ELSEVIER

Fuzzy Sets and Systems 81 (1996) 311–319

FUZZY sets and systems

Forecasting enrollments based on fuzzy time series

Shyi-Ming Chen

Department of Computer and Information Science, National Chiao Tung University, Hsinchu, Taiwan, ROC

Received March 1994; revised June 1995

Abstract

This paper presents a new method to forecast university enrollments based on fuzzy time series. The data of historical enrollments of the University of Alabama shown in Song and Chissom (1993a, 1994) are adopted to illustrate the forecasting process of the proposed method. The robustness of the proposed method is also tested. The proposed method not only can make good forecasts of the university enrollments, but also can make robust forecasts when the historical data are not accurate. The proposed method is more efficient than the one presented in Song and Chissom (1993a) due to the fact that the proposed method uses simplified arithmetic operations rather than the complicated max–min composition operations presented in Song and Chissom (1993a).

Keywords: Fuzzy time series; Enrollments; Forecasting; Fuzzy set; Linguistic value; Linguistic variable

1. Introduction

In [3, 4], Song and Chissom presented the definition of fuzzy time series and outlined its modeling by means of fuzzy relational equations and approximate reasoning [6, 9]. Fuzzy time series is a new concept which can be used to deal with forecasting problems in which historical data are linguistic values. In [3], Song and Chissom have applied the fuzzy time series model to forecast the enrollments of the University of Alabama, where a first-order time invariant model is developed and a step-by-step procedure is provided. In [3], they also pointed out that Bintley [1] has successfully applied fuzzy logic and approximate reasoning to a practical case of forecasting, but the concept of fuzzy time series was not applied on the method presented in [1]. The method presented by Song and Chissom [3] uses the following model for forecasting university enrollments:

$$A_i = A_{i-1} \circ R, \tag{1}$$

where A_{i-1} is the enrollment of year $i-1$ represented by a fuzzy set [2, 7, 8], A_i is the forecasted enrollment of year i represented by a fuzzy set, "$\circ$" is the max–min composition operator, and R is a fuzzy relation indicating fuzzy relationships between fuzzy time series. However, the forecasted method presented in [3] requires a large amount of computations to derive the fuzzy relation R of (1), and the max–min composition operations of (1) will take a large amount of computation time when the fuzzy relation R of (1) is very big. Thus, we must

SSDI 0165-0114(95)00220-0

develop a new method to forecast the enrollments of the university in a more efficient manner.

In this paper, we propose a new method to forecast university enrollments based on fuzzy time series [3,4]. The data of historical enrollments of the University of Alabama shown in [3,5] are adopted to illustrate the forecasting process. The robustness of the proposed method is also tested. The proposed method is more efficient than the one presented in [3] due to the fact that the proposed method uses simplified arithmetic operations rather than the complicated max–min composition operations presented in [3].

The rest of this paper is organized as follows. In Section 2, the concepts of fuzzy time series are reviewed from [3,4]. In Section 3, we propose an efficient method to forecast the university enrollments based on fuzzy time series. In Section 4, the robustness of the proposed method is tested. The conclusions are discussed in Section 5.

2. Fuzzy time series

In this section, we briefly review some concepts of fuzzy time series from [3,4]. The main difference between the fuzzy time series and conventional time series is that the values of the former are fuzzy sets [7] while the values of the latter are real numbers. Roughly speaking, a fuzzy set is a class with fuzzy boundaries. Let U be the universe of discourse, $U = \{u_1, u_2, \ldots, u_n\}$. A fuzzy set A of U is defined by

$$A = f_A(u_1)/u_1 + f_A(u_2)/u_2 + \cdots + f_A(u_n)/u_n, \qquad (2)$$

where f_A is the membership function of A, $f_A : U \to [0, 1]$, and $f_A(u_i)$ indicates the grade of membership of u_i in A, where $f_A(u_i) \in [0, 1]$ and $1 \leqslant i \leqslant n$. The definitions of fuzzy time series are reviewed as follows.

