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含積分式控制器之設計
On the Design of Controller with Integral Mode

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ABSTRACT — An optimal or a sub-optimal control law with integral mode is developed by the introduction of a multiplier into the Kalman control law and the proper modification, while not changing the original quadratic performance index. The resulting control yields an output transient response which is reasonable and satisfactory for both undisturbed and disturbed cases. The result is also compared with that obtained by the classical Cohen-Coon method and a smaller value of the performance index is obtained.

I. Introduction

The optimal control of linear systems with quadratic performance index is well Known [1]. However, in practice, very few works have been done on the application of optimal control theory to the design of the control systems of industrial processes. The gap between optimal control theory and industrial applications is still wide and open. The restrictions of the practical usefulness of optimal control law are that all the state variables can not be measured and the system disturbances are ignored.

Kalman [2] had shown that the optimization based on a quadratic performance functional for all initial states of an n-th order , linear regulator system requires that all n states be avilable for measurement. However , it is often not possible to measure every state variable in most of the chemical plants, and hence, the performance of any system with inaccessible state, in general, will be sub-optimal. The investigations of the problems concerned with the optimization of linear regulator systems with some inaccessible state variables are available for elsewhere [3-13].

The optimal control theory gives a control law which is only proportional feedback of the state, i.e. no integral action is obtained by the theory. This lack of integral action leads to an offset if an external disturbances occur or if there is some modelling error. To avoid offset from a permanent change in the load, Koppel [14] suggested that an amount of integral action in parallel with the optimal control should be added in the design of a digital process controller.

For a linear plant with a single input and an external disturbance, Johnson [15-16] obtained the optimal feedback control by minimizing not the usual quadratic performance index but one which penalizes the rate of change of control rather than the control itself. By the introduction of the integral of the output variable as a new state variable, shih [17] obtained the optimal linear feedback control law for the second-order system to the conventional proportional-integral-derivative (PID) control by changing the weighting factors of the quadratic criterion. In order to meet the requirement of $X(\infty)=0$, Shih's solution also had an additional restriction, i.e.

$$\int_0^\infty C^*(t)dt \equiv 0 \ ,$$

Several investigator's considerations are also available in the previous literatures [18-20]. However, their approaches are all based on the change of the performance index, which are equivalent to the consideration of the entire different problems, or they require the complete a prior knowledge of the disturbance.

The author [6] has presented a technique which introduces integral mode into the optimal control law and does not require the measurement of all of the state variables during the dynamic period. In this paper, a further result is presented and a method which incorporates an integral mode without changing the original quadratic performance index for a single-input linear system is developed.

II. Construct the Control Law with an Integral Mode

Consider the controllable linear time-invariant system

$$\dot{x}(t)=A\ x(t)+B\ u(t) \tag{1}$$

and the quadratic performance index

$$J(u)=\frac{1}{2}\int_0^\infty[x^TQx+u^TRu]dt \tag{2}$$

where u(t) is not constrained, Q is a positive semidefinite matrix, and R is a positive definite matrix.

Then an optimal control exists, is unique, and is given by the equation

$$u(t)=-R^{-1}B^TKx(t) \tag{3}$$

where the optimal gain matrix, K, is the constant nxn positive definite matrix which can be obtained from the nonlinear matrix algebraic equation [1].

$$-KA-A^TK+KBR^{-1}B^T\bar{K}-Q=0 \tag{4}$$

For single-input-single-output systems, the control can be written as

$$u(t)=-k_1c-k_2\frac{dc}{dt} - \ldots\ldots -k_n\frac{d^{n-1}c}{dt^{n-1}}, \quad (5)$$

Thus, for a second-order system, the optimal control theory gives a control law which is equivalent to a proportional-derivative control. No integral action is obtained by the theory. In order to have controllers with integral mode, suitable transformations and modifications of Kalman control are investigated.

A second-order system is chosen for detailed discussions because the physical realization of the optimal control can be demonstrated easily. However, the technique can be easily extended to more general systems.

