, the calculated the portion of DMs opposing the group preference is 66.67% which significantly exceeds the preset rigid disagreement threshold of 25%. Employing cluster analysis in Step 6, the distance maps among the majority position and all DMs positions for dimensions, criteria, and strategies are presented in Figs. 5, 6, and 7.

Since it is extremely common, an acceptable consensus frequently is not achieved initially. Therefore, marketing DMs of Company T is going to the second cycle of the proposed interactive GDM solution process. Step 7 is to let DMs adjust their opinions and formulate the problem again as Model 7, then go to Step 3. Repeat Steps 3 to 6, the computed proportion of DMs opposing to the group choice is 33.33% which is obviously still higher than the predefined rigid disagreement threshold of 25%. Consequently, Company T goes to the third cycle of the proposed process to resolving conflicting judgments. Likewise, in Step 7, DMs amend their judgments and model the problem again, then proceed through Steps 3 to 6 once more. Lastly, the generated proportion of DMs opposing to the group choice is down to 22.22% which satisfies the pre-specified rigid disagreement requirement. Consequently, following Step 8, after executing sensitivity analysis and confirming the rigid majority opinion for five marketing strategies, Strategy A is selected by Company T, and the rigid and stable majority of marketing department of Company T focuses on supporting its execution. The evolution

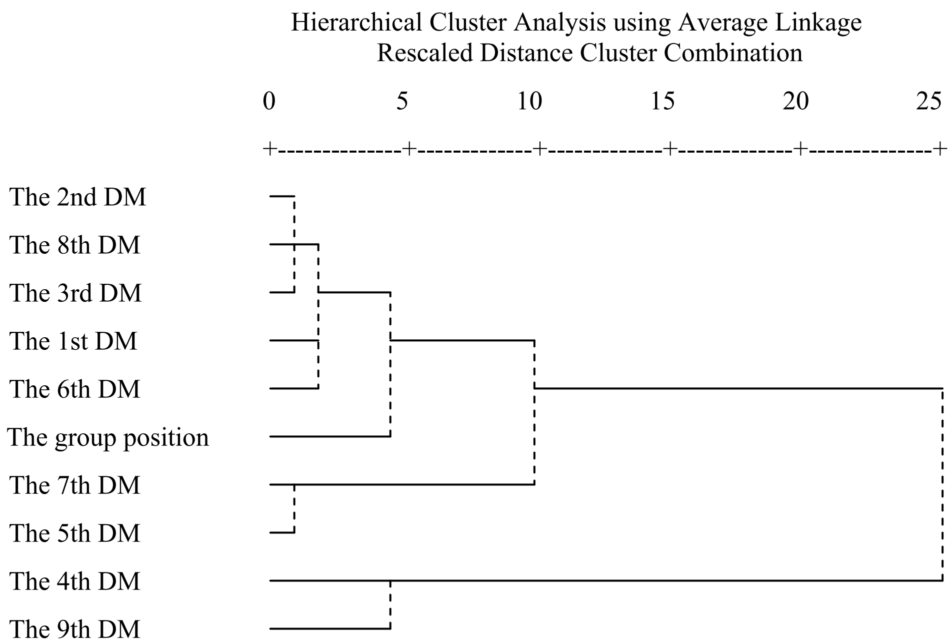

Fig. 5 Horizontal Tree Map for dimension priority
(The dissimilar distance among DMs and the aggregated majority position)