Definition 2.1. Let $Y(t)$ $(t = \ldots, 0, 1, 2, \ldots)$, a subset of $\mathbb{R}$, be the universe of discourse on which fuzzy sets $f_i(t)$ $(i = 1, 2, \ldots)$ are defined and let $F(t)$ be a collection of $f_i(t)$ $(i = 1, 2, \ldots)$. Then, $F(t)$ is called a fuzzy time series on $Y(t)$ $(t = \ldots, 0, 1, 2, \ldots)$.

From Definition 2.1, we can see that $F(t)$ can be regarded as a linguistic variable [9] and $f_i(t)$ $(i = 1, 2, \ldots)$ can be viewed as possible linguistic values of $F(t)$, where $f_i(t)$ $(i = 1, 2, \ldots)$ are represented by fuzzy sets. We also can see that $F(t)$ is a function of time t, i.e., the values of $F(t)$ can be different at different times due to the fact that the universe of discourse can be different at different times. According to [3], if $F(t)$ is caused by $F(t-1)$ only, then this relationship is represented by $F(t-1) \to F(t)$.

Definition 2.2. Let $F(t)$ be a fuzzy time series. If for any time t, $F(t) = F(t-1)$ and $F(t)$ only has finite elements, then $F(t)$ is called a time-invariant fuzzy time series. Otherwise, it is called a time-variant fuzzy time series.

In [4], Song and Chissom have used the following two examples to explain the concepts of fuzzy time series:

(1) Observe the weather of a certain place in North America, begin from the first day and ending with the last day of a year, where the common daily words (i.e., "good", "very good", "quite good", "very very good", "cool", "very cool", "quite cool", "hot", "very hot", "cold", "very cold", "quite cold", "very very cold", … , etc.) are used to describe the weather conditions and these words are represented by fuzzy sets.

(2) Observe the mood of a person with normal mental conditions during a period of time, where the mood of a person can be expressed according to his own feeling using fuzzy sets "good", "very good", "very very good", "really good", "bad", "not bad", "not too bad", … , etc.

In [4], Song and Chissom also pointed out that the above two examples are dynamic processes and their observations are fuzzy sets; the conventional time series models are no longer applicable to describe these processes. For more details, please refer to [3,4].

3. A new method for forecasting enrollments with fuzzy time series

In [3], Song and Chissom applied the fuzzy times series model to propose a step-by-step procedure for forecasting the enrollments of the University of

Alabama. The method presented in [3] uses the following model for forecasting the enrollments:

$$A_i = A_{i-1} \circ R,$$

where A_{i-1} is the enrollment of year $i - 1$ in terms of a fuzzy set and A_i is the forecasted enrollment of year i in terms of a fuzzy set, R is a fuzzy relation indicating fuzzy relationships between fuzzy time series, and $\circ$ is the max–min composition operator. However, the derivation of the fuzzy relation R is a very tedious work, and the max–min composition operations will take a large amount of time when the fuzzy relation R is very big. For example, let $A_i \to A_{i+1}$ denote the fuzzy logical relationship "If the enrollment of year i is A_i, then that of year $i + 1$ is A_{i+1}", where A_i and A_{i+1} are fuzzy sets, and let us consider the following fuzzy logical relationships shown in [3]:

$$\begin{aligned} &A_1 \to A_1, \quad A_1 \to A_2, \quad A_2 \to A_3, \quad A_3 \to A_3,\\ &A_3 \to A_4, \quad A_4 \to A_4, \quad A_4 \to A_3, \quad A_4 \to A_6, \qquad (3)\\ &A_6 \to A_6, \quad A_6 \to A_7. \end{aligned}$$

