Purification of single qubits by collinear photons

Satoru Odate and Yuishi Takeno
Core Research for Evolutional Science and Technology (CREST), Japan Science and Technology Corporation (JST), Department of Physics, Graduate School of Science, The University of Tokyo, Hongo 7-3-1, Bunkyo, Tokyo 113-0033, Japan

Takayoshi Kobayashi
Core Research for Evolutional Science and Technology (CREST), Japan Science and Technology Corporation (JST), Department of Physics, Graduate School of Science, The University of Tokyo, Hongo 7-3-1, Bunkyo, Tokyo 113-0033, Japan; Department of Applied Physics and Chemistry, Institute of Laser Research, The University of Electro-Communications, Chofugaoka 1-5-1, Chofu, Tokyo 182-8585, Japan; Institute of Laser Engineering, Osaka University, Yamada-oka 2-6, Suita, Osaka 565-0971, Japan; and Department of Electrophysics, National Chiao-Tung University, Hsinchu 3005, Taiwan

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We have demonstrated purification of a single qubit by two collinear photons, one of which was time-delayed. Our method can be applied to a single qubit many times, and the qubit can be purified to an arbitrarily high degree of purity. Thus, by repeating this method we can make the statistical error that single qubits incur along a transmission channel much smaller.

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When a qubit in a pure state is affected by a noisy transmission channel, the state gets mixed. Once a state of the qubit becomes a mixed state, its information will not be transmitted correctly. Such incorrect transmission of information has been known as one of the issues which limits the abilities of the quantum information technology. To solve this issue, the quantum error correcting (QEC) technique was studied [1,2]. Aside from this technique, the purification protocol has been studied, which distills purity of the qubit [3]. Cirac *et al.* have theoretically shown that a single qubit can be purified by using N single qubits in equal states. They suggested that the larger N is, the higher the purity of the single qubit attained. Ricci *et al.* have experimentally demonstrated a case of $N=2$ using the purifying method of Cirac [4]. In their method, incoming single qubits were injected noncollinearly, and outgoing single qubits were launched collinearly. Because a noncollinear photon pair was used for the input, it was difficult to extend their method to the case $N>2$. In this paper, we report on a more convenient method to purify a single qubit, which uses collinear photons for the input. Our method can be extended to the case $N>2$, by repeating the corresponding method for the case $N=2$. Namely, the purity of the single qubit is distilled toward the value of 1 by iterating the $N=2$ process. Our technique is applicable to photons transmitted through a optical fiber because the input photons in our method are collinear photons while those in Ricci's method were noncollinear photons.

Let us think about the density operator of a single qubit. The density operator of a single qubit in an arbitrary state can be expressed as follows:

$$\varepsilon_\xi(\rho_{\vec{n}}) = \xi\rho_{\vec{n}} + \frac{1-\xi}{2}I \quad (0<\xi<1), \tag{1}$$

where $\rho_{\vec{n}}=\frac{1}{2}(I+\vec{n}\cdot\vec{\sigma})$ is a density operator defined for a pure state ($|\vec{n}|=1$). The purity of this density operator is calculated by $\mathrm{Tr}[\varepsilon^2]=\frac{1}{2}(1+\xi^2)$. Thus, the pure state of a single qubit becomes a mixed state by transmitting through a depolarizing channel [5]. A purification scheme of the case $N=2$ is shown in Fig. 1. Two single qubits in states $\varepsilon_\xi(\rho)$ and $\varepsilon_\eta(\rho)$ go through a controlled-NOT (CNOT) gate, then the target qubit goes through a Hadamard gate. After that, the Pauli-Z operation is performed on a source qubit only when the measurement result of a target qubit is $|1\rangle$. Thus the normalized density operator of the source qubit at the output port is given as

$$\varepsilon_p(\rho_{\vec{n}}) = p\rho_{\vec{n}} + \frac{1-p}{2}I, \tag{2}$$

where the parameter p is defined as

$$p = \frac{\xi+\eta}{1+\xi\eta}. \tag{3}$$

In this case $\eta=\xi$, since the parameter p is larger than ξ, namely, the purity of the output qubit is larger than that of the input qubit. Note that this quantum circuit is identical to the half side of the quantum circuit used in the entanglement distillation protocol [6,7]. From the truth-value table of the CNOT gate, we can find that the input states at the target and source qubits were equivalent, when $|0\rangle$ is measured at the target output port. A nonpolarizing half-beam splitter (HBS)

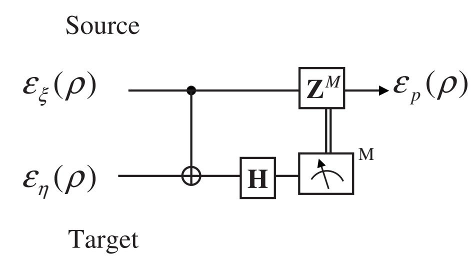

FIG. 1. Schematic representation of a quantum circuit for the purification.

