Elastic Constants Identification of Composite Materials Using Single Angle-Ply Laminate

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Abstract: A method is presented for identification of elastic constants for composite materials using three measured strains of a single angle-ply laminate subjected to tensile testing. In the proposed method, the trial material constants of the angle-ply laminate are used to predict the corresponding strains in the laminate. An error function is established to measure the difference between the experimental and theoretical predictions of the strains. The identification of material constants is then formulated as a constrained minimization problem in which the material constants are determined to make the error function a global minimum. The accuracy and capability of the proposed method are demonstrated by means of a number of examples of the identification of material constants of angle-ply laminates with different lay-ups. Experimental data obtained from static tensile tests of several angle-ply laminates are used to identify the material constants of the laminates. The excellent results obtained in the experimental investigation have validated the applicability of the proposed method.

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Introduction

In recent years, the amount of composite materials used in fabricating structural and mechanical parts has been greatly increased. To facilitate the manufacturing process and increase the production rate, different manufacturing techniques have also been developed to fabricate various types of composite material parts. In general, the techniques using different manufacturing/curing processes to fabricate composite parts may produce different mechanical properties in the parts, and furthermore, the material constants determined using standard specimens tested in the laboratory may deviate significantly from those of the actual composite parts fabricated in the factory.

Thus a quality control program may become futile if the material constants of the fabricated parts are not determined properly; on the other hand, composite parts that have been in service for a long period of time may experience material degradation that will lower the magnitude of the material constants or even lead to failure of the composite parts. Therefore reliability assessments of the existing composite parts may become unrealistic if the material constants of the composite parts are not estimated properly.

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In view of the aforementioned difficulties in the determination of material constants, it is clear that techniques are urgently needed that can determine the realistic material constants of fabricated or existing composite parts in an efficient and effective way. This determination has thus become an important topic of research, and many researchers have been working in this area. For instance, Marin et al. (2004) used the boundary element method and the strain/displacement measurements on the boundary to identify the material constants of isotropic materials.

Wang and Kam (2001) proposed a method to identify five material constants of shear deformable laminated composite plates using measured strains and/or displacements. In their method, the problem of identifying material constants was treated as an optimization problem in which a constrained minimization technique together with a set of measured strains/displacements was used to identify the material constants. The proposed method was successfully applied to the identification of material constants of laminated composite plates subjected to uniformly distributed or concentrated loads.

Grédiac et al. (2002) proposed a method based on the principle of virtual work to identify the material stiffness coefficients of an orthotropic laminate using measured deformational data of the laminate. Shin and Pande (2003) developed a two-step method to identify the material constants of orthotropic materials. In their method, the monitored data of a structure are used in the first step to recursively train a neural network-based constitutive model embedded in a finite-element code, and in the second step, the required material parameters are computed from the trained neural network-based constitutive model. Recently, vibration data such as measured natural frequencies and mode shapes have been used to identify material constants of structural parts. For instance, Ip et al. (1998), Wilde and Sol (1987), Rikards et al. (1999), and Mota Soares et al. (1993) used 12 to 16 experimental eigenfrequencies to identify material properties of laminated composites.

In this paper, the previously proposed optimization method (Wang and Kam 2001) is modified and extended to the identifi-

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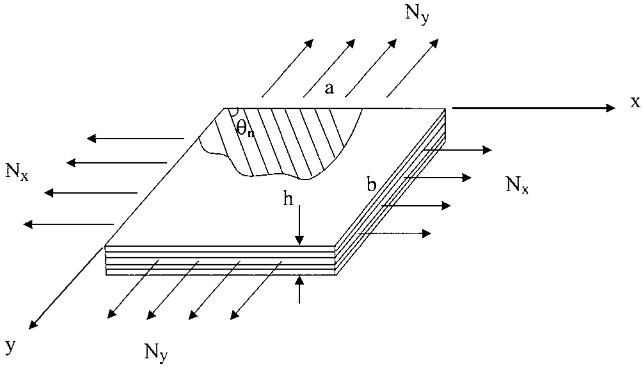

Fig. 1. Geometry and loading condition of laminate

cation of material constants of angle-ply laminates subjected to in-plane loads. The identification of material constants of an angle-ply laminate is treated as an optimization problem in which the theoretically and experimentally predicted axial, lateral, and shear strains of the angle-ply laminate are used to construct the objective function of the optimization problem.

The objective function that measures the sum of the differences between the experimental and theoretical predications of the axial, lateral, and shear strains of the angle-ply laminate is minimized using a multistart global minimization technique to identify the material constants of the laminate. The accuracy and feasibility of the present method are studied by means of a number of numerical examples. Experimental investigation of the identification of material constants of several angle-ply laminates is performed to demonstrate the applications of the present method.