The method proposed by Song and Chissom [3] for deriving the fuzzy relation R of (1) is summarized as follows. Firstly, Song and Chissom defined an operator "×" of two vectors. Let D and B be row vectors of dimension m and let $C = (c_{ij}) = D^{\mathrm{T}} \times B$, then the element c_{ij} of matrix C at row i and column j is defined as $c_{ij} = \min(D_i, B_j)$ $(i, j = 1, \ldots, m)$, where D_i and B_j are the ith and the jth elements of D and B, respectively, and D^{T} is the transpose of D. Then, based on the above fuzzy logical relationships and the "×" operator, the following relations are obtained:

$$\begin{aligned} &R_1 = A_1^{\mathrm{T}} \times A_1, \quad R_2 = A_1^{\mathrm{T}} \times A_2,\\ &R_3 = A_2^{\mathrm{T}} \times A_3, \quad R_4 = A_3^{\mathrm{T}} \times A_3,\\ &R_5 = A_3^{\mathrm{T}} \times A_4, \quad R_6 = A_4^{\mathrm{T}} \times A_4, \qquad (4)\\ &R_7 = A_4^{\mathrm{T}} \times A_3, \quad R_8 = A_4^{\mathrm{T}} \times A_6,\\ &R_9 = A_6^{\mathrm{T}} \times A_6, \quad R_{10} = A_6^{\mathrm{T}} \times A_7. \end{aligned}$$

Finally, the fuzzy relation R is obtained by performing the following operations:

$$R = \bigcup_{i=1}^{10} R_i, \qquad (5)$$

where "$\bigcup$" is the union operator. It is obvious that the derivation of the fuzzy relation R is a very tedious work, and the max–min composition operations will take a large amount of time when the fuzzy relation R is very big. Thus, we must develop a new method to forecast university enrollments in a more efficient manner.

In the following, we propose a new method to efficiently forecast university enrollments with fuzzy time series based on [3,4], where the data of historical enrollments of the University of Alabama shown in [3,5] are used to illustrate the forecasting process of the proposed method. The method is essentially a modification of the one presented in [3]. It is more efficient than the one presented in [3] due to the fact that it uses simplified arithmetic operations rather than the complicated max–min composition operations presented in [3].

Let $D_{\min}$ and $D_{\max}$ be the minimum enrollment and the maximum enrollment of known historical data. Based on $D_{\min}$ and $D_{\max}$, we define the universe of discourse U as $[D_{\min} - D_1, D_{\max} + D_2]$, where D_1 and D_2 are two proper positive numbers. According to the data of actual enrollments of the University of Alabama shown in [5], we can see that $D_{\min} = 13\,055$ and $D_{\max} = 19\,337$. Thus, initially, we let $D_1 = 55$ and $D_2 = 663$. Thus, in this paper, the universe of discourse $U = [13\,000, 20\,000]$. The method for forecasting the university enrollments is presented as follows.

Step 1: Partition the universe of discourse $U = [D_{\min} - D_1, D_{\max} + D_2]$ into even lengthy and equal length intervals $u_1, u_2, \ldots, u_m$. In this paper, we partition $U = [13\,000, 20\,000]$ into seven intervals u_1, u_2, u_3, u_4, u_5, u_6, and u_7, where $u_1 = [13\,000, 14\,000]$, $u_2 = [14\,000, 15\,000]$, $u_3 = [15\,000, 16\,000]$, $u_4 = [16\,000, 17\,000]$, $u_5 = [17\,000, 18\,000]$, $u_6 = [18\,000, 19\,000]$, and $u_7 = [19\,000, 20\,000]$.

Step 2: Let $A_1, A_2, \ldots, A_k$ be fuzzy sets which are linguistic values of the linguistic variable "enrollments". Define fuzzy sets $A_1, A_2, \ldots, A_k$ on the universe of discourse U as follows:

$$\begin{aligned} A_1 &= a_{11}/u_1 + a_{12}/u_2 + \cdots + a_{1m}/u_m,\\ A_2 &= a_{21}/u_1 + a_{22}/u_2 + \cdots + a_{2m}/u_m, \qquad (6)\\ &\vdots\\ A_k &= a_{k1}/u_1 + a_{k2}/u_2 + \cdots + a_{km}/u_m, \end{aligned}$$

