High Frequency Transistor amplifier Design using In Paramcters

by Teh Hua Chang M. S., (張 德 驊)
Associate Professor of Chiao-Tung university.

Part 1 Basic Concepts

1 General Analysis:

Analysis of Power Flow

Let $I_1=1+jo$ (1) and

$$E_2=a+jb=(L+jM)\frac{-h_{21}}{2h_{22r}} \qquad (2)$$

$$E_1=I_1h_{11}+E_2h_{12} \qquad (3) \qquad\qquad I_2=I_1h_{21}+E_2h_{22} \qquad (4)$$

$$\text{Power out}=P_o=R_e(-E_2^*\ I_2) \qquad (5)$$

$$I_2=h_{21}(1+jo)+(L+jM)\frac{-h_{21}}{2h_{22}}\ h_{22} \qquad (6)$$

Put Eqs (2) and (6) in Eq. (5)

$$P_o=R_e\frac{h_{21}^*\ (L-jM)}{2h_{22r}}\left[h_{21}+\frac{(L+jM)\ (-h_{21}\ h_{22})}{2h_{22r}}\right]$$

$$=L\ \frac{|h_{21}|^2}{2h_{22r}}-\frac{(L^2+M^2)\ |h_{21}|^2}{4h_{22r}} \qquad (7)$$

$$L^2+M^2-2L+\frac{4h_{22r}}{|h_{21}|^2}\ P_o=0$$

$$(L-1)^2+M^2=1-\frac{4h_{22r}}{|h_{21}|^2}P_o \qquad (8)$$

This is an equation of parabo loid with its peak at L=1 M=0 it intersects the LM plane on the unit circle centered at L=1. M=0, The value of P_o at L=1, M=0, is

$$P_{oo}=\frac{|h_{21}|^2}{4h_{22r}} \qquad \text{(Fig. 1)}$$

Eq (8) becomes

$$(L-1)^2+M^2=1-\frac{P_o}{P_{oo}} \qquad (9)$$

This is an equation of concentric circles centered at L=1, M=0, On

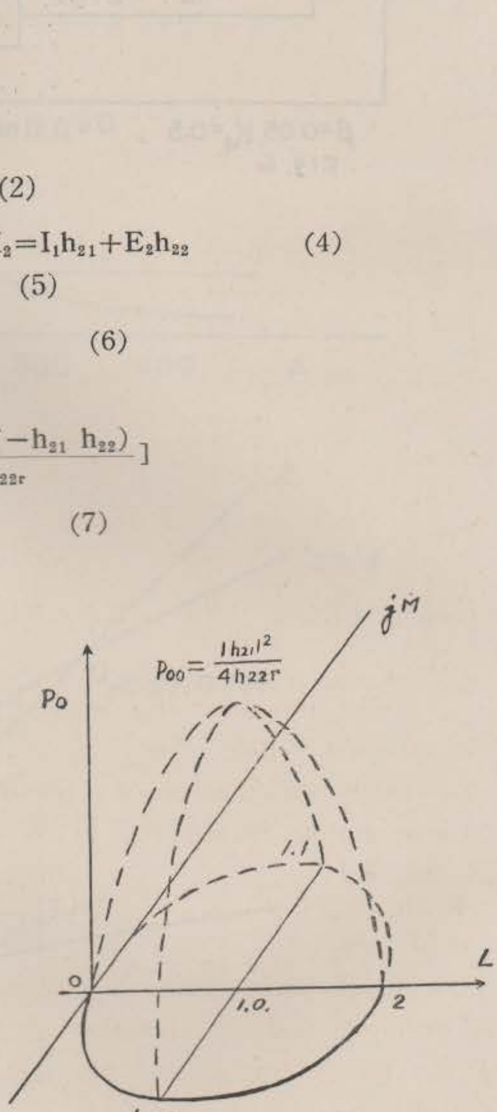

Fig.1. Power out put in a function of L and M

the LM plane are contours of constant power output. The unit circle is the locus of zero power output (Fig.2)

If the M axis is moved one unit to the right. Eq. (9) becomes

$$L^{12}+M^2=1-\frac{P_o}{P_{oo}} \quad (10) \text{ origin at } (1.0)$$

$$P_o=P_{oo}\ (1-L^{12}-M^2) \quad (11) \text{ origin at } (1.0)$$

The power input eguations are derived as follows

$$\text{powes input}=p_i=R_e\ E_1^*\ I_1=R_e\ E_1^* \quad (12)$$

$$E_1=I_1 h_{11}+E_2 h_{12}=h_{11}(1+jo)+(L+jM)\frac{-h_{21}}{2h_{22r}}\ h_{12} \quad (13)$$

$$P_i=h_{11r}+L\ R_e\frac{-h_{12}\ h_{21}}{2h_{22r}}+M\ Im\frac{h_{12}\ h_{21}}{2h_{22r}} \quad (14)$$

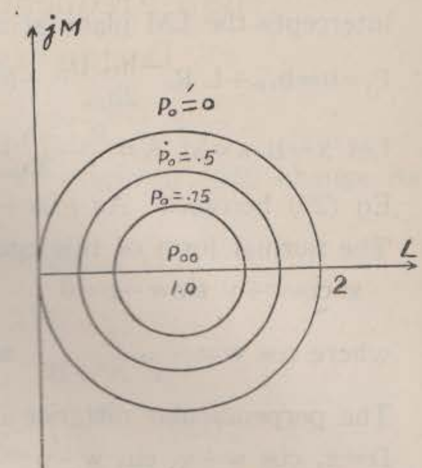

Fig.2 Constant Power output Circles in L.M plane

For the origin at L=1, M=0

$$P_i=h_{11r}+(L'+1)\ R_e\frac{-h_{12}\,h_{21}}{2h_{22r}}+M\ Im\,\frac{h_{12}\,h_{21}}{2h_{22r}}$$

$$=h_{11r}-\frac{R_e(h_{12}\ h_{21})}{2h_{22r}}+L^1\ Re\,\frac{-h_{12}\,h_{21}}{2h_{22r}}+M\ Im\,\frac{h_{12}\ h_{21}}{2h_{22r}} \quad (15)$$

