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利用步進響應匹配簡化分離時間傳遞函數
Simplification of Discrete-Time Transfer Functions Via Step-Response Matching

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Abstract — A computer-aided method for linear discrete-time system model reduction via step-response matching is presented in this paper. Golub's algorithm for solving least squares problem is used to find the optimum coefficients of the reduced model. The advantage of this method is that both the time response and frequency response within the bandwidth region of reduced model are very close to that of original model. Several illustrative examples are provided.

I. Introduction

Since the exact analysis of most of the systems of higher order is both tedious and costly, model reduction techniques have been the subject of many recent investigations. There are two ways of simplifying the systems: (1) time-domain approach, and (2) frequency-domain approach. The purpose of model reduction technique is to use a low order model to replace the original high order system for easy analysis and design of control systems such that both the time response and frequency response of the new model is very close to those of the original system. Most of the methods presented usually satisfy the system specifications in one domain but fail in the other domain. Whenever time-domain approach is used, the frequency response, specially in the bandwidth region, of the reduced model should be as close as that of the original system. A number of methods [1, 2, 3, 4, 5, 6] for reduction of discrete-time transfer functions have been presented in the literature. Shih et al. [1] used bilinear transformation and continued fraction method to reduce the order of the discrete-time transfer function; the initial output response of the reduced model may not be zero, although that of the original system is zero. The method proposed by Shamash [2] may have stability problems. Shih et al. [5] used moments matching and retaining dominant eigenvalues technique to reduce the order of the discrete-time transfer function, and the result seems to be better than that of the results of other methods.

In this paper, a new method based on minimization of square errors of step response of the reduced model and that of the original system is presented. Golub's algorithm [7, 8] for solving least squares problem is used iteratively to find the optimum coefficients of the reduced model. The whole procedures can be programmed by using FORTRAN IV or any other high-level language. The time response and frequency response within the bandwidth of the reduced model are very close to those of the original system.

II. Review of Golub's Algorithm for Least Squares Errors Problems

Golub's algorithm is a numerical algorithm for solving least squares problem in a highly accurate manner. Breen et al. [7] and Kao [8] have applied this algorithm in circuit optimization successfully. The concept of this algorithm for least squares problems is described briefly as follows.

Consider the system of linear equations

$$\underline{A}\,\underline{X} = \underline{B} \tag{1}$$

where $\underline{A}$ is a given (n x m) real matrix of rank m, $\underline{B}$ is a given (n x 1) real column vector, and $\underline{X}$ is an unknown (m x 1) real column vector. Equation (1) has unique solution if n = m. If the number of unknows is less than the number of equation (i. e. m < n), the unknown vector $\underline{X}$ can be obtained by linear multiregression:

$$\underline{X} = (\underline{A}^T\,\underline{A})^{-1}\,\underline{A}^T\,\underline{B} \tag{2}$$

where $\underline{A}^T$ is the transpose of $\underline{A}$. Although the rank of $\underline{A}$ is m, the rank of $\underline{A}^T\,\underline{A}$ may be less than m and the solution of $\underline{X}$ can not be found from Equation (2) [7]. Golub's algorithm is another algorithm to find the unknown $\underline{X}$ of Equation (1) and avoids the disadvantage of linear multiregression method. This algorithm is to find a vector $\hat{\underline{X}}$ such that the euclidean norm of $\underline{B} - \underline{A}\,\underline{X}$, $||\,\underline{B} - \underline{A}\underline{X}\,||$, is minimum. Then we will choose on orthogonal matrix $\underline{Q}$ such that

$$\underline{Q}\,\underline{A} = \underline{R} = \begin{bmatrix} \tilde{\underline{R}} \\ \cdots\cdots \\ \underline{O} \end{bmatrix} \begin{matrix} m \times m \\ \\ (n-m) \times m \end{matrix} \tag{3}$$

and $\tilde{\underline{R}}$ is an upper triangular matrix as

$$\tilde{\underline{R}} = \begin{bmatrix} r_{11} & r_{12} \cdots\cdots & r_{1m} \\ 0 & r_{22} \cdots\cdots & r_{2m} \\ & \ddots & \vdots \\ 0 & & r_{mm} \end{bmatrix} \tag{4}$$