where $a_{ij} \in [0,1]$, $1 \leqslant i \leqslant k$, and $1 \leqslant j \leqslant m$. The value of a_{ij} indicates the grade of membership of u_j in the fuzzy set A_i, where $a_{ij} \in [0,1]$, $1 \leqslant i \leqslant k$, and $1 \leqslant j \leqslant m$. Find out the degree of each year's enrollments belonging to each A_i ($i = 1, 2, \ldots, m$). If the maximum membership of one year's enrollment is under A_k, then the fuzzified enrollment for this year is treated as A_k. Then, fuzzy logical relationships are derived based on the fuzzified historical enrollments. In this study, the following linguistic values A_1 = (not many), A_2 = (not too many), A_3 = (many), A_4 = (many many), A_5 = (very many), A_6 = (too many), and A_7 = (too many many) are adopted from [3], where

$$\begin{aligned}
A_1 &= 1/u_1 + 0.5/u_2 + 0/u_3 \\
&\quad + 0/u_4 + 0/u_5 + 0/u_6 + 0/u_7, \\
A_2 &= 0.5/u_1 + 1/u_2 + 0.5/u_3 \\
&\quad + 0/u_4 + 0/u_5 + 0/u_6 + 0/u_7, \\
A_3 &= 0/u_1 + 0.5/u_2 + 1/u_3 \\
&\quad + 0.5/u_4 + 0/u_5 + 0/u_6 + 0/u_7, \\
A_4 &= 0/u_1 + 0/u_2 + 0.5/u_3 \\
&\quad + 1/u_4 + 0.5/u_5 + 0/u_6 + 0/u_7, \qquad (7) \\
A_5 &= 0/u_1 + 0/u_2 + 0/u_3 \\
&\quad + 0.5/u_4 + 1/u_5 + 0.5/u_6 + 0/u_7, \\
A_6 &= 0/u_1 + 0/u_2 + 0/u_3 \\
&\quad + 0/u_4 + 0.5/u_5 + 1/u_6 + 0.5/u_7, \\
A_7 &= 0/u_1 + 0/u_2 + 0/u_3 \\
&\quad + 0/u_4 + 0/u_5 + 0.5/u_6 + 1/u_7.
\end{aligned}$$

The fuzzified historical enrollments of the enrollments of the University of Alabama are shown in Table 1 which are adopted from [5].

Then, the fuzzy logical relationships of the enrollments can be obtained from Table 1 which are shown in Table 2, where the fuzzy logical relationship $A_j \rightarrow A_k$ means "If the enrollment of year i is A_j, then that of year $i+1$ is A_k", where A_j is called the current state of the enrollment, and A_k is called the next state of the enrollment (note: the repeated relationships are counted only once).

Table 1
Fuzzified historical enrollments (data source: [5])

Year	Actual enrollment	Fuzzified enrollment
1971	13055	A_1
1972	13563	A_1
1973	13867	A_1
1974	14696	A_2
1975	15460	A_3
1976	15311	A_3
1977	15603	A_3
1978	15861	A_3
1979	16807	A_4
1980	16919	A_4
1981	16388	A_4
1982	15433	A_3
1983	15497	A_3
1984	15145	A_3
1985	15163	A_3
1986	15984	A_3
1987	16859	A_4
1988	18150	A_6
1989	18970	A_6
1990	19328	A_7
1991	19337	A_7
1992	18876	A_6

Table 2
Fuzzy logical relationships of the enrollments

$A_1 \rightarrow A_1$	$A_1 \rightarrow A_2$	$A_2 \rightarrow A_3$	$A_3 \rightarrow A_3$
$A_3 \rightarrow A_4$	$A_4 \rightarrow A_4$	$A_4 \rightarrow A_3$	$A_4 \rightarrow A_6$
$A_6 \rightarrow A_6$	$A_6 \rightarrow A_7$	$A_7 \rightarrow A_7$	$A_7 \rightarrow A_6$

Table 3
Fuzzy logical relationship groups

Group 1:	$A_1 \rightarrow A_1$	$A_1 \rightarrow A_2$	
Group 2:	$A_2 \rightarrow A_3$		
Group 3:	$A_3 \rightarrow A_3$	$A_3 \rightarrow A_4$	
Group 4:	$A_4 \rightarrow A_4$	$A_4 \rightarrow A_3$	$A_4 \rightarrow A_6$
Group 5:	$A_6 \rightarrow A_6$	$A_6 \rightarrow A_7$	
Group 6	$A_7 \rightarrow A_7$	$A_7 \rightarrow A_6$	

Step 3: Divide the derived fuzzy logical relationships into groups based on the current states of the enrollments of fuzzy logical relationships. Thus, based on Table 2, we can obtain six fuzzy logical relationship groups shown in Table 3.