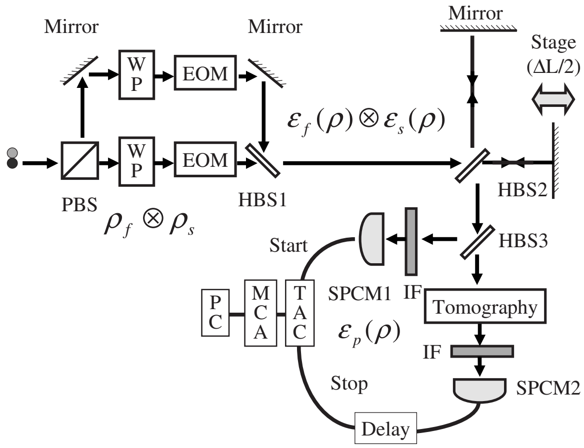

FIG. 2. Schematic diagram of the experimental setup. PBS: polarizing beam splitter, WP: wave plate, EOM: electro-optic modulator. HBS1-3: nonpolarizing beam splitter, IF: interference filter, SPCM1-2: single-photon counting module, TAC/SCA: time-to-amplitude converter and single channel analyzer, MCA: multichannel pulse-height analyzer.

acts similar to this operation. If two photons are simultaneously injected into each input port of the HBS and both are detected at one of the output ports, it is highly probable that the detected photons are in the same polarization state due to the bunching effect [8]. Thus, the HBS has a partial function of the CNOT gate. In the case of purification using the HBS as the CNOT gate, the density operator at one of output ports of the HBS is calculated as follows:

$$\varepsilon_p(\rho_{\vec{n}}) = \frac{4\xi}{3+\xi^2}\rho_{\vec{n}} + \frac{1-4\xi/(3+\xi^2)}{2}I. \tag{4}$$

The success probability of this scheme is $(3+\xi^2)/16$. Note that the photon pair is assumed to be injected into two input ports of the HBS noncollinearly, and to come out from one of the output ports of the HBS. Therefore it is difficult to apply the purification protocol repeatedly to the collinearly launched photon pair from the output. In the following, we will experimentally show that purification can be accomplished in two collinear photons by adding a time difference between the photons.

The schematic diagram of the experiment is illustrated in Fig. 2. Orthogonally polarized photon pairs were generated by a type-II spontaneous parametric down conversion (SPDC) in a 1-mm-thick beta-barium borate (BBO) crystal pumped by the second harmonic of a cw Ti:sapphire laser operated at 860 nm with a power of 130 mW. Two dichroic mirrors were used to remove the pump light from the SPDC. An interference filter of the center wavelength at 860 nm and the FWHM bandwidth 10 nm was set at the front of each single photon counting module (SPCM). A vertically polarized (V-polarized) photon was merged to a horizontally polarized (H-polarized) photon by a HBS1 with 1 ns delay. In order to compensate for the spatial modes between the two arms of the unbalanced Mach-Zehnder (MZ) interferometer, two aspherical lenses were placed along the longer arm. Using two wave plates (WPs) along each arm of the interferometer, two single qubits in the same polarization state with 1 ns time difference were prepared. Then, two identical single qubits had the same time difference. For simplicity, the single qubit prepared in this experiment was in the H-polarized state.

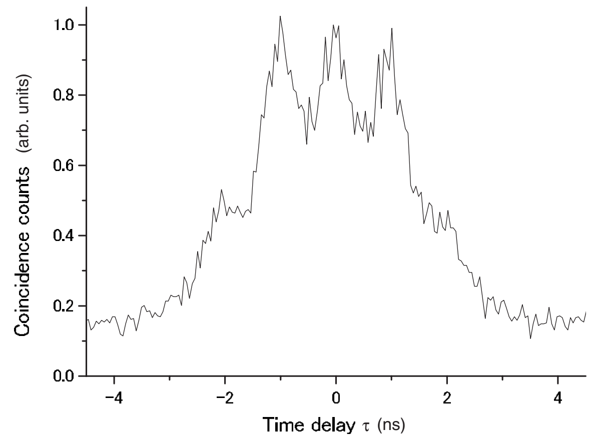

FIG. 3. Coincidence counts at the HBS2 output against delay time τ of the TAC.