Strain Analysis of Angle-Ply Laminate

The strains of the angle-ply laminate with lay-up of $[\theta/-\theta/\theta/\cdots/-\theta/\theta]$ subjected to the stress resultants N_x and N_y in Fig. 1 are determined in the strain analysis of the laminate using the following stress resultant and strain relations:

$$\begin{Bmatrix} N_x \\ N_y \\ 0 \end{Bmatrix} = \begin{bmatrix} A_{xx} & A_{xy} & A_{xs} \\ A_{xy} & A_{yy} & A_{ys} \\ A_{xs} & A_{ys} & A_{ss} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{1}$$

where ε_x, ε_y and γ_{xy}=axial, lateral, and shear strains, respectively. The laminate in-plane stiffness coefficients A_{ij} are expressed as

$$A_{ij} = \int_{-h/2}^{h/2} \bar{Q}_{ij}^{(m)} dz \quad (i,j = x,y,s) \tag{2}$$

where h=thickness of the laminate and $\bar{Q}_{ij}^{(m)}(i,j=x,y,s)$=transformed material stiffness constants of the mth layer with an arbitrary fiber angle. For an orthotropic lamina, the original material stiffness constants are expressed as

$$\underset{\sim}{Q} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \tag{3}$$

with

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}}, \quad Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}}$$

$$Q_{12} = Q_{21} = \frac{\nu_{21}E_1}{1-\nu_{12}\nu_{21}} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}}$$

$$Q_{66} = G_{12} \tag{4}$$

where E_1, E_2=Young's moduli in the fiber and matrix directions, respectively; ν_{ij}=Poisson's ratio for transverse strain in the j-direction when stressed in the i-direction; and G_{12}=shear modulus in the 1–2 plane. The relations between $\bar{Q}_{ij}$ and Q_{ij} can be found in the literature (Swanson 1997). The inversion of Eq. (1) gives

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} a_{xx} & a_{xy} & a_{xs} \\ a_{xy} & a_{yy} & a_{ys} \\ a_{xs} & a_{ys} & a_{ss} \end{bmatrix} \begin{Bmatrix} N_x \\ N_y \\ 0 \end{Bmatrix} \tag{5}$$

where a_{ij} $(i,j=x,y,s)$=in-plane compliance coefficients that are functions of the material constants $(E_1,E_2,G_{12},\nu_{12})$ of the laminate. Expansion of Eq. (5) in algebraic form gives

$$\varepsilon_x = a_{xx}N_x + a_{xy}N_y$$

$$\varepsilon_y = a_{xy}N_x + a_{yy}N_y$$

$$\gamma_{xy} = a_{xs}N_x + a_{ys}N_y \tag{6}$$

Note that the strains $(\varepsilon_x,\varepsilon_y,\gamma_{xy})$ can be determined directly from the above equations when the material constants of the composite laminate are available. In the present problem of identification of material constants, an attempt is made to determine the material constants from the above equations for a given set of stress resultants and their associated strains. In view of the fact that the relations of Eq. (6) are highly nonlinear and the material constants outnumber the available equations, the material constants cannot be determined directly from the above equation using any conventional technique. The previously proposed optimization method is used here to overcome this difficulty. The material constants of the angle-ply laminate will be identified using three measured strains in the optimization problem, as will be described in the following section.

Identification of Material Constants

The problem of identification of material constants of an angle-ply laminate is formulated as an optimization problem. In mathematical form it is stated as

$$\text{Minimize} \quad e(\underset{\sim}{x}) = [(\varepsilon_x - \varepsilon_x^*)^2 + (\varepsilon_y - \varepsilon_y^*)^2 + (\gamma_{xy} - \gamma_{xy}^*)^2] \cdot \xi$$

$$\text{Subject to} \quad x_i^L \leqslant x_i \leqslant x_i^U;\ i = 1-4 \tag{7}$$

where $e(\underset{\sim}{x})$=objective function measuring the differences between the predicted and measured strains; $\underset{\sim}{x}=[x_1=E_1,x_2=E_2,x_3=G_{12},x_4=\nu_{12}]$=vector containing the estimates of the material constants; ε_x, ε_y, and γ_{xy}=respectively, the predicted axial, lateral, and shear strains; ε_x^*, ε_y^*, and γ_{xy}^*=measured axial, lateral, and shear strains, respectively; ξ=amplification factor whose value is determined based on experience and whose best chosen value is in the range from 10^5 to 10^6; and x_i^L, x_i^U=lower and upper bounds of the material constants.

The predicted strains in the above equation are determined from Eq. (6) for a given set of trial material constants. Eq. (7)

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is then converted into an unconstrained minimization problem by creating the following general augmented Lagrangian (Vanderplaats 1984):

$$\bar{\Psi}(\tilde{\underline{x}},\underline{\mu},\underline{\eta},r_p)=e(\underline{x})+\sum_{j=1}^{4}[\mu_j z_j+r_p z_j^2+\eta_j\phi_j+r_p\phi_j^2] \qquad (8)$$

with

$$z_j=\max\left[g_j(\tilde{x}_j),\frac{-\mu_j}{2r_p}\right]$$

$$g_j(\tilde{x}_j)=\tilde{x}_j-\tilde{x}_j^U\leqslant 0$$

$$\phi_j=\max\left[H_j(\tilde{x}_j),\frac{-\eta_j}{2r_p}\right]$$

$$H_j(\tilde{x}_j)=\tilde{x}_j^L-\tilde{x}_j\leqslant 0;\quad j=1-4 \qquad (9)$$

where μ_j,η_j,r_p=multipliers and max $[*,*]$ takes on the maximum value of the numbers in brackets. The modified design variables $\tilde{\underline{x}}$ are defined as

$$\tilde{\underline{x}}=\left[\frac{E_1}{\alpha_1},\frac{E_2}{\alpha_2},\frac{G_{12}}{\alpha_3},\frac{\nu_{12}}{\alpha_4}\right] \qquad (10)$$