Step 4: Calculate the forecasted outputs. The calculations are carried out by the following principles:

(1) If the fuzzified enrollment of year i is A_j, and there is only one fuzzy logical relationship in the fuzzy logical relationship groups derived in Step 3 in which the current state of the enrollment is A_j, which is shown as follows:

$$A_j \to A_k,$$

where A_j and A_k are fuzzy sets and the maximum membership value of A_k occurs at interval u_k, and the midpoint of u_k is m_k, then the forecasted enrollment of year $i+1$ is m_k.

(2) If the fuzzified enrollment of year i is A_j, and there are the following fuzzy logical relationships in the fuzzy logical relationship groups derived in Step 3 in which the current states of the fuzzy logical relationships are A_j, respectively, which is shown as follows:

$$A_j \to A_{k1},$$
$$A_j \to A_{k2},$$
$$\vdots$$
$$A_j \to A_{kp}.$$

where $A_j, A_{k1}, A_{k2}, \ldots, A_{kp}$ are fuzzy sets, and the maximum membership values of $A_{k1}, A_{k2}, \ldots, A_{kp}$ occur at intervals $u_1, u_2, \ldots, u_p$, respectively, and the midpoints of $u_1, u_2, \ldots, u_p$ are $m_1, m_2, \ldots, m_p$, respectively, then the forecasted enrollment of year $i+1$ is $(m_1 + m_2 + \cdots + m_p)/p$.

(3) If the fuzzified enrollment of year i is A_j, and there do not exist any fuzzy logical relationship groups whose current state of the enrollment is A_j, where the maximum membership value of A_j occurs at interval u_j, and the midpoint of u_j is m_j, then the forecasted enrollment of year $i+1$ is m_j.

Thus, based on Tables 1 and 3, we can forecast the enrollments of the University of Alabama from 1972 to 1992 by the proposed method. In the following, we only illustrate the forecasting process of the years 1972, 1975, 1976, 1980, 1989, and 1991. The same procedure can be applied on the years 1973, 1974, 1977, 1978, 1979, 1981, ..., 1988, 1990, and 1992.

[1972]: Because the fuzzified enrollment of 1971 shown in Table 1 is A_1, and from Table 3, we can see that there are the following fuzzy logical relationships in group 1 of Table 3 in which the current states of the fuzzy logical relationships are A_1, respectively, which is shown as follows:

$$A_1 \to A_1, \qquad A_1 \to A_2,$$

where the maximum membership values of the fuzzy sets A_1 and A_2 occur at intervals u_1 and u_2, respectively, where $u_1 = [13\,000, 14\,000]$ and $u_2 = [14\,000, 15\,000]$, and the midpoints of the intervals u_1 and u_2 are 13 500 and 14 500, respectively. Thus, the forecasted enrollment of 1972 is equal to $\frac{1}{2}(13\,500 + 14\,500) = 14\,000$.

[1975]: Because the fuzzified enrollment of 1974 shown in Table 1 is A_2, and from Table 3, we can see that there is the following fuzzy logical relationship in group 2 of Table 3 in which the current state of the fuzzy logical relationship is A_2, which is shown as follows:

$$A_2 \to A_3,$$

and the maximum membership value of the fuzzy set A_3 occurs at interval u_3, where $u_3 = [15\,000, 16\,000]$, and the midpoint of u_3 is 15 500. Thus, the forecasted enrollment of 1975 is equal to 15 500.

