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Identification of mechanical properties of elastically restrained laminated composite plates using vibration data

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Abstract

A nondestructive evaluation method using vibration data to determine mechanical properties (material and spring constants) of elastically restrained laminated composite plates is presented. The Rayleigh–Ritz method in which a set of Legendre's polynomials is adopted to approximate the plate deflection is used to determine the theoretical natural frequencies of the elastically restrained laminated composite plates. A number of natural frequencies extracted from the impulse vibration test data of the laminated composite plates supported by elastic restraints at both the edges and centers of the plates are used in the present method to determine the mechanical properties of the plates. The sum of the squared differences function which measures the differences between the experimentally and theoretically predicted natural frequencies of the elastically restrained laminated composite plates is established. The identification of the plate mechanical properties is then formulated as a constrained minimization problem in which the mechanical properties are determined by making the sum of the squared differences function a global minimum. The feasibility and accuracy of the proposed method are studied by means of several numerical examples on the mechanical properties identification of elastically restrained laminated composite plates with different layups made of various composite materials. Experimental investigation of the mechanical properties identification of several elastically restrained laminated composite plates has been performed to illustrate the applications of the present method. It has been shown that the present method can produce good estimates of the mechanical properties of the elastically restrained laminated composite plates in an efficient and effective way.

1. Introduction

Owing to their many advantageous properties, the fiber-reinforced composite plates have been increasingly used in the aeronautical and aerospace industry as well as many other fields of modern technology. The attainment of the actual behavioral predictions of such structures usually depends on the correctness of the system parameters of the structures such as the elastic constants of the materials constituting the structures and the stiffnesses of the supports restraining the structures. As is well known, composite structures fabricated by different methods or curing processes may possess different mechanical properties and the material

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constants of the structures in service will change due to structural and material degradations. Therefore, the material properties determined from standard specimens tested in laboratory in general may deviate from those of the laminated composite components manufactured in factory or the existing composite structures. In recent years, the determination of realistic material constants of structural components/structures has become an important topic of research and different techniques for elastic constants identification of beam and plate types of structures have been proposed. For instance, Castagnède et al. [1] determined the elastic constants of thick composite plates via a quantitative ultrasonic approach. Fallstrom and Jonsson [2] determined the material constants of anisotropic plates using the frequencies and mode shapes measured by a real-time TV-holography system. Nielsenand and Toftegaard [3] used the ultrasonic measurement approach to obtain the elastic constants of fiber-reinforced polymer composites under the influence of absorbed moisture. Berman and Nagy [4] used measured normal modes and natural frequencies to improve an analytical mass and stiffness matrix model of a structure. Kam and his associates [5–10] developed methods to identify the element bending stiffnesses of beam structures using measured natural frequencies and mode shapes and determine elastic constants of shear deformable laminated composite plates using measured strains and/or displacements obtained from static testing of the plates. Recently, a number of researchers have used experimental natural frequencies to identify the elastic constants of laminated composite plates with free boundary conditions [11–18]. For instance, Moussu and Nivoit [14] used the method of superposition to determine the elastic constants of free rectangular plates from the measured experimental natural frequencies of the plates. Wilde and Sol [15] used the method of Bayesian estimation to study the identification of elastic constants from the experimental natural frequencies of free rectangular composite plates. Araujo et al. [16,17] used an optimization method to determine the elastic constants of free composite plates using the measured natural frequencies of the plates. In general, the previously proposed methods were only applicable for plates with simple boundary conditions and might require the use of 12–16 natural frequencies in the elastic constants identification process if obtaining results with satisfactory accuracy was desired. In view of the fact that the dynamical behaviors of plates with elastic restraints are very different from those with simple boundary conditions, when using vibration data to identify the mechanical properties of a flexibly supported plate, it is expected that the elastic restraints of the plate will play an important role in the identification. Therefore, if realistic mechanical properties of the plate are to be determined nondestructively, the effects of the elastic restraints on the identified properties must be taken into consideration. Although the system identification of plates with flexible supports is an important topic of research, so far not much work has been devoted to this area.

In this paper, a nondestructive evaluation method is presented for the identification of mechanical properties of laminated composite plates elastically restrained both at the edges and in the interior of the plates. The Rayleigh–Ritz method together with an appropriate set of characteristic functions is used to predict the natural frequencies of the flexibly supported laminated composite plates. Vibration tests of the flexibly supported laminated composite plates are performed to extract the natural frequencies of the plates from the measured vibration data. The sum of the squared differences function which measures the differences between the experimental and theoretical predictions of natural frequencies of the laminated composite plates is established. The identification of mechanical properties is then formulated as a constrained minimization problem in which the mechanical properties are determined by making the sum of the squared differences function a global minimum. A multi-start global minimization method is used to search for the global minimum and a normalization technique for normalizing the design variables is adopted to increase the convergence rate of the solution. A number of examples of the mechanical properties identification of elastically restrained laminated composite plates with different layups made of different composite materials are given to illustrate the accuracy and feasibility of the proposed method. Several flexibly supported laminated composite plates are subjected to impulse vibration testing. The measured natural frequencies of the composite plates are used in the present method to identify the mechanical properties of the plates.

2. Plate vibration analysis

Without loss of generality, consider the elastically restrained rectangular symmetrically laminated composite plate of area $a_0 \times b_0$ and constant thickness h composed of a finite number of orthotropic layers of same material properties and thickness in Fig. 1. The x and y coordinates of the plate are taken in the mid-plane of the plate. The plate is supported continuously around the edges by flexible strip-type pads of cross-sectional dimensions $b_e \times h_e$ and at the center by an annulus-type flexible restraint with inner radius r_i. For the flexible supports considered in this study, it is further assumed that the dimensions, b_e and r_i, of the elastic supports are much smaller than the plate dimensions, a_0 and b_0, so that the edge flexible supports of the plate can be modeled by longitudinal and torsional springs while the center support by a longitudinal spring as shown in Fig. 2. It is noted that the plate size used in the vibration analysis is $a \times b$ in which $a = a_0 - b_e$ and $b = b_0 - b_e$. For free vibration, the plate vertical displacement $w(x, y, t)$ is assumed to be of the form

$$w(x, y, t) = W(x, y) \sin \omega t, \tag{1}$$

where $W(x, y)$ is the deflection function and ω is the angular frequency. According to the classical lamination theory with the neglect of the rotary inertia effect, the maximum strain energy U_P and maximum kinetic energy T of the plate are expressed as [19]

$$U_P = \frac{1}{2}\int_0^b \int_0^a \left[D_{11}\left(\frac{\partial^2 W}{\partial x^2}\right)^2 + 2D_{12}\left(\frac{\partial^2 W}{\partial x^2}\right)\left(\frac{\partial^2 W}{\partial y^2}\right) + D_{22}\left(\frac{\partial^2 W}{\partial y^2}\right)^2 + 4D_{66}\left(\frac{\partial^2 W}{\partial x \partial y}\right)^2 + 4D_{16}\left(\frac{\partial^2 W}{\partial x^2}\right)\left(\frac{\partial^2 W}{\partial x \partial y}\right) + 4D_{26}\left(\frac{\partial^2 W}{\partial y^2}\right)\left(\frac{\partial^2 W}{\partial x \partial y}\right)\right] \mathrm{d}x\,\mathrm{d}y \tag{2}$$

and

$$T = \frac{1}{2}\rho h \omega^2 \int_0^b \int_0^a W^2 \,\mathrm{d}x\,\mathrm{d}y, \tag{3}$$