The normalization factors α_i are chosen so that the modified design variables are less than 10. Note that the modified design variables $\tilde{\underline{x}}$ are only used in the minimization algorithm while the original design variables $\underline{x}$ are used in the strain analysis of the laminate. The update formulas for the multipliers μ_j, η_j, and r_p are

$$\mu_j^{n+1}=\mu_j^n+2r_p^n z_j^n$$

$$\eta_j^{n+1}=\eta_j^n+2r_p^n\phi_j^n;\quad j=1-4$$

$$r_p^{n+1}=\begin{cases}\gamma_0 r_p^n & \text{if } r_p^{n+1}<r_p^{\max}\\ r_p^{\max} & \text{if } r_p^{n+1}\geqslant r_p^{\max}\end{cases} \qquad (11)$$

where the superscript n denotes the iteration number; γ_0=constant; and $r_p^{\max}$=maximum value of r_p. Based on experience, the parameters μ_j^0, η_j^0, r_p^0, γ_0, and $r_p^{\max}$ are chosen as

$$\mu_j^0=1.0,\quad \eta_j^0=1.0,\quad j=1-4$$

$$\gamma_0=2.5,\quad r_p^{\max}=100,\quad r_p^0=0.4 \qquad (12)$$

The constrained minimization problem of Eq. (7) has thus become the solution of the following unconstrained optimization problem:

$$\text{Minimize}\quad \bar{\Psi}(\tilde{\underline{x}},\underline{\mu},\underline{\eta},r_p) \qquad (13)$$

The straightforward solution of the above unconstrained optimization problem is to use the previously proposed unconstrained multistart stochastic global optimization algorithm (Snyman and Fatti 1987). In the adopted optimization algorithm, the objective function of Eq. (8) is treated as the potential energy of a traveling particle, and the search trajectories for locating the global minimum are derived from the equation of motion of the particle in a conservative force field. The design variables—the material constants that make the potential energy of the particle (objective function) the global minimum—constitute the solution of the problem.

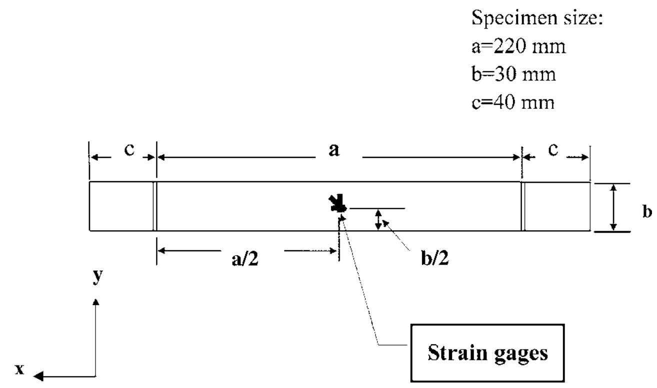

Fig. 2. Dimensions of laminated specimen for tensile test

In the minimization process, the side constraints in Eq. (7) are observed and a series of starting points for the design variables of Eq. (10) are selected at random from the region of interest. The lowest local minimum along the search trajectory initiated from each starting point is determined and recorded. A Bayesian argument is then used to establish the probability of the current overall minimum value of the objective function being the global minimum, given the number of starts and the number of times this value has been achieved.

The multistart optimization procedure is terminated once the condition that a target probability, typically 0.995, has been exceeded is satisfied. Note that the two-stage multistart global optimization algorithm in the previous paper (Wang and Kam 2001) for identifying five elastic constants has been simplified to the present one-stage multistart global optimization algorithm for identifying four elastic constants.

Experimental Investigation

A number of angle-ply laminates with lay-ups of $[(30°/-30°)_4/30°]$ and $[(45°/-45°)_4/45°]$ were fabricated for the experimental study of material constants identification of composite laminates. The dimensions of the angle-ply laminates are shown in Fig. 2. The laminates comprised nine laminae and the average thickness of each lamina was 0.125 mm. The laminates were made of graphite/epoxy prepreg tapes supplied by Toray Co., Japan. The material constants of the graphite/epoxy lamina were first determined using the standard specimens in accordance with the ASTM (1990) specifications, and their average values and coefficients of variation are given as follows:

$$E_1=146.5\text{ GPa }(0.7\%)$$

$$E_2=9.22\text{ GPa }(1.2\%)$$

$$G_{12}=6.84\text{ GPa }(3.2\%)$$

$$\nu_{12}=0.3\ (0.19\%) \qquad (14)$$

In the above equation, the values in parentheses denote the coefficients of variation (COVs). The average values in the above equation are treated herein as the "actual" values of the composite material.

