

Linear relations of high energy absorption/emission amplitudes of D-brane

Jen-Chi Lee^{a,*}, Yi Yang^{a,b}

^a Department of Electrophysics, National Chiao-Tung University, Hsinchu, Taiwan, ROC

^b Physics Division, National Center for Theoretical Sciences, Hsinchu, Taiwan, ROC

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Abstract

We calculate the absorption amplitudes of a closed string state at arbitrary mass level leading to two open string states on the D-brane at high energies. As in the case of Domain-wall scattering we studied previously, this process contains only one kinematic variable. However, in contrast to the power-law behavior of Domain-wall scattering, its form factor behaves as exponential fall-off in the high energy limit. After identifying the geometric parameter of the kinematic, we derive the linear relations (of the kinematic variable) and ratios among the high energy amplitudes corresponding to absorption of different closed string states for each fixed mass level by D-brane. This result is consistent with the coexistence of the linear relations and exponential fall-off behavior of high energy string/D-brane amplitudes.

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It is well-known that there are two fundamental characteristics of high energy string scattering amplitudes, which make them very different from those of field theory scatterings. The first one is the softer exponential fall-off behavior of the form factors of string scatterings in the high-energy limit [1] in contrast to the power-law behavior of point-particle field theory scatterings. The second one is the existence of Regge-pole structure in the high energy string scattering amplitudes [2] due to the infinite number of resonances in the string spectrum.

Recently high-energy, fixed angle behavior of string scattering amplitudes [3–5] was intensively reinvestigated for massive string states at arbitrary mass levels [2,6–12]. An infinite number of linear relations, or stringy symmetries, among string scattering amplitudes of different string states were obtained. An important new ingredient of these calculations is the zero-norm states (ZNS) [13–15] in the old covariant first quantized (OCFQ) string spectrum. The existence of these infinite linear relations constitutes the third fundamental characteristics of high energy string scatterings, which is not shared by the usual point-particle field theory scatterings. These linear relations persist for string scattered from generic Dp -brane [16] except D-instanton and Domain-wall. For the scattering of D-instanton, the form factor exhibits the well-known power-law behavior without Regge-pole structure, and thus resembles a field theory amplitude. For the special case of Domain-wall (D24-brane for the case of bosonic string) scattering, it was discovered [17] recently that, in contrast to the common wisdom of exponential fall-off behavior [18], its form factor behaves as *power-law* with Regge-pole structure. This discovery makes Domain-wall scatterings an unique example of a hybrid of string and field theory scatterings. Moreover, it was shown [17] that the linear relations break down for the Domain-wall scattering due to this unusual power-law behavior. This result gives a strong evidence that the existence of the infinite linear relations, or stringy symmetries, of high-energy string scattering amplitudes is responsible for the softer, exponential fall-off high-energy string scatterings than the power-law field theory scatterings. It is crucial to note that there is only one kinematic variable for the Domain-wall scatterings in contrast to two for other generic Dp -brane scatterings with $p \geq 0$. This is one of the main reasons that force the high energy behavior of Domain-wall scattering to be the unusual power-law one.

* Corresponding author.

E-mail addresses: jclee@cc.nctu.edu.tw (J.-C. Lee), yyang@phys.cts.nthu.edu.tw (Y. Yang).

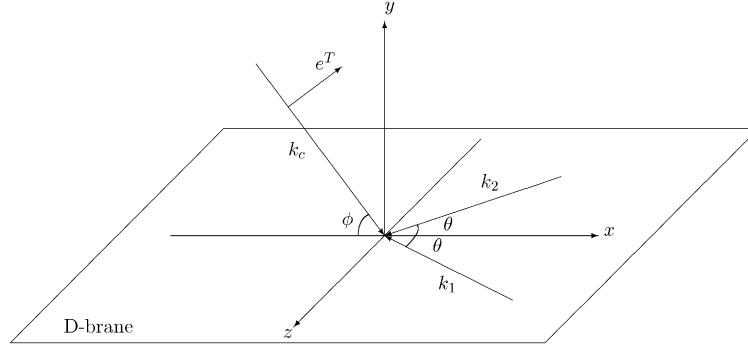


Fig. 1. Kinematic setting up.