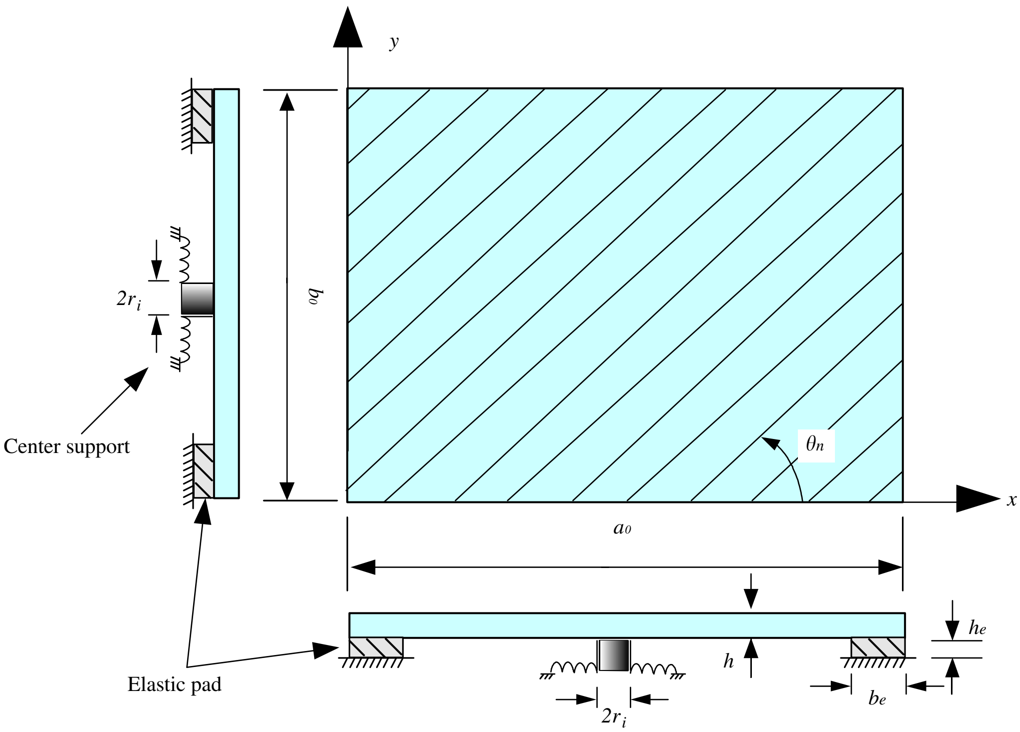

Fig. 1. Elastically restrained laminated composite plate.

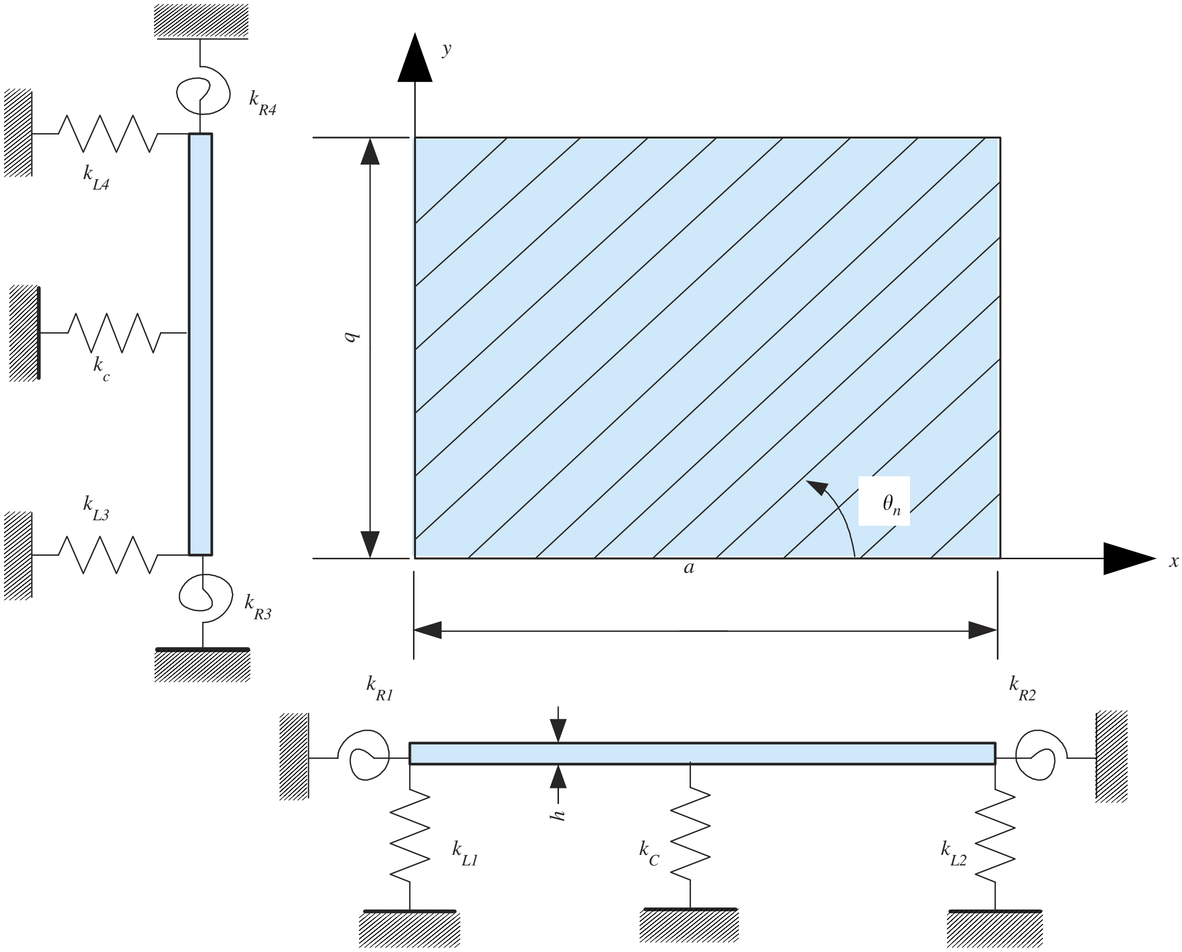

Fig. 2. Mathematical model of elastically restrained composite plate.

where D_{ij} are bending stiffness coefficients and ρ is material density. The bending stiffness coefficients are given by

$$D_{ij} = \int_{-h/2}^{h/2} \bar{Q}_{ij}^{(m)} z^2 \, \mathrm{d}z \quad (i,j = 1, 2, 6). \tag{4}$$

The transformed lamina stiffness coefficients $\bar{Q}_{ij}^{(m)}$ depend on the material properties and fiber orientation of the mth layer. For a layer with zero fiber angle, the lamina stiffness coefficients are expressed as

$$\begin{aligned} &Q_{11} = \frac{E_1}{1 - v_{12}v_{21}}, \quad Q_{12} = \frac{v_{12}E_2}{1 - v_{12}v_{21}} = Q_{21}, \quad Q_{22} = \frac{E_2}{1 - v_{12}v_{21}}, \\ &Q_{66} = G_{12} \quad \text{with } \frac{v_{12}}{E_1} = \frac{v_{21}}{E_2}, \end{aligned} \tag{5}$$

where E_1, E_2 are Young's moduli in the fiber and transverse directions, respectively; v_{ij} is the Poisson's ratio for transverse strain in the j-direction when stressed in the i-direction; G_{12} is shear modulus in the 1–2 plane.

For the plate with spring-type elastic supports, additional strain energy stored in the supporting springs exists. The maximum strain energy of the flexible restraints is

$$U_B = \frac{k_{L1}}{2}\left[\int_0^b W^2\,\mathrm{d}y\right]_{x=0} + \frac{k_{L2}}{2}\left[\int_0^b W^2\,\mathrm{d}y\right]_{x=a} + \frac{k_{L3}}{2}\left[\int_0^a W^2\,\mathrm{d}y\right]_{y=0} + \frac{k_{L4}}{2}\left[\int_0^a W^2\,\mathrm{d}y\right]_{y=b}$$
$$+ \frac{k_{R1}}{2}\left[\int_0^b \left(\frac{\partial W}{\partial x}\right)^2 \mathrm{d}y\right]_{x=0} + \frac{k_{R2}}{2}\left[\int_0^b \left(\frac{\partial W}{\partial x}\right)^2 \mathrm{d}y\right]_{x=a} + \frac{k_{R3}}{2}\left[\int_0^a \left(\frac{\partial W}{\partial y}\right)^2 \mathrm{d}y\right]_{y=0}$$
$$+ \frac{k_{R4}}{2}\left[\int_0^a \left(\frac{\partial W}{\partial y}\right)^2 \mathrm{d}y\right]_{y=b} + \frac{k_C}{2}[W^2]_{x=a/2,y=b/2}, \tag{6}$$

where k_{Li} and k_{Ri} $(i = 1, \ldots, 4)$ are spring constants per unit length of the edge longitudinal and torsional springs, respectively; k_C is spring constant of the center spring. The integrals in the brackets of the above equation are evaluated at the four edges of the plate. Herein, the equivalent translational and rotational spring constants of the flexible strip-type pad with cross-sectional area of $b_e \times h_e$ and Young's modulus E_e used as an edge support as shown in Fig. 3 are to be approximated via the mechanics of materials approach. In the determination of the translational spring constant, it is assumed that the load is distributed uniformly on the top surface of the edge support where the top surface after deformation remains plane and horizontal as shown in Fig. 3a. Hence, when treating the flexible pad of unit length as an axial member, the translational spring constant per unit length is obtained as

