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ORIGINAL ARTICLE

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Precision measures for processes with multiple manufacturing lines

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Abstract Process capability indices C_p, and C_{pk} have been popularly used in the manufacturing industry for providing measures of process potential and performance. The precision index C_p is used to measure product quality consistency, while the yield index C_{pk} is used to provide measures of percentage of product conforming to manufacturing specifications. Properties of C_p for processes with a single manufacturing line have been investigated extensively. However, research on properties of C_p for processes with multiple manufacturing lines have been neglected. In this paper, we consider the precision index C_p for processes with three manufacturing lines. We develop a practical procedure for process precision testing to determine whether a process meets the precision requirement preset in the factory. An application example is given.

1 Introduction

Process capability indices C_p, and C_{pk} have been popularly used in the manufacturing industry to provide numerical measures of process potential and performance. Examples include the manufacturing of semiconductor products [8], front-end alignment for automobiles [6], head/gimbals assembly for memory storage systems [23], jet-turbine engine components [9], flip-chips and chip-on-board [13], piston rings for automobile engineering [4], cable locking terminals for automobile ignition system [4], speaker drivers [16], electrolytic capacitors [17], voltage level translator [12], chip resistors [18], linear regulators [19], low-drop-out linear regulators [20], power distribution switch [21], high-speed buffer amplifiers [22], couplers and wavelength division multiplexers [25] and many others. The two basic indices are defined as:

$$C_p = \frac{USL - LSL}{6\sigma},$$

$$C_{pk} = \min\left\{\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right\},$$

where LSL and USL are the lower and upper specification limits, μ is the process mean, and σ is the process standard deviation.

The index C_p measures the magnitude of process variation relative to the spread of the specification limits which reflect product quality consistency. For processes with two-sided specification limits, the process yield can be calculated as F(USL) − F(LSL), where F(·) is the cumulative distribution function of the quality characteristic X. On the assumption of normality, the process yield can be expressed as $\text{Yield} = \Phi\{(USL-\mu)/\sigma\} - \Phi\ (USL-\mu)/\sigma\}$, where $\Phi(\cdot)$ is the cumulative function of the standard normal distribution. If the process is perfectly centered $(\mu = (USL + LSL/2)$, then the process yield can be expressed alternatively as Yield=$2\Phi(3C_p)-1$. Thus, the index C_p provides an exact measure (one-to-one correspondence) of the actual process yield. Since C_p measures the magnitude of process variation, C_p may be viewed as a process precision index. Table 1 displays various values of C_p, the corresponding σ levels, and the percentages of non-conformity (defect rate) in parts-per-million (ppm) for single processes with normal distribution, where $d = USL - LSL)/2$, and $m = (USL + LSL)/2$. For the requirement commonly used in the electronic industry, 200 ppm (parts per million), the corresponding C_p value approximately equals 1.24. We

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Table 1 Various C_p values with corresponding number of non-conformities in ppm (r=3.71)

μ	σ	σ Level	C_p	ppm
m	d	1	0.33	317311
m	d/2	2	0.67	45500
m	d/3	3	1.00	2700
m	d/r	r	1.24	200
m	d/4	4	1.33	63
m	d/5	5	1.67	0.57
m	d/6	6	2.00	0.002

note, however, for such measures to be valid, the process must be perfectly centered. That is, μ=m.

The natural estimators of C_p and C_{pk} can be defined below, where $X = (\sum_{i=1}^{n} X_i)/n$ and $S = \{(n-1)^{-1} \sum_{i=1}^{n} (X_i - X)^2\}^{1/2}$ are conventional estimators of the process parameters, μ, and σ.

$$\widehat{C}_p = \frac{USL - LSL}{6S},$$

$$\widehat{C}_{pk} = \min\left\{\frac{USL - \overline{X}}{3S}, \frac{\overline{X} - LSL}{3S}\right\}.$$

Statistical properties of the estimator $\widehat{C}_p$ for processes with single manufacturing line (particularly for normal distributions) have been investigated extensively [1, 5, 14]. However, it is an exception that investigation of the estimator $\widehat{C}_p$ for processes with multiple manufacturing lines has been neglected [15].

2 The contamination model

Recently, Kocherlakota et al. [10], and Kotz and Johnson [11] investigated the behavior of the natural estimator of C_p for contaminated distributions and mixtures of multiple distributions. The contamination model is useful, particularly for processes with multiple manufacturing lines where the equipment or workmanship may not be identical in precision for each manufacturing line, or cases where multiple suppliers are involved in providing raw materials for manufacturing. Such situations often result in productions with inconsistent precision in quality characteristics. Use of the contaminated model to characterize the population would, therefore, be appropriate.