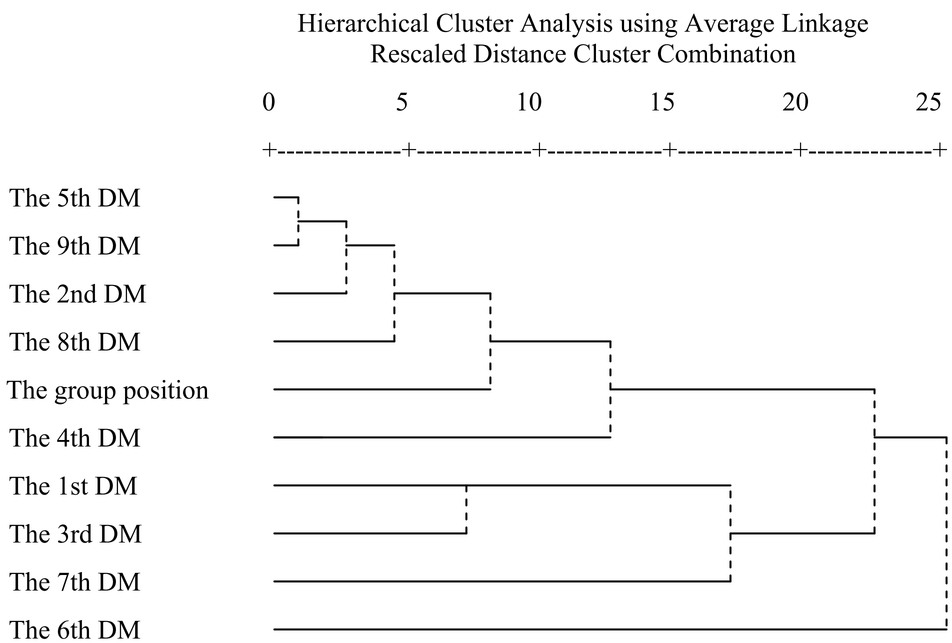

Fig. 6 Horizontal Tree Map for criteria priority
(The dissimilar distance among DMs and the aggregated majority position)

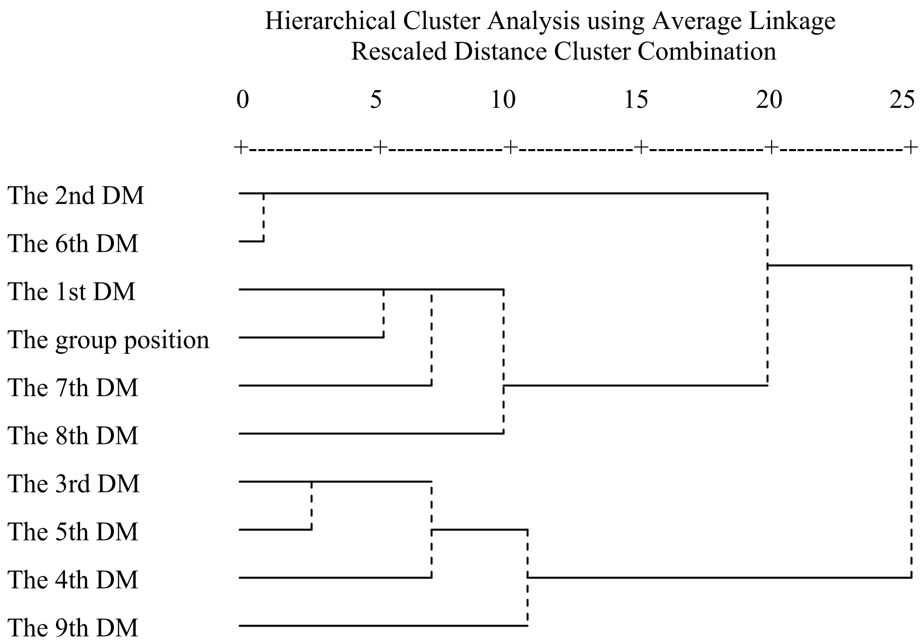

Fig. 7 Horizontal Tree Map for strategy priority

(The dissimilar distance among DMs and the aggregated majority position)

of group consensuses on dimensions, criteria, and strategies over three-cycle interactive GDM solution processes is displayed in Fig. 8, and the derived each DM and the majority "consensus" preferred scores of five strategies are summarized in Tale 6.