$$k_L = \frac{E_e b_e}{h_e}. \tag{7}$$

To determine the rotational spring constant, it is assumed that the moment-induced load is distributed linearly across the width of the support where the top surface of the pad remains plane after rotation as shown in Fig. 3b. Hence, when treating the top surface of the flexible pad of unit length as a beam section, the rotational spring constant per unit length is obtained as

$$k_R = \frac{E_e b_e^3}{12 h_e}. \tag{8}$$

In view of Eqs. (2) and (6), the total strain energy, U, is then written as

$$U = U_P + U_B. \tag{9}$$

Based on the Rayleigh–Ritz method, the deflection function expressed in the nondimensional form is

$$W(\xi, \eta) = \sum_{i=1}^{I}\sum_{j=1}^{J} C_{ij}\phi_i(\xi)\varphi_j(\eta), \tag{10}$$

where C_{ij} are undetermined displacement coefficients, $\phi_i(\xi)$ and $\phi_j(\eta)$ are the characteristic functions. In this study, the Legendre's orthogonal polynomials with $\xi = (2x/a) - 1$ for $-1 \leqslant \xi \leqslant 1$ and $\eta = (2y/b) - 1$ for $-1 \leqslant \eta \leqslant 1$ are chosen to formulate the characteristic functions. In terms of the Legendre's orthogonal polynomials, for instance, $\phi_i(\xi)$ can be written as

$$\phi_1(\xi) = 1,$$
$$\phi_2(\xi) = \xi$$

and if $n \geqslant 3$,

$$\phi_n(\xi) = [(2n-3)\xi \times \phi_{n-1}(\xi) - (n-2) \times \phi_{n-2}(\xi)]/(n-1). \tag{11}$$

It is noted that the above characteristic functions $\phi_i(\xi)$ satisfy the orthogonality condition

$$\int_{-1}^{1} \phi_n(\xi)\phi_m(\xi)\,\mathrm{d}\xi = \begin{cases} 0 & \text{if } n \neq m, \\ \frac{2}{(2n-1)} & \text{if } n = m. \end{cases} \tag{12}$$

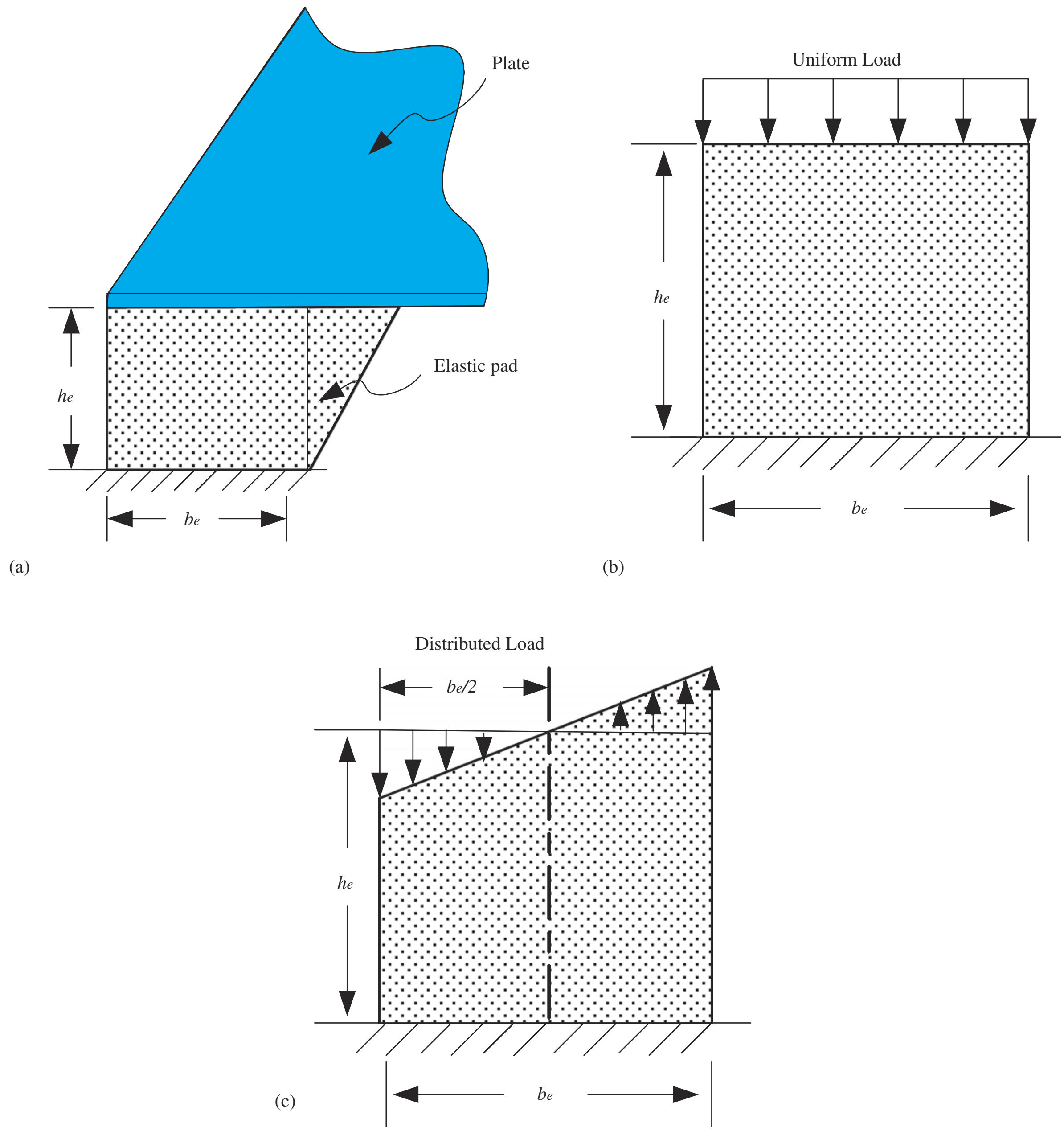

Fig. 3. Model of edge support made of elastic pad.

Extremization of the functional Π which is defined as $\Pi = U - T$ with respect to the displacement coefficients C_{ij} leads to the following eigenvalue problem:

$$([\mathbf{K}] - \lambda^2[\mathbf{M}])\{\mathbf{C}\} = 0 \tag{13}$$

with $\mathbf{K} = \mathbf{K}_P + \mathbf{K}_B$ where $\lambda = \sqrt{\rho h \omega^2 a^4 / D_0}$, the nondimensionalized natural frequencies; $\{\mathbf{C}\}$ is the displacement coefficient vector; $D_0 = E_1 h^3/[12(1 - \nu_{12}\nu_{21})]$; $\mathbf{K}$ is the structural stiffness matrix of the flexibly supported plate; $\mathbf{K}_P$ and $\mathbf{K}_B$ are portions of the structural stiffness matrix contributed by the stiffnesses of the laminated plate and edge restraints, respectively. The elements of $\mathbf{K}_P$, $\mathbf{K}_B$, and $\mathbf{M}$ are obtained, respectively, as

$$[K_P]_{mnij} = \frac{16}{D_0}\{D_{11}E^{22}_{mi}F^{00}_{nj} + \alpha^2 D_{12}(E^{02}_{mi}F^{20}_{nj} + E^{20}_{mi}F^{02}_{nj}) + \alpha^4 D_{22}E^{00}_{mi}F^{22}_{nj} + 2\alpha D_{16}(E^{21}_{mi}F^{01}_{nj} + E^{12}_{mi}F^{10}_{nj}) + 2\alpha^3 D_{26}(E^{01}_{mi}F^{21}_{nj} + E^{10}_{mi}F^{12}_{nj}) + 4\alpha^2 D_{66}E^{11}_{mi}F^{11}_{nj}\}, \tag{14}$$

$$[K_B]_{mnij} = 2 \times \{K_1 F^{00}_{nj}\phi_m(-1)\phi_i(-1) + K_2 F^{00}_{nj}\phi_m(1)\phi_i(1) + \alpha^4(K_3 E^{00}_{mi}\varphi_n(-1)\varphi_j(-1) + K_4 E^{00}_{mi}\varphi_n(1)\varphi_j(1)) + 4 \times [R_1 F^{00}_{nj}\phi'_m(-1)\phi'_i(-1) + R_2 F^{00}_{nj}\phi'_m(1)\phi'_i(1) + \alpha^4(R_3 E^{00}_{mi}\varphi'_n(-1)\varphi'_j(-1) + R_4 E^{00}_{mi}\varphi'_n(1)\varphi'_j(1))]\} + 4\alpha^3 K\phi_m(0)\phi_i(0)\varphi_n(0)\varphi_j(0) \tag{15}$$