Kocherlakota et al. [10] considered the contamination of two normal populations, with probability p for population I distributed as $N(\mu_1, \sigma^2)$, and probability 1-p for population II distributed as $N(\mu_2, \sigma^2)$. The probability density function of the contaminated normal distributions may be expressed as:

$$f(x) = p\phi_x(\mu_1, \sigma^2) + (1-p)\phi_x(\mu_2, \sigma^2),$$

where $0 \le p \le 1$ and

$$\phi_x(\mu_i, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{\frac{1}{2}\left(\frac{x-\mu_i}{\sigma}\right)^2\right\}, i = 1, 2.$$

If p=1, then the contaminated normal distribution reduces to the normal distribution $N(\mu_1, \sigma^2)$. If p=0, then the contaminated normal distribution reduces to the other normal distribution $N(\mu_2, \sigma^2)$. For p=0.5, the contaminated distribution is symmetric. On the other hand, for p<0.5 with small values of a, the contaminated distribution tends to be skewed [15]. Kocherlakota et al. [10] investigated the natural estimator C_p and obtained the exact distribution. They also derived the exact formulae for the expected value and variance of C_p. Kotz and Johnson [11] extended the contamination model to include k normal distributions. The probability density function of the extended contaminated model can be expressed as the following, where p_i>0, 1≤i≤ k, and $p_1+p_2+\ldots+p_k=1$.

$$f(x) = \sum_{i=1}^{k} p_i \cdot \phi_x(\mu_i, \sigma^2).$$

Clearly, for k=2, the model reduces to one considered by Kocherlakota et al. and Pearn and Chang [10, 15], and for k=3 the model reduces to the theoretical investigation in Kotz and Johnson [11].

2.1 Contamination model with k=3

Kotz and Johnson [11] considered the natural estimator $\widehat{C}_p$ of C_p, for the contamination model of three (k=3) normal distributions. The probability density function of the contamination model considered is:

$$f(x) = p_1\phi_x(\mu_1, \sigma^2) + p_2\phi_x(\mu_2, \sigma^2) + (1 - p_1 - p_2)\,\phi_x(\mu_3, \sigma^2),$$

where $0 \le p_1, p_2 \le 1$ and $\phi_x(\mu, \sigma 2)$ as defined earlier. If we define $a_1 = (\mu_1 - \mu_3)/\sigma$, $a_2 = (\mu_2 - \mu_3)/\sigma$, and $\mu_3 = 0$, then the probability density function can be alternatively written as the following:

$$\frac{1}{\sqrt{2\pi}\sigma}\left\{p_1 \times \exp\left[\frac{-(x - a_1\sigma)^2}{2\sigma^2}\right] + p_2 \times \exp\left[\frac{-(x - a_2\sigma)^2}{2\sigma^2}\right] + (1 - p_1 - p_2) \times \exp\left[\frac{-x^2}{2\sigma^2}\right]\right\}.$$

It is easy to verify that the mean of the contaminated distribution $\mu_c = (p_1 a_1 + p_2 a_2)\,\sigma$, and the variance of the contaminated distribution $(\sigma_c)^2 = [(1 + p_1 a_1^2 + p_2 a_2^2) - (p_1 a_1 + p_2 a_2)^2]\,\sigma^2$. Fig. 1a–f and Fig. 2a–f display various non-normal distributions modeled by the contamination of three normal distributions, $N(\mu_1, \sigma^2)$ and $N(\mu_2, \sigma^2)$, and $N(\mu_3, \sigma^2)$ for (a_1, a_2, p_1, p_2)=(−1.00, 2.00, 0.1, 0.2), (−1.00, 3.00, 0.1, 0.1), (−1.00, 2.50, 0.1, 0.2), (−1.00, 2.50, 0.3, 0.3), (−1.00, 2.50, 0.1, 0.3), (−1.00, 2.00, 0.3, 0.3), respectively. Table 2 display the values of a_1, a_2, p_1, and p_2 corresponding to the distributions depicted in Fig. 1a–f and Fig. 2a–f.

Table 3 displays the nonconformities (defective items) in parts-per-million (ppm_1) for the contaminated distributions shown in Fig. 1a–f and Fig. 2a–f, with C_p=1.00. The corresponding nonconformities for the standard normal distribution N(0,1), expressed as ppm_2, is also tabulated in Table 3. Sommerville and Montgomery [24] considered the nonconformities for various centered non-normal processes with C_p=1.00, and showed significant errors in nonconformity measure can occur if normality assumptions are not met. For processes with multiple manufacturing lines, the distributions tend to be non-normal. The nonconformity estimations using the contamination approach would greatly reduce the errors.

3 Estimating process precision

Kotz and Johnson [11] showed that for populations with contamination of three normal distributions, the conditional distribution of the natural estimator $\widehat{C}_p$ based on the sample data (n_1, n_2, n_3) can be expressed as:

$$\hat{C}_p\big|_{(n1,\,n2,\,n3)} \sim \frac{C_p\sqrt{n-1}}{\sqrt{\chi^2_{n-1}\left(\lambda_{(n1,n2,n3)}\right)}}.$$

The notation (n1, n2, n3) represents the random sample (N_1, N_2, N_3)=(n1, n2, n3) with $n1 + n2 + n3 = n$ observations, where n1 sample observations are taken from manufacturing line 1, n2 sample observations are taken from manufacturing line 2, n3 sample observations are taken from manufacturing line 3, and $\chi^2_{n-1}\left(\lambda_{(n1,n2,n3)}\right)$ is a non-central Chi-squared distribution with n−1 degrees of freedom and non-centrality parameter $\lambda_{(n1,\,n2,\,n3)}$:

$$\lambda_{(n1,n2,n3)} = \sum_{j=1}^{3} n_j(\mu_j - \overline{\mu})^2,$$

where $\overline{\mu}_{(n1,n2,n3)} = \frac{n1\mu_1 + n2\mu_2 + n3\mu_3}{n}$.