$$\text{Let } P_{io}=\frac{2h_{11r}\ h_{22r}-R_e\ (h_{12}\ h_{21})}{2h_{22r}},\ A=R_e\frac{-h_{12}\ h_{21}}{2h_{22r}} \quad B=Im\frac{h_{12}\ h_{21}}{2h_{22r}}$$

$$\text{then } A^2+B^2=\frac{|h_{12}\ h_{21}|^2}{4h_{22r}^{\ 2}}=|G_2|^2 \quad (16)$$

$$G_{oo}=\frac{P_{oo}}{P_{io}}=\frac{|h_{21}|^2}{4h_{11r}\ h_{22r}-2R_e\ (h_{12}\ h_{21})} \quad (17)$$

$$P_i=P_{io}+AL'+BM \quad (18)$$

Eq (18) is the equation of a plane wih 3 intersecting points: h_{11r} at P_i axis, $\frac{2h_{11r}\ h_{22r}}{R_e\ (h_{12}\ h_{21})}$ at L axis, and $\frac{-2h_{11r}\ h_{22r}}{Im\ (h_{12}\ h_{21})}$ at M axis.

The input power plane is defines by its gradient and its elevation at L=1 M=0, The elevation at this point is

$$P_{io}=\frac{2h_{11r}\ h_{22r}-R_e\ (h_{12}\ h_{21})}{2h_{22r}} \quad (19)$$

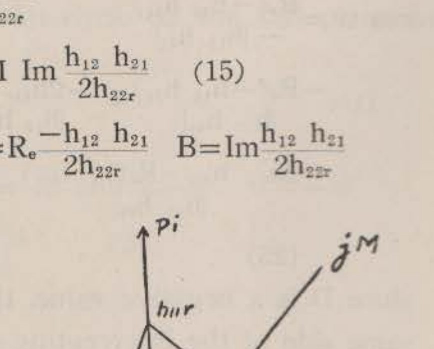

Fig 3. Input Power diagram

2 The gradient is derived in the following manner The input power plane intercepts the LM plane at a line whose eguation is

$$P_i=0=h_{11r}+L\ R_e\frac{-h_{12}\ h_{21}}{2h_{22r}}+M\ Im\frac{h_{12}\ h_{21}}{2h_{22r}} \quad (20)$$

Let x=L x=M $A=\frac{R_e(-h_{12}\ h_{21})}{2h_{22r}}$ $B=Im\frac{h_{12}\ h_{21}}{2h_{22r}}$ $C=h_{11r}$

Eq (20) becomes $Ax+By+c=0$ (21)

The normal form of this eguotion is

$$x\ \cos w+y\ \sin w-p=0 \quad (22)$$

where $\cos w=\frac{A}{-\sqrt{A^2+B^2}}$ $\sin w=\frac{B}{-\sqrt{A^2+B^2}}$ $-p=\frac{C}{-\sqrt{A^2+B^2}}$ (23)

The perpeudicular distance D from a point (x_1, y_1) to this lins is

$$D=x_1\ \cos w+y_1\ \sin w-p \quad (24)$$

$$=\cos w-p \text{ at } x_1=1,\ y_1=0$$

$$-p=\frac{C}{-\sqrt{A^2+B^2}}=\frac{-2h_{11r}\ h_{22r}}{|h_{12}\ h_{21}|}$$

$$\cos w=\frac{R_e(-h_{12}\ h_{21})}{-|h_{12}\ h_{21}|}$$

$$\therefore D=\frac{-R_e(-h_{12}\ h_{21})}{|h_{12}\ h_{21}|}+\frac{-2h_{11r}\ h_{22r}}{|h_{12}\ h_{21}|}$$

$$=-\frac{2h_{11r}\ h_{22r}-R_e(h_{12}\ h_{21})}{|h_{12}\ h_{21}|}=-\frac{1}{C}$$

(25)

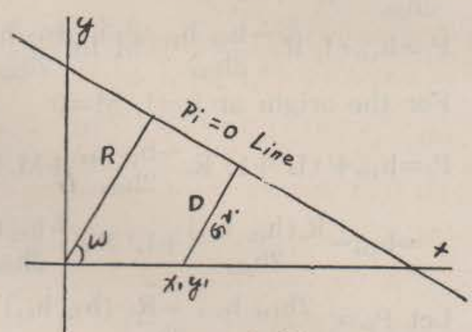

Fig. 4 Gradient Line Diagram

since D is a negative value, the origin and the point (1.0) are at the same side of the intercepting line.

The gradicnt of the power input plane is defines as

$$\text{Gradient}=G_2=\frac{pio}{|D|}=\frac{|h_{12}\ h_{21}|}{2h_{22r}} \quad (26)$$

Let the line $A'x+B'y+C'=0$ (27)

pass the point (1.0) and perpendiculav to line

$$Ax+By+C=0 \quad (21)$$

then $A'+C'=0$ $A'=-C'$ and $m=-\frac{1}{m'}$

m is the slope of line (21). m' is the slope of the line (27)

$$m=\frac{A}{B} \quad m'=\frac{A'}{B'} \quad \therefore \frac{A}{B}=-\frac{B'}{A'}$$

The equation of line (27) which is perpendicular to line (21)

is $A'x+B'y-A'=0 \quad x+\frac{B'}{A'}y-1=0$

or $x-\frac{A}{B}y-1=0$

$\therefore Bx-Ay-B=0$ (28) gradient line with its origin at (0.0) change its origin from (0.0) to (1.0)

$B(x'+1)-Ay-B=0 \quad Bx'-Ay=0$ (29) origin at (1.0)

$$\mathrm{Tan}\theta=\frac{y}{x'}=\frac{B}{A}=-\frac{\mathrm{Im}(-h_{12}h_{21})}{R_e(-h_{12}h_{21})} \qquad (30)$$

$$\theta=-\mathrm{Ang}(-h_{12}h_{21}) \qquad (31)$$

$\therefore$ The Gradient of power input $G_r=\left|\frac{h_{12}h_{21}}{2h_{22r}}\right|e^{j\theta}$ (32)