[1976]: Because the fuzzified enrollment of 1975 shown in Table 1 is A_3, and from Table 3, we can see that there are the following fuzzy logical relationships shown in group 3 of Table 3 in which the current states of the fuzzy logical relationships are A_3, respectively, which is shown as follows:

$$A_3 \to A_3, \qquad A_3 \to A_4,$$

where the maximum membership values of the fuzzy sets A_3 and A_4 occur at intervals u_3 and u_4, respectively, where $u_3 = [15\,000, 16\,000]$ and $u_4 = [16\,000, 17\,000]$, and the midpoints of the intervals u_3 and u_4 are 15 500 and 16 500, respectively. Thus, the forecasted enrollment of 1976 is equal to $\frac{1}{2}(15\,500 + 16\,500) = 16\,000$.

[1980]: Because the fuzzified enrollment of 1979 shown in Table 1 is A_4, and from Table 3, we can see that there are the following fuzzy logical relationships shown in group 4 of Table 3 in which the current states of the fuzzy logical relationships are

Table 4
A comparison of the forecasted methods

Year	Actual enrollment (data sources [5]):	Forecasted enrollment by Song and Chissom [3]	Forecasted enrollment by the proposed method
1971	13055		
1972	13563	14000	14000
1973	13867	14000	14000
1974	14696	14000	14000
1975	15460	15500	15500
1976	15311	16000	16000
1977	15603	16000	16000
1978	15861	16000	16000
1979	16807	16000	16000
1980	16919	16813	16833
1981	16388	16813	16833
1982	15433	16709	16833
1983	15497	16000	16000
1984	15145	16000	16000
1985	15163	16000	16000
1986	15984	16000	16000
1987	16859	16000	16000
1988	18150	16813	16833
1989	18970	19000	19000
1990	19328	19000	19000
1991	19337	19000	19000
1992	18876	Not forecasted	19000

A_4, respectively, which is shown as follows:

$$A_4 \rightarrow A_4, \quad A_4 \rightarrow A_3, \quad A_4 \rightarrow A_6,$$

where the maximum membership values of the fuzzy sets A_4, A_3, and A_6 occur at intervals u_4, u_3, and u_6, respectively, where $u_4 = [16\,000, 17\,000]$, $u_3 = [15\,000, 16\,000]$ and $u_6 = [18\,000, 19\,000]$, and the midpoints of u_4, u_3, and u_6 are 16 500, 15 500, and 18 500, respectively. Thus, the forecasted enrollment of 1980 is equal to $\frac{1}{3}(16\,500 + 15\,500 + 18\,500) = 16\,833$.

[1989]: Because the fuzzified enrollment of 1988 shown in Table 1 is A_6, and from Table 3, we can see that there are the following fuzzy logical relationships shown in group 5 of Table 3 in which the current states of the fuzzy logical relationships are A_6, respectively, which is shown as follows:

$$A_6 \rightarrow A_6, \qquad A_6 \rightarrow A_7,$$

where the maximum membership values of the fuzzy sets A_6 and A_7 occur at intervals u_6 and u_7, respectively, where $u_6 = [18\,000, 19\,000]$ and $u_7 = [19\,000, 20\,000]$, and the midpoints of u_6 and u_7 are 18 500 and 19 500, respectively. Thus, the forecasted enrollment of 1989 is equal to $\frac{1}{2}(18\,500 + 19\,500) = 19\,000$.

[1991]: Because the fuzzified enrollment of 1990 shown in Table 1 is A_7, and from Table 3, we can see that there are the following fuzzy logical relationships shown in group 6 of Table 3 in which the current states of the fuzzy logical relationships are A_7, respectively, which is shown as follows:

$$A_7 \rightarrow A_7, \qquad A_7 \rightarrow A_6,$$

where the maximum membership values of the fuzzy sets A_7 and A_6 occur at intervals u_7 and u_6, respectively, where $u_7 = [19\,000, 20\,000]$ and $u_6 = [18\,000, 19\,000]$, and the midpoints of u_7 and u_6 are 19 500 and 18 500, respectively. Thus, the forecasted enrollment of 1991 is equal to $\frac{1}{2}(19\,500 + 18\,500) = 19\,000$.