For reference, the "actual" material constants in Eq. (14) are used to compute the "actual" strains of the angle-ply laminates. The "actual" strains of the laminates determined from Eq. (6) are

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Table 1. Actual Strains of Graphite/Epoxy $[(\theta°/-\theta°)_4/\theta°]$ Laminate Subject to N_x=16.667 kN/m and N_y=0

Fiber angle θ (degrees)	Strain $\varepsilon_x(10^{-4})$	$\varepsilon_y(10^{-4})$	$\gamma_{xy}(10^{-4})$
15	1.240	−1.198	−0.2544
30	2.614	−3.277	−0.2185
45	6.337	−4.492	−0.1874
60	11.34	−3.277	−0.1676

tabulated in Table 1 for future comparison. The fabricated laminates were subjected to tensile tests in which three strain gauges were used to measure the axial, lateral, and 45° direction strains $(\varepsilon_x^*, \varepsilon_y^*, \varepsilon_{45}^*)$ at the midspan of each of the laminates.

The tensile tests of the laminates were performed using the Hung Ta Instrument Co. LTD (Type HT-9102A) with test speed less than 0.01 mm/s. A pair of mechanical rather than hydraulic type grips were used to clamp the two ends of each laminate during testing. Note that the aspect (length to width) ratio of the laminates is 8.67. The large aspect ratio of the laminate when coupled with the use of the mechanical grips could reduce St. Venant's end effects in such a way that the deformations at the laminate center were similar to the shearing and extension of an unrestrained laminate. The strain gauges used in the tests were produced by KYOWA, Japan, and had a gauge length of 3 mm and gauge factor of 2.08±1.0%. Any strain component can be derived from the measured strains using the strain transformation relation. The derived shear strain γ_{xy}^* thus obtained is

$$\gamma_{xy}^* = 2\varepsilon_{45}^* - \varepsilon_x^* - \varepsilon_y^* \tag{15}$$

Herein, the derived shear strain is also treated as the measured shear strain when used in the present method. In the tensile testing of each type of angle-ply laminate, three specimens with the same lay-up were tested, and the load–strain relations of the specimens were constructed to produce the strain statistics for material constant identification. For instance, Fig. 3 shows the typical load-strain curves for the $[(30°/-30°)_4/30°]$ laminates. The average values and coefficients of variation of the measured axial, lateral, and shear strains of the $[(30°/-30°)_4/30°]$ and $[(45°/-45°)_4/45°]$ laminates are listed in Tables 2 and 3, respectively. Note that the COVs of the measured strains are less than or equal to 3.2% and the differences between the "actual" and (average) measured strains are less than or equal to 8.6%. The experimental (average) strains were then used in the present method to identify the material constants of the plates.