Then

$$||\,\underline{B} - \underline{A}\,\underline{X}\,|| = ||\,\underline{Q}\,\underline{B} - \underline{Q}\,\underline{A}\,\underline{X}\,|| = ||\,\underline{C} - \underline{R}\,\underline{X}\,|| \tag{5}$$

where $\underline{C} = \underline{Q}\,\underline{B}$

Equation (5) can be rewritten as

$$\begin{aligned} ||\,\underline{B} - \underline{A}\,\underline{X}\,|| &= [(c_1 - r_{11}x_1 - r_{12}x_2 \cdots\cdots - r_{1m}x_m)^2 \\ &\quad + (c_2 - r_{22}x_2 \cdots\cdots - r_{2m}x_m)^2 \\ &\quad + c_{m+1} + c_{m+2} + \cdots\cdots + c_n]^{1/2} \end{aligned} \tag{6}$$

Thus $||\,\underline{B} - \underline{A}\,\underline{X}\,||$ is minimized when

$$\left.\begin{array}{l} r_{11}\,x_1 + r_{12}\,x_2 + \dots\dots + r_{1m}\,x_m = c_1 \\ \qquad r_{22}\,x_2 + \dots\dots + r_{2m}\,x_m = c_2 \\ \qquad\qquad \ddots \qquad \vdots \\ \qquad\qquad\qquad r_{mm}\,x_m = c_m \end{array}\right\} \tag{7}$$

and the minimized value is

$$\| \underline{B} - \underline{A}\,\underline{X} \| = (c_{m+1} + c_{m+2} + \dots + c_n)^{1/2} \tag{8}$$

The decomposition of Equation (3) can be realized by Householder transformations. The details of decomposition can be found in the papers of Kao [8], Householder [9], and Wilkinson [10].

From the above analysis, we see that Golub's algorithm can be applied to least squares errors problems. Assume that the cost function F is defined as

$$F(\underline{d}) = \sum_{i=1}^{n} e^2_i(\underline{d}, t_i) \tag{9}$$

where $t_i \in T$ represents the independent variable sampled in T, $\underline{d}$ is a dependent variable vector with m elements, e_i is the error function at t_i, and n is the total number of sampled points in T. The purpose is to find the optimum parameter vector $\underline{d}$ such that the cost function F is minimized. The partial derivative of $e_i(\underline{d})$ with respect to $\underline{d}$ is $\underline{e}'_i(\underline{d}) = \frac{\partial e_i(\underline{d})}{\partial \underline{d}}$, and the Taylor's series of $e_i(\underline{d})$ is

$$e_i(\underline{d} + \Delta \underline{d}) = e_i(\underline{d}) + \underline{e}'_i(\underline{d})\, \Delta\underline{d} + \frac{1}{2!}\,\underline{e}''_i(\underline{d})\,(\Delta\underline{d})^2 + \dots\dots \tag{10}$$

If $\Delta\underline{d}$ is very small, the second-order and higher-order terms can be neglected, and we get

$$e_i(\underline{d} + \Delta \underline{d}) \cong e_i(\underline{d}) + \underline{e}'_i(\underline{d})\, \Delta\underline{d} \tag{11}$$

In order to minimize the cost function F ($\underline{d}$), the direction vector $\Delta\underline{d}$ should be in a direction so that $e_i(\underline{d} + \Delta\underline{d}) = 0$. Hence $\Delta\underline{d}$ can be determined by the equation

$$e_i(\underline{d} + \Delta\underline{d}) \cong e_i(\underline{d}) + \underline{e}'_i(\underline{d})\,\Delta\underline{d} = 0, \; i = 1, 2, \dots, n \tag{12}$$

or

$$e'_i(\underline{d})\,\Delta\underline{d} = -\,e_i(\underline{d}), \; i = 1, 2, \dots, n \tag{13}$$

which can be rewritten as:

$$\begin{bmatrix} \frac{\partial e_1}{\partial d_1} & \frac{\partial e_1}{\partial d_2} & \dots\dots & \frac{\partial e_1}{\partial d_m} \\ \frac{\partial e_2}{\partial d_1} & \frac{\partial e_2}{\partial d_2} & \dots\dots & \frac{\partial e_2}{\partial d_m} \\ \dots & \dots & \dots & \dots \end{bmatrix} \begin{bmatrix} \Delta d_1 \\ \Delta d_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} -e_1 \\ -e_2 \\ \vdots \end{bmatrix} \tag{14}$$

$$\begin{bmatrix} \cdots\cdots\cdots & & \cdots\cdots\cdots \\ \frac{\partial e_n}{\partial d_1} & \frac{\partial e_n}{\partial d_2} \cdots\cdots & \frac{\partial e_n}{\partial d_m} \end{bmatrix} \begin{bmatrix} \vdots \\ \Delta d_m \end{bmatrix} \begin{bmatrix} \vdots \\ -e_n \end{bmatrix}$$

Since $n > m$, we can apply Golub's algorithm to the equation (14) and find the values of $\Delta \underline{d}$.

But due to the approximation made in Equation (12) and the fact that $\Delta \underline{d}$ is only a least square solution, the new parameter vector $\underline{d} + \Delta \underline{d}$ does not, in general, minimize the cost function $F(\underline{d})$. The direction of $\Delta \underline{d}$ only shows that if $\underline{d}$ is changed infinitesimally in the direction of $\Delta \underline{d}$, the cost function $F(\underline{d})$ will be reduced. This means

$$F(\underline{d} + \alpha \underline{d}) < F(\underline{d}) \tag{15}$$

where α in called the step size and $0 < \alpha \leq 1$. The cost function $F(\underline{d} + \alpha \Delta \underline{d})$ vs. α can be plotted as shown in Fig. 1. The value of α should be chosen properly; otherwise, $F(\underline{d} + \alpha \Delta \underline{d})$ will be greater than $F(\underline{d})$, or the convergence rate is too slow. There are many algorithms to find the optimum α; we omit the details here.

Given the initial $\underline{d}$ as $\underline{d}^o$, we may evaluate the value of the cost function $F(\underline{d})$ as F^o. Using abovementioned procedures to find $\Delta \underline{d}$ and α, we get new value of $\underline{d}$ as $\underline{d}^1$ which results in $F(\underline{d}^o) > F(\underline{d}^1)$. We repeat the above procedures until $F(\underline{d} + \alpha \Delta \underline{d})$, or the difference between $F(\underline{d} + \alpha \Delta \underline{d})$ and $F(\underline{d})$ is within a reasonable range. Then the optimum parameter vector can be obtained. All the step of the above procedures can be programmed using FORTRAN IV or any other high-level language and we may get the results from the computer rather easily.

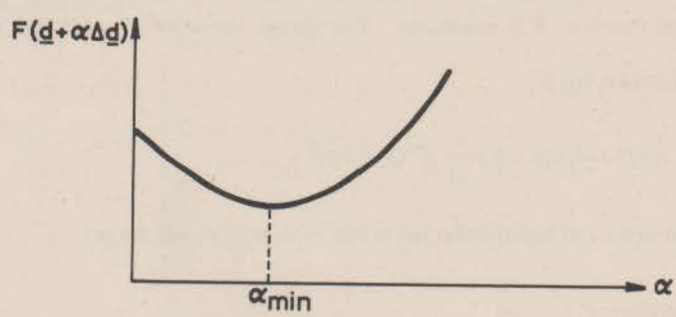

Fig. 1. The cost function $F(\underline{d} + \alpha \Delta \underline{d})$ vs. the step size α .