3 The derivation of the stability factor c is as follows The gradient line equation or LM plane with its origin at (0.0) from Eq (28)

$$\mathrm{Im}\frac{(h_{12}h_{21})}{2h_{22r}}(L-1)+\mathrm{Re}\frac{(h_{12}h_{21})}{2h_{22r}}M=0 \qquad (33)$$

To find the intercepting points of the unit circle on the LM plane by the gradient line

From Eq (10) with $p_o=0$

$$(L-1)^2+M^2=1 \qquad (34)$$

From Eq. (33) and Eq. (34) to solve for M and L

$$M=\pm\frac{\mathrm{Im}(h_{12}h_{21})}{|h_{12}h_{21}|} \qquad L=1\mp\frac{R_e(h_{12}h_{21})}{|h_{12}h_{21}|} \qquad (35)$$

Take (−) sign (at lower point)

$$M=-\frac{\mathrm{Im}(h_{12}h_{21})}{|h_{12}h_{21}|} \qquad L=1+\frac{R_e(h_{12}h_{21})}{|h_{12}h_{21}|}$$

Put into Eq (14)

$$p_i=h_{11r}-\frac{R_e(h_{12}h_{21})}{2h_{22r}}-\frac{|h_{12}h_{21}|}{2h_{22r}}=p_{io}-\frac{|h_{12}h_{21}|}{2h_{22r}} \qquad (36)$$

$$C=\frac{p_{io}-p_i}{p_{io}}=1-\frac{p_i}{p_{io}}=\frac{|h_{12}h_{21}|}{2h_{11r}h_{22r}-R_e(h_{12}h_{21})}=\frac{2p_{oo}}{p_{io}}\left|\frac{h_{12}}{h_{21}}\right| \qquad (37\text{-a})$$

Take (+) sign (at upper poivt)

$$L=1-\frac{R_e(h_{12}h_{21})}{|h_{12}h_{21}|} \qquad M=\frac{\mathrm{Im}(h_{12}h_{21})}{|h_{12}h_{21}|}$$

$$p_i=\frac{2h_{11r}h_{22r}-R_e(h_{12}h_{21})}{2h_{22r}}+\frac{|h_{12}h_{21}|}{2h_{22r}}=p_{io}+\frac{|h_{12}h_{21}|}{2h_{22r}}$$

$$C=\frac{p_i-p_{io}}{p_{io}}=\frac{|h_{12}\ h_{21}|}{2h_{11r}\ h_{22r}-R_e\ (h_{12}\ h_{21})} \qquad (37\text{-b})$$

$$G_{oo}=\frac{p_{oo}}{p_{io}}=\frac{C}{2}\left|\frac{h_{21}}{h_{12}}\right| \qquad (38)$$

From Eq. (18) (26) and (37)

we get $|G_r|=CP_{io}$ or $G_r=CP_{io}\ e^{j\theta}$ (39)

4 The derivation of the maximum gain is as follows

The maximum power output occurs at the apex of the paraboloid. but the maximum gain does not occur at this ponit. since the slope of the input power plane near the ponit (1.0) greater than that of the paraboloid. There must be a point at which the maximum gain will occur.

$$\left.\begin{aligned} p_o&=p_{oo}\ (1-L'^2-M^2) \quad (11) \\ p_i&=AL'+BM+p_{io} \quad (18)\end{aligned}\right\}\ \text{origin at } (1.0)$$

$$G=\frac{p_o}{p_i}=\frac{p_{oo}\ (1-L'^2-M^2)}{AL'+BM+p_{io}} \qquad (40)$$

$$\frac{\partial G}{\partial L'}=0 \qquad \partial L'\ (L'A+BM+p_{io})+A(1-L'^2-M^2)=0 \qquad (41)$$

$$\frac{\partial G}{\partial M}=0 \qquad \partial M\ (L'A+BM+p_{io})+B(1-L'^2-M^2)=0 \qquad (42)$$

Solve Eq (41) and Eq (42) we get

$$AM=BL' \qquad \frac{B}{A}=\frac{M}{L'}=\tan\theta \qquad (43)$$

and $(A^2+B^2)L'^2+2Ap_{io}\ L'+A^2=0$ (44)

$$L'=\frac{-2Ap_{io}\pm\sqrt{4A^2\ p_{io}^2-4A^2\ (A^2+B^2)}}{2(A^2+B^2)}=A\frac{-p_{io}+\sqrt{p_{io}^2-(A^2+B^2)}}{(A^2+B^2)}$$

$$=-A\frac{p_{io}-p_{io}\sqrt{1-c^2}}{(p_{io}\ c)^2}=-A\frac{1-\sqrt{1-c^2}}{p_{io}\ c^2}=\frac{-R_e\ (h_{12}\ h_{21})}{|h_{12}\ h_{21}|}\ \frac{1-\sqrt{1-c^2}}{c}$$

$$=-d\cos\theta \qquad (45)$$

$$M=-B\frac{1-\sqrt{1-c^2}}{p_{io}\ c^2}=\frac{-B}{|G_r|}\ \frac{1-\sqrt{1-c^2}}{c}=\frac{\text{Im}\ (h_{12}\ h_{21})}{|h_{12}\ h_{21}|}\ \frac{1-\sqrt{1-c^2}}{c}$$

$$=-d\sin\theta \qquad (46)$$

$$\therefore\ (L'^2+M^2)^{\frac{1}{2}}=-\frac{1-\sqrt{1-c^2}}{c}=-d \qquad (47)$$