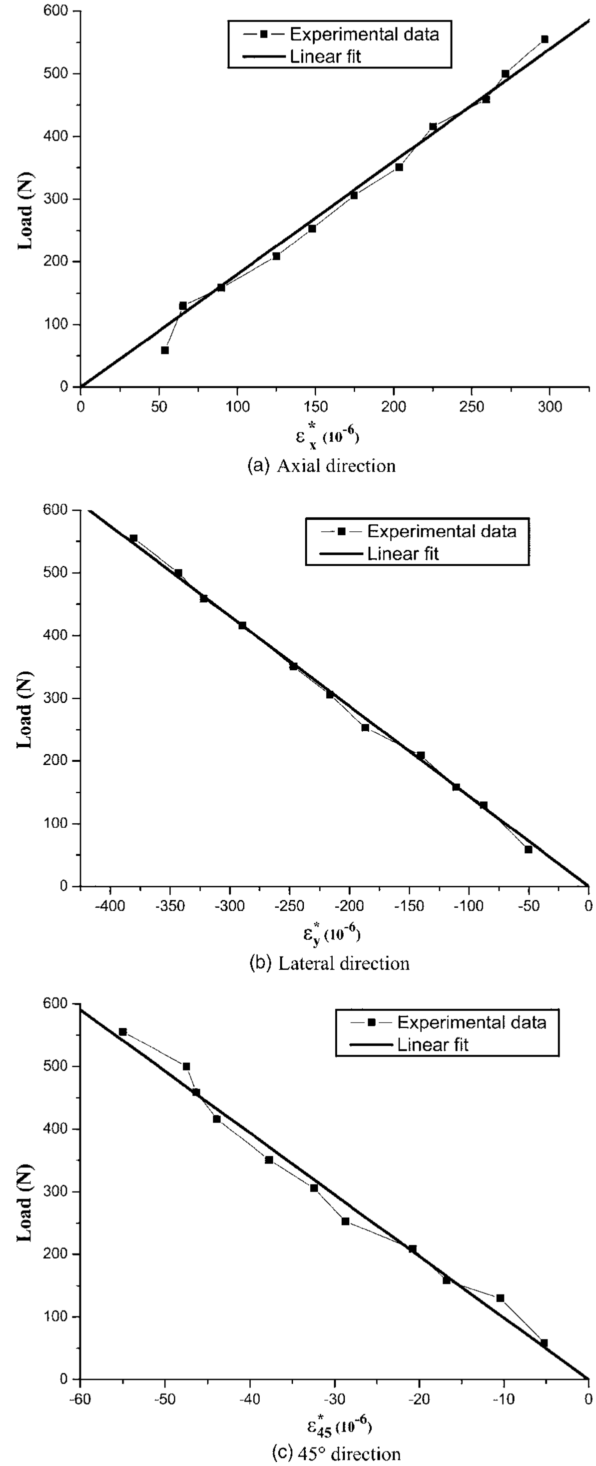

Fig. 3. Experimental load–strain relation of $[(30°/-30°)_4/30°]$ laminate

Results and Discussion

The aforementioned optimization method will first be applied to the theoretical material characterization study of angle-ply laminates made of graphite/epoxy or glass/epoxy materials. The material constants of the graphite/epoxy laminates are the same as those given in Eq. (14), while those of glass/epoxy laminates are E_1=38.6 GPa, E_2=8.27 GPa, G_{12}=4.14 GPa, and ν_{12}=0.26 (Swanson 1997). The upper and lower bounds of the material constants of the composite materials are chosen to be reasonably large.

$$\text{Graphite/epoxy:}\quad 0 < E_1 < 310\text{ GPa},\ 0 < E_2 < 20\text{ GPa},\ 0 < G_{12} < 20\text{ GPa},\ 0.1 < \nu_{12} < 0.5 \tag{16a}$$

$$\text{Glass/epoxy:}\quad 0 < E_1 < 60\text{ GPa},\ 0 < E_2 < 16\text{ GPa},\ 0 < G_{12} < 16\text{ GPa},\ 0.05 < \nu_{12} < 0.5 \tag{16b}$$

The modified design variables of Eq. (10) are obtained via use of the following normalization factors:

$$\text{Graphite/epoxy:}\quad \alpha_1 = 1{,}000,\quad \alpha_2 = 100,\quad \alpha_3 = 10,\quad \alpha_4 = 1 \tag{17a}$$

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Table 2. Statistics of Measured Strains of Graphite/Epoxy $[(30°/-30°)_4/30°]$ Laminates Subject to N_x=16.667 kN/m and N_y=0

Specimen number	Measured strain $\varepsilon_x^*(10^{-4})$	Measured strain $\varepsilon_y^*(10^{-4})$	Measured strain $\varepsilon_{45}^*(10^{-4})$	Derived strain $\gamma_{xy}^*(10^{-4})$
1	2.687	−3.361	−0.4481	−0.2222
	(+2.8%)[a]	(+2.6%)	(+1.7%)	(+1.7%)
2	2.772	−3.467	−0.4658	−0.2366
	(+6%)	(+5.8%)	(+5.7%)	(+8.3%)
3	2.713	−3.431	−0.4747	−0.2314
	(+3.8%)	(+4.7%)	(+7.7%)	(+5.9%)
Average	2.724	−3.420	−0.4629	−0.2301
	(+4.2%)	(+4.7%)	(+5.0%)	(+5.3%)
COV	1.6%	1.6%	2.9%	3.2%

[a]Values in parentheses denote percentage difference between actual and measured strains.

$$\text{Glass/epoxy:}\quad \alpha_1 = 100,\quad \alpha_2 = 100,\quad \alpha_3 = 10,\quad \alpha_4 = 1 \qquad (17b)$$

The value of the amplification factors ξ in Eq. (7) is set to be 10^6. Note that use of the above values for the normalization and amplification factors can help increase the convergence rate of the solution. A number of numerical examples are given to study the accuracy and feasibility of the present method for identifying the material constants of angle-ply laminates subjected to in-plane loads.

In the numerical study of graphite/epoxy angle-ply laminates subjected to the stress resultants of N_x=16.667 kN/m and N_y=0, the actual strains of the laminates with different fiber angles in Table 1 are treated as the "measured" strains for identifying the material constants given in Eq. (14). The $[(45°/-45°)_4/45°]$ laminate is first used as an example to show the process of material constant identification in the solution of the optimization problem.

In this case, five starting points are randomly generated in obtaining the global minimum with probability exceeding 0.995. The randomly generated starting points of $(E_1, E_2, G_{12}, \nu_{12})$ are (243.45 GPa, 3.19 GPa, 4.95 GPa, 0.19), (163.82 GPa, 9.19 GPa, 13.60 GPa, 0.38), (309.33 GPa, 15.17 GPa, 16.72 GPa, 0.31), (67.47 GPa, 2.77 GPa, 15.32 GPa, 0.34), and (150.33 GPa, 9.49 GPa, 5.69 GPa, 0.44). The average number of iterations required for identifying the lowest local minima for the starting points is around 12.

Table 3. Statistics of Measured Strains of Graphite/Epoxy $[(45°/-45°)_4/45°]$ Laminates Subject to N_x=10 kN/m and N_y=0

Specimen number	Measured strain $\varepsilon_x^*(10^{-4})$	Measured strain $\varepsilon_y^*(10^{-4})$	Measured strain $\varepsilon_{45}^*(10^{-4})$	Derived strain $\gamma_{xy}^*(10^{-4})$
1	4.095	−2.971	0.5045	−0.1150
	(+7.7%)[a]	(+10.2%)	(+1.4%)	(+2.3%)
2	4.084	−2.921	0.5233	−0.1184
	(+7.4%)	(+8.4%)	(+5.0%)	(+5.3%)
3	4.028	−2.888	0.5121	−0.1158
	(+5.9%)	(+7.2%)	(+3.0%)	(+3%)
Average	4.069	−2.927	0.5130	−0.1164
	(+7.0%)	(+8.6%)	(+3.2%)	(+3.6%)
COV	0.9%	1.4%	1.7%	1.5%

[a]Values in parentheses denote percentage difference between actual and measured strains.