Of course, some corresponding marketing events also were used in accordance with Strategy A. Examples of such events include inviting a healthy and famous movie star to be the spokesman of the new brand image, using advertising to tell the public that the brand of milk comes from a very clean and fresh ranch, and so on. Unlike in the past, based on traditional statistic techniques alone the selected solution is often plagued since whatever strategy a company adopts, it will always get support by part of marketing members. Hence, every year the selected strategy is stuck in a pestering and harassing working environment, and the entire marketing strength of Company T cannot be fully fired. Since using the proposed method lets each DM within the marketing department feels respected, is given a fair opportunity to express their individual their views freely, and the interactive solution process is completely transparent, the selected marketing solution thus can obtain full support from all marketing members and be executed smoothly with the minimal internal disruption and maximal group concord.

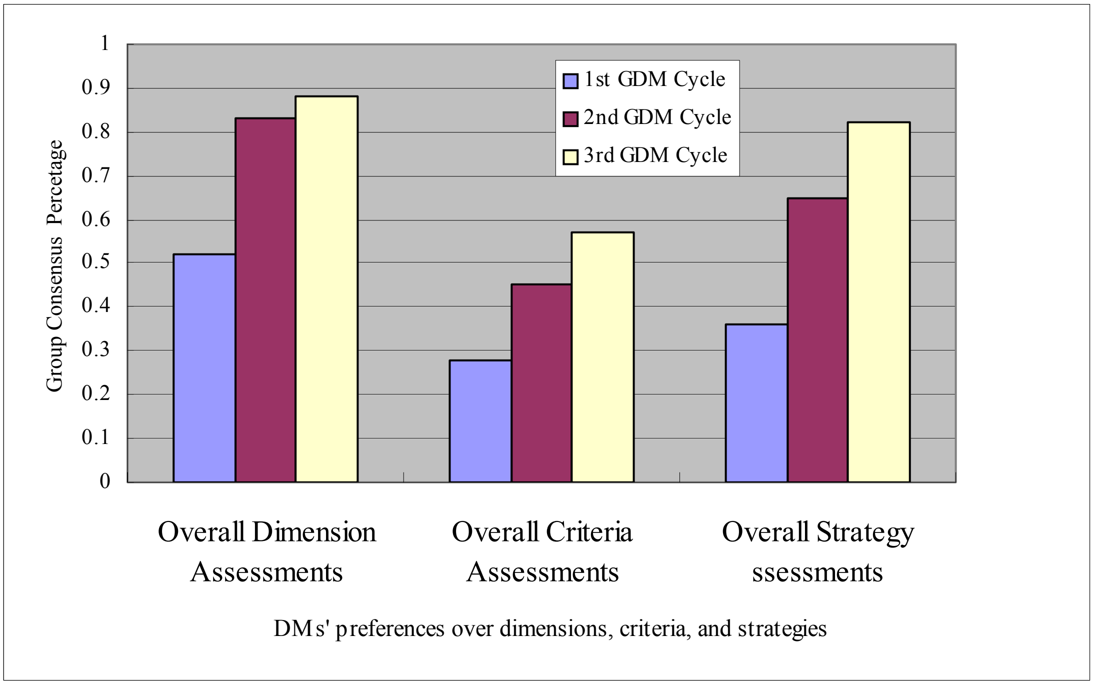

Fig. 8　The evolution of group consensuses via the interactive solution process

Table 6　The final group consensus and each DM preferred scores

Strategy	Aggregated Group Judgments	Individual DM's Judgment (Rank) 1	2	3	4	5	6	7	8	9
A	0.110 (5)	0.122 (4)	0.090 (5)	0.103 (4)	0.116 (4)	0.101 (5)	0.080 (5)	0.108 (5)	0.126 (4)	0.173 (2)
B	0.383 (1)	0.369 (1)	0.418 (1)	0.422 (1)	0.434 (1)	0.399 (1)	0.445 (1)	0.221 (3)	0.220 (3)	0.372 (1)
C	0.120 (4)	0.121 (5)	0.093 (4)	0.129 (3)	0.154 (2)	0.112 (4)	0.098 (4)	0.140 (4)	0.094 (5)	0.157 (3)
D	0.229 (2)	0.235 (2)	0.256 (2)	0.249 (2)	0.185 (3)	0.224 (3)	0.233 (2)	0.251 (2)	0.242 (2)	0.146 (5)
E	0.158 (3)	0.154 (3)	0.143 (3)	0.097 (5)	0.110 (5)	0.164 (2)	0.143 (3)	0.281 (1)	0.318 (1)	0.152 (4)

10. Concluding Remarks

Due to that every marketing strategy is always effective to a limited extent and its effectiveness remains uncertain and is subject to changing markets and the responses of competitors, the strong conflicting and contradictory argues frequently occur in the marketing department of any company. Therefore, how to treat ambiguous, uncertain and conflicting judgments is a challenge task to marketing managers. Due to that existing fuzzy GDM methods usually require complicated and tedious fuzzy mathematical operations, real-world management practices very few utilize those methods to treat practical cases. Since the simplicity, flexibility, and intuitive appeal of the proposed model, as well as its ability to trade-off group consensus, individual preference and judgment consistency in the same GP decision framework, the proposed method is an attractive, promising and worthwhile alternative to current fuzzy GDM methods.