This point ($L'=-d\cos\theta$, $M=-d\sin\theta$) whose distance from the axis at $L=1$ $M=0$, is $-d$ gives the maximum power gain. as shown in Fig. 5 The power output at the point is

$$p_o = p_{oo}(1 - L'^2 - M^2) = p_{oo}\left(1 - \frac{(1-\sqrt{1-c^2})}{c^2}\right) = p_{oo}(1-d^2) \qquad (48)$$

$$p_j = p_{io} + AL' + BM = p_{io} - A^2\frac{1-\sqrt{1-c^2}}{p_{io}\ c^2} - B^2\frac{1-\sqrt{1-c^2}}{p_{io}\ c^2}$$

$$= p_{io}[1-(1-\sqrt{1-c^2})] = p_{io}\sqrt{1-c^2} \qquad (49)$$

p_i may be written in the form

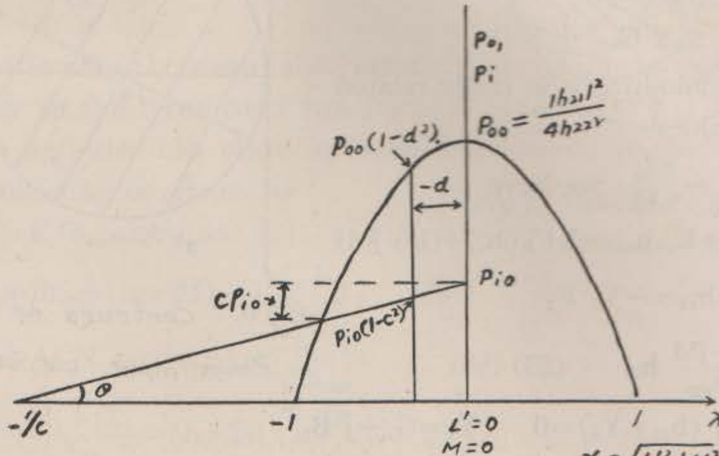

Fig. 5 power Relation along the gradient line

$$p_i = p_{io}\left(1 + \frac{A}{p_{io}}L' + \frac{B}{p_{io}}M\right)$$

$$= p_{io}\left(1 + \frac{\sqrt{A^2+B^2}}{p_{io}}\sqrt{L'^2+M^2}\right)$$

$$= p_{io}\left(1 + \frac{|G_r|}{p_{io}}x\right) = p_{io}(1+cx) \qquad (50)$$

and $G_r = \left|\frac{h_{12}\ h_{21}}{2h_{22r}}\right| e^{j\theta}$ (32)

From Eqs (32) and Eq (50) for The particular gradient which corresponds to angle θ, the contours of constant power input are shown in Fig. 6 The optimum gain is

$$\text{Gopt} = \frac{p_o}{p_i} = \frac{p_{oo}\ (1-d^2)}{p_{io}\ (1-c^2)^{\frac{1}{2}}} = \frac{p_{oo}}{p_{io}}\frac{2}{c^2}(1-\sqrt{1-c^2})$$

$$= G_{oo}\frac{2}{c^2}(1-\sqrt{1-c^2}) \qquad (51)$$

$$K_G = \frac{G_{opt}}{G_{oo}} = \frac{2}{c^2}(1-\sqrt{1-c^2}) = \frac{2}{c} d \quad (52)$$

$$d = \frac{c}{2} K_G \quad (53)$$

when c approaches 1, d approaches

$$1, \frac{d}{c} = 1 - \frac{\sqrt{1-c^2}\ (1-\sqrt{1-c^2})}{c^2} \leq 1$$

$$\therefore K_G \leq 2$$

$$\therefore G_{opt} \leq 2\ G_{oo} \quad (54)$$

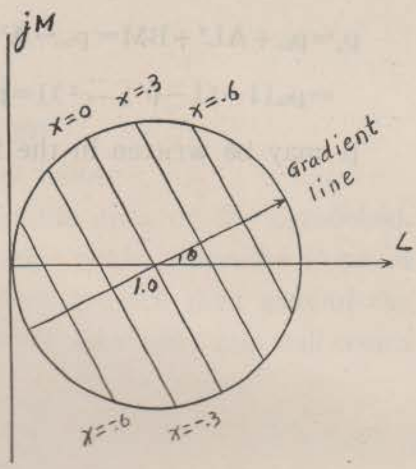

Fig 6. Contours of Constant Power input at θ angle

5 The load admittance is casily related to LM values

Y_L being $-\frac{I_2}{E_2}$ we have

$$I_2 = I_1\ h_{21} + E_2\ h_{22} = (1+jo)h_{21} + (L+jM)$$

$$\frac{-h_{21}}{2h_{22r}}\ h_{22} = -Y_L\ E_2$$

$$= Y_L \frac{L+jM}{2h_{22r}}\ h_{21} \quad (55)$$

$$I_1\ h_{21} + E_2\ (h_{22}+Y_L) = 0 \qquad Y_L = G_L + j\ B_L$$

$$h_{22} + Y_L = -I_1\ \frac{h_{21}}{E_2} = -\frac{h_{21}}{E_2} = \frac{2h_{22r}}{L+jM} = G_2 + j\ B_2 \quad (56)$$

$$G_2 + j\ B_2 = \frac{2h_{22r}\ L}{L^2+M^2} - j\frac{2h_{22r}\ M}{L^2+M^2} \quad (57)$$

$$G_2 = \frac{2h_{22r}\ L}{L^2+M^2} \qquad \therefore\ L^2+M^2 - \frac{2h_{22r}}{G_2}\ L = 0$$

$$\therefore\ (L - \frac{h_{22r}}{G_2})^2 + M^2 = (\frac{h_{22r}}{G_2})^2 \quad (58)$$

This is an equation of constant G_2 circle

$$B_2 = \frac{-2h_{22r}\ M}{L^2+M^2} \qquad L^2+M^2+\frac{2h_{22r}\ M}{B_2} = 0$$

$$L^2 + (M + \frac{h_{22r}}{B_2})^2 = (\frac{h_{22r}}{B_2})^2 \quad (59)$$

This is an equation of constant B_2 circle

$$G_2 = h_{22r} + G_L \qquad B_2 = h_{22i} + B_L \quad (60)$$

From Eqs (58) and (59) we can draw loci of constant real and imaginary parts of $G_2 + j\ B_2$ whicl are mutually orthogonal circles in the LM plane as in smith Chart.