In summary, a comparison of the actual enrollments of the University of Alabama, the forecasted enrollments by Song–Chissom method [3], and the forecasted enrollment by the proposed method is shown in Table 4.

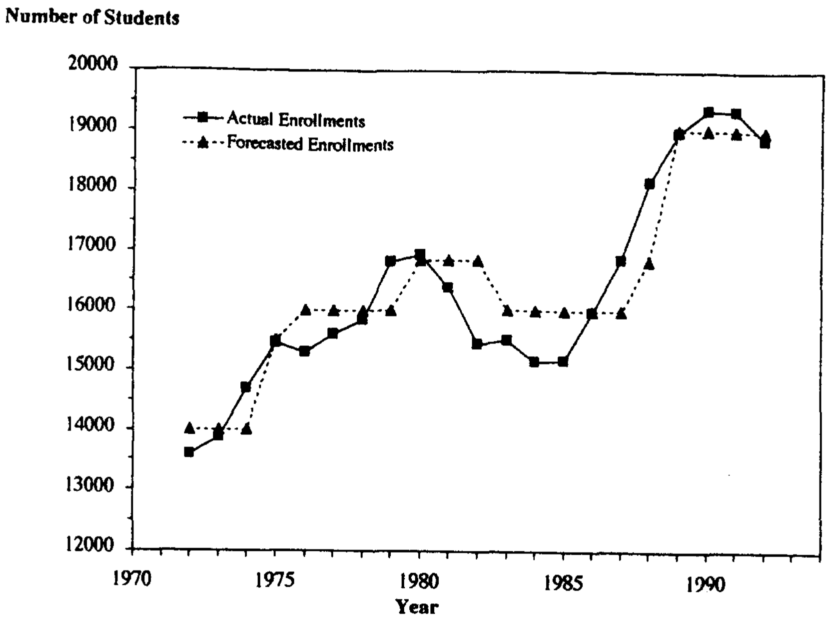

Fig. 1. Forecasted enrollments and actual enrollments.

From Table 4, we can see that the forecasted results of the proposed method is very close to that of the Song–Chissom method presented in [3]. The curves of the actual enrollments and the forecasted enrollments of the proposed method are shown in Fig. 1, where the solid line is the actual enrollment and the dashed line is the forecasted enrollment. It is obvious that the proposed method is more efficient than the one presented in [3] due to the fact that the proposed method uses simplified arithmetic operations rather than the complicated max–min composition operations presented in [3].

4. Robustness of the proposed method

In this section, we test the robustness of the proposed method to show the proposed method can still yield good forecasting results when the historical data are not accurate. In order to test the robustness of the proposed method, we adopted the example shown in [3, 5] to increase a few year's enrollment data by 5% with the rest of the data unchanged, i.e., the university enrollments of 1974, 1978, 1985, and 1990 are increased by 5%. In this case, the fuzzy logical relationships obtained from the changed historical enrollment data are shown in Table 5.

Table 5
Fuzzy logical relationships derived from the changed historical enrollment data

$A_1 \to A_1$	$A_1 \to A_3$	$A_3 \to A_3$
$A_3 \to A_4$	$A_4 \to A_4$	$A_4 \to A_3$
$A_4 \to A_6$	$A_6 \to A_6$	$A_6 \to A_7$
$A_7 \to A_7$	$A_7 \to A_6$	

By applying the proposed method, the forecasted enrollments from 1972 to 1992 can be evaluated. The curves of the actual enrollments and the forecasted enrollments are shown in Fig. 2, where the solid line is the actual enrollment and the dashed line is the forecasted enrollment. The forecasting errors range from 0.10% to 9.07%, and the average error is 3.23%. The average error of the proposed method is smaller than the one presented in [3]. From Fig. 2, we can see that as time increases, the forecasting error decreases. This shows that even if the historical data are not accurate, the proposed method can still make good forecasts.