Table 4. Identified Material Constants of Graphite/Epoxy $[(\theta°/-\theta°)_4/\theta°]$ Laminates Using Actual Strains (N_x=16.667 kN/m, N_y=0)

Fiber angle θ (degrees)	Identified material constant E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}
15	146.47	9.23	6.84	0.30
	(0.02%)[a]	(0.1%)	(0%)	(0%)
30	146.59	9.26	6.84	0.30
	(0.06%)	(0.4%)	(0%)	(0%)
45	146.51	9.20	6.84	0.30
	(0.007%)	(0.2%)	(0%)	(0%)
60	142.14	9.35	6.79	0.30
	(2.9%)	(1.4%)	(0.7%)	(0%)

[a]Values in parentheses denote percentage difference between identified and actual data.

It has been shown that all the starting points can produce the same estimates of the material constants with errors less than or equal to 0.2%. The identified material constants and their associated errors for the angle-ply laminates with different fiber angles are listed in Table 4. Note that overall the uses of these laminates in the present method can produce excellent estimates of the material constants with errors less than or equal to 2.9%. The fact that for any of the angle-ply laminates, all the starting points can produce the global minimum with merely the use of around 12 iterations has demonstrated the efficiency and effectiveness of the present optimization method.

Next consider the material constants identification of the graphite/epoxy angle-ply laminates subjected to biaxial loads. The actual strains of the laminates under different loading conditions are listed in Table 5, and the identified material constants together with their associated errors for the laminates in Table 6. Note that except for the $[(45°/-45°)_4/45°]$ laminate subjected to the biaxial loads of N_x=N_y=10 kN/m or N_x=$-N_y$=10 kN/m, excellent results have been obtained for the laminates irrespective

Table 5. Actual Strains of Graphite/Epoxy $[(\theta°/-\theta°)_4/\theta°]$ Laminates Subject to Different Biaxial In-Plane Loads

Stress resultant N_x (kN/m)	Stress resultant N_y (kN/m)	Fiber angle θ (degrees)	Strain $\varepsilon_x(10^{-5})$	Strain $\varepsilon_y(10^{-5})$	Strain $\gamma_{xy}(10^{-5})$
10	1	15	6.720	1.756	−1.622
		30	13.72	−12.86	−1.412
		45	35.33	−23.15	−1.237
		60	66.07	−18.09	−1.137
10	10	15	0.2529	82.23	−2.485
		30	−3.977	48.83	−2.317
		45	11.07	11.07	−2.248
		60	48.38	−3.977	−2.317
10	−1	15	8.157	−16.13	−1.431
		30	17.65	−26.47	−1.211
		45	40.72	−30.76	−1.012
		60	70.00	−21.23	−0.8745
10	−10	15	14.62	−96.60	−0.5678
		30	35.35	−87.70	−0.3057
		45	64.98	−64.98	0
		60	87.70	−35.35	0.3057

Table 6. Identified Material Constants Using Graphite/Epoxy $[(\theta°/-\theta°)_4/\theta°]$ Laminates Subject to Different Biaxial Loads

Stress resultant		Fiber angle θ (degrees)	Identified material constant			
N_x (kN/m)	N_y (kN/m)		E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}
10	1	15	146.50 (0%)[a]	9.23 (0.1%)	6.84 (0%)	0.30 (0%)
		30	146.54 (0.03%)	9.28 (0.7%)	6.84 (0%)	0.30 (0%)
		45	146.46 (0.03%)	9.21 (0.1%)	6.84 (0%)	0.30 (0%)
		60	141.34 (3.5%)	9.40 (2%)	6.77 (1%)	0.307 (2.3%)
10	10	15	143.75 (1.9%)	9.24 (0.2%)	6.72 (1.8%)	0.317 (5.7%)
		30	147.56 (0.7%)	9.14 (0.9%)	6.89 (0.7%)	0.29 (3.3%)
		45	Multiple global minima			
		60	147.56 (0.7%)	9.14 (0.9%)	6.89 (0.7%)	0.29 (3.3%)
10	−1	15	146.60 (0.07%)	9.24 (0.2%)	6.83 (0.1%)	0.305 (1.7%)
		30	146.68 (0.01%)	9.25 (0.3%)	6.83 (0.1%)	0.30 (0%)
		45	146.61 (0.08%)	9.17 (0.5%)	6.84 (0%)	0.30 (0%)
		60	139.43 (4.8%)	9.40 (2%)	6.78 (0.9%)	0.30 (0%)
10	−10	15	148.59 (1.4%)	9.23 (0.1%)	6.83 (0.1%)	0.317 (5.7%)
		30	148.20 (1.1%)	9.23 (0.1%)	6.84 (0%)	0.31 (3.3%)
		45	Multiple global minima			
		60	148.20 (1.1%)	9.24 (0.2%)	6.84 (0%)	0.31 (3.3%)

[a]Values in parentheses denote percentage difference between identified and actual data.