III. Using Golub's Algorithm in Model Reduction of Linear Time-Invariant Discrete-Time Systems

Consider a linear time-invariant causal discrete-time system with the following transfer function

$$G(Z) = \frac{b_M Z^M + b_{M-1} Z^{M-1} + b_{M-2} Z^{M-2} + \cdots\cdots + b_1 Z + b_o}{Z^N + a_{N-1} Z^{N-1} + a_{N-2} Z^{N-2} + \cdots\cdots + a_1 Z + a_o} \qquad N \geq M \tag{16}$$

Equation (16) may also be written in a nonrecursive form as

$$G(Z) = r_o + r_1 Z^{-1} + r_2 Z^{-2} + \cdots\cdots \tag{17}$$

where r_i is equivalent to g (iT), the impulse response of this discrete-time system at sampling instant iT. Since the z-transform of unit-step function is

$$R(Z) = \frac{1}{1-Z^{-1}} = 1 + Z^{-1} + Z^{-2} + Z^{-3} + \dots\dots\dots \qquad (18)$$

The step-response of G(Z) can be written as

$$\begin{aligned} Y(Z) &= G(Z)\,R(Z) \\ &= (r_o + r_1 Z^{-1} + r_2 Z^{-2} + \dots)(1 + Z^{-1} + Z^{-2} + Z^{-3} + \dots) \\ &= r_o + (r_o + r_1) Z^{-1} + (r_o + r_1 + r_2) Z^{-2} + \dots \\ &= \beta_o + \beta_1 Z^{-1} + \beta_2 Z^{-2} + \dots\dots\dots \end{aligned} \qquad (19)$$

where $\beta_i = \sum_{j=0}^{i} r_j$, and β_i is the step-response of discrete system at sampling instant iT.

The transfer function expression in Eqn. (16) is approximated to a low-order transfer function of prespecified dimension by

$$H(Z) = \frac{d_m Z^m + d_{m-1} Z^{m-1} + d_{m-2} Z^{m-2} + \dots + d_1 Z + d_o}{Z^n + c_{n-1} Z^{n-1} + c_{n-2} Z^{n-2} + \dots\dots\dots + c_1 Z + c_o}, \quad N > n \geq m \qquad (20)$$

The problem now is to find the coefficient vectors $\underline{C} = (c_o, c_1, c_2, \dots c_{n-1})$ and $\underline{D} = (d_o, d_1, \dots d_m)$ so that the performance of the model H (Z) is very close to that of the original system G (Z).

Suppose both systems G (Z) and H (Z) are stable and the step-responses at sampling instant iT of both systems are denoted as $y_1(iT)$ and $y_2(iT)$. The sum of squares of the errors of these step-responses can be respresented by cost function $F_1(\underline{C}, \underline{D})$ as

$$F_1(\underline{C}, \underline{D}) = \sum_{i=0}^{\infty} e_i^2 = \sum_{i=0}^{\infty} [y_1(iT) - y_2(iT, \underline{C}, \underline{D})]^2 . \qquad (21)$$

Since G (Z) and H (Z) are stable, the step-responses of these two systems will reach steady states after a finite time interval. The closeness of these two step-responses can also be justified by the cost function F $(\underline{C}, \underline{D})$ as

$$F(\underline{C}, \underline{D}) = \sum_{i=0}^{K} e_i^2 = \sum_{i=0}^{K} [y_1(iT) - y_2(iT, \underline{C}, \underline{D})]^2 \qquad (22)$$

where KT is much greater than the settling time of both the systems. Since the order of H (Z) should be as low as possible, so K is always greater n+m+1, and Golub's algorithm can be applied in this case also.

Guess the initial values of coefficient vectors $\underline{C}$ and $\underline{D}$ of the reduced model as $\underline{C}^o$ and $\underline{D}^o$ which will make the resulting system stable. We may use Eqn. (19) to find the step-responses of G (Z) and H (Z). Using the proceduces described in Section II, we may get coefficient correction vectors $\Delta\underline{C}^o$ and $\Delta\underline{D}^o$, and optimumstep size α^o. The new coefficient vecotors $\underline{C}^1$ and $\underline{D}^1$ are

$$\underline{C}^1 = \underline{C}^o + \alpha^o \Delta\underline{C}^o$$

$$\underline{D}^1 = \underline{D}^o + \alpha^o \ \Delta \underline{D}^o \tag{23}$$

snd $F(\underline{C}^1, \underline{D}^1) < F(\underline{C}^o, \underline{D}^o)$. Repeat above procedures iteratively until $|F(\underline{C}^j, \underline{D}^j)|$, or $|F(\underline{C}^j, \underline{D}^j) - F(\underline{C}^{j-1}, \underline{D}^{j-1})|$ is within a reasonable range, and the optimum coefficients of the new model are found. All the reduction procedures can again be programmed in FORTRAN IV or in other high-level languages and hence the optimum coefficients of the reduced model can be found very easily.