and

$$[M]_{mnij} = E^{00}_{mi}F^{00}_{nj}, \quad m,i = 1,2,3,\ldots M,I; \quad n,j = 1,2,3,\ldots N,J; \quad \alpha = a/b \tag{16}$$

with

$$E^{rs}_{mi} = \int_{-1}^{1}\left[\frac{\mathrm{d}^r\phi_m(\xi)}{\mathrm{d}\xi^r}\frac{\mathrm{d}^s\phi_i(\xi)}{\mathrm{d}\xi^s}\right]\mathrm{d}\xi; \quad F^{rs}_{nj} = \int_{-1}^{1}\left[\frac{\mathrm{d}^r\varphi_n(\eta)}{\mathrm{d}\eta^r}\frac{\mathrm{d}^s\varphi_j(\eta)}{\mathrm{d}\eta^s}\right]\mathrm{d}\eta; \quad r,s = 0,1,2, \tag{17}$$

$$(K_1, K_2, K_3, K_4) = (k_{L1}a^3/D_0, k_{L2}a^3/D_0, k_{L3}b^3/D_0, k_{L4}b^3/D_0), \tag{18}$$

$$(R_1, R_2, R_3, R_4) = (k_{R1}a/D_0, k_{L2}a/D_0, k_{L3}b/D_0, k_{L4}b/D_0) \tag{19}$$

and

$$K = k_C b^2/D_0. \tag{20}$$

The solution of Eq. (13) gives the theoretical natural frequencies of the flexibly supported laminated composite plate. The theoretically predicted natural frequencies may deviate from the actual natural frequencies of the flexibly supported laminated composite plate if incorrect mechanical properties such as E_1, E_2, G_{12}, v_{12}, E_e and k_C are used in the frequency analysis of the plate. In the following section, a method is presented to identify the mechanical properties of flexibly supported laminated composite plates by minimizing the differences between the theoretical and experimental predictions of natural frequencies of the plates.

3. The inverse problem

The problem of mechanical properties identification of elastically restrained laminated composite plates is formulated as a minimization problem. In mathematical form it is stated as

$$\begin{aligned} &\text{Minimize} \quad e(\mathbf{x}) = (\boldsymbol{\omega}^*)^t(\boldsymbol{\omega}^*), \\ &\text{Subject to} \quad x_i^L \leqslant x_i \leqslant x_i^U, \quad i = 1 - N, \end{aligned} \tag{21}$$

where $\mathbf{x} = [E_1, E_2, G_{12}, v_{12}, E_e, k_C]$ the vector containing the design variables used to denote the mechanical properties of the elastically restrained laminated composite plates; $\boldsymbol{\omega}^*$ is an $N \times 1$ vector containing the differences between the measured and predicted values of the natural frequencies; $e(\mathbf{x})$ is the sum of the squared differences function measuring the differences between the predicted and measured data; x_i^L, x_i^U are the lower and upper bounds of the design variables. The elements in $\boldsymbol{\omega}^*$ are expressed as

$$\omega_i^* = \frac{\omega_{pi} - \omega_{mi}}{\omega_{mi}}, \quad i = 1{-}N, \tag{22}$$

where ω_{pi}, ω_{mi} are predicted and measured values of the natural frequencies, respectively. The above problem of Eq. (21) is then converted into an unconstrained minimization problem by creating the following general augmented Lagrangian [20]:

$$\bar{\boldsymbol{\Psi}}(\tilde{\mathbf{x}}, \boldsymbol{\mu}, \boldsymbol{\eta}, r_p) = e(\tilde{\mathbf{x}}) + \sum_{j=1}^{6}[\mu_j z_j + r_p z_j^2 + \eta_j \phi_j + r_p \phi_j^2] \tag{23}$$

with

$$z_j = \max\left[g_j(\tilde{x}_j), \frac{-\mu_j}{2r_p}\right], \quad g_j(\tilde{x}_j) = \tilde{x}_j - \tilde{x}_j^U \leqslant 0,$$
$$\phi_j = \max\left[H_j(\tilde{x}_j), \frac{-\eta_j}{2r_p}\right], \quad H_j(\tilde{x}_j) = \tilde{x}_j^L - \tilde{x}_j \leqslant 0, \quad j = 1-6, \tag{24}$$

where μ_j, η_j, γ_p are multipliers; max [*,*] takes on the maximum value of the numbers in the bracket. The modified design variables $\tilde{\mathbf{x}}$ are defined as

$$\tilde{\mathbf{x}} = \left[\frac{E_1}{\alpha_1}, \frac{E_2}{\alpha_2}, \frac{G_{12}}{\alpha_3}, v_{12}, \frac{E_e}{\alpha_4}, \frac{k_C}{\alpha_5}\right], \tag{25}$$

where α_i are normalization factors. It is noted that the values of α_i can affect the search direction and properly selected values of α_i can help expedite the convergence of the solution. In general, the values of $\tilde{x}_i (i = 1, \ldots, 4)$ are best chosen to be greater than 0 and less than 10. The modified design variables $\tilde{\mathbf{x}}$ are only used in the minimization algorithm while the original variables $\mathbf{x}$ are used in the Rayleigh–Ritz method to determine the natural frequencies of the plate. The updated formulas for the multipliers μ_j, η_j, and γ_p are

$$\mu_j^{n+1} = \mu_j^n + 2r_p^n z_j^n, \quad \eta_j^{n+1} = \eta_j^n + 2r_p^n \phi_j^n, \quad j = 1-6,$$
$$r_p^{n+1} = \begin{cases} \gamma_0 r_p^n & \text{if } r_p^{n+1} < r_p^{\max}, \\ r_p^{\max} & \text{if } r_p^{n+1} \geqslant r_p^{\max}, \end{cases} \tag{26}$$

where the superscript n denotes iteration number; γ_0 is a constant; $r_p^{\max}$ is the maximum value of r_p. The parameters μ_j^0, η_j^0, r_p^0, γ_0, $r_p^{\max}$ chosen based on experience are

$$\mu_j^0 = 1.0, \quad \eta_j^0 = 1.0, \quad r_p^0 = 0.4, \quad \gamma_0 = 2.5, \quad r_p^{\max} = 100. \tag{27}$$

The constrained minimization problem of Eq. (23) has thus become the solution of the following unconstrained optimization problem:

$$\text{Minimize } \bar{\boldsymbol{\Psi}}(\tilde{\mathbf{x}}, \boldsymbol{\mu}, \boldsymbol{\eta}, r_p). \tag{28}$$

The above unconstrained optimization problem is to be solved using a multi-start global optimization algorithm. In the adopted optimization algorithm, the objective function is treated as the potential energy of a traveling particle and the search trajectories for locating the global minimum are derived from the equation of motion of the particle in a conservative force field [21,22]. The design variables, i.e., the plate elastic constants, Young's modulus of the edge elastic restraints, and spring constant of the interior support, that make the potential energy of the particle, i.e., objective function, the global minimum constitute the solution of the problem. In the minimization process, a series of starting points for the design variables of Eq. (25) are selected at random from the region of interest. The lowest local minimum along the search trajectory initiated from each starting point is determined and recorded. A Bayesian argument is then used to establish the probability of the current overall minimum value of the objective function being the global minimum, given the number of starts and the number of times this value has been achieved. The multi-start optimization procedure is terminated when a target probability, typically 0.99, has been exceeded.