Table 7. Actual Strains of Glass/Epoxy $[(\theta°/-\theta°)_4/\theta°]$ Laminates Subject to Different In-Plane Biaxial Loads

Stress resultant		Fiber angle θ (degrees)	Strain		
N_x (kN/m)	N_y (kN/m)		$\varepsilon_x(10^{-5})$	$\varepsilon_y(10^{-5})$	$\gamma_{xy}(10^{-5})$
3	0.1	15	7.804	−2.393	−0.9613
		30	11.99	−6.914	−1.074
		45	20.90	−10.23	−0.8447
		60	28.90	−7.477	−0.4604
3	3	15	4.465	28.40	−1.099
		30	4.365	21.28	−1.485
		45	10.33	10.33	−1.635
		60	21.28	4.365	−1.485
3	−0.1	15	8.035	−4.516	−0.9518
		30	12.51	−8.858	−1.046
		45	21.63	−11.65	−0.7902
		60	29.42	−8.294	−0.3897
3	−3	15	11.37	−35.31	−0.8140
		30	20.14	−37.05	−0.6350
		45	32.21	−32.21	0
		60	37.05	−20.14	0.6350

Table 8. Identified Material Constants of Glass/Epoxy $[(\theta°/-\theta°)_4/\theta°]$ Laminates Subject to Different Biaxial Loads

Stress resultant		Fiber angle θ (degrees)	Identified material constant			
N_x (kN/m)	N_y (kN/m)		E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}
3	0.1	15	38.60 (0%)[a]	8.00 (3.3%)	4.15 (0.2%)	0.26 (0%)
		30	38.51 (0.2%)	8.00 (3.3%)	4.16 (0.5%)	0.266 (2.3%)
		45	38.63 (0.08%)	8.27 (0%)	4.14 (0%)	0.26 (0%)
		60	39.24 (1.7%)	8.26 (0.1%)	4.15 (0.2%)	0.26 (0%)
3	3	15	39.3 (1.8%)	8.29 (0.2%)	4.22 (1.9%)	0.25 (3.8%)
		30	38.98 (1%)	8.26 (0.1%)	4.19 (1.2%)	0.25 (3.8%)
		45	Multiple global minima			
		60	38.98 (1%)	8.26 (0.1%)	4.19 (1.2%)	0.25 (3.8%)
3	−0.1	15	38.60 (0%)	8.24 (0.4%)	4.14 (0%)	0.26 (0%)
		30	38.6 (0%)	8.23 (0.5%)	4.14 (0%)	0.26 (0%)
		45	38.64 (0.1%)	8.27 (0%)	4.14 (0%)	0.26 (0%)
		60	39.54 (2.4%)	8.25 (0.2%)	4.15 (0.2%)	0.26 (0%)
3	−3	15	38.6 (0%)	8.27 (0%)	4.14 (0%)	0.26 (0%)
		30	38.45 (0.4%)	8.26 (0.1%)	4.14 (0%)	0.26 (0%)
		45	Multiple global minima			
		60	39.45 (0.4%)	8.26 (0.1%)	4.14 (0%)	0.26 (0%)

[a]Values in parentheses denote percentage difference between identified and actual data.

of the magnitudes and directions of the applied loads and the errors of the estimated material constants are less than or equal to 4.8, 2, 1, and 5.7% for E_1, E_2, G_{12}, and ν_{12}, respectively.

The $[(45°/-45°)_4/45°]$ laminate when subjected to the biaxial loads of $N_x=N_y=10$ kN/m or $N_x=-N_y=10$ kN/m produced multiple global minima, which led to erroneous estimates of the material constants. The incapability of the $[(45°/-45°)_4/45°]$ laminate to produce good identification is due to the equality of the absolute values of the axial and lateral strains, which reduces the number of equations in Eq. (6) from 3 to 2. The material characterization of glass/epoxy angle-ply laminates with different fiber angles is also studied by means of several examples.

The actual strains of the laminates under different loading conditions are shown in Table 7, and the estimates of the material constants obtained in the solutions of the optimization problems for the laminates are listed in Table 8. Again, note that except for the $[(45°/-45°)_4/45°]$ laminate subjected to the biaxial loads of $N_x=N_y=3$ kN/m or $N_x=-N_y=3$ kN/m, excellent estimates of the material constants can be obtained for the glass/epoxy angle-ply laminates with errors less than or equal to 1.8, 3.3, 1.2, and 3.8% for E_1, E_2, G_{12}, and ν_{12}, respectively.