In this Letter, we calculate the absorption amplitudes of a closed string state at arbitrary mass level leading to two open string states on the D-brane at high energies. The corresponding simple case of absorption amplitude for massless closed string state was calculated in [19] (the discussion on massless string states scattered from D-brane can be found in [18,20]). The inverse of this process can be used to describe Hawking radiation in the D-brane picture. As in the case of Domain-wall scattering discussed above, this process contains *one* kinematic variable (energy E) and thus occupies an intermediate position between the conventional three-point and four-point amplitudes. However, in contrast to the power-law behavior of high energy Domain-wall scattering which contains only one kinematic variable (energy E), its form factor behaves as exponential fall-off at high energies. It is thus of interest to investigate whether the usual linear relations of high energy amplitudes persist for this case or not. As will be shown in this Letter, after identifying the geometric parameter of the kinematic, one can derive the linear relations (of the kinematic variable) and ratios among the high energy amplitudes corresponding to absorption of different closed string states for each fixed mass level by D-brane. This result is consistent with the coexistence [17] of the linear relations and exponential fall-off behavior of high energy string/D-brane amplitudes.

We first briefly review the high energy scatterings of four open string states. At a fixed mass level $M_{op}^2 = 2(n - 1)$ of 26D open bosonic string theory, it was shown that [9,10] a four-point function is at the leading order at high-energy limit only for states of the following form

$$|n, 2m, q\rangle \equiv (\alpha_{-1}^T)^{n-2m-2q} (\alpha_{-1}^L)^{2m} (\alpha_{-2}^L)^q |0, k\rangle, \quad (1)$$

where $n \geq 2m + 2q, m, q \geq 0$. Note that, in the high energy limit, the scattering process becomes a plane scattering. The state in Eq. (1) is arbitrarily chosen to be the second vertex of the four-point function. The other three points can be any string states. We have defined the normalized polarization vectors of the second string state to be [6,7]

$$e^P = \frac{1}{M_{op}}(E_2, k_2, 0) = \frac{k_2}{M_{op}}, \quad (2)$$

$$e^L = \frac{1}{M_{op}}(k_2, E_2, 0), \quad (3)$$

$$e^T = (0, 0, 1) \quad (4)$$

in the CM frame contained in the plane of scattering. By using the decoupling of two types of ZNS,

$$\text{Type I: } L_{-1}|x\rangle, \text{ where } L_1|x\rangle = L_2|x\rangle = 0, L_0|x\rangle = 0 \quad \text{and} \quad (5)$$

$$\text{Type II: } \left(L_{-2} + \frac{3}{2}L_{-1}^2\right)|\tilde{x}\rangle, \text{ where } L_1|\tilde{x}\rangle = L_2|\tilde{x}\rangle = 0, (L_0 + 1)|\tilde{x}\rangle = 0, \quad (6)$$

in the high energy limit, it was shown that there exists infinite linear relations among string scattering amplitudes [9,10]

$$\mathcal{T}^{(n,2m,q)} = \left(-\frac{1}{M_{op}}\right)^{2m+q} \left(\frac{1}{2}\right)^{m+q} (2m-1)!! \mathcal{T}^{(n,0,0)}. \quad (7)$$

Moreover, these linear relations can be used to fix the ratios among high energy scattering amplitudes of different string states at each fixed mass level algebraically. Eq. (7) explicitly shows that there is only one independent high-energy scattering amplitudes at each fixed mass level.

To study the high energy process of Dp brane ($2 \leq p \leq 24$) absorbs (emits) a massive closed string state leading to two open strings on the Dp brane, we set up the kinematic for the massive closed string state to be

$$e^P = \frac{1}{M}(E, k_c \cos \phi, -k_c \sin \phi, 0) = \frac{k_c}{M},$$

$$\begin{aligned}
e^L &= \frac{1}{M}(k_c, E \cos \phi, -E \sin \phi, 0), \\
e^T &= (0, \sin \phi, \cos \phi, 0), \\
k_c &= (E, k_c \cos \phi, -k_c \sin \phi, 0).
\end{aligned} \tag{8}$$

For simplicity, we chose the open string excitation to be two tachyons with momenta (see Fig. 1)

$$k_1 = \left(-\frac{E}{2}, -\frac{k_{op}}{2} \cos \theta, 0, -\frac{k_{op}}{2} \sin \theta \right), \tag{9}$$

$$k_2 = \left(-\frac{E}{2}, -\frac{k_{op}}{2} \cos \theta, 0, +\frac{k_{op}}{2} \sin \theta \right). \tag{10}$$