4. Experimental investigation

A number of elastically restrained square laminated composite plates with layups $[0°]_8$, $[0°/90°]_{2S}$, and $[45°/-45°/45°]_S$ were fabricated for experimental investigation. The laminated composite plates were supported by strip-type elastic pads with cross-sectional dimensions of $b_e = 5.0$ mm and $h_e = 2.1$ mm around the peripheries of the plates with or without an annulus-type flexible support at the plate centers. The materials used to fabricate the laminated composite plates were T300/2500 graphite/epoxy prepreg tapes produced by Torayca Co., Japan. The elastic constants of the cured graphite/epoxy laminates were first determined experimentally using the standard testing procedure in accordance with the relevant ASTM specifications [23]. The means and coefficients of variation (c.o.v.) of the elastic constants determined using three standard specimens for each test are as follows:

$$E_1 = 146.503\,\text{GPa}(0.72\%), \quad E_2 = 9.223\,\text{GPa}(1.19\%), \quad G_{12} = 6.836\,\text{GPa}(3.16\%),$$
$$v_{12} = 0.306(0.19\%). \tag{29}$$

The values in the parentheses in the above equation denote the c.o.v.'s of the elastic constants of the composite material. The average layer thickness and mass density of the laminated composite plates were 0.125 mm and 1543 kg/m^3, respectively. The elastic constant E_e of the edge supporting pads was also determined following the standard ASTM tensile testing procedure. The mean and c.o.v. of E_e are 2.028 MPa and 2.3%, respectively. The center annulus support, which was made of corrugate fabric as shown in Fig. 4, was connected to the plate via a hollow cylindrical tube with negligible mass. The inner and outer radii of the center annulus support were $r_i = 12.5$ mm and $r_0 = 16$ mm, respectively. The translational spring constant of the center annulus support determined from static testing was $k_C = 3.865$ kN/m.

The elastically restrained laminated composite plates were subjected to impulse vibration testing using the experimental setup shown in Fig. 5. In the vibration testing, a hand-held impulse hammer (Kistler 9722A500, Kistler Instrument, USA) was used to excite the composite plate at different points on the plate, a force transducer (Kistler 9904A, Kistler Instrument, USA) attached to the hammer's head to measure the input forces, an accelerometer (AP19, APTechnology, Netherland) of mass 0.14 g, which is about 0.2% of the plate weight, located at different points on the plate to pick up the vibration response data, and a data acquisition and analysis system (B&K 3560C and B&K Pulse Labshop Version 6.1) to process the vibration data from which the natural frequencies of the composite plates were extracted. A series of tests had shown that the light accelerometer weight had negligible effects on the measured natural frequencies. Each flexibly supported composite plate was then tested for 15 times and each test produced a set of vibration data for constructing the frequency response spectrum of the plate. In general, the modal damping ratios of the plates were small, less than 2%, for the first seven modes of the plates. Therefore, without loss of generality, it is assumed that the effects of damping on the natural frequencies of the plate were negligible and not taken into consideration when extracting the natural frequencies from the frequency response spectrum of the plate. Herein, the first seven natural frequencies were extracted directly from the corresponding peaks in the frequency response spectra of the plates. For illustration purpose, Fig. 6 shows a typical frequency response spectrum of the

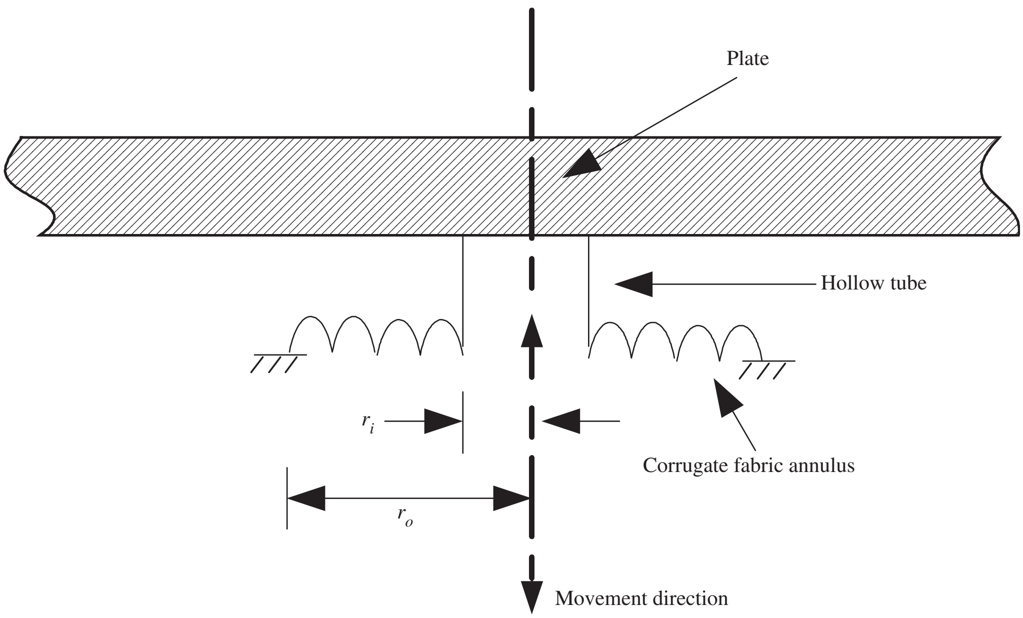

Fig. 4. Schematic description of center support.

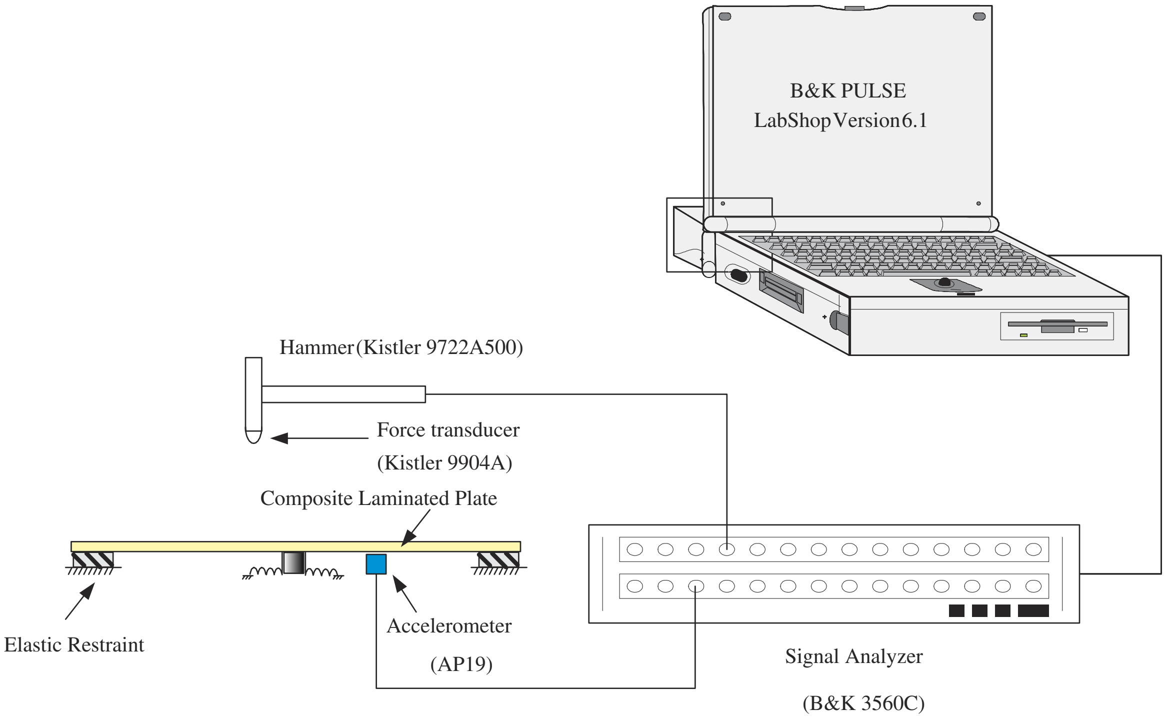

Fig. 5. Experimental setup for impulse vibration testing.