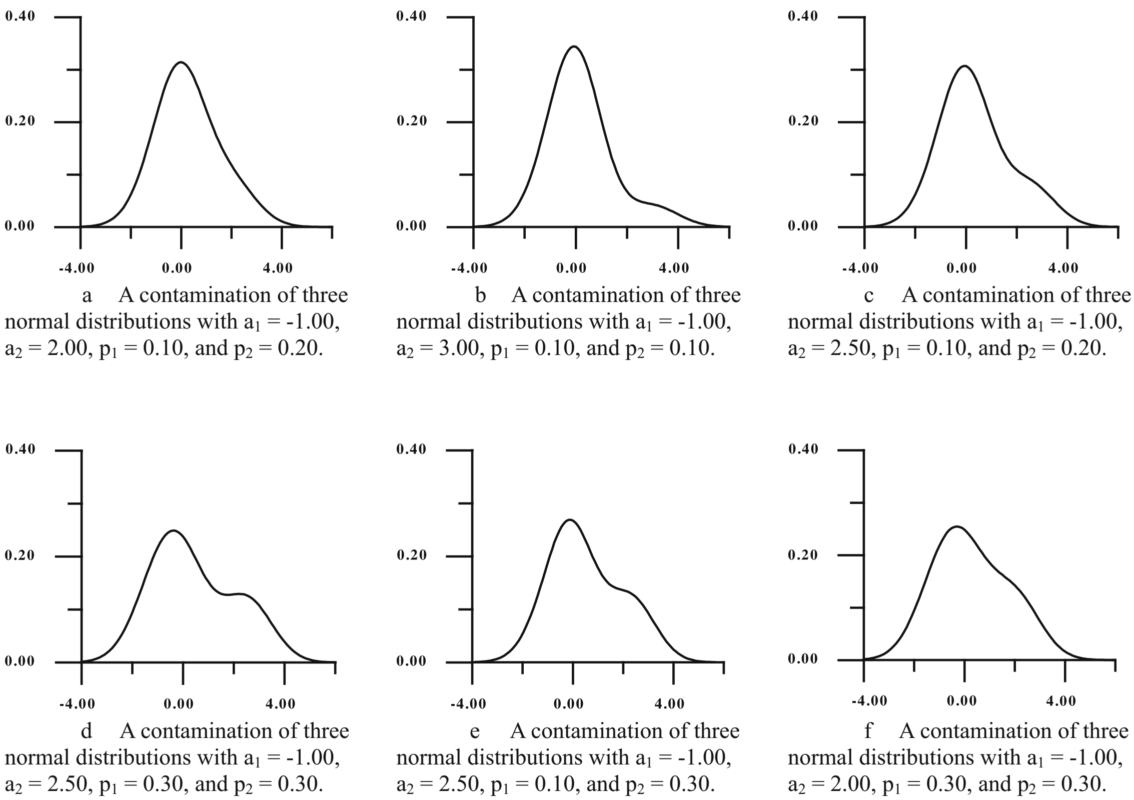
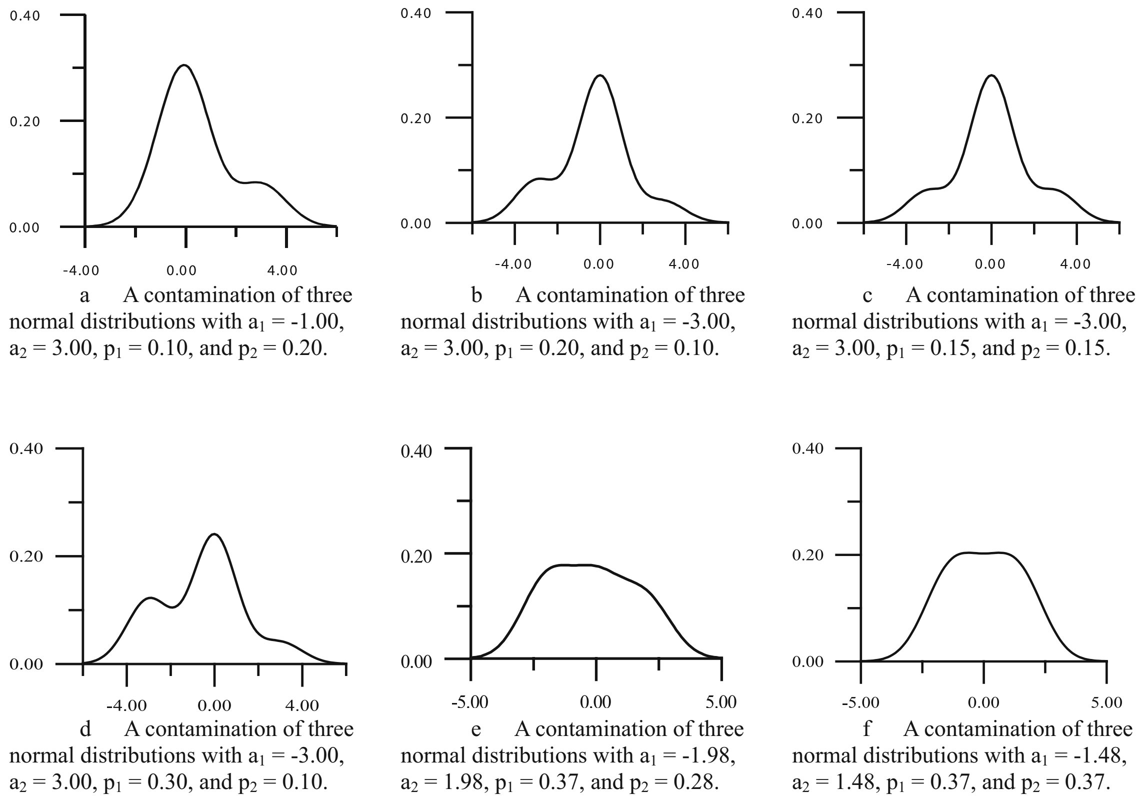