The present method is now used to identify the material constants of the angle-ply laminates that have been tested. The measured axial, lateral, and 45° direction strains of different

Table 9. Identified Material Constants Obtained from Different Sets of Measured Strains of Graphite/Epoxy $[(30°/-30°)_4/30°]$ Laminates

	Measured strain			Identified material constant			
Case	$\varepsilon_x^*(10^{-4})$	$\varepsilon_y^*(10^{-4})$	$\gamma_{xy}^*(10^{-4})$	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}
1	2.687	−3.361	−0.2222	141.18 (3.6%)[a]	8.34 (9.5%)	6.78 (0.9%)	0.30 (0%)
2	2.772	−3.467	−0.2366	139.91 (4.5%)	9.89 (7.3%)	6.27 (8.3%)	0.30 (0%)
3	2.713	−3.431	−0.2314	144.70 (1.2%)	10.24 (11.1%)	6.31 (7.7%)	0.30 (0%)
4[b]	2.724	−3.420	−0.2301	141.92 (3.1%)	9.50 (3.0%)	6.45 (5.7%)	0.30 (0%)

[a]Values in parentheses denote percentage difference between identified and actual data.
[b]This case uses average measured strains for identification.

Table 10. Identified Material Constants Obtained from Different Sets of Measured Strains of Graphite/Epoxy $[(45°/-45°)_4/45°]$ Laminates

	Measured strain			Identified material constant			
Case	$\varepsilon_x^*(10^{-4})$	$\varepsilon_y^*(10^{-4})$	$\gamma_{xy}^*(10^{-4})$	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}
1	4.095	−2.971	−0.1150	145.42 (0.7%)[a]	8.41 (8.8%)	6.29 (8%)	0.30 (0%)
2	4.084	−2.921	−0.1184	139.8 (4.6%)	8.56 (7.2%)	6.34 (7.3%)	0.29 (3.3%)
3	4.028	−2.888	−0.1158	142.28 (2.9%)	8.93 (3.1%)	6.43 (6%)	0.30 (0%)
4[b]	4.069	−2.927	−0.1164	142.55 (2.7%)	8.61 (6.6%)	6.35 (7.2%)	0.30 (0%)

[a]Values in parentheses denote percentage difference between identified and actual data.
[b]This case uses average measured strains for identification.

$[(30°/-30°)_4/30°]$ laminates as well as their average values in Table 2 are used in different cases to identify the material constants of the laminates. The estimates of the material constants obtained in the cases using different sets of measured strains are listed in Table 9. Note that all the cases have produced excellent estimates of errors the material constants for which are less than or equal to 4.5, 11.1, 8.3, and 0% for E_1, E_2, G_{12}, and ν_{12}, respectively.

In particular, the errors of the estimates of E_1, E_2, G_{12}, and ν_{12} obtained for Case 4 where the average measured strains have been used in the identification process are 3.1, 3.0, 5.7, and 0%, respectively. The measured strains of different $[(45°/-45°)_4/45°]$ laminates as well as their average values in Table 3 are also used in different cases to identify the material constants. The estimates of the material constants obtained in the cases using different sets of measured strains are listed in Table 10. Again, note that for all the cases, excellent estimates of E_1, E_2, G_{12}, and ν_{12} have been obtained, with errors less than or equal to 4.6, 8.8, 8, and 3.3%, respectively. In particular, the errors of the estimates of the material constants obtained for Case 4 where the average measured strains have been used in the identification process are 2.7, 6.6, 7.2, and 0% for E_1, E_2, G_{12}, and ν_{12}, respectively.

Conclusions

The material constants determination of a composite angle-ply laminate has been treated as an optimization problem in which three measured strains of the angle-ply laminate have been used to construct the objective function of the optimization problem. A constrained global minimization technique has been adopted to solve the optimization problem and identify the material constants of the composite laminate by minimizing the objective function. Appropriate values of the parameters used in the adopted minimization technique have been selected to expedite the convergence rate of the solution.

The feasibility and applications of the proposed material constants identification method have been studied via both theoretical and experimental approaches. In the theoretical study, a number of numerical examples on the material constants identification of graphite/epoxy and glass/epoxy angle-ply laminates subjected to different in-plane loads have been given to illustrate the efficiency and accuracy of the proposed method.

It has been shown that, for the case of the $[(45°/-45°)_4/45°]$ laminates subjected to biaxial loads with the same magnitude and arbitrary directions, material constants of the laminates cannot be identified due to the existence of multiple global minima. As for the other cases, excellent estimates of the material constants can be identified for graphite/epoxy and glass/epoxy angle-ply laminates, regardless of the loading conditions and fiber angles of the laminates.

In the experimental study, a number of graphite/epoxy angle-ply laminates with lay-ups of $[(45°/-45°)_4/45°]$ and $[(30°/-30°)_4/30°]$ have been subjected to tensile tests in which three strains of each of the laminates have been measured. The measured strains as well as their average values have been used to identify the material constants of the laminates. The percentage errors of the identified material constants of the $[(30°/-30°)_4/30°]$ laminate are 3.1, 3.0, 5.7, and 0% for E_1, E_2,

G_{12}, and ν_{12}, respectively, if the average measured strains are used in the identification process.

On the other hand, with the use of the average measured strains in the identification process, the percentage errors of the identified material constants of the $[(45°/-45°)_4/45°]$ laminate are 2.7, 6.6, 7.2, and 0% for E_1, E_2, G_{12}, and ν_{12}, respectively. The small errors obtained in the experimental study have validated the applicability of the proposed method.

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