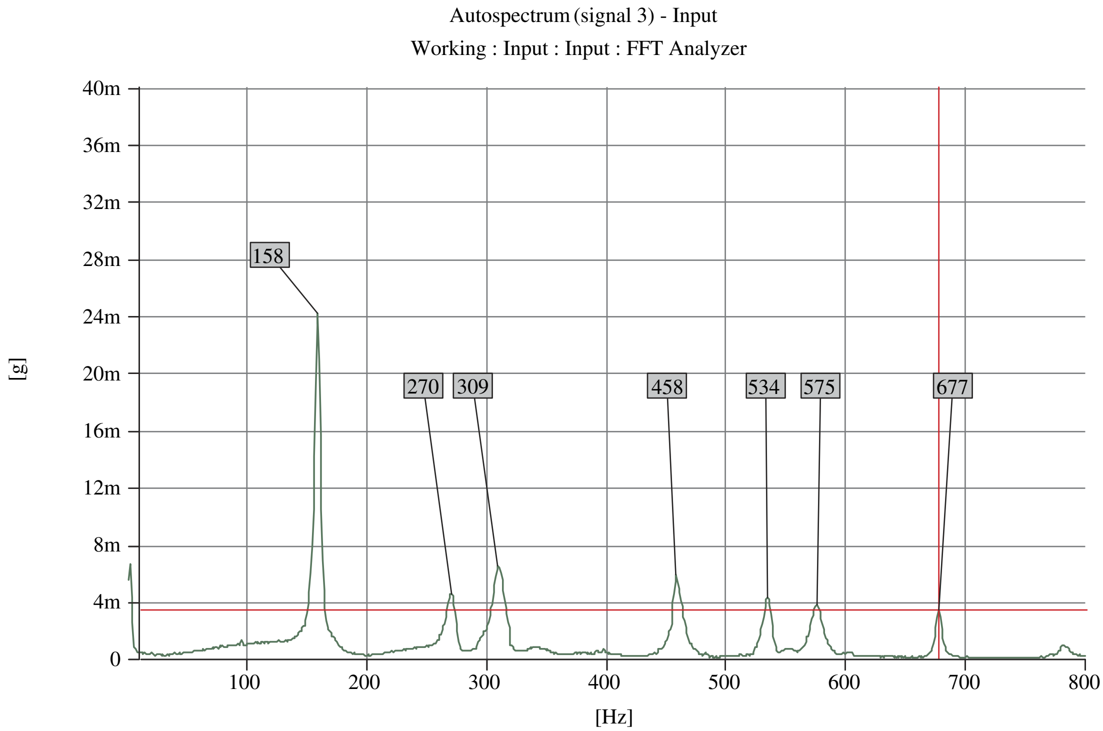

Fig. 6. Frequency response spectrum of the [45°/−45°/45°]$_S$ plate restrained peripherally and centrally.

Table 1
Measured natural frequencies of peripherally and elastically restrained square composite plates with or without center support

Layup	Plate dimensions and weight		Center support k_C (kN/m)	Natural frequency						
				1st	2nd	3rd	4th	5th	6th	7th
$[0°]_8$	Length (cm)	20.5	0	120	187	311	417	467	490	552
	Thickness	1		(0.45%)[a]	(0.24%)	(0.27%)	(0.55%)	(0.70%)	(0.18%)	(0.28%)
	(mm)		3.865	143	183	310	418	459	474	549
	Weight (g)	64.63		(0.49%)	(0.77%)	(0.61%)	(0.66%)	(0.39%)	(0.37%)	(0.51%)
$[0°/90°]_{2S}$	Length (cm)	20.5	0	122	281	364	469	574	710	773
	Thickness	1		(0.67%)	(0.43%)	(0.75%)	(0.11%)	(0.47%)	(0.57%)	(0.59%)
	(mm)		3.865	145	284	367	467	582	707	781
	Weight (g)	64.72		(0.24%)	(0.37%)	(0.35%)	(0.28%)	(0.53%)	(0.62%)	(0.06%)
$[45°/-45°/45°]_S$	Length (cm)	19.5	0	126	261	301	444	523	554	660
	Thickness	0.75		(0%)	(0%)	(0.18%)	(0.12%)	(0.22%)	(0.15%)	(0.25%)
	(mm)		3.865	158	270	309	458	534	575	677
	Weight (g)	44.23		(0.74%)	(0.33%)	(0.67%)	(0.96%)	(0.32%)	(0.27%)	(0.62%)

[a]The values in parentheses denotes the coefficient of variation of the measured natural frequency.

$[45°/-45°/45°]_S$ plate restrained peripherally and centrally. It is noted that the first seven natural frequencies of the $[45°/-45°/45°]_S$ plate can be easily identified from the peaks of the frequency response spectrum as shown in the figure. The means and c.o.v.'s of the first seven measured natural frequencies of the peripherally restrained composite plates with or without center elastic supports determined from the impulse vibration testing of the plates are listed in Table 1. It is noted that the c.o.v.'s of the measured natural frequencies are less than or equal to 0.96%. In the elastic constants identification of the plates as will be described in the following section, the means of measured natural frequencies will be treated as the measured natural frequencies in Eq. (22) for identifying the mechanical properties of the plates.

5. Results and discussion

Before proceeding to the mechanical properties identification of the elastically restrained laminated composite plates which have been tested, the present method in predicting natural frequencies and identifying mechanical properties of elastically restrained composite plates made of different materials is worth studying. The present method is first used to predict the natural frequencies of several laminated composite plates with different boundary conditions. A convergence study has shown that the numbers of the characteristic functions in Eq. (10) being $I = J = 10$ are sufficient to make the solutions of the flexibly supported plates with or without center supports to converge. Therefore, the number of terms of $I \times J = 10 \times 10$ for the characteristic functions in the Rayleigh–Ritz method is chosen to evaluate the natural frequencies of the plates under consideration. The results obtained by the present method are listed in Table 2 in comparison with those available in the literature [24,25] or obtained in the finite element analyses of the plates using the commercial code ANSYS [26]. For the cases with infinite k_L, the value of k_L is chosen as $10^8\,\mathrm{KN/m^2}$ in the analyses when using the present method or ANSYS to solve the problems. It is noted that the present method can predict excellent natural frequencies for the laminated composite plates with or without center elastic supports. Next study the capability of the present method in mechanical properties identification of various elastically restrained laminated composite plates made of graphite/epoxy (Gr/ep) or glass/epoxy (Gl/ep) composite materials. The sizes of the square and rectangular plates are $200\,\mathrm{mm} \times 200\,\mathrm{mm}$ and $200\,\mathrm{mm} \times 100\,\mathrm{mm}$, respectively. The elastic constants of the Gr/ep and Gl/ep composite materials are as follows:

$$
\begin{aligned}
Gr/ep: &\quad E_1 = 131\,\mathrm{GPa},\ E_2 = 11.2\,\mathrm{GPa},\ G_{12} = 6.55\,\mathrm{GPa},\ \nu_{12} = 0.28,\ \rho = 1550\,\mathrm{kg/m^3},\\
Gl/ep: &\quad E_1 = 43.5\,\mathrm{GPa},\ E_2 = 11.5\,\mathrm{GPa},\ G_{12} = 3.45\,\mathrm{GPa},\ \nu_{12} = 0.27,\ \rho = 2000\,\mathrm{kg/m^3}.
\end{aligned} \tag{30}
$$

Table 2
Natural frequencies of square plates predicted by different methods

Layup	Edge support k_L	Edge support k_R	Center support k_C	Method	Natural frequency λ 1st	2nd	3rd	4th	5th	6th	7th
[0°/90°/0°]	2 (MN/m^2)	800 (N)	5 (kN/m)	Present[a]	28.95	29.68	48.89	57.18	61.02	63.11	69.93
				ANSYS [26]	28.90	29.46	48.53	57.00	60.84	63.05	69.76
[45°/−45°/45°]	100 (kN/m^2)	100 (N)	1 (kN/m)	Present	17.60	22.93	28.95	31.33	37.59	39.79	44.58
				ANSYS [26]	17.49	22.83	28.89	31.24	37.57	39.72	44.55
[0°/90°/0°]	∞	0	0	Present	13.95	21.76	38.64	51.21	55.79	63.99	66.98
				ANSYS [26]	13.94	21.74	38.61	51.17	55.74	63.88	66.90
				Masoud [24]	13.95	—	—	—	—	—	—
				Reddy [25]	13.948	—	—	—	—	—	—
		20	0	Present	25.90	33.33	49.59	69.39	74.04	74.79	84.85
				ANSYS [26]	25.71	33.16	49.41	68.92	73.58	74.46	84.40
				Masoud [24]	25.91	—	—	—	—	—	—
		20	0	Present	31.24	38.64	55.73	82.94	84.51	88.81	99.34
				ANSYS [26]	30.96	38.39	55.48	82.53	83.80	88.12	98.67
				Masoud [24]	31.24	—	—	—	—	—	—

[a]Material property and definition of normalized natural frequency for the analysis: $E_1 = 200\,\text{GPa}$, $E_2 = 10\,\text{GPa}$, $G_{12} = 6\,\text{GPa}$, $\nu_{12} = 0.25$, $\lambda = \sqrt{\rho h \omega^2 a^4 / D_0}$.