Consider the second-order system which has been treated by Shih [17].

$$\tau_1\tau_2 \frac{d^2c}{dt^2} + (\tau_1+\tau_2) \frac{dc}{dt} + c=k_pu \quad (6)$$

According to Equation (5), the feedback control law can be written as

$$u(t)=-k_1c-k_2\frac{dc}{dt} \quad (7)$$

Now, integrate Equations (6) and (7) with respect to time to obtain

$$\tau_1\tau_2\frac{dc}{dt}+(\tau_1+\tau_2)c+\int_0^t cd\tau=k_p\int_0^t ud\tau+[\tau_1\tau_2 \frac{dc(0)}{dt}+(\tau_1+\tau_2)c(0)] \quad (8)$$

and

$$\int_0^t ud\tau=-k_1\int_0^t cd\tau-k_2c+k_2c(0) \quad (9)$$

Substituting Equation (9) into Equation (8), we obtain

$$\tau_1\tau_2 \frac{dc}{dt}+(\tau_1+\tau_2+k_2k_p)c+(1+k_1k_p)\int_0^t cd\tau=\tau_1\tau_2\frac{dc(0)}{dt}$$

$$+(\tau_1+\tau_2+k_2k_p)c(0) \quad (10)$$

Define

$$f(t)\equiv\frac{dc}{dt}+a_2c+a_1\int_0^t cd\tau-a_0 \quad (11)$$

where

$$a_0=\frac{dc(0)}{dt}+a_2c(0) \quad (11a)$$

$$a_1=\frac{1+k_1k_p}{\tau_1\tau_2} \quad (11b)$$

$$a_2=\frac{\tau_1+\tau_2+k_2k_p}{\tau_1\tau_2} \quad (11c)$$

then, by Equation (10), we have

$$f(t)\equiv 0 \tag{12}$$

Consider, with further a prior justification, the function termed the augmented control function,

$$U(t)=u(t)-\lambda f(t)$$

where λ is an arbitrary parameter, or multiplier. In actuality, since by Equation (12), $f(t)\equiv 0$, we have

$$U(t)\equiv u(t)$$

and therefore, the control equation can be written as

$$u(t)=-k_1c-k_2\frac{dc}{dt}-\lambda\left[\frac{dc}{dt}+a_2c+a_1\int_o^t cd\tau-a_o\right]$$

or

$$u(t)=-\left\{(\lambda+k_2)\frac{dc}{dt}+(a_2\lambda+k_1)c+a_1\lambda\int_o^t cd\tau\right\}+a_o\lambda \tag{13}$$

This is a modified conventional proportional-integral-derivative (PID) control. That is, for a suitable transformation, an integral mode can be introduced in the Kalman control. However, the control as defined in Equation (13) is no longer function of output only, it also depends upon the state initial conditions except the limitting case, say,

$$a_o=\frac{dc(0)}{dt}+a_2c(0)=0$$

or

$$\frac{dc(0)}{dt}+\frac{\tau_1+\tau_2+k_2k_p}{\tau_1\tau_2}c(0)=0. \tag{14}$$

Since Equation (13) is undesired in the controller design, therefore, a modification must be made to obtain a control such that the controller is independent of any state initial conditions, and contains an integral mode, i.e. assuming $a_o=0$, then, Equation (13) becomes

$$u(t)=-\left\{(\lambda+k_2)\frac{dc}{dt}+(a_2\lambda+k_1)c+a_1\lambda\int_o^t cd\tau\right\} \tag{15}$$

or

$$u(t)=-\left\{K_2\frac{dc}{dt}+K_1c+K_o\int_o^t cd\tau\right\} \tag{16}$$

where

$$K_2=k_2+\lambda \tag{16a}$$

$$K_1=k_1+(\tau_1+\tau_2+k_2K_p)\lambda/\tau_1\tau_2 \tag{16b}$$

$$K_o=(1+k_1K_p)\lambda/\tau_1\tau_2 \tag{16c}$$

For the case that the initial conditions are satisfied by Equation (14), Equation (16) is an optimal control. For the general case, Equation (16) is a sub-optimal control. However, since λ can be assign arbitrarily, we can make the output trajectory arbitrarily close to the optimal Kalman trajectory by a suitable choice of λ.

THEOREM 1: For the second-order systems, Equation (6), an optimal or a sub-optimal controller with integral mode, Equation (16), is obtained and the control coefficients, K_2, K_1 and K_o, are determined in terms of k_1, k_2 and system parameters which are given by Equations (16a), (16b) and (16c) respectively.