Furthermore, compared with other group consensus-reaching methods, the method proposed here has the following advantages: (a) DMs can evaluate alternatives individually, anywhere, and anytime, reducing the need to gather all DMs for meetings; (b) DMs use AHP to conduct evaluation and then use a GP weighted average method to aggregate group opinions, reducing processing time and tedious discussions; (c) the rigid disagreement threshold can assist in modifying extreme preferences and unstable group consensus; (d) the proposed fuzzy AHP-based GDM model can generate a defuzzified priority vector following a GP computation, implying that tedious calculation or iterative procedures for defuzzifying fuzzy priorities, required by traditional fuzzy GDM methods, are unnecessary; and (e) the methodology presented herein can also be applied to other economic, social, political and management sciences problems.

Appendix A

Proof of Lemma 1. Based on Proposition 1 in the literature (Yu and Li, 2001), a triangular membership function $\mu(a_d)$ as shown in Fig. 1 can be interpreted by

$$\mu(a_d) = s_{d,L}(a_d - a_{d1}) + \frac{s_{d,R} - s_{d,L}}{2}(|a_d - a_{d2}| + a_d - a_{d2}).$$

Consider a corollary expressed below:

PP1: Maximize $Z = s(|a - x| + a - x)$

where s is a negative value and x is a non-negative number.

can be linearized as PP2 below:

PP2: Maximize $ZZ = 2s(a - x + \delta)$

Subject to: $a - x + \delta \geq 0$, $\delta \geq 0$,

where s is a negative value and x is a non-negative number.

This corollary can be examined as follows:

(i) If $a - x \geq 0$, the optimal solution ZZ will force $\delta = 0$, which results in $ZZ = -2(a - x) = Z$.

(ii) If $a - x < 0$, the optimal solution ZZ will force $\delta = x - a$, which results in $ZZ = 0 = Z$.

Based on the corollary above, $\mu(a_d) = s_{d,L}(a_d\text{-}a_{d1}) + \frac{s_{d,R} - s_{d,L}}{2}(|a_d\text{-}a_{d2}|+a_d\text{-}a_{d2})$ can be reformulated as follows:

$$\mu(a_d)=s_{d,L}(a_d-a_{d1})+(s_{d,R}-s_{d,L})(a_d-a_{d2}+\delta)=s_{d,R}\times a_d-(s_{d,R}-s_{d,L})a_{d2}+(s_{d,R}-s_{d,L})\delta-s_{d,L}\times a_{d1}$$

where $a_d\text{-}a_{d2}+\delta \geq 0$ and $\delta \geq 0$.