The frequency response of G (Z) is written as

$$G(e^{j\omega T}) = \frac{b_M e^{jM\omega T} + b_{M-1} e^{j(M-1)\omega T} + b_{M-2} e^{j(M-2)\omega T} + \ldots + b_1 e^{j\omega T} + b_o}{e^{jN\omega T} + a_{N-1} e^{(N-1)\omega T} + a_{N-2} e^{j(N-2)\omega T} + \ldots + a_1 e^{j\omega T} + a_o} \tag{24}$$

or can be evaluated from Eqn. (17) as

$$G(e^{j\omega T}) = r_o + r_1 e^{-j\omega T} + r_2 e^{-j\omega T} + \ldots \tag{25}$$

When $\omega = o$, the value of $G(e^{joT})$ is

$$G(e^{jo}) = r_o + r_1 + r_2 + \ldots = \sum_{i=0}^{\infty} r_i \tag{26}$$

which is equal to the steady-state value of the step-response. So matching the steady state values of step-responses of the reduced model and of the original system is the same as matching the frequency responses at zero frequency of both the systems. Since the sampling frequency $\omega_s = \frac{2\pi}{T}$, Eqn. (25) can be rewritten as

$$\begin{aligned} G(e^{j\omega T}) &= G(e^{j2\pi\frac{\omega}{\omega_s}}) = r_o + r_1 e^{-j2\pi\frac{\omega}{\omega_s}} + r_2 e^{-j2\pi\frac{2\omega}{\omega_s}} + \ldots \\ &= [r_o + r_1 \cos(2\pi\frac{\omega}{\omega_s}) + r_2 \cos(2\pi\frac{2\omega}{\omega_s}) + \ldots] \\ &\quad - j[r_1 \sin(2\pi\frac{\omega}{\omega_s}) + r_2 \sin(2\pi\frac{2\omega}{\omega_s}) + \ldots]. \end{aligned} \tag{27}$$

At low frequency, $\cos \omega T$ approaches 1, and $\sin \omega T$ approacheso,so that the real part of $G(e^{j\omega T})$ dominates the frequency response and the value is approximately equal to $\sum_{i=0}^{\infty} r_i$. From Eqn. (19), we see that matching of step-responses of both the systems is equivalent to matching of impulse responses of both the systems. But from experimental study, we see that the frequency response of the reduced model via matching step-response is better than that via matching impulse response.

IV. Numerical Examples:

Example 1:

Consider the following eighth-order discrete-time transfer function which is used in the paper by Shih et al. [1,5]

$$G(Z) = \frac{280.333Z^7 + 186Z^6 - 35Z^5 + 25.333Z^4 - 86Z^3 - 43.666Z^2 + 7.333Z - 1}{666Z^8 - 280.333Z^7 - 186Z^6 + 35Z^5 - 25.333Z^4 + 86Z^3 + 43.666Z^2 - 7.333Z + 1} \tag{28}$$

and the sampling period of this system $T = \sqrt{0.5}$. Use the procedures described in Section III and select the first 30 points. The optimum coefficients of the second-order and third-order reduced models can be obtained as:

$$H_2(Z) = \frac{0.460997Z - 0.303206}{Z^2 - 1.530156Z + 0.687127} \tag{29}$$

and

$$H_3(Z) = \frac{0.42604Z^2 - 0.304414Z + 0.000989}{Z^3 - 1.722563Z^2 + 0.991368Z - 0.146425} \tag{30}$$

The second-order reduced model obtained by Shih et al. [5] is

$$H(Z) = \frac{0.4981Z - 0.34194}{Z^2 - 1.50189Z + 0.65805} \tag{31}$$

The step responses of G (Z), $H_2(Z)$, $H_3(Z)$ and H (Z) are listed in Table 1.