Fig. 1 **a** A contamination of three normal distributions with a_1=−1.00, a_2=2.00, p_1=0.10, and p_2=0.20. **b** A contamination of three normal distributions with a_1=−1.00, a_2=3.00, p_1=0.10, and p_2=0.10. **c** A contamination of three normal distributions with a_1=−1.00, a_2=2.50, p_1=0.10, and p_2=0.20. **d** A contamination of three normal distributions with a_1=−1.00, a_2=2.50, p_1=0.30, and p_2=0.30. **e** A contamination of three normal distributions with a_1=−1.00, a_2=2.50, p_1=0.10, and p_2=0.30. **f** A contamination of three normal distributions with a_1=−1.00, a_2=2.00, p_1=0.30, and p_2=0.30



a A contamination of three normal distributions with a_1 = -1.00, a_2 = 3.00, p_1 = 0.10, and p_2 = 0.20.

b A contamination of three normal distributions with a_1 = -3.00, a_2 = 3.00, p_1 = 0.20, and p_2 = 0.10.

c A contamination of three normal distributions with a_1 = -3.00, a_2 = 3.00, p_1 = 0.15, and p_2 = 0.15.

d A contamination of three normal distributions with a_1 = -3.00, a_2 = 3.00, p_1 = 0.30, and p_2 = 0.10.

e A contamination of three normal distributions with a_1 = -1.98, a_2 = 1.98, p_1 = 0.37, and p_2 = 0.28.

f A contamination of three normal distributions with a_1 = -1.48, a_2 = 1.48, p_1 = 0.37, and p_2 = 0.37.

Fig. 2 **a** A contamination of three normal distributions with a_1=−1.00, a_2=3.00, p_1=0.10, and p_2=0.20. **b** A contamination of three normal distributions with a_1=−3.00, a_2=3.00, p_1=0.20, and p_2=0.10. **c** A contamination of three normal distributions with a_1=−3.00, a_2=3.00, p_1=0.15, and p_2=0.15. **d** A contamination of three normal distributions with a_1=−3.00, a_2=3.00, p_1=0.30, and p_2=0.10. **e** A contamination of three normal distributions with a_1=−1.98, a_2=1.98, p_1=0.37, and p_2=0.28. **f** A contamination of three normal distributions with a_1=−1.48, a_2=1.48, p_1=0.37, and p_2=0.37

Kotz and Johnson [11] also showed that the *r*-th moment (about zero) of the natural estimator $\widehat{C}_p$, can be expressed as:

$$E\left[\widehat{C}_p^r\Big|_{(n1,n2,n3)}\right] = \left[C_p^r(n-1)^{r/2}\exp\left(-\frac{1}{2}\lambda_{(n1,n2,n3)}\right)\right] \times \left[\sum_{i=0}^{\infty}\left\{\frac{\left[\frac{1}{2}\lambda_{(n1,n2,n3)}\right]^i}{i!}\right\}\frac{\Gamma\left(\frac{n-1}{2}+i-\frac{r}{2}\right)}{2^{r/2}\Gamma\left(\frac{n-1}{2}+i\right)}\right].$$

Table 2 The corresponding values of a_1, a_2, and p_1, p_2 for Fig. 1a–f and Fig. 2a–f

Figure	a_1	a_2	p_1	p_2
1(a)	−1.00	2.00	0.10	0.20
1(b)	−1.00	3.00	0.10	0.10
1(c)	−1.00	2.50	0.10	0.20
1(d)	−1.00	2.50	0.30	0.30
1(e)	−1.00	2.50	0.10	0.30
1(f)	−1.00	2.00	0.30	0.30
2(a)	−1.00	3.00	0.10	0.20
2(b)	−3.00	3.00	0.20	0.10
2(c)	−3.00	3.00	0.15	0.15
2(d)	−3.00	3.00	0.30	0.10
2(e)	−1.98	1.98	0.37	0.28
2(f)	−1.48	1.48	0.37	0.37