Lemma 1 is then verified.

References

Aczel, J. and Saaty, T. L., "Procedures for synthesizing ratio judgments," *Journal of Mathematical Psychology*, Vol. 27, 1983, pp. 93-102.

Author, N., "Survey on Taiwan's Milk Market," A report by Taiwan's Food Research and Development Institute, Hsinchu, Taiwan, 2001.

Author N, "Annals of Taiwan's Food Industry," A report by Taiwan's Food Research and Development Institute, Hsinchu, Taiwan, 2001.

Badri, M. A., "A combined AHP-GP model for quality control systems," *International Journal of Production Economic*, Vol. 72, 2001, pp. 27-40.

Bryson, N. "A goal programming method for generating priority vectors," *Journal of Operational Research Society*, Vol. 46, 1995, pp. 461-468.

Bryson, N., "Group decision-making and the analytic hierarchy process: exploring the consensus-relevant information content," *Computer and Operational Research*, Vol. 23, 1996, pp. 27-35.

Bryson, N., Mobolurin, A. and Ngwenyama, O., "Modeling pairwise comparisons on ratio scales," *European Journal of Operational Research*, Vol. 83, 1995, pp. 639-654.

Bult, J. R. and Foekens, E. W., "An analytic approach to marketing decisions," *International Journal of Research in Marketing*, Vol. 10 (4), 1993, pp. 407-410.

Chen, S. J. and Hwang, C. L., *Fuzzy Multiple Attribute Decision Making*, Berlin: Springer, 1992.

Davies, M. A. P., "Using the AHP in marketing decision making," *Journal of Marketing Management*, Vol. 10, 1994, pp. 57-73.

Dyer, R. F. and Forman, E. H., *An Analytic Application to Marketing Decisions*, New Jersey: Prentice-Hall, 1991.

Dyer, R. F., Forman, E. H. and Mustafa, M. A., "Decision support for media selection using the analytic hierarchy process," *Journal of Advertising*, Vol. 21 (1), 1992, pp. 59-62.

Hwang, C. L. and Yoon, K., *Multiple Attribute Decision Making: Methods and Applications*, Berlin: Spring-Verlag, 1981.

Kacprzyk, J., Fedrizzi, M. and Nurmi, H., "Group decision making and consensus under fuzzy preferences and fuzzy majority," *Fuzzy Sets and Systems*, Vol. 49, 1992, pp. 21-31.

Fichtner, J., "On deriving priority vectors from matrices of pairwise comparisons," *Social Economic Planning Sciences*, Vol. 20 (6), 1986, pp. 341-345.

Keane, J. G., "Some observations on marketing research in top management decision making," *Journal of Marketing*, Vol. 33 (4), 1969, pp. 10-16.

Li, H. L., Yu, C. S. and Hung, T. W., "A goal programming method for solving AHP problems," *Management Research*, Vol. 1, No. 1, 2001, pp. 33-54.

Meade, L. M. and Presley, A., "R&D project selection using the analytic network process," *IEEE Transactions on Engineering Management*, Vol. 49 (1), 2002, pp. 59-66.

Numi, H., "Imprecise notions in individual and group decision theory: resolution of allies' paradox and related problems," *Stochastic*, Vol. 6, 1982, pp. 283-303.

Numi, H., "Voting procedures: a summary analysis," *British Journal of Political Science*, Vol. 13, 1983, pp. 181-208.

Numi, H., "Problems of finding optimal voting and representation systems," *European Journal of Operational Research*, Vol. 24 (1), 1986, pp. 91-98.

Saaty, T. L., *Multi Criteria Decision Making: the Analytic Hierarchy Process*, Pittsburgh: University of Pittsburgh Press, 1988.

Yu, C. S. and Li, H. L., "An algorithm for generalized fuzzy binary linear programming problems," *European Journal of Operational Research*, Vol. 133 (3), 2001, pp. 42-57.