Loci of constant G_2 and loci of constant B_2 in the LM plane are shown is the Figure 7

The optimum load admittance is

$$Y_L\ opt=-h_{22}-I_1\frac{h_{21}}{E_2}=-h_{22}+\frac{2h_{22r}}{L+jM}$$

$$=-h_{22}+\frac{2h_{22r}}{(1\text{-}d\cos\theta\text{-}j\ d\sin\theta)}=-h_{22}+\frac{2h_{22r}}{1\text{-}de^{j\theta}}$$

$$=-h_{22}+\frac{2h_{22r}}{1-\frac{ck_g}{2}e^{j\theta}} \qquad (61)$$

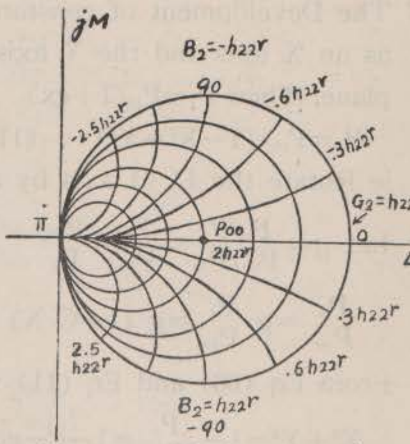

Fig 7. Constant G_2 and B_2 Contour

6 The relationship between the input impedance of the terminated two port structure and the LM plane is found by the following development

$E_1=I_1\ h_{11}+E_2\ h_{12}=2_{in},\ I_1=1+jo$ (1) and (3)

$$2_{in}=(1+jo)h_{11}+(L+jM)\frac{-h_{12}\ h_{21}}{2h_{22r}} \qquad (61)$$

$$2_{in}-h_{11}=R_1+jX_1=(L+jM)\frac{-h_{12}\ h_{21}}{2h_{22r}} \qquad (62)$$

$$\frac{R_1+jX_1}{\left|\frac{h_{12}\ h_{21}}{2h_{22r}}\right|}=\frac{R_1+jX_1}{|G_r|}=(L+jM)\ e^{-j\theta} \qquad (63)$$

The rectangular grid in shown in Fig. 8 this figure on a transparent overlay is convenient for reading $\frac{R_1}{|G_r|}+j\frac{X_1}{|G_r|}$ for any point in the LM plane

The L axis is put at $-\theta$ with respect to the R_1 axis

The optimum source impedance is

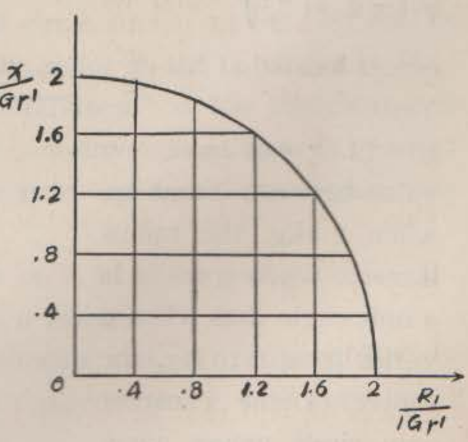

Fig 8 Input Impedance overlay

$$2_{s\ opt}=2_{in}{}^*=[h_{11}+(L+jM)\frac{-h_{12}\ h_{21}}{2h_{22r}}]^*$$

$$=[h_{11}+(1+L'+jM)\frac{-h_{12}\ h_{21}}{2h_{22r}}]^*$$

$$=[h_{11}-\frac{h_{12}\ h_{21}}{2h_{22r}}(1-d\cos\theta-jd\sin\theta)]$$

$$=[h_{11}-\frac{h_{12}\ h_{21}}{2h_{22r}}(1-de^{\theta j})]^*$$

$$=[h_{11}-\frac{h_{12}\ h_{21}}{2h_{22r}}(1-\frac{ck_g}{2}e^{j\theta})]^* \qquad (64)$$

7 The Development of constant gain circles If we use the gradient line as an X axis and the Y axis perpendicular to it at L=1 M=0 on L M plane, Then $P_i=P_{io}(1+cx)$ (50)

$P_o=P_{oo}\ (1-X^2-Y^2)$ (11)

ie Rotate the L′ M axis by an angle θ to get X Y axis

Let $g=\frac{P_o/P_i}{P_{oo}/P_{io}}=\frac{P_o}{P_{oo}}\ \frac{P_{io}}{P_i}$ (65)

$\frac{P_o}{P_{oo}}=g\ \frac{P_i}{P_{io}}=g\ (1+C\ X)$ (66)

From Eq (66) and Eq (11) we get

$X^2+Y^2=1-\frac{P_o}{P_{oo}}=1-g-gcx$

$1-g=X^2+gcx+Y^2\quad 1-g+\left(\frac{gc}{2}\right)^2=X^2+gcx+\left(\frac{gc}{2}\right)^2+Y^2$

$\therefore\ \left(x+\frac{gc}{2}\right)^2+Y^2=1-g+\frac{gc}{2}=(\text{Radius})^2$ (67)

This is the equation for constant gain circle with its radius equal to $\sqrt{1-g+\left(\frac{gc}{2}\right)^2}$ and its center located at $X=-\frac{gc}{2}$, $Y=0$,

If $c<1$, g can have any value between 0 and kg, when g=kg, the radius is zero, when g=0 it is a unit circle thus when g varies from 0 to kg, the center of the constant gain cirele moves away along the x axis from the center of the unit cirele to $-\frac{gc}{2}$ at which point its radins is reduced to zero and The sensitivty curves can be constructed on the L M plane, sensitivity is definied as the ratio of the percentage change in input immittance to the percentage change in

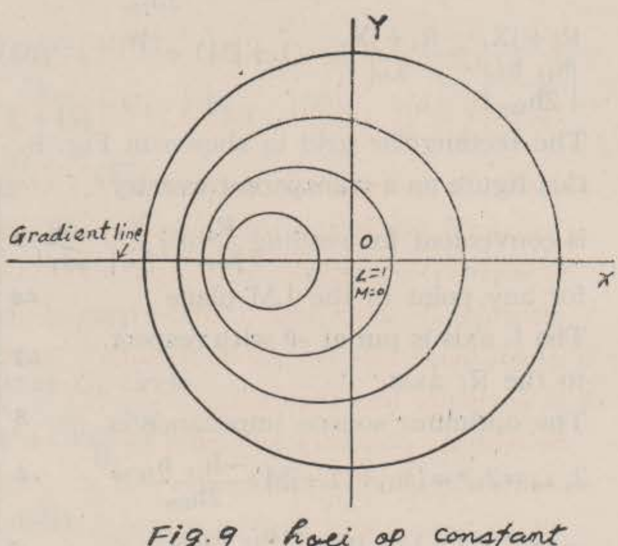

Fig. 9 loci of constant gain circles

load immittance

$$\delta=\frac{\frac{dY_{in}}{Y_{in}}}{\frac{dY_L}{Y_L}}=\left|\frac{Y_L}{y_{22}+Y_L}\right|\left|\frac{y_{11r}}{y_{11}}\right|\left|\frac{\frac{y_{12}\ y_{21}}{y_{11r}\ y_{22r}}}{\frac{y_{22}+Y_L}{y_{22r}}+\frac{y_{11r}}{y_{11}}\frac{y_{12}\ y_{21}}{y_{11r}\ y_{22r}}e^{j\theta}}\right|$$