Table 3 The corresponding nonconformities in parts-per-million (ppm_1) for processes with C_p=1

Figure	σ_c	C_p	ppm_1	ppm_2
1(a)	1.35	1.00	2,330	2,700
1(b)	1.40	1.00	8,240	2,700
1(c)	1.48	1.00	2,068	2,700
1(d)	1.72	1.00	299	2,700
1(e)	1.60	1.00	582	2,700
1(f)	1.55	1.00	587	2,700
2(a)	1.63	1.00	1,754	2,700
2(b)	1.90	1.00	1,090	2,700
2(c)	1.92	1.00	840	2,700
2(d)	2.06	1.00	522	2,700
2(e)	1.88	1.00	98	2,700
2(f)	1.62	1.00	272	2,700

By setting r=1, we may obtain

$$E\left[\widehat{C}_p|_{(n1,n2,n3)}\right] = \left[C_p(n-1)^{1/2}\exp\left(-\frac{1}{2}\lambda_{(n1,n2,n3)}\right)\right] \times \left[\sum_{i=0}^{\infty}\left\{\frac{\left[\frac{1}{2}\lambda_{(n1,n2,n3)}\right]^i}{i!}\right\}\frac{\Gamma\left(\frac{n}{2}+i-1\right)}{\sqrt{2}\Gamma\left(\frac{n-1}{2}+i\right)}\right],$$

and therefore

$$E\left[\widehat{C}_p\right] = \sum_{(n1,n2,n3)}\left[\frac{n!p_1^{n1}p_2^{n2}p_3^{n3}}{n1!n2!n3!}\right] \times \left[C_p\sqrt{n-1}\exp\left(-\frac{1}{2}\lambda_{(n1,n2,n3)}\right)\right] \times \left[\sum_{i=0}^{\infty}\left\{\frac{\left[\frac{1}{2}\lambda_{(n1,n2,n3)}\right]^i}{i!}\right\}\frac{\Gamma\left(\frac{n}{2}+i-1\right)}{\sqrt{2}\Gamma\left(\frac{n-1}{2}+i\right)}\right].$$

Table 4(a)–(c) display the expected values $E[\widehat{C}_p]$, and the variances $Var[\widehat{C}_p]$, for the cases of C_p=1 with a_1=a_2=0.005, 0.01, 0.125, 0.25, 0.5, 1.00, p_1=p_2=0.05, 0.25, and σ=1 for various sample sizes n=10(10)380. If we define:

$$\widehat{\Theta} = \sum_{(n1,n2,n3)}\left[\frac{n!p_1^{n1}p_2^{n2}p_3^{n3}}{n1!n2!n3!}\right] \times \left[\sqrt{n-1}\exp\left(-\frac{1}{2}\widehat{\lambda}_{(n1,n2,n3)}\right)\right] \times \left[\sum_{i=0}^{\infty}\left\{\frac{\left[\frac{1}{2}\widehat{\lambda}_{(n1,n2,n3)}\right]^i}{i!}\right\}\frac{\Gamma\left(\frac{n}{2}+i-1\right)}{\sqrt{2}\Gamma\left(\frac{n-1}{2}+i\right)}\right],$$

$$\text{where } \widehat{\lambda}_{(n1,n2,n3)} = \sum_{j=1}^{3} n_j\left(\overline{x}_j-\overline{x}\right)^2,$$

Therefore, in practice we may use the conventional estimator $\widetilde{C}_p = \widehat{C}_p\widehat{\Theta}$ to calculate C_p. That is,

$$E\left[\widetilde{C}_p\right] = E\left[\frac{\widehat{C}_p}{\widehat{\Theta}}\right] \approx C_p.$$

We note that $\frac{(n-1)S^2}{\sigma^2}\Big|_{(n1,n2,n3)} \sim \chi^2_{n-1}\left(\lambda_{(n1,n2,n3)}\right)$, and $S^2\big|_{(n1,n2,n3)} \sim \frac{\sigma^2}{n-1}\chi^2_{n-1}\left(\lambda_{(n1,n2,n3)}\right)$. Thus, the asymptotically confidence interval of C_p may be established as the following:

$$p\left[\frac{\sigma^2\chi^2_{n-1,\alpha/2}\left(\lambda_{(n1,n2,n3)}\right)}{n-1} \le S^2 \le \frac{\sigma^2\chi^2_{n-1,1-\alpha/2}\left(\lambda_{(n1,n2,n3)}\right)}{n-1}\Big|_{(n1,n2,n3)}\right] = 1-\alpha,$$

$$\sum_{(n1,n2,n3)}\left[\frac{n!p_1^{n1}p_2^{n2}p_3^{n3}}{n1!n2!n3!}\right] \times p\left[\frac{(n-1)S^2}{\chi^2_{n-1,1-\alpha/2}\left(\lambda_{(n1,n2,n3)}\right)} \le \sigma^2 \le \frac{(n-1)S^2}{\sigma^2\chi^2_{n-1,\alpha/2}\left(\lambda_{(n1,n2,n3)}\right)}\right] = 1-\alpha,$$