Our final results, however, will remain the same for arbitrary two open string excitation at high energies. Conservation of momentum on the D-brane implies

$$\underbrace{\frac{1}{2}(k_c + D \cdot k_c)}_{(k_c)_\parallel} + k_1 + k_2 = 0 \Rightarrow k_c \cos \phi = k_{op} \cos \theta, \tag{11}$$

where $D_{\mu\nu} = \text{diag}\{-1, 1, -1, 1\}$. It is crucial to note that, in the high energy limit, $k_c = k_{op}$ and the scattering angle θ is identical to the incident angle ϕ . One can calculate θ

$$\begin{aligned}
e^T \cdot k_1 &= e^T \cdot k_2 = e^T \cdot D \cdot k_1 = e^T \cdot D \cdot k_2 = -\frac{k_{op} \cos \theta \sin \phi}{2} = -\frac{k_c \sin \phi \cos \phi}{2}, \\
e^L \cdot k_1 &= e^L \cdot k_2 = e^L \cdot D \cdot k_1 = e^L \cdot D \cdot k_2 = \frac{1}{M} \left[\frac{k_c E}{2} - \frac{k_{op} E}{2} \cos \theta \cos \phi \right] = \frac{k_c E}{2M} \sin^2 \phi, \\
e^T \cdot D \cdot k_c &= 2k_c \sin \phi \cos \phi, \\
e^L \cdot D \cdot k_c &= -\frac{2k_c E}{M} \sin^2 \phi,
\end{aligned} \tag{12}$$

which will be useful for later calculations. We can define the kinematic invariants

$$t \equiv -(k_1 + k_2)^2 = M_1^2 + M_2^2 - 2k_1 \cdot k_2 = -2(2 + k_1 \cdot k_2) = 2k_1 \cdot k_c = 2k_2 \cdot k_c, \tag{13}$$

$$s \equiv 4k_1 \cdot k_2 = 2M_1^2 + 2M_2^2 + 2(k_1 + k_2)^2 = -2(4 + t), \tag{14}$$

and calculate the following identities

$$k_1 \cdot k_c + k_2 \cdot D \cdot k_c = k_2 \cdot k_c + k_1 \cdot D \cdot k_c = t, \tag{15}$$

$$k_c \cdot D \cdot k_c = M^2 - 2t. \tag{16}$$

Note that there is only one kinematic variable as s and t are related in Eq. (14) [19]. On the other hand, since the scattering angle θ is fixed by the incident angle ϕ , ϕ and θ are not the dynamical variables in the usual sense.

Following Eq. (1), we consider an incoming high energy massive closed state to be [16,17] $(\alpha_{-1}^T)^{n-m-2q} (\alpha_{-1}^L)^m (\alpha_{-2}^L)^q \otimes (\tilde{\alpha}_{-1}^T)^{n-m'-2q'} (\tilde{\alpha}_{-1}^L)^{m'} (\tilde{\alpha}_{-2}^L)^{q'} |0\rangle$ with $\underline{m} = \underline{m}' = 0$. The amplitude of the absorption process can be calculated to be

$$\begin{aligned}
A &= \int dx_1 dx_2 d^2 z \cdot (x_1 - x_2)^{k_1 \cdot k_2} (z - \bar{z})^{k_c \cdot D \cdot k_c} (x_1 - z)^{k_1 \cdot k_c} \\
&\quad \times (x_1 - \bar{z})^{k_1 \cdot D \cdot k_c} (x_2 - z)^{k_2 \cdot k_c} (x_2 - \bar{z})^{k_2 \cdot D \cdot k_c} \\
&\quad \times \exp\left\{ \left[[ik_1 X(x_1) + ik_2 X(x_2) + ik_c \tilde{X}(\bar{z})] [(n-2q)\varepsilon_T^{(1)} \partial X^T + iq\varepsilon_L^{(1)} \partial^2 X^L](z) \right] \right. \\
&\quad \left. + \left[[ik_1 X(x_1) + ik_2 X(x_2) + ik_c X(z)] [(n-2q')\varepsilon_T^{(2)} \bar{\partial} \tilde{X}^T + iq'\varepsilon_L^{(2)} \bar{\partial}^2 \tilde{X}^L](\bar{z}) \right]_{\text{linear terms}} \right\} \\
&= (-1)^{q+q'} \int dx_1 dx_2 d^2 z \cdot (x_1 - x_2)^{k_1 \cdot k_2} (z - \bar{z})^{k_c \cdot D \cdot k_c} (x_1 - z)^{k_1 \cdot k_c} \\
&\quad \times (x_1 - \bar{z})^{k_1 \cdot D \cdot k_c} (x_2 - z)^{k_2 \cdot k_c} (x_2 - \bar{z})^{k_2 \cdot D \cdot k_c} \\
&\quad \times \left[\frac{ie^T \cdot k_1}{x_1 - z} + \frac{ie^T \cdot k_2}{x_2 - z} + \frac{ie^T \cdot D \cdot k_c}{\bar{z} - z} \right]^{n-2q} \cdot \left[\frac{ie^T \cdot D \cdot k_1}{x_1 - \bar{z}} + \frac{ie^T \cdot D \cdot k_2}{x_2 - \bar{z}} + \frac{ie^T \cdot D \cdot k_c}{z - \bar{z}} \right]^{n-2q'} \\
&\quad \times \left[\frac{e^L \cdot k_1}{(x_1 - z)^2} + \frac{e^L \cdot k_2}{(x_2 - z)^2} + \frac{e^L \cdot D \cdot k_c}{(\bar{z} - z)^2} \right]^q \cdot \left[\frac{e^L \cdot D \cdot k_1}{(x_1 - \bar{z})^2} + \frac{e^L \cdot D \cdot k_2}{(x_2 - \bar{z})^2} + \frac{e^L \cdot D \cdot k_c}{(z - \bar{z})^2} \right]^{q'}.
\end{aligned} \tag{17}$$