Table 3
Actual natural frequencies of the *Gr*/*ep* and *Gl*/*ep* plates supported by elastic restraints with different rigidities

Material	Layup	Shape	Edge support E_e (MPa)	Center support k_C (kN/m)	Natural frequency 1st	2nd	3rd	4th	5th	6th	7th
Gr/*ep*	$[0°/90°/0°]_S$	Square	1	1	106.159	210.290	287.920	360.594	424.408	531.693	612.360
				10	163.290	210.290	287.920	360.594	444.188	531.693	627.521
		Rectangular	15	1	310.451	446.202	755.975	852.199	941.920	1158.707	1213.974
				10	368.062	446.202	783.529	852.199	941.920	1158.707	1213.974
	$[45°/-45°]_{2S}$	Square	1	1	152.074	317.208	348.261	526.847	611.322	630.898	777.177
				10	189.412	317.208	348.261	530.342	611.322	647.641	777.177
		Rectangular	15	1	411.270	638.073	969.090	1145.644	1394.426	1432.504	1840.852
				10	445.426	638.073	984.205	1145.644	1394.426	1432.566	1840.852
Gl/*ep*	$[0°/90°/0°]_S$	Square	1	1	71.817	132.677	161.183	215.186	266.810	329.155	338.956
				10	120.301	132.677	161.183	215.186	284.263	329.155	363.741
		Rectangular	15	1	218.688	288.550	455.579	562.911	618.972	696.449	739.844
				10	274.087	288.550	494.273	562.911	618.972	696.449	739.844
	$[45°/-45°]_{2S}$	Square	1	1	92.877	190.224	202.845	319.178	367.807	374.863	482.650
				10	132.318	190.224	202.845	321.233	367.807	400.575	482.650
		Rectangular	15	1	265.755	387.479	580.410	705.413	831.212	860.262	1089.689
				10	304.749	387.479	601.264	705.413	831.212	860.303	1089.689

Each laminated composite plate is peripherally supported by same strip-type elastic pads and centrally supported by an elastic spring. Different values for the Young's modulus E_e of the elastic pads and spring constant k_C of the center elastic spring are adopted in the study. The first seven actual natural frequencies of the elastically restrained composite plates under consideration are listed in Table 3. The actual natural frequencies in Table 3 will be treated as "measured" natural frequencies and used in the numerical study to identify the mechanical properties of the elastically restrained plates. The square $[45^\circ/-45^\circ]_{2S}$ plate made of *Gr/ep* material supported by edge elastic pads with $E_e = 1.0\,\text{MPa}$ and center support with spring constant $k_C = 1\,\text{kN/m}$ is used as an example to demonstrate the identification process of the present method. The upper and lower bounds of the mechanical properties adopted in solving the identification problem are chosen based on experience

$$0 \leqslant E_1 \leqslant 400\,\text{GPa}, \quad 0 \leqslant E_2 \leqslant 40\,\text{GPa}, \quad 0 \leqslant G_{12} \leqslant 20\,\text{GPa},$$
$$0 \leqslant v_{12} \leqslant 0.5, \quad 0 \leqslant E_e \leqslant 20\,\text{MPa}, \quad 0 \leqslant k_C \leqslant 20\,\text{kN/m}. \tag{31}$$

The modified design variables of Eq. (25) are obtained via the use of the following normalization factors

$$\alpha_1 = 100 \tag{32a}$$

and

$$\alpha_i = 10 \quad (i = 2 - 5). \tag{32b}$$

It is noted that the use of the above normalization factors can adjust the search direction in such a way that the convergence of the solution can be expedited. The randomly generated starting points, the lowest local minima for the starting points, numbers of iterations required to obtain the lowest local minima, and the global minimum for the *Gr/ep* $[45^\circ/-45^\circ]_{2S}$ plate using 5 and 6 "measured" natural frequencies in identifying the mechanical properties are listed in Tables 4 and 5, respectively. For the cases under consideration, six starting points are sufficient to find the global minima and around seven iterations to obtain the lowest local minima for the starting points during the minimization process. A further study has shown that the actual mechanical properties can definitely be identified when more than six "measured" natural frequencies are used in the present method. Similarly, the mechanical properties of the other elastically restrained composite plates in Table 3 can be identified using the same identification procedure. Herein, six "measured" natural frequencies have been used in the identification process to identify the mechanical properties of the elastically restrained *Gr/ep* and *Gl/ep* plates. The results have shown that the actual mechanical properties of the plates can be obtained for all the cases under consideration irrespective of the rigidities of the flexible supports. It is worth noting that if the number of spring constants and elastic constants

Table 4
Mechanical properties identification of the square *Gr/ep* $[45^\circ/-45^\circ]_{2S}$ plate using five "measured" natural frequencies

Starting point no.	Stage	Mechanical property							
		E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	v_{12}	E_e (MPa)	k_C (kN/m)	Sum of squared differences	Number of iterations
1	Initial	310.387	27.119	13.090	0.1464	14.643	0.978	2.2251E+0	5
	Final	132.894	11.008	7.637	0.0322	1.016	0.980	1.40E−16	
2	Initial	188.674	23.736	7.596	0.2094	17.884	0.274	7.2722E−1	10
	Final	132.895	11.007	7.638	0.0320	1.016	0.980	1.072E−16	
3	Initial	130.031	23.142	16.456	0.0698	9.797	16.291	4.5275E−1	9
	Final	132.894	11.007	7.638	0.0321	1.016	0.980	1.139E−16	
4	Initial	193.400	6.969	5.425	0.3481	14.904	9.830	6.9981E−1	8
	Final	102.814	9.063	7.957	0.2908	7.471	0.291	5.1718E−4	
5	Initial	130.976	11.236	18.951	0.0447	10.671	0.077	1.1503E−1	9
	Final	132.894	11.007	7.638	0.0321	1.016	0.980	1.45E−16	
6	Initial	149.957	32.578	12.079	0.3394	5.183	2.197	3.4467E−1	9
	Final	132.895	11.007	7.638	0.0320	1.016	0.980	1.073E−16	
Global minimum		132.895 (1.45%)[a]	11.007 (−1.72%)	7.638 (16.61%)	0.0320 (−88.57%)	1.016 (1.56%)	0.980 (−1.97%)		

[a]The values in parentheses denote percentage difference between predicted and measured data.

Table 5
Mechanical properties identification of the square *Gr/ep* [45°/−45°]$_{2S}$ plate using six "measured" natural frequencies

Starting point no.	Stage	Mechanical property							
		E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	v_{12}	E_e (MPa)	k_C (kN/m)	Sum of squared differences	Number of iterations
1	Initial	174.214	1.738	8.137	0.3526	10.378	16.747	6.2286E−1	6
	Final	131.000	11.200	6.550	0.2800	1.000	1.000	1.499E−16	
2	Initial	69.822	29.244	9.650	0.1984	7.450	6.899	3.6506E−2	7
	Final	131.000	11.200	6.550	0.2800	1.000	1.000	1.1476E−16	
3	Initial	285.288	3.545	6.267	0.0411	0.362	0.045	1.1060E−1	5
	Final	131.000	11.200	6.550	0.2800	1.000	1.000	1.0456E−16	
4	Initial	331.548	33.018	7.028	0.3314	3.440	17.652	2.7603E+0	10
	Final	131.000	11.200	6.550	0.2800	1.000	1.000	1.2525E−16	
Global minimum		131.000 (0%)[a]	11.200 (0%)	6.550 (0%)	0.2800 (0%)	1.000 (0%)	1.000 (0%)		

[a]The values in parentheses denote percentage difference between predicted and measured data.

of the elastic supports to be identified is larger than two, six "measured" natural frequencies will be insufficient and more natural frequencies will be required in the present method to identify the mechanical properties of the plate.

Now the present method is applied to the mechanical properties identification of the elastically estrained laminated composite plates which have been tested. The measured frequencies of the [0°]$_8$ plate with or without a center support in Table 1 are first used to illustrate the identification process. Tables 6 and 7 list the randomly generated starting points, the lowest local minima obtained for the starting points, the numbers of iterations required for getting the lowest local minima, and the global minimum for the plate with or without a center support, respectively, using different numbers of measured natural frequencies in the identification processes. In view of the results in Table 6, due to the existence of noise in the measurements, the use of the first six measured natural frequencies in the identification process is unable to produce acceptable result for the [0°]$_8$ plate with a center support while the use of seven measured natural frequencies can produce satisfactory estimates of the mechanical properties with percentage differences less than or equal to 6.48%. It is worth noting that for the case with the use of seven measured natural frequencies, only four starting points are needed to obtain the global minimum with probability exceeding 0.99 and around eight iterations to find the lowest local minima for the starting points during the minimization process. As for the case without a center support, the results in Table 7 show that the use of five rather than four measured natural frequencies in the identification process can produce better estimates of the mechanical properties with percentage differences less than or equal to 5.99% for the [0°]$_8$ plate. It is also noted that the use of five measured natural frequencies only requires four starting points to find the global minimum and around seven iterations to obtain the lowest local minima for the starting points in the minimization process. The mechanical properties of the other elastically restrained composite plates which have been tested and listed in Table 1 are then identified using the present method with the use of seven and five measured natural frequencies for the cases with or without a center support, respectively. The identified mechanical properties and their associated percentage differences of the plates are listed in Table 8. Again, it is noted that very good estimates of the mechanical properties with percentage differences less than or equal to 7.63% have been obtained for the plates. In view of the small percentage differences between the actual and identified mechanical properties obtained for the plates, the neglect of the damping effects on the measured natural frequencies is found to be acceptable. It is also worth pointing out that if the number of the unknown spring constants and elastic constants of the elastic supports are larger than two, it is required to use more than seven measured natural frequencies to identify the mechanical properties of the plates.