$$\sum_{(n1,n2,n3)}\left[\frac{n!p_1^{n1}p_2^{n2}p_3^{n3}}{n1!n2!n3!}\right] \times p\left[\frac{\sqrt{\chi^2_{n-1,\alpha/2}\left(\lambda_{(n1,n2,n3)}\right)}}{\sqrt{n-1}\left(\widetilde{C}_p\widehat{\Theta}\right)^{-1}} \le C_p \le \frac{\sqrt{\chi^2_{n-1,1-\alpha/2}\left(\lambda_{(n1,n2,n3)}\right)}}{\sqrt{n-1}\left(\widetilde{C}_p\widehat{\Theta}\right)^{-1}}\right] \approx 1-\alpha.$$

The estimated conditional length, given the samples (n_1, n_2, n_3), $l|_{(n1,n2,n3)}$, of the α-level (equal-tail) asymptotic confidence interval of C_p can be obtained as:

$$l|_{(n1,n2,n3)} = \frac{\widetilde{C}_p\widehat{\Theta}|_{(n1,n2,n3)}}{\sqrt{n-1}}\left[\sqrt{\chi^2_{n-1,1-\alpha/2}\left(\lambda_{(n1,n2,n3)}\right)} - \sqrt{\chi^2_{n-1,\alpha/2}\left(\lambda_{(n1,n2,n3)}\right)}\right],$$

Table 4 $E(l)$for C_p=1 , α=0.05 for various *n*

Sample size	$E(l)$	Sample size	$E(l)$
10	1.100	160	0.199
20	0.628	180	0.187
30	0.490	200	0.177
40	0.415	220	0.169
50	0.367	240	0.161
60	0.332	260	0.155
70	0.306	280	0.149
80	0.285	300	0.144
90	0.268	320	0.140
100	0.253	340	0.135
120	0.231	360	0.131
140	0.213	380	0.128

and the length of the asymptotically estimated confidence interval can be found as:

$$l = \sum_{(n1,n2,n3)} \left[\frac{n!p_1^{n1}p_2^{n2}p_3^{n3}}{n1!n2!n3!}\right] \times \frac{\widetilde{C}_p\Theta|_{(n1,n2,n3)}}{\sqrt{n-1}} \left[\sqrt{\chi^2_{n-1,1-\alpha/2}\left(\lambda_{(n1,n2,n3)}\right)} - \sqrt{\chi^2_{n-1,\alpha/2}\left(\lambda_{(n1,n2,n3)}\right)}\right].$$

Therefore, the asymptotically expected length, $E(l)$, of C_P is:

$$E(l) = \sum_{(n1,n2,n3)} \left[\frac{n!p_1^{n1}p_2^{n2}p_3^{n3}}{n1!n2!n3!}\right] \times \frac{E\left[\widehat{C}_p|_{(n1,n2,n3)}\right]}{\sqrt{n-1}} \times \left[\sqrt{\chi^2_{n-1,1-\alpha/2}\left(\lambda_{(n1,n2,n3)}\right)} - \sqrt{\chi^2_{n-1,\alpha/2}\left(\lambda_{(n1,n2,n3)}\right)}\right].$$

For large sample size n, the following approximation may be applied [7]:

$$\begin{aligned} &p\left[\chi^2_{n-1}\left(\lambda_{(n1,n2,n3)}\right) \le \chi^2_{n-1,\alpha}\left(\lambda_{(n1,n2,n3)}\right)\right] \\ &\cong p\left[Z \le \frac{\chi^2_{n-1,\alpha}\left(\lambda_{(n1,n2,n3)}\right) - (n-1) - \lambda_{(n1,n2,n3)} + 1}{\sqrt{2\left((n-1) + 2\lambda_{(n1,n2,n3)}\right)}}\right] \\ &= p[Z \le z_\alpha] = \alpha. \end{aligned}$$

Thus, $\frac{\chi^2_{n-1,\alpha}\left(\lambda_{(n1,n2,n3)}\right) - (n-1) - \lambda_{(n1,n2,n3)} + 1}{\sqrt{2\left((n-1)+2\lambda_{(n1,n2,n3)}\right)}} \cong z\alpha$, or equivalently,

$$\sqrt{\chi^2_{n-1,\alpha}\left(\lambda_{(n1,n2,n3)}\right)} \cong \left\{z_\alpha\sqrt{2\left((n-1) + 2\lambda_{(n1,n2,n3)}\right)} + (n-1) + \lambda_{(n1,n2,n3)} - 1\right\}^{1/2}.$$

Therefore, the asymptotically expected length of the confidence interval, $E(l)$, of C_p can be approximated by:

$$\sum_{(n1,n2,n3)} \left[\frac{n!p_1^{n1}p_2^{n2}p_3^{n3}}{n1!n2!n3!}\right] \times \frac{E\left[\widehat{C}_p|(n1,n2,n3)\right]}{\sqrt{n-1}} \times \left[\begin{array}{l} \left\{z_{1-\alpha/2}\sqrt{2\left((n-1) + 2\lambda_{(n1,n2,n3)}\right)} + (n-1) + \lambda_{(n1,n2,n3)} - 1\right\}^{1/2} \\ -\left\{z_{-\alpha/2}\sqrt{2\left((n-1) + 2\lambda_{(n1,n2,n3)}\right)} + (n-1) + \lambda_{(n1,n2,n3)} - 1\right\}^{1/2} \end{array}\right].$$

For the contaminated distribution depicted in Fig. 1a with parameters a_1=−1.00, a_2=2.00, p_1=0.10, and p_2=0.20, the expected length $E(l)$, of the confidence interval for C=1.00 and α=0.05, with various sample sizes n=10(10) 100, and 120(20)380 are displayed in Table 4.