Using two electro-optic modulators (EOMs) set on each arm of the interferometer, the depolarizing channel was artificially made. When the H-polarized photon was injected into the EOM whose fast axis was rotated by 45° to the horizontal axis and to which the $\lambda/2$ voltage was applied, the polarization state was converted to the orthogonal state at the output of the EOM. In our case, applied voltage waveforms were rectangular pulses with a height of $\lambda/2$ voltage, a duration time τ, and a repetition time T. By this modulation, the state of single-qubit became mixed,

$$\rho_{\vec{n}} \mapsto \nu\rho_{\vec{n}} + (1-\nu)\rho_{\vec{n}}^{\perp}, \tag{5}$$

where the duty ration ν is τ/T and the orthogonal state of the single qubit in a pure state is defined as $\rho_{\vec{n}}^{\perp}=\rho_{-\vec{n}}$. The relation between ξ and ν is $\xi=(1-\nu)/2$, using the identity $I=\rho_{\vec{n}}+\rho_{\vec{n}}^{\perp}$. Therefore, adjusting the duty ratio ν, an arbitrary mixed state could be prepared. The setup explained above can be regarded as a situation which provided two identical single qubits with time differences transmitted collinearly through a noisy channel.

This prepared photon pair was injected into an unbalanced Michelson interferometer. The photon pair with 1 ns time difference got another 1 ns time difference at this interferometer. We compensated the spatial mode of the interferometer by setting one aspherical lens on the longer arm. A photon pair was launched into the input port of a HBS3. Figure 3 shows the histogram of coincidence counts against the delay time τ acquired by a time-to-amplitude converter (TAC) single channel analyzer (SCA) and a multichannel pulse-height analyzer (MCA) without $\lambda/2$ voltage applied to the EOMs. Peaks of the coincidence were counted at ±1 ns which corresponds to the time delay by the MZ interferometer, and at ±2 ns and 0 ns by the Michelson interferometer. Restricting the time window of the TAC from −0.5 ns to 0.5 ns enabled us to detect only the two photons

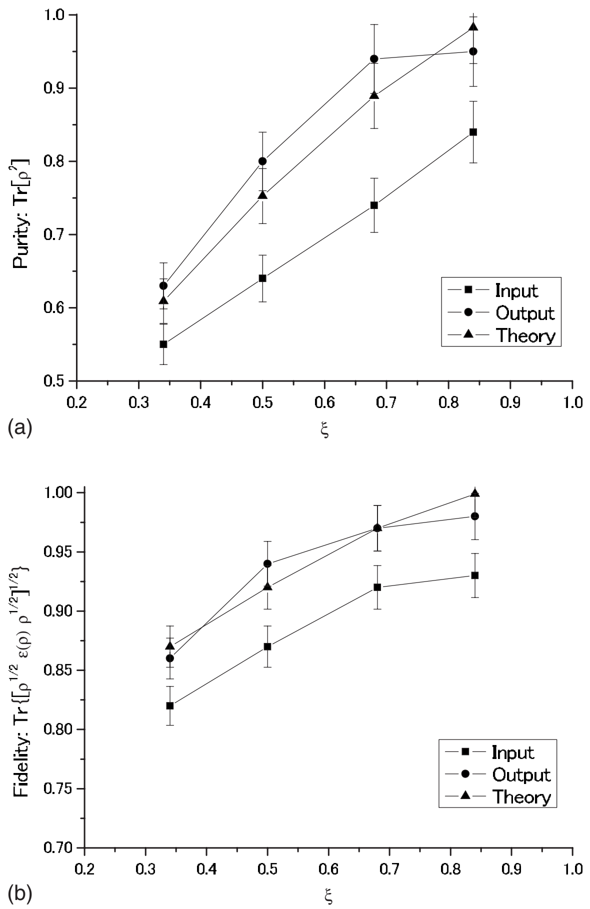

FIG. 4. Experimental results of the purification. (a) The purities at the input (filled rectangle) and the output (filled circle) of the HBS2 are plotted. The theoretically calculated purities at the output (filled triangle) are also shown. (b) The fidelities at the input (filled rectangle) and the output (filled circle) of the HBS2 are plotted. The theoretically calculated fidelities at the output (filled triangle) are also shown.

which were transmitted by different arms of the Michelson interferometer and combined at HBS2 simultaneously [10]. Placing a HBS3 between the HBS1 and a HBS2, and measuring the coincidence counts between SPCM1 and a SPCM set at the other output port of the HBS3, we could observe the Hong-Ou-Mandel (HOM) dip [8,9]. The visibility of the HOM dip was about 75.1%. From this measurement, the path difference ΔL of the Michelson interferometer could be fixed at the center of the HOM dip. A longer path difference ΔL would lead to better temporal separation of the five peaks in Fig. 3. In this situation, the visibility of the HOM dip is expected to be improved and to enhance the spatial mode difference of the photon pair. So we chose a time delay of 1 ns.