The frequency responses of these systems are also plotted in Fig. 3 we see from Fig. 2 that the frequency response within the bandwidth region of $H_2(Z)$ or $H_3(Z)$ is very close to that of G (Z).

Table 1. The step-responses of G(Z), $H_2(Z)$, $H_3(Z)$ and H (Z) of Example 1.

Time instant T	Original system G (Z)	Second-order model $H_2(Z)$	Third-order model $H_3(Z)$	Second-order model of Shih et al. H (Z)
0.	0.	0.	0.	0.
0.707	0.420920	0.460997	0.426040	0.498100
1.414	0.877374	0.863189	0.855506	0.904251
2.121	1.13451	1.16184	1.17392	1.18647
2.828	1.38613	1.34247	1.35902	1.34307
3.535	1.42675	1.41365	1.42510	1.39154
4.242	1.39806	1.39845	1.40204	1.36380
4.949	1.31835	1.32628	1.32391	1.28807
5.656	1.22423	1.22630	1.22187	1.19326
6.363	1.12044	1.12289	1.12017	1.10068
7.070	1.03397	1.03337	1.03471	1.02404
7.777	0.973932	0.967444	0.973383	0.969856
8.484	0.938627	0.928074	0.937568	0.938904
9.191	0.924598	0.913133	0.924162	0.928078
9.898	0.927437	0.917323	0.927594	0.932184
10.605	0.941339	0.934001	0.941553	0.945477
11.312	0.959799	0.956624	0.960233	0.962739
12.019	0.978469	0.979826	0.979073	0.979916
12.726	0.994455	0.999744	0.995053	0.994356
13.433	1.00621	1.01429	1.00664	1.00473

14.140	1.01326	1.02287	1.01351	1.01083
14.847	1.01614	1.02599	1.01620	1.01315
15.554	1.01584	1.02488	1.01572	1.01262
16.261	1.01349	1.02103	1.01323	1.01030
16.968	1.01014	1.01590	1.00981	1.00716
17.675	1.00667	1.01071	1.00632	1.00398
18.382	1.00366	1.00628	1.00334	1.00126
19.089	1.00141	1.00307	1.00115	0.999283
19.796	1.00003	1.00120	0.999835	0.998088
20.503	0.999425	1.00055	0.999298	0.997600

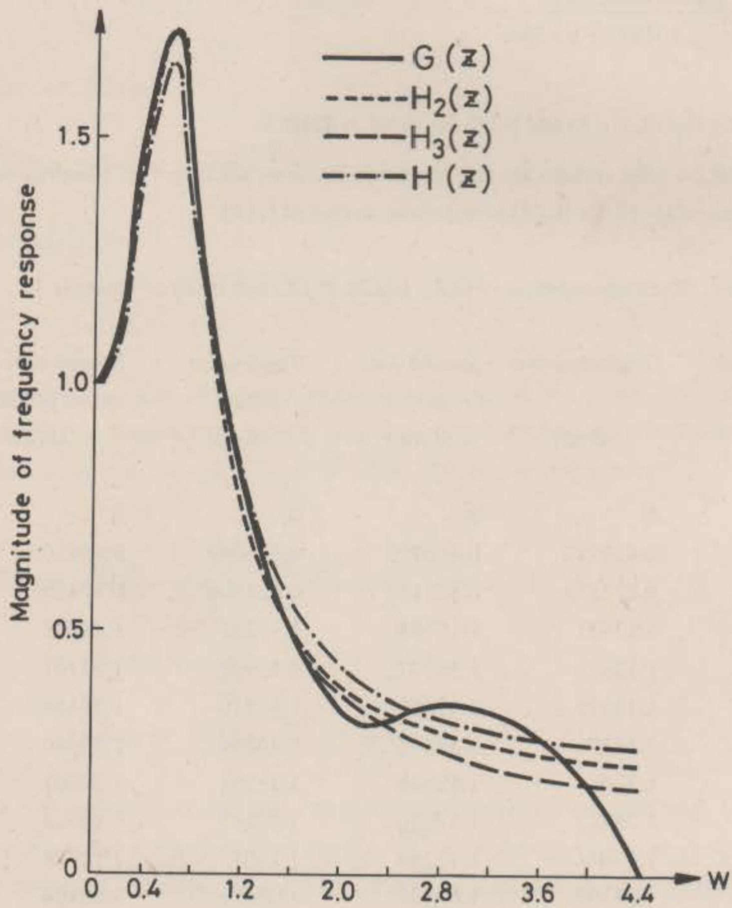

Fig. 2. Gain plots of frequency response of G (Z), H_2 (Z), H_3 (Z) and H (Z) of example 1.