Example 1

As an example consider the control system which has been considered by Liou et. al. [3]

$$\frac{d^2c}{dt^2}+\frac{3}{2}\frac{dc}{dt}+c=u$$

an the performance index

$$J(u)=\frac{1}{2}\int_o^\infty[c^2(t)+\frac{1}{8}u^2(t)]dt$$

with given conditions

$$c(0)=c_o,\quad \frac{dc(0)}{dt}=0$$

Since the system is controllable, the optimal control law can be obtained as

$$k_1=2,\quad k_2=1$$

Using Theorem 1, the control law is given by

$$u(t)=-\{(1+\lambda)\frac{dc}{dt}+(2+2.5\lambda)c+3\lambda\int_o^t c\,d\tau\} \tag{E1}$$

It is noted that the control becomes the Kalman control when $\lambda=0$. When $\lambda\neq0$, this is only a sub-optimal. For the case that no disturbance has been introduced, it is clearly that a smaller value of λ is desirable. For the case that a constant disturbance has been introduced into the system, a best value of λ can be obtained.

Since the value of λ can be assigned arbitrarily, the output response also depends on the choice of λ. The transient response without load disturbance for different values of λ is shown in Figure 1. The optimal Kalman transient response is also shown in the figure. The transient response with a constant load disturbance for different value of λ and the optimal Kalman transient res-

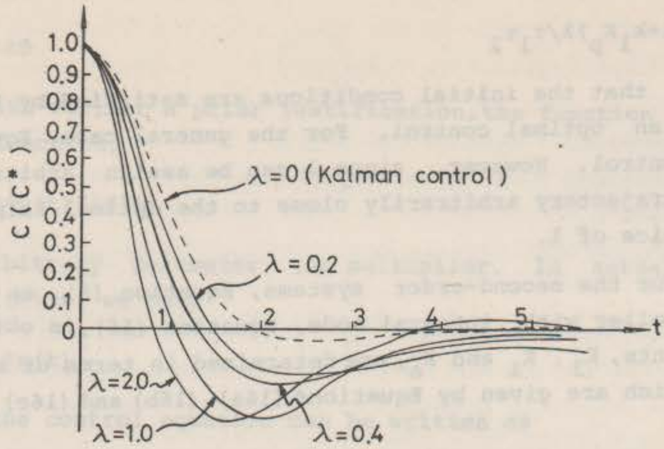

Fig. 1. Comparison of output transient response to non-zero initial state and no disturbance.

ponses with and without disturbance are shown in Figure 2. By the investigation of these two figures, we can find that the best value of λ is 0.4 according to Figure 2; however, according to Figure 1, one can see that the best value of λ is as smaller as possible; therefore, we can choose λ such that the transient response will be reasonable and satisfactory to both cases of with and without disturbance. This is discussed in the next section.

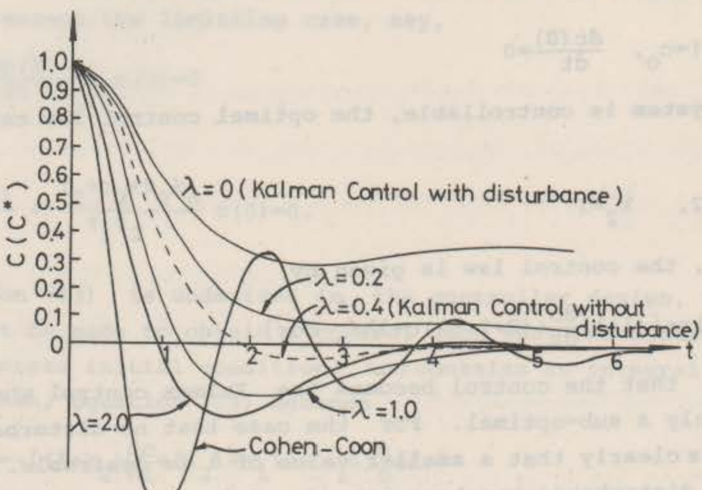

Fig. 2. Comparison of output transient response to non-zero initial state and a step disturbance

III. Selection of Arbitrary Parameter or Multiplier, λ.

An important factor to be considered is the value of objective function which depends upon the choice of control law, and hence, depends upon the choice of λ. However, this is equivalent to the non-disturbance case. When a constant disturbance is introduced, the deviation from the optimal Kalman trajectory, i.e. an error function, is also to be considered.