By the above procedure, we prepared two single qubits, a decoherence channel, and a Michelson interferometer needed for the purification experiment. The density matrices of the initial mixed state at the output of the decoherence channel and the purified state at the output of the interferometer can be estimated by a quantum state tomography (QST) method [11]. We measured them with several different settings of the

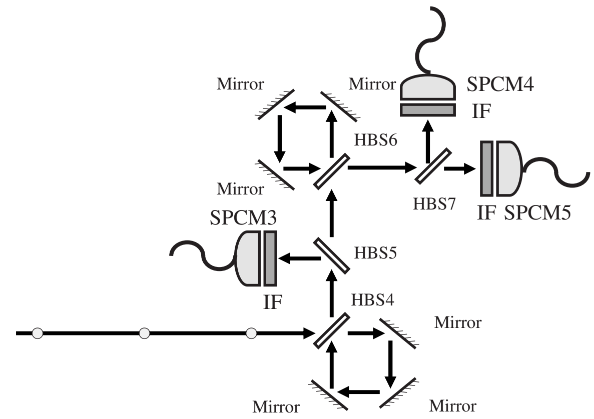

FIG. 5. Schematic diagram of the repetition being two. HBS4-7: nonpolarizing beam splitter, IF: interference filter, SPCM3-5: single-photon counting module.

duty ratio of the EOM. In this measurement, SPCM1 was used as a trigger detector. The maximum measured coincidence count rare was about 360 per minute. From these density matrices, we calculated the purities and the fidelities for the comparison between the output state and the input state, where the fidelity is defined as $\mathrm{Tr}\{[\rho^{1/2}\varepsilon(\rho)\rho^{1/2}]^{1/2}\}$. The measured purities and fidelities are shown in Fig. 4, where the horizontal axis is ξ, corresponding to the duty ratio of the EOM. For the error analysis of each plot, we assumed each detection probability in the QST measurement to be a Poisson distribution. The error bars were calculated by using the variances of their distributions. The theoretical predictions of the purities and the fidelities at the output are plotted by substituting the density matrixes at the input into Eq. (4). From the value of the visibility in the HOM experiment, we evaluated accidental coincidence counts due to the imperfection of the timing selection. The accidental coincidence counts were also included for the theoretical predictions of the density matrixes. We regarded errors of rotation angles of the waveplates as negligibly small. The experimental data were in good agreement with theoretical data.

By preparing additional single qubits, our purification protocol can be performed repeatedly to distill the input qubit much more, although the success probability is degraded gradually. As an example, let us describe this procedure in the case of two repetitions. The schematic diagram of this example is shown in Fig. 5. An additional single qubit with another time delay is collinearly transmitted through the first purification setup (HBS4). The purified single qubit from the output of the HBS4 and the additional single qubit are utilized for the second purification setup at the HBS6. SPCM3 and SPCM4 are utilized as trigger detectors, one of which is arranged at the first purification setup. In this situation, the density operator at the output can be calculated as

$$\varepsilon_p(\rho_{\vec{n}}) = \frac{14\xi + 2\xi^3}{9 + 7\xi^2}\rho_{\vec{n}} + \frac{1 - (14\xi + 2\xi^3)/(9 + 7\xi^2)}{2}I. \quad (6)$$

Noting the inequality $4\xi/(3+\xi^2) \leqslant (14\xi+2\xi^3)/(9+7\xi^2)$, we find that the purification can be accomplished twice. However, the success probability is degraded to $(3+\xi^2)^2/256$. Extending this procedure, one can apply the procedure to many collinear photons with different time delays. The single qubit passing through the depolarization channel is distilled toward a pure state by iterating this process.

In conclusion, we have demonstrated and proposed a procedure of the purification of a single qubit with collinear multiphotons. By using collinear multiphotons, single qubits can overcome the statistical errors affected by the noisy channel.

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