Example 2.

Consider a continuous-time system with seventh-order transfer function [11] as

$$F(S) = \frac{(S+0.5)(S+10)(S+14)\,[(S+4)^2+4^2]}{(S+1)\,[(S+2)^2+2^2]\,[(S+2)^2+4^2]\,[(S+3)^2+3]^2} \tag{32}$$

Assume that a sampling period T is chosen as .3 sec; the resulting discrete-time system may be evaluated by impulse invariant transformation as

$$G(Z) = \frac{0.269541Z^6 - 0.160640Z^5 - 0.077199Z^4 + 0.027885Z^3 - 0.013412Z^2}{Z^7 - 2.549914Z^6 + 3.127139Z^5 - 2.418528Z^4 + 1.259320Z^3 - 0.442880Z^2}$$

$$\frac{+\,0.000861Z}{+\,0.097047Z - 0.011109} \tag{33}$$

Use the procedures described in Section III and select the first 24 points, the optimum coefficients of the third-order reduced model can now be obtained as

$$H(Z) = \frac{0.251345Z^2 + 0.139610Z - 0.310523}{Z^3 - 1.721266Z^2 + 1.134987Z - 0.309154} \tag{34}$$

The time responses and frequency responses of both the systems are listed and plotted as shown in Table 2 and Fig. 3, respectively.

Table 2. The step-response of original model and reduced model of Example 2.

Time	Original model	Reduced model
0.3	0.2695	0.2513
0.6	0.7862	0.8236
0.9	1.2191	1.2129
1.2	1.3302	1.3109
1.5	1.2120	1.2149
1.8	1.0429	1.0588
2.1	0.9247	0.9292
2.4	0.8652	0.8537
2.7	0.8375	0.8226
3.0	0.8209	0.8147
3.3	0.8087	0.8130
3.6	0.7996	0.8094
3.9	0.7928	0.8029
4.2	0.7875	0.7950
4.5	0.7831	0.7878
4.8	0.7797	0.7823
5.1	0.7771	0.7787
5.4	0.7753	0.7764
5.7	0.7739	0.7748
6.0	0.7729	0.7736
⋮	⋮	⋮
15.0	0.7701	0.7701

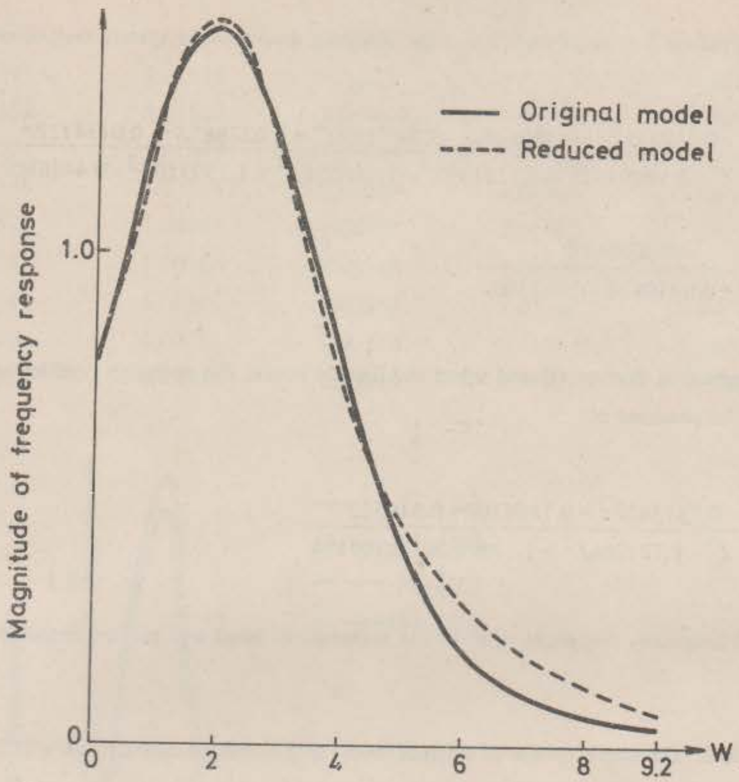

Fig. 3. Gain plots of frequency responses of original model and reduced model of example 2.