1. Effect of multiplier λ on the performance index

The control variable u(t) and the output variable c(t) are assumed to be related by the second-order equation, Equation (6), with initial conditions

$$c(0)=c_o, \quad \frac{dc(0)}{dt}=0,$$

The purpose of the control is to drive the output variable to the final steady state values, i.e.

$$c(\infty)=0, \quad \frac{dc(\infty)}{dt}=0.$$

Suppose the control law is given by Equation (16). The objective is to investigate how the multiplier be effected on the performance index.

$$J(u)=\frac{1}{2}\int_o^\infty [c^2(t)+\rho u^2(t)]dt$$

Take Laplace transform, Equations (6) and (16) yield

$$[\tau_1\tau_2 s^2+(\tau_1+\tau_2)s+1]c(s)=K_p u(s)+(\tau_1\tau_2 s+\tau_1+\tau_2)c_o \quad (17)$$

$$u(s)=-(K_2 s+K_1+\frac{K_o}{s})c(s)+K_2 c_o \quad (18)$$

Combination of Equations (17) and (18), we can obtain

$$c(s)=(s^2+b_2 s)c_o/P_1 \quad (19)$$

$$u(s)=-\{K_1 s^2+(K_1 b_2+K_o-K_2 b_1)s+(K_o b_2-K_2 b_o)\}c_o/P_1 \quad (20)$$

where

$$P_1=s^3+b_2 s^2+b_1 s+b_o$$

$$b_2=(\tau_1+\tau_2+K_2 K_p)/\tau_1\tau_2 \quad (20a)$$

$$b_1=(1+K_1 K_p)/\tau_1\tau_2 \quad (20b)$$

$$b_o=K_o K_p/\tau_1\tau_2 \quad (20c)$$

Applying Parseval's theorem [21], one can obtain

$$J_1=\int_o^\infty c^2(t)dt=(b_1+b_2^2)b_o c_o^2/P_2 \quad (21)$$

$$J_2=\int_o^\infty \rho u^2(t)dt=\{(K_1^2 b_1+d_1^2-2K_1 d_o)b_o+d_o^2 b_2\}\rho c_o^2/P_2 \quad (22)$$

where

$$P_2=2b_o(b_1b_2-b_o) \tag{22a}$$

$$d_1=K_1b_2+K_o-K_2b_1 \tag{22b}$$

$$d_o=K_ob_2-K_2b_o \tag{22c}$$

then, the value of performance index is given by

$$J=\frac{C_o^2}{2p_2}\{(b_1+b_2^2)b_o+\rho[(K_1^2b_1+d_1^2-2K_1d_o)b_o+d_o^2b_2]\}$$

or

$$J=\frac{1}{2}(J_1+J_2) \tag{23}$$

It is noted that the performance index, Equation (23), is function of λ, and there is a minimum value at $\lambda=0$. Numerical results are illustrated in Example 2.

2. Effect of multiplier λ on the error function

Consider the system, Equation (6), with the control, Equation (16), the output transient response to a load change at t=0 is the solution of the equation:

$$\tau_1\tau_2\frac{d^2c}{dt^2}+(\tau_1+\tau_2)\frac{dc}{dt}+c+K_p\{K_2\frac{dc}{dt}+K_1c+K_o\int_o^t cd\tau\}-w=0$$

where w is a constant load disturbance.

Let c* be an optimal transient and define the deviation variable

$$\Delta c=c^*-c,$$

then

$$\frac{d^2\Delta c}{dt^2}+b_2\frac{d\Delta c}{dt}+b_1\Delta c+b_o\int_o^t\Delta cd\tau+(\frac{w-a_o\lambda}{\tau_1\tau_2})=0 \tag{24}$$

and

$$\frac{d\Delta c(0)}{dt}=\Delta c(0)=0$$

Let w be a unit step function, Laplace transformation of Equation (24), yields

$$\Delta c(s)=(a_o\lambda-1)/\tau_1\tau_2P_1 \tag{25}$$

It is noted that Equation (25) has shown that $\Delta c(t)\to 0$ as $t\to\infty$, and hence there will be no offset. Using the stability criterion, it is simple to show that $\lambda\geq 0$ always yields a stable system. The output response to a step disturbance for the system considered in Example 1, is illustrated in Figure 3.

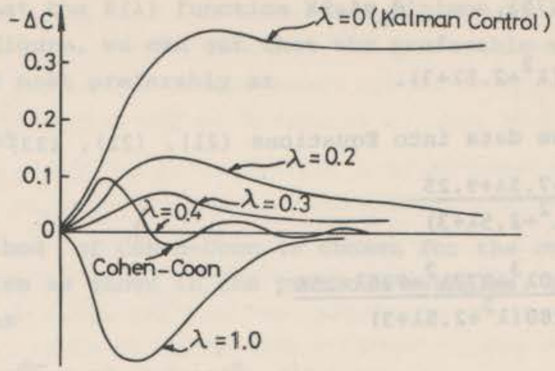

Fig. 3. Comparison of output response to a step disturbance.