4 Testing process precision

Due to the complexity and difficulty in applying the analytical results obtained by the statisticians, practitioners have taken unreliable approaches in evaluating their process capability by simply looking at the index values calculated from the sample data. In the following, we take into account the sampling errors and develop a procedure similar to those described in [2, 3, 15] using an asymptotic estimator of C_p for processes with three manufacturing lines. To judge whether the process meets the capability requirement preset in the factory and runs under the desired quality condition, we can consider the following statistical testing hypothesis. A given process meets the preset capability (quality) requirement if C_p>C, and fails to meet the preset capability requirement if $C_p \le$C.

H_0: $C_p \le$C (process is incapable); H_1: C_p>C (process is capable). By recognizing that given the samples (n1, n2, n3) the statistic W=

$$p\left\{\widehat{C}_p\big/\widehat{\Theta} > c_0|C_p = C\right\} \approx \alpha,$$

$(C_p)^2(n-1)\big/\left(\widehat{C}_p\right)^2$ is distributed as $\chi^2_{n-1}\left(\lambda_{(n1,n2,n3)}\right)$,

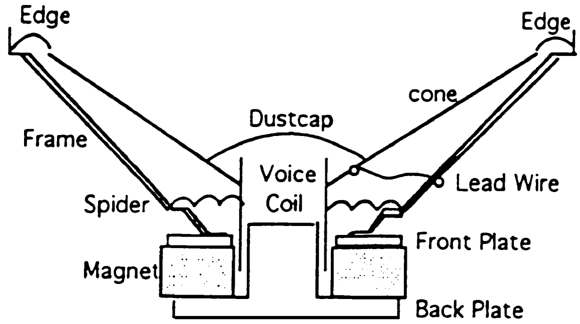


Fig. 3 A subwoofer driver

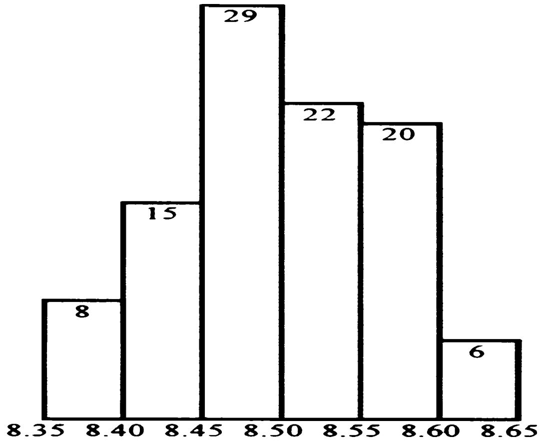


Fig. 4 Histogram of the data

the critical value c_0 can be obtained by solving the equation $p\left\{\widetilde{C}_p > c_0 \middle| C_p = C\right\} \approx \alpha$

$$\sum_{(n1,n2,n3)} \left[\frac{n!p_1^{n1}p_2^{n2}p_3^{n3}}{n1!n2!n3!}\right] \times p\left\{\frac{C_p\sqrt{n-1}}{\sqrt{W}} > \widehat{\Theta}c_0 \middle| C_p = C, (n1, n2, n3)\right\} \approx \alpha,$$

$$\sum_{(n1,n2,n3)} \left[\frac{n!p_1^{n1}p_2^{n2}p_3^{n3}}{n1!n2!n3!}\right] \times p\left\{W < \left[\frac{C\sqrt{n-1}}{\widehat{\Theta}c_0}\right]^2 \Big|_{(n1,n2,n3)}\right\} \approx \alpha,$$

$$\sum_{(n1,n2,n3)} \left[\frac{n!p_1^{n1}p_2^{n2}p_3^{n3}}{n1!n2!n3!}\right] \times p\left\{\chi^2_{n-1}\left(\lambda_{(n1,n2,n3)}\right) < \left[\frac{C\sqrt{n-1}}{\widehat{\Theta}c_0}\right]^2\right\} \approx \alpha,$$

Obviously, the computation for finding the critical value c_0 is rather complicated. To simplify the computations, some approximation methods must be applied (which will not be discussed in this paper), and the equation has to be solved via some numerical methods. To apply this approach, we first decide the values of C, α-risk (the chance of incorrectly concluding an incapable process capable), and calculate $\widehat{C}_p$, $a_1 = \left(\overline{X}_2 - \overline{X}_3\right)/S$ and $a_2 = \left(\overline{X}_2 - \overline{X}_3\right)/S$, where S estimates the standard deviation σ. We then compute the estimators, $\widehat{\Theta}$, $\widetilde{C}_p$, and c_0. If $\widetilde{C}_p$ is greater than c_0, then we may conclude that the process is capable (C_p>C). Otherwise, we do not have enough information to conclude this.