$$\begin{aligned}
&\quad \times \left[\frac{e^L \cdot k_1}{(x_1 - z)^2} + \frac{e^L \cdot k_2}{(x_2 - z)^2} + \frac{e^L \cdot D \cdot k_c}{(\bar{z} - z)^2} \right]^q \cdot \left[\frac{e^L \cdot D \cdot k_1}{(x_1 - \bar{z})^2} + \frac{e^L \cdot D \cdot k_2}{(x_2 - \bar{z})^2} + \frac{e^L \cdot D \cdot k_c}{(z - \bar{z})^2} \right]^{q'}.
\end{aligned} \tag{18}$$

Set $\{x_1, x_2, z\} = \{-x, x, i\}$ to fix the $SL(2, R)$ gauge and use Eq. (12), we have

$$\begin{aligned}
 A &= (-1)^{n+M^2/2+t/2} 2^{M^2-2-5t/2} \cdot \int_{-\infty}^{+\infty} dx \cdot x^{-t/2-2} (1-ix)^{t+1} (1+ix)^{t+1} \\
 &\times \left[\frac{-k_c \sin \phi \cos \phi}{1-ix} + \frac{-k_c \sin \phi \cos \phi}{1+ix} + \frac{2k_c \sin \phi \cos \phi}{2} \right]^{n-2q} \\
 &\times \left[\frac{-k_c \sin \phi \cos \phi}{1+ix} + \frac{-k_c \sin \phi \cos \phi}{1-ix} + \frac{2k_c \sin \phi \cos \phi}{2} \right]^{n-2q'} \\
 &\times \left[\frac{k_c E}{2M} \frac{\sin^2 \phi}{(1-ix)^2} + \frac{k_c E}{2M} \frac{\sin^2 \phi}{(1+ix)^2} + \frac{-2k_c E}{M} \frac{\sin^2 \phi}{4} \right]^q \cdot \left[\frac{k_c E}{2M} \frac{\sin^2 \phi}{(1+ix)^2} + \frac{k_c E}{2M} \frac{\sin^2 \phi}{(1-ix)^2} + \frac{-2k_c E}{M} \frac{\sin^2 \phi}{4} \right]^{q'}
 \end{aligned} \tag{19}$$

$$\begin{aligned}
 &= (-1)^{n+M^2/2+t/2} 2^{M^2-2-5t/2} \cdot (k_c \sin \phi \cos \phi)^{2n-2(q+q')} \left(-\frac{k_c E \sin^2 \phi}{2M} \right)^{q+q'} \\
 &\times \int_{-\infty}^{+\infty} dx \cdot x^{-t/2-2} (1+x^2)^{t+1} \left[\frac{x^2}{1+x^2} \right]^{2n-2(q+q')} \left[1 - \frac{2(1-x^2)}{(1+x^2)^2} \right]^{q+q'}.
 \end{aligned} \tag{20}$$

By using the binomial expansion, we get

$$\begin{aligned}
 A &= (-1)^{n+M^2/2+t/2} 2^{M^2-2-5t/2} \cdot (E \sin \phi \cos \phi)^{2n} \left(-\frac{1}{2M \cos^2 \phi} \right)^{q+q'} \\
 &\times \sum_{i=0}^{q+q'} \sum_{j=0}^i \binom{q+q'}{i} \binom{i}{j} (-2)^i (-1)^j \\
 &\times \int_0^\infty d(x^2) \cdot (x^2)^{-t/4-3/2+2n-2(q+q')+j} (1+x^2)^{t+1-2n+2(q+q')-2i}.
 \end{aligned} \tag{21}$$