Define the error function, E,

$$E \equiv \int_o^{\infty} (\Delta c)^2 dt$$

then, using Parseval's theorem, one can obtain

$$E=[(a_o\lambda-1)/\tau_1\tau_2]^2 b_2/P_2 \qquad (26)$$

The E function can be considered as a measure of deviation from the optimal Kalman trajectory. It is also noted that E is function of λ. If we choose λ such that

$$\lambda=1/a_o \quad \text{(provided } a_o\neq 0) \qquad (27)$$

then, $\Delta c(t)\equiv 0$ and $E(\lambda)\equiv 0$, Equation (27) gives the best value of λ when a unit step disturbance, w, appears at t=0.

Equations (23) and (26) are the two important rules to determine the best value of λ, and this is illustrated in Example 2.

Example 2

Consider the same system as in Example 1, then

$$\tau_1\tau_2=1.0, \qquad \tau_1+\tau_2=1.5, \qquad K_p=1.0$$

$$\rho=0.125, \qquad C_o=1.0$$

The sub-optimal controller gains are given by Equation (E1),

$$K_2=1+\lambda, \qquad K_1=2+2.5\lambda, \qquad K_o=3\lambda.$$

and the constants are given by Equations (20) and (22)

$$b_2=2.5+\lambda, \qquad b_1=3+2.5\lambda, \qquad b_o=3\lambda,$$

$$d_1=2+5.75\lambda, \qquad d_o=4.5\lambda$$

and $$p_2=15\lambda(\lambda^2+2.5\lambda+3).$$

Substitute these data into Equations (21), (22), (23) and (26), we obtain

$$J_1=\frac{\lambda^2+7.5\lambda+9.25}{5(\lambda^2+2.5\lambda+3)} \tag{E2}$$

$$J_2=\frac{250\lambda^3+977\lambda^2+990\lambda+256}{8\text{x}80(\lambda^2+2.5\lambda+3)} \tag{E3}$$

$$J=\frac{25}{128}(\lambda+1.92) \tag{E4}$$

$$E=\frac{(\lambda+2.5)(2.5\lambda-1)^2}{15\lambda(\lambda^2+2.5\lambda+3)} \tag{E5}$$

Equations (E4) and (E5) show the effect of multiplier on the performance index and error function respectively. The effects of increasing λ on the J_1 J_2, J and E are shown in Figure 4.

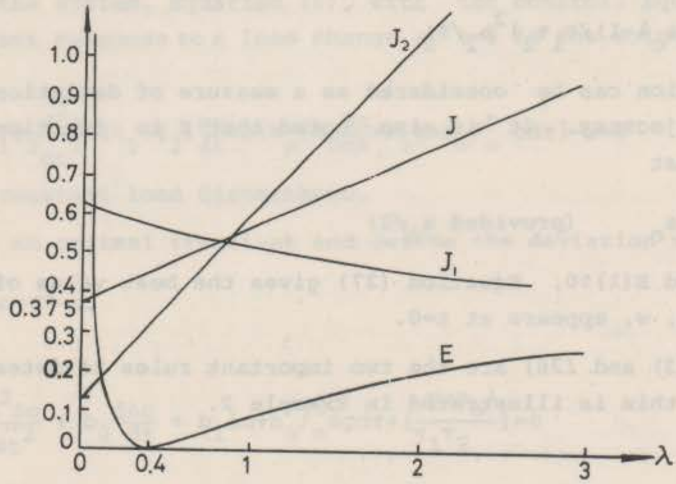

Fig. 4. Effects of λ on J and E function

It is important to notice that $J(\lambda)$ is a linear increasing function, and hence the minimum value of $J(\lambda)$ is given by

$$\text{Min } J(\lambda)=J^*(0)=0.375$$

and it is reduced to Kalman optimal control case. The normalized performance index can be obtained

$$\frac{J(\lambda)-J^*(0)}{J^*(0)}=0.521\lambda$$

It is also noted that the $E(\lambda)$ function has a minimum at $\lambda=0.4$, and $E(0.4)=0$. Referring to the figure, we can get that the preferable value of λ is between 0.2 and 0.4, and most preferably at

$$0.25\leq\lambda\leq 0.35$$

Example 3

The classical method of Cohen-Coon is chosen for the comparison purpose. Consider the same system as shown in the previous examples, Cohen-Coon method gives the controller as

$$u(t)=-\{1.10\frac{dc}{dt}+9.56c+12.65\int_o^t c d\tau\} \quad \text{(E6)}$$

The formulation of Equation (E6) is shown in Appendix.