Example 3.

Consider another continuous-time system with ninth-order transfer function [11] as

$$F(S) = \frac{S^4 + 35S^3 + 291S^2 + 1093S + 1700}{(S+1)[(S+1)^2+1^2]\,[(S+1)^2+2^2]\,[(S+1)^2+3^2]\,[(S+1)^2+4^2]} \tag{35}$$

With the sampled procedure as described in Example 2, and that the sampling period T is chosen as .5 sec; the resulting discrete-time function is

$$G(Z) = \frac{0.004038Z^8+0.062137Z^7+0.109120Z^6+0.028222Z^5-0.000742Z^4}{Z^9-1.907509Z^8+2.194341Z^7-1.921284Z^6+1.381388Z^5-0.837855Z^4}$$

$$\frac{+0.001571Z^3-0.000182Z^2+0.000017Z}{+0.408693Z^3-0.180123Z^2+0.057602Z-0.011109} \tag{36}$$

The optimum coefficients of the third-order reduced model can be obtained as

$$H(Z) = \frac{0.009567Z^2+0.037979Z+0.199744}{Z^3-1.352284Z^2+0.790448Z-0.191039} \tag{37}$$

The step-responses and frequency responses of G (Z) and H (Z) are listed and plotted as shown in Table 3 and Fig. 4, respectively.

Table 3. The step-response of original model and reduced model of Example 3.

Time	Originl model	reduced model
0.5	0.0040	0.0096
1.0	0.0739	0.0605
1.5	0.3074	0.3215
2.0	0.6354	0.6361
2.5	0.8768	0.8649
3.0	0.9743	0.9754
3.5	0.9952	1.0042
4.0	0.9978	0.9995
4.5	0.9983	0.9914
5.0	0.9983	0.9898
5.5	0.9983	0.9930
6.0	0.9983	0.9972
6.5	0.9983	1.0000
7.0	0.9983	1.0001
7.5	0.9986	1.0001
8.0	0.9992	1.0001
8.5	0.9997	1.0001
9.0	1.0000	1.0000
9.5	1.0000	1.0000
10.0	1.0000	1.0000

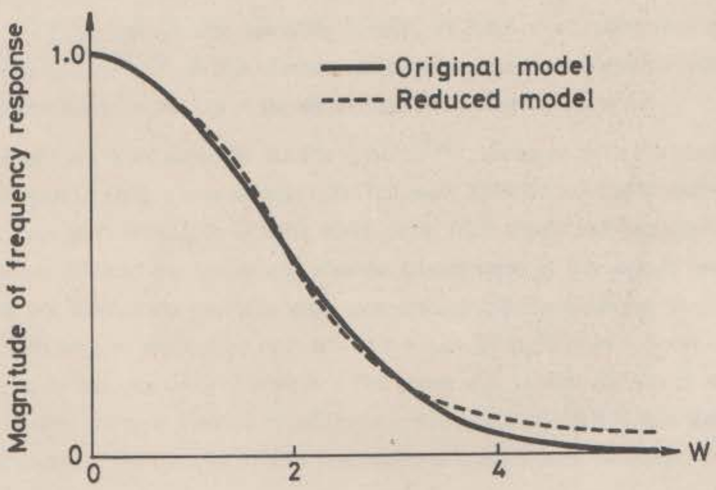

Fig. 4. Gain plots of frequency responses of original model and reduced model of example 3.

V. Conclusions:

A new approach to simplify high-order discrete-time transfer functions to low-order discrete-time transfer functions via step-response matching has been proposed. The advantage of this method is that both the time response and frequency response within the bandwidth region of the new low-order model are very close to those of the orignal system.

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