Finally, to reduce the integral to the standard beta function, we do the linear fractional transformation $x^2 = \frac{1-y}{y}$ to get

$$\begin{aligned}
 A &= (-1)^{n+M^2/2+t/2} 2^{M^2-2-5t/2} \cdot (E \sin \phi \cos \phi)^{2n} \left(-\frac{1}{2M \cos^2 \phi} \right)^{q+q'} \\
 &\times \sum_{i=0}^{q+q'} \sum_{j=0}^i \binom{q+q'}{i} \binom{i}{j} (-2)^i (-1)^j \int_0^1 dy \cdot y^{-3t/4-3/2+2i-j} \cdot (1-y)^{-t/4-3/2+2n-2(q+q')+j} \\
 &= (-1)^{n+M^2/2+t/2} 2^{M^2-2-5t/2} \cdot (E \sin \phi \cos \phi)^{2n} \left(-\frac{1}{2M \cos^2 \phi} \right)^{q+q'} \\
 &\times \frac{\Gamma(-\frac{3t}{4}-\frac{1}{2})\Gamma(-\frac{t}{4}-\frac{1}{2})}{\Gamma(-t-1)} \sum_{i=0}^{q+q'} \sum_{j=0}^i \binom{q+q'}{i} \binom{i}{j} (-2)^i (-1)^j \left(\frac{3}{4}\right)^{2i-j} \left(\frac{1}{4}\right)^{2n-2(q+q')+j} \\
 &= (-1)^{n+M^2/2+t/2} 2^{M^2-2-5t/2} \cdot \left(\frac{E \sin \phi \cos \phi}{4} \right)^{2n} \\
 &\times \left(-\frac{2}{M \cos^2 \phi} \right)^{q+q'} \frac{\Gamma(-\frac{3t}{4}-\frac{1}{2})\Gamma(-\frac{t}{4}-\frac{1}{2})}{\Gamma(-t-1)}.
 \end{aligned} \tag{22}$$

In addition to an exponential fall-off factor, the energy E dependence of Eq. (22) contains a pre-power factor in the high energy limit. To obtain the linear relations for the amplitudes at each fixed mass level, we rewrite Eq. (22) in the following form

$$\frac{\mathcal{T}^{\mathcal{T}^{n-2q} L^q, \mathcal{T}^{n-2q'} L^{q'}}}{\mathcal{T}^{\mathcal{T}^n, \mathcal{T}^n}} = \left(-\frac{2}{M \cos^2 \phi} \right)^{q+q'}. \tag{23}$$

One first notes that Eq. (23) does not contradict with Eq. (7), which predict the ratios $(-\frac{1}{2M})^{q+q'}$. This is because for the absorption process we are considering, there is only one kinematic variable and the usual Ward identity calculations do not apply. To compare

Eq. (23) with the “ratios” of the Domain-wall scattering [17]

$$\frac{\mathcal{T}^{T^{n-2q}L^q, T^{n-2q'}L^{q'}}}{\mathcal{T}^{T^n, T^n}} \Big|_{\text{Domain}} = \left(\frac{E \sin \phi}{M \sqrt{|M_1^2 - 2M^2 - 1|} \cos^2 \phi} \right)^{q+q'}, \quad (24)$$

one sees that, in addition to the incident angle ϕ , there is an energy dependent power factor within the bracket of $q + q'$. Thus there is no linear relations for the Domain-wall scatterings. On the contrary, Eq. (23) gives the linear relations (of the kinematic variable E) and ratios among the high energy amplitudes corresponding to absorption of different closed string states for each fixed mass level n by D-brane. Note that since the scattering angle θ is fixed by the incident angle ϕ , ϕ is not a dynamical variable in the usual sense. Another way to see this is through the relation of s and t in Eq. (14). We will call such an angle a *geometrical parameter* in contrast to the usual dynamical variable. This kind of geometrical parameter shows up in closed string state scattered from generic Dp -brane (except D-instanton and D-particle) [16,17]. This is because one has only two dynamical variables for the scatterings, but needs more than two variables to set up the kinematic due to the relative geometry between the D-brane and the scattering plane at high energies. We emphasize that our result in Eq. (23) is consistent with the coexistence [17] of the linear relations and exponential fall-off behavior of high energy string/D-brane amplitudes. That is, linear relations of the amplitudes are responsible for the softer, exponential fall-off high-energy string/D-brane scatterings than the power-law field theory scatterings.

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