$$=\left|\frac{Y_L}{Y_L+h_{22}}\right|\left|\frac{h_{11r}}{h_{11}}\right|\left|\frac{h_{12}\ h_{21}/h_{11r}\ h_{22r}}{\frac{h_{22}+Y_L}{h_{22r}}+\frac{h_{11r}}{h_{11}}\frac{h_{12}\ h_{21}}{h_{11r}\ h_{22r}}e^{j\theta}}\right| \qquad (68)$$

$$\doteq\frac{|y_{12}\ y_{21}|}{|y_{11}|\ |Y_L|}\doteq\frac{|h_{12}\ h_{21}|}{|h_{11}|\ |Y_L|} \qquad (69) \qquad \text{when } Y_L\gg h_{22}\ (\text{or } y_{22})$$

$$Y_L=\frac{|h_{12}\ h_{21}|}{|h_{11}|\ \delta} \qquad Y_L+h_{22}=\frac{|h_{12}\ h_{21}|}{|h_{11}|\ \delta}+h_{22}=\frac{2h_{22r}}{L+jM} \qquad (56)$$

$$\text{Let } G\delta+jB\delta=|Y_L|+h_{22}=\frac{|h_{12}\ h_{21}|}{|h_{11}|\ \delta}+h_{22}=\frac{2h_{22r}\ L}{L^2+M^2}-j\frac{2h_{22r}\ M}{L^2+M^2} \qquad (69)$$

$$\therefore\ G\delta=\frac{2h_{22}\ L}{L^2+M^2} \qquad L^2+M^2-\frac{2h_{22}\ L}{G\delta}=0$$

$$\left(L-\frac{h_{22r}}{G\delta}\right)^2+M^2=\left(\frac{h_{22r}}{G\delta}\right)^2 \qquad (70)$$

$$G\delta=\frac{|h_{12}\ h_{21}|}{|h_{11}|\ \delta}+h_{22r} \qquad B\delta=h_{22i} \qquad (71)$$

Eq (70) is an equaton of constant $G\delta$ circle similar to Eq (58) the loci of constant $G\delta$ in the L M plane is the sauce as the G_2 circles in Fig. 7

sensitivity is a measure of non unilateralness″ it has significance in multistage IF strip design. Sensitivity contourt define the area in which a given loade can be placed to satisfy gain and sensitivity requirments, δ is normally less than 0.3.

Sensitivity contours and gain circles on L M plane neatly relate gain, stability alignability and bandwidth at a glance

all h paramcters in the above formulas can be replaced by the corresponding y parameters and 2g is replaced by Yg, then carry on all calculations.

The following examples arc shown for its design technigue for $c<1$ and $c>1$

Reference: Transistors and active circuits by Linvill and Gibbons 1961 Mc Graw Hill Book co.

Part 2. Examples

a $c<1$ stable condition

use TI transistor 2N>43 in C E connection at 60mc (V_{CE}=5v I_C=5ma) its h parameters are

$h_{11}=81.6-j73.1=109.5\ \underline{/-41.9}$

$h_{12}=.0592+j.0661=.0887\ \underline{/48.1}$

$h_{21}=-0.493-j6.08=6.08\ \underline{/-94.65}$

$h_{22}=(6.16+j1.52)10^{-3}=6.35\ 10^{-3}\ \underline{/13.85}$

After calculation we get following data

$c=.85\ \ \theta=-133.45\ \ d=.557\ \ (d=\frac{cg}{2}$ at rad.=0)

P_{io}=51.5w 17.12 db, P_{oo}=1500 w, 31.77 db, G_r=43.75

G_{oo}=29.1, 14.64 db K_g=1.31 G_{max}=38.1, 15.82 db

If we use the maximnm gain at the point circle, Gmax=15.82 db at d=.557, then $G_2=1.34\ h_{22r}=8.25\ 10^{-3}$ $B_2=-0.4\ h_{22r}=-2.46\ 10^{-3}$ and $Y_L=8.25\ 10^{-3}-6.16\ 10^{-3}=2.09\ 10^{-3}$ The sensitivity is too high $\delta=2.35$, therefore we use $\delta=.3$ and we get $Y_L\geq 16.4\ 10^{-3}$ and $G_2\geq h_{22r}+|Y_L|$ $G_2\geq 3.66\ h_{22r}=22.56\ 10^{-3}$ Then find a constant gain circle which is tangent to $G_L=3.66\ h_{22r}$ circle, we get g=0.61 circle (G=17.7 12.5 db), its center is at $-\frac{gc}{2}=-.26$ and its radius is 0.676, At the tangent point $G_2=3.66\ h_{22r}$ $B_2=-0.4\ h_{22r}$ This gain circle g=0.61 (G=17.7, 12.5 db) is only 2.1 db less than G_{oo} (14.64db), Then draw-3db constant gain circle i.e. 9.5 db (i.c. g=.306 G=8.9) constant gain circle, its center is at $-\frac{gc}{2}=-0.13$ and its radius is 0.844. This constant gain circle g=.306 intersects the constant $G_2=3.66\ h_{22r}$ circle at two points, we get $B_{2c}=-4h_{22r}$, $B_{2b}=3.3\ h_{22r}$, then $\triangle B=7.3\ h_{22r}=45\ 10^{-3}$, the total load capacitance is $C_L=\frac{\triangle B}{2\triangle w}$, Assuming a 3db bandwidth of 3mc for both input and output networks, c_L=119.4 pf, $B_L=wc_L-\frac{1}{wL_2}$ $=B_2-h_{22i}$, $B_L=-2.46\ 10^{-3}-1.52\ 10^{-3}=-3.98\ 10^{-3}$ $\therefore$ L_2=.054uh $2_{in}=h_{11}-\frac{h_{12}\ h_{21}}{h_{22}+Y_L}=66.35-j54.7=86\ \underline{/-39.5}$ r at $Y_L=16.4\ 10^{-3}$ If we use the input impedence overlay, we get $\frac{R_1}{|G_r|}=-.39$ $\frac{X_1}{|G_r|}=0.35$ at the tangent point $G_2=3.66\ h_{22r}$ $B_2=-0.4\ h_{22r}$ and $-\theta=133.45$, $2_{in}=h_{11}+R_1+jx_1=81.6$