4.1 Test procedures

STEP1

Decide the definition of "capable" (C, normally set to 1.00, 1.33, 1.50, or 2.00), and the α-risk (normally set to 0.01, 0.025, or 0.05).

Table 5 The collected sample data of 100 observations

Manufacturing line I									
8.37	8.51	8.57	8.45	8.46	8.53	8.49	8.44	8.51	8.48
8.48	8.41	8.62	8.38	8.50	8.38	8.52	8.52	8.47	8.47
8.44	8.58	8.41	8.36	8.53	8.40	8.45	8.56	8.43	
Manufacturing line II									
8.43	8.55	8.58	8.56	8.59	8.51	8.55	8.55	8.57	8.44
8.41	8.47	8.52	8.48	8.62	8.45	8.46	8.39	8.57	8.47
8.54	8.48	8.47	8.56	8.50	8.63	8.59	8.49	8.51	8.48
8.54	8.65								
Manufacturing line III									
8.36	8.44	8.56	8.54	8.44	8.55	8.36	8.53	8.51	8.53
8.55	8.45	8.45	8.57	8.45	8.59	8.46	8.42	8.58	8.56
8.45	8.49	8.62	8.50	8.56	8.60	8.51	8.46	8.56	8.60
8.54	8.43	8.39	8.47	8.47	8.50	8.45	8.55	8.49	

STEP2

Calculate $\widehat{C}_p$ and $a_1 = (\overline{X}_1 - \overline{X}_2)/S$, and $a_2 = (\overline{X}_2 - \overline{X}_3)/S$, where S estimates the standard deviation σ.

STEP3

Calculate $\widehat{\Theta}$, $\widetilde{C}_p = \widehat{C}_p / \widehat{\Theta}$, and c_0 with input C, a_1, a_2, p_1, p_2, and n.

STEP4

Conclude that the process is capable if $\widetilde{C}_p$ value is greater than the critical value c_0. Otherwise, we do not have enough information to conclude that the process is capable.

5 Pulux surround manufacturing

We consider a real example of a manufacturing process making speaker drivers including 3-inch tweeters, 3-inch and 4-inch full-ranges, 5-inch mid-ranges, 6.5-inch woofers, 8-inch, 10-inch, 12-inch, 15-inch, and 18-inch subwoofers. A standard woofer or subwoofer driver, depicted in Fig. 3, consists of the following components: edge, cone, dust cap, spider (also called damper), voice coil, lead wire, frame, magnet, front plate, and back plate (also called T-york). The edge (on the top) and the spider (on the bottom) are glued onto the frame to hold the cone during the piston movements, and the dust cap is glued onto the center top of the cone to cover the voice coil, which decouples the noise from the musical signals.

One component that critically determines the bass performance, musical image, clarity and cleanness of the sound, transparence, and compliance (excursion movement) of the mid-range, full range, woofer, and subwoofer driver units is the edge (rubber surround). Some key factors determining the rubber surround include the hardness, thickness, and the weight. The weight of the rubber surround is essential to the quality of the driver. As the advancement of the material science and technology, the newly developed Pulux has gradually become an excellent choice of material for replacing the edge with the traditional rubber edge. We consider the following sample data of the Pulux-surround weight collected from the factory. The specifications, LSL and USL for a particular model were set to 8.25 and 8.75 (in grams, see Fig. 4). The 100 observations taken from a combined process with three manufacturing lines are displayed in Fig. 5 with probability p=0.3 for line I, and p=0.3 for line II, and p=0.4 for line III.

Table 5 displays the histogram of the collected data of 100 observations. The factory has defined a process "capable" if C_p>1.00. Clearly, at the current stage the factory's effort in quality improvement has mainly focused on the reduction of process variation. We first calculated $\widehat{C}_p = 1.24$, a_1=−0.44, and a_2=0.29. We then calculated $\widehat{\Theta} = 1.00$, and c_0=1.14 with input C=1.00, a_1=−0.44, a_2=0.29, p_1=0.3, p_2=0.3, and the sample size n=100 (assuming the α-risk is 0.05). Since $\widetilde{C}_p = C_p / \widehat{\Theta} = 1.23$ is greater than the critical value c_0, we conclude that the process is capable.

6 Conclusions

The precision index C_p for processes with a single manufacturing line has been investigated extensively. However, research on C_p for processes with multiple manufacturing lines with non-normal distributions has been comparatively neglected. For processes with multiple manufacturing lines, the raw materials, or equipment, or workmanship may not be identical in precision for each manufacturing line. In this paper, we considered the precision index C_p for processes with three manufacturing lines. We investigated the nonconformities for various manufacturing conditions, established confidence intervals of C_p, and computed their expected lengths. We also considered a hypothesis testing, which can be used to judge whether a process meets the precision requirement. An application example for Pulux Edge manufacturing with three manufacturing lines is given.

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