Table 6
Identified mechanical properties of the $[0°]_8$ plate with a center support using different numbers of measured natural frequencies

No. of measured natural frequencies	Starting point no.	Stage	Identified mechanical property							
			E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	E_e (MPa)	k_C (N/m)	Sum of squared differences	Number of iterations
6	1	Initial	72.738	10.611	10.343	0.23114	6.272	15.033	1.2473E−1	6
		Final	136.34	8.563	5.59	0.3	3.061	3.643	7.7620E−7	
	2	Initial	394.832	34.915	7.625	0.37145	15.168	6.887	3.8838E+0	6
		Final	136.343	8.563	5.593	0.3	3.058	3.644	7.7605E−7	
	3	Initial	278.223	8.977	18.075	0.31906	7.187	16.213	1.0200E+0	10
		Final	165.22	6.164	1.925	0.29764	1.951	18.007	2.4735E−2	
	4	Initial	272.79	38.101	4.232	0.39451	8.252	10.868	2.5804E+0	9
		Final	136.343	8.563	5.593	0.3	3.058	3.644	7.7603E−7	
	5	Initial	23.313	21.975	8.017	0.43736	4.375	12.574	1.8799E−1	5
		Final	136.342	8.563	5.592	0.3	3.059	3.644	7.7604E−7	
	6	Initial	315.04	7.485	17.158	0.14178	9.933	2.138	8.0614E−1	8
		Final	136.343	8.563	5.593	0.3	3.058	3.644	7.7604E−7	
	Global minimum		136.343 (−6.94%)[a]	8.563 (−7.16%)	5.593 (−18.18%)	0.3 (−1.96%)	3.058 (50.79%)	3.644 (−5.72%)		
7	1	Initial	175.047	2.213	13.979	0.31602	14.829	11.284	1.8501E−1	6
		Final	138.492	8.625	6.625	0.3	2.118	3.811	2.5864E−5	
	2	Initial	294.196	15.885	7.743	0.29797	1.583	0.695	5.9002E−1	8
		Final	138.495	8.626	6.626	0.3	2.118	3.811	2.5864E−5	
	3	Initial	146.693	26.308	18.35	0.16604	15.901	15.63	1.9222E+0	6
		Final	138.492	8.625	6.625	0.30001	2.118	3.811	2.5864E−5	
	4	Initial	245.851	1.123	13.69	0.24554	6.003	17.796	2.4924E−1	8
		Final	138.491	8.625	6.625	0.3	2.119	3.811	2.5864E−5	
	Global minimum		138.492 (−5.47%)	8.625 (−6.48%)	6.625 (−3.09%)	0.30001 (−1.96%)	2.118 (4.44%)	3.811 (−1.40%)		

[a]The values in parentheses denote percentage difference between predicted and measured data.

Table 7
Identified mechanical properties of the $[0°]_8$ plate without a center support using different numbers of measured natural frequencies

No. of measured natural frequencies	Starting point no.	Stage	Identified mechanical property						
			E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	E_e (MPa)	Sum of squared differences	Number of iterations
4	1	Initial	187.593	8.076	10.861	0.16856	19.587	2.8635E−1	8
		Final	137.128	9.943	6.195	0.2861	1.999	1.5681E−4	
	2	Initial	239.461	22.073	10.155	0.02748	13.113	8.0578E−1	9
		Final	137.047	9.936	6.135	0.2983	1.999	1.5671E−4	
	3	Initial	57.795	16.601	9.77	0.29726	1.813	3.9507E−2	7
		Final	137.036	9.934	6.128	0.29998	1.999	1.5670E−4	
	4	Initial	321.878	19.596	14.711	0.20862	8.341	1.1344E+0	7
		Final	137.074	9.938	6.152	0.29476	1.999	1.5674E−4	
	Global minimum		137.036 (−6.46%)[a]	9.934 (7.71%)	6.128 (−10.36%)	0.29998 (−1.97%)	1.999 (−1.43%)		
5	1	Initial	224.312	1.477	8.309	0.30108	7.357	1.7129E−1	7
		Final	137.728	9.645	6.727	0.30001	1.997	3.8830E−4	
	2	Initial	76.698	26.265	18.626	0.46979	12.917	6.0113E−1	7
		Final	137.728	9.645	6.726	0.30001	1.997	3.8830E−4	
	3	Initial	275.026	36.965	17.584	0.43226	0.409	8.5835E−1	9
		Final	137.726	9.645	6.727	0.30001	1.997	3.8830E−4	
	4	Initial	347.444	29.501	11.229	0.16032	1.476	1.2201E+0	6
		Final	137.724	9.645	6.728	0.30001	1.997	3.8830E−4	
	Global minimum		137.724 (−5.99%)	9.645 (4.58%)	6.728 (−1.58%)	0.30001 (−1.96%)	1.997 (−1.53%)		

[a]The values in parentheses denote percentage difference between predicted and measured data.

Table 8
Identified mechanical properties of flexibly supported laminated composite plates using measured natural frequencies

Layup	Support condition	Identified mechanical property					
		E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	E_e (MPa)	k_C (N/m)
$[0°/90°]_{2S}$	Edge and center	142.181 (−2.95%)[a]	8.76 (−5.02%)	6.439 (−5.81%)	0.3 (−1.96%)	2.126 (4.83%)	3.834 (−0.80%)
	Edge	139.72 (−4.63%)	8.818 (−4.39%)	6.818 (−0.26%)	0.30002 (−1.95%)	1.998 (−1.48%)	—
$[45°/-45°/45°]_S$	Edge and center	147.149 (0.44%)	9.927 (7.63%)	6.833 (−0.04%)	0.3 (−1.96%)	2 (−1.38%)	3.777 (−2.28%)
	Edge	138.925 (−5.17%)	9.158 (−0.70%)	7.158 (4.71%)	0.30001 (−1.96%)	1.999 (−1.43%)	—

[a]The values in parentheses denote percentage difference between predicted and measured data.

6. Conclusions

The nondestructive evaluation of mechanical properties of a number of laminated composite plates elastically restrained at the centers and peripheries of the plates using measured natural frequencies extracted from the vibration data of the plates have been studied via both theoretical and experimental approaches. The nondestructive evaluation method used for the mechanical properties identification of the plates has been established on the basis of the Rayleigh–Ritz method together with a multi-start global minimization method. The theoretical natural frequencies which are obtained in the Rayleigh–Ritz method using trial mechanical properties and the measured natural frequencies of the plates have been used to construct the sum of the squared differences function for measuring the differences between the theoretical and experimental natural frequencies of the plates. The multi-start global minimization method together with several measured natural frequencies has been used to identify the mechanical properties of each of the plates by making the sum of the squared differences function of the plate a global minimum. A normalization technique has also been used in the identification process to expedite the convergence of the solution. In the theoretical study, the mechanical properties identifications of several peripherally and centrally restrained plates made of *Gr/ep* or *Gl/ep* composite materials with different layups and dimensions have been performed to demonstrate the capability and accuracy of the present method. It has been shown that the use of six actual natural frequencies, which are treated as measured ones, can identify the actual mechanical properties of the plates with peripheral and central elastic supports in an efficient and effective way. In the experimental study, several flexibly supported laminated composite plates have been fabricated and subjected to impulse vibration testing. For the plates with peripheral and central elastic supports, seven measured natural frequencies have been used to identify the plate mechanical properties of which the percentage differences are less than or equal to 7.63%. For the plates with only peripheral elastic supports, five measured natural frequencies have been used to identify the plate mechanical properties of which the percentage differences are less than or equal to 5.99%. The experimental investigation has demonstrated the applications and validated the capability of the present method.

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