$-j73.1-17.1+j15.3=64.5-j57.8$, $Y_{in}=8.97\ 10^{-3}+j7.4\ 10^{-3}=11.63\ 10^{-3}\ \underline{/39.5}$ $Y_g=8.97\ 10^{-3}-j7.4\ 10^{-3}$, $Y_{out}=h_{22}-\dfrac{h_{12}\ h_{21}}{h_{11}+2_g}=3.36\ 10^{-3}+j3.82\ 10^{-3}$ $Y_{out}=5.1\ 10^{-3}$ $\underline{/48.7}$, $C_{out}=\dfrac{3.82\ 10^{-3}}{2\pi.\ 610^2}=10.4$pf. (transistor output capacitance) Then calculate the parallel equivalent of the conpling condenser c_{2a} which is in series with R_2 (50r) as shown in Fig. 10. $c_{2a}=\dfrac{1}{w\theta_2\ R_2}$ $\theta_L=\sqrt{\dfrac{1}{G_L\ R_2}-1}$ $G_L=|Y_L|=16.4\ 10^{-3}$ $\theta_L=0.47$ (to get 3db bandwidth) $c_{2a}=112.8$ pf (use 110 pf) $c_{2a}'=c_{2a}\left(\dfrac{1}{1+\dfrac{1}{\theta_L^2}}\right)=19.9$ pf $\therefore$ c_2 tank $=119.4-10.4-19.9=89.1$ pf (variable one) The transistor input capacitance is $c_{in}=\dfrac{7.4\ 10^{-3}}{2\pi.\ 610^7}=19.6$ pf The total source capacitance is $c_g=\dfrac{2\theta_s\ R_e\ (Y_{in})}{w}=143$ pf with $\theta_s=3$, $c_{1a}=\dfrac{1}{w\ \theta_{in}\ R_1}$ $\theta_{in}=\sqrt{\dfrac{1}{R_e\ (Y_{in})\ R_1}-1}=1.11$ at $R_1=50\Omega$ $c_{1a}=47.8$ pf (use 47 pf) $c_{1a}'=c_{1a}\dfrac{1}{1+\dfrac{1}{\theta_{in}^2}}=25.9$ pf c_1 tank $=143-19.6-25.9=97.5$ pf (variable one) $L_1=\dfrac{1}{w^2\ c_g}=.0492$uh

The process is shown in the figure 10 and the circuit diagram is shown in Fig. 11

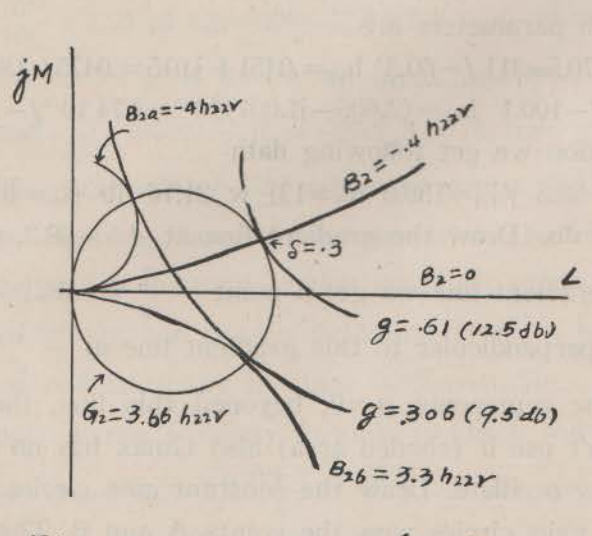

Fig. 10 Linvill's chart

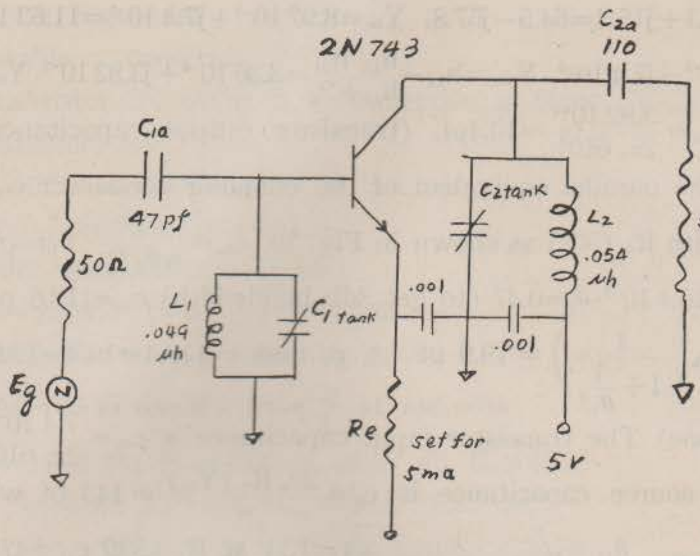

Fig 11 amplifier circuit

b. Example 2 c>1 potentially unstable condition

The IF transistors uses to have c>1, since c decreases with increasing frequency, If we use one series of transistors for the tuner set we get this problem.

use G. E. silicon transistor IN 3855 at 10.7mc (FM, IF, at $V_{CE}=10$v, $I_c=3$ma) its h parameters are