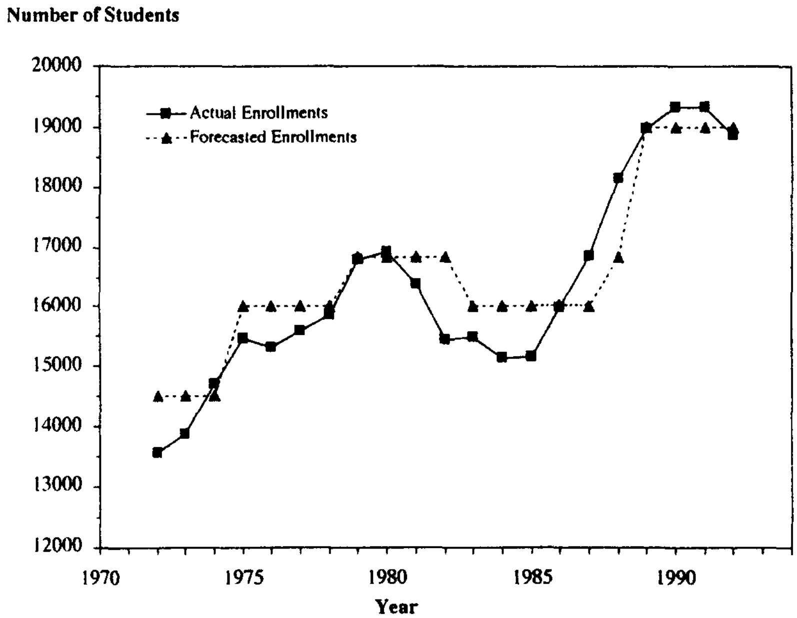

Fig. 2. Curves of forecasted enrollments and actual enrollments.

5. Conclusions

In this paper, we present a new method to forecast university enrollments based on fuzzy time series, where the data of historical enrollments of the University of Alabama are adopted from [3, 5] to illustrate the forecasting processes, and the robustness of the proposed method is tested. From the illustrative example, we can see that the proposed method not only can make good forecasts of the university enrollments, but also can make robust forecasts when the historical enrollment data are not accurate. The proposed method is more efficient than the one presented in [3] due to the fact that the proposed method uses simplified arithmetic operations rather than the complicated max–min composition operations presented in [3].

Acknowledgements

The author would like to thank the referees for providing very helpful comments and suggestions. Their insight and comments led to a better presentation of the ideas expressed in this paper. This work was supported in part by the National Science Council, Republic of China, under Grant NSC 85-2213-E-009-123.

References

[1] H. Bintley, Time series analysis with REVEAL, *Fuzzy Sets and Systems* **23** (1987) 97–118.

[2] A. Kaufmann and M.M. Gupta, *Fuzzy Mathematical Models in Engineering and Management Sciences* (North-Holland, Amsterdam, 1988).

[3] Q. Song and B.S. Chissom, Forecasting enrollments with fuzzy time series – part I, *Fuzzy Sets and Systems* **54** (1993) 1–9.

[4] Q. Song and B.S. Chissom, Fuzzy time series and its models, *Fuzzy Sets and Systems* **54** (1993) 269–277.

[5] Q. Song and B.S. Chissom, Forecasting enrollments with fuzzy time series – part II, *Fuzzy Sets and Systems* **62** (1994) 1–8.

[6] W. Wu, Fuzzy reasoning and fuzzy relational equations, *Fuzzy Sets and Systems* **20** (1986) 67–78.

[7] L.A. Zadeh, Fuzzy sets, *Inform. and Control* **8** (1965) 338–353.

[8] L.A. Zadeh, Outline of a new approach to the analysis of complex systems and decision processes, *IEEE Trans. Systems Man Cybernet.* **3** (1973) 28–44.

[9] L.A. Zadeh, The concept of a linguistic variable and its application to approximate reasoning, parts 1–3, *Inform. Sci.* **8** (1975) 199–249; **8** (1975) 301–357; **9** (1975) 43–80.

[10] Li Zuoyoung, Chen Zhenpei and Li Jitao, A model of weather forecast by fuzzy grade statistics, *Fuzzy Sets and Systems* **26** (1988) 275–281.