$h_{11}=154.2-j270.5=311\,\underline{/-60.3}$ $h_{12}=.0451+j.015=.0475\underline{/\ 18.4}$ $h_{21}=-4.24-j23.85=24.2\,\underline{/-100.1}$ $h_{22}=(3.605-j1.007)10^{-3}=3.74\,10^{-3}\underline{/-15.6}$

After calculation we get following data

$c=1{,}218$ $\theta=-98.3$ $|G_r|=159.6$ $p_{lo}=131$ w 21.16 db $p_{oo}=40600$ w 46.07 db $G_{oo}=310$, 24.9 db. Draw the gradient line at $\theta=-98.3$, on the negative side of this gradient line we get a point $-\frac{1}{c}=-.821$

draw a line perpendicular to this gradient line at $-\frac{1}{c}$

point this line represents $p_i=0$, beyoned this line, the region is for $p_i<0$, we can't use it (shaded area) also Gmax has no meaning as the transistor may oscillate. Draw the constant gain circles as in the usual way, all the gain circles pass the points A and B. The g=1 (24.9 db) circle passes the origin and A. B points

Assume $\delta = .3$ we get $|Y_L| \geq 12.36\ 10^{-3}$ and $G_2 \geq 3.47\ h_{22r}$ use $G_2 = 3.5\ h_{22r}$ circle, To find the gain circle which tangents this $G_2 = 3.5\ h_{22r}$ circle, we get $g = .821$ $(-.86$ db) $G = 255$ 24.07 db, its center is at $-\frac{cg}{2} = -.5$ its radius is 0.656, The tangent point is at $G_2 = 3.5\ h_{22r} = 12.62\ 10^{-3}$ $B_2 = -1.1\ h_{22r} = -3.9\ 10^{-3}$, Then draw $g = .412$, 21.07 db $G = 128$ circle with its center

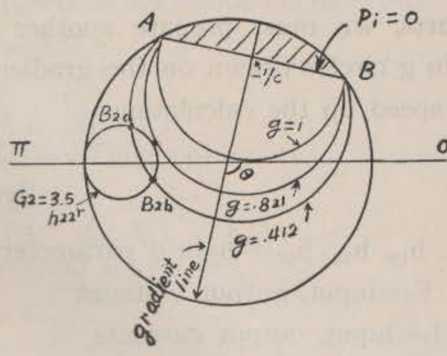

Fig. 12 Gain circle Diagram

at .25 and radius equal to 0.807 The intersecting points of $G_2 = 3.5\ h_{22r}$ and 21.07 db gain circle are at $B_{2a} = -4.2\ h_{22r}$, $B_{2b} = 2.2\ h_{22r}$ $\therefore \triangle B = 6.4\ h_{22r} = 0.96\ 10^{-3}$, $c_L = \frac{\triangle B}{2\triangle w} = 306$ pf with bandwidth 250 kc (FM, IF band width), $B_L = wc_L - \frac{1}{wL_2} = B_2 - h_{22i} = -1.1\ h_{22} - h_{22i} = -2.953\ 10^{-3}$ $\therefore L_2 = .634$ μh $2_{in} = 139.34 - j200 = 244\ \underline{/-55.1}$ with $Y_L = 12.4\ 10^{-3}$ $Y_{in} = 4.1 10^{-3}\ \underline{/55.1}$ $c_{in} = \frac{3.36\ 10^{-3}}{67.2\ 10^{6}} = 50.1$ pf $G_{in} = 2.35\ 10^{-3}$ $R_{in} = 426\Omega$ $Y_g = (2.35 - j3.364)\ 10^{-3}$ $Y_{out} = (2.19 + j2.54)\ 10^{-3} = 2.36\ 10^{-3}\ \underline{/49.2}$ with $Y_g = 4.1\ 10^{-3}\ \underline{/-55.1}$ $\therefore$ cont $= 37.8$ pf $(\frac{2.54\ 10^{-3}}{67.2\ 10^{6}} = 37.8$ pf $C_{2b} = 306 - 37.8 = 268.2$ pf (tank) $G_{out} = 2.19\ 10^{-3}$ $R_{out} = 456 s_L$ c_1 tank $= 268.2$ pf If we use the input impedance overlay we get $\frac{R_1}{|G_r|} = -.185$ and $\frac{X_1}{|G_r|} = 0.49$ $\therefore R_1 = -29.5$ $X_1 = 78$ then $2_{in} = 154.2 - j270.5 - 29.5 + j78 = 124.7 - j192.5 = 230\ \underline{/-57.1}$ when we design the IF stage of FM receiver, we select high h_{11r} transistor, because when the freguency swings to lower side, the load is inductive which by Miller effect turns out more negative input resistance and causes oscillation, If h_{11r} is high enoguh, we get position input resistance the IF stage is then stable,

Finally we apply sterns stabilety formnla to check the stability of the stages $(y_{11r} + G_g)\ (y_{22r} + G_L) = k\ (\frac{u+v}{2})$ If $k > 1$ the system is stable if $k < 1$ the system is unstable G_g is the source conductance, G_L is the load conductance $u = |y_{12}\ y_{21}|$ $v = R_e\ (y_{12}\ y_{21})$ y_{11r}, y_{22r} are real parts of y_{11} and y_{22}

To do high frequency transistor amplifier design by using Linvill's charts, we must prepare another two transparent overlays of constant gain g circles drawn on the gradient line and constant sensitivity δ chart to speed up the calculation

Symbols

h_{11}, h_{12}, h_{21} h_{22}="hybrid" paramcters
E_1, E_2=input, output voltages
I_1, I_2=input, output currents
p_i=Input power p_{io}=Inpnt power at $L=1$, $M=0$ point
p_o=output power p_{oo}=power outpnt at $L=1$ $M=0$ point
G_r=Gradient θ=angle of gradient line
m, m'=slope of a line
c=Linvill's stability factor
R_e=Real part of Im=Imaginary part of
h_{11r}, h_{12r}, h_{21r}, h_{22r},=real part of h_{11}, h_{12}, h_{21}, h_{22}
G_{oo}=power gain at $L=1$, $M=0$
G=power gain, G_{opt}=max, power gain.
k_g=power ratio
Y_L=load admittance, G_L=load conductance
B_L=load susceptance
δ=sensitivity factor