Using the controller obtained by Equation (E6) and following the method presented above, we obtain

$$J_1=0.587$$

$$J_2=5.69$$

and $$J=\frac{1}{2}(J_1+\frac{1}{8}J_2)=3.14$$

It is noted that the minimum value of J is only 0.375. In this view point, Cohen-Coon controller setting only yields a sub-optimal, and the value of the performance index is about 8.37 times greater than the minimum value of the performance index. A comparison result is shown in Table 1

Table 1

Control Type	Control Law u(t)	J_1	J_2	J	$\frac{J-J^*}{J^*}$	E	off-set
Kalman	$-\{\frac{dc}{dt}+2c\}$	0.617	0.133	0.375	0	∞	33.3%
This Work (λ=0.2)	$-\{1.2\frac{dc}{dt}+2.5c+0.6\int_o^t cd\tau\}$	0.610	0.218	0.414	0.104	0.062	0
This Work (λ=0.4)	$-\{1.4\frac{dc}{dt}+3\frac{dc}{dt}+1.2\int_o^t cd\tau\}$	0.597	0.309	0.453	0.208	0	0
This Work (λ=0.53)	$-\{1.63\frac{dc}{dt}+3.32\frac{dc}{dt}+1.59\int_o^t cd\tau\}$	0.587	0.368	0.477	0.276	0.012	0
Cohen-Coon	$-\{1.10\frac{dc}{dt}+9.56c+12.65\int_o^t cd\tau\}$	0.587	5.69	3.14	7.37	-	0

IV. Summary and Conclusion

Optimal control theory gives a control law which is only proportional state feedback. Since the complete feedback of the states is not often possible in most of the chemical plants, and there is no integral mode in the controller, practical application of optimal control theory to the design of the industrial process control systems is very restricted. Therefore, a sub-optimal controller with an integral mode is requested in the design of controllers.

The method presented here satisfies the above requirements, while not changing the ordinary quadratic performance index as considered by Kalman. The method is based on the introduction of a multiplier into the optimal Kalman control law and the proper transformations and modifications, and thus obtains an optimal or a sub-optimal control law which incorporates an integral mode. The control coefficients are determined in terms of Kalman control gains and system parameters only, and, are independent of the initial values of the system.

Since an integral action has been introduced in the controller design , an offset is eliminated. The resulting control yields an output transient response which is satisfactory for both cases that the disturbance is absent or introduced. For the conventional undisturbed problem, the responses are illustrated in Figure 2. Referring these two figures, we can find a proper value of λ, for instance, $\lambda=0.2$ for the illustrated system.

For comparison purpose, the classical method of Cohen-Coon is investigated. The output transient response to non-zero initial state and a step disturbance is show in Figure 2. The output transient response to zero state and a step disturbance is shown in Figure 3. The value of ordinary performance index is also shown in Table 1.

Referring Table 1, the values of J_1 are almost the same,however ,the values of J_2 have great differences, and the method presented here yields a smaller value of the performance index than that obtained by Cohen-Coon method.

Appendix

Controller Setting by using Cohen-Coon Method

Consider the system:

$$\frac{d^2c}{dt^2}+\frac{3}{2}\frac{dc}{dt}+c=u \tag{A1}$$

Step response (u=M) of the second-order system of Equation (A1) is taken as the process reaction curve.

$$c(t)=M[1-e^{-at}(\cos bt+\frac{a}{b}\sin bt)]$$

where $a=\frac{3}{4}$ and $b=\sqrt{7}/4$

Now, we can estimate the values of K_p, T and T_d from the process reaction curve. Since

$$\frac{d^2c(t)}{dt^2} = \frac{M}{b}e^{-at}[b\ \cos bt - a\ \sin bt]$$

Then, the inflection point, t_i, is

$$t_i = \frac{1}{b}\tan^{-1}\left(\frac{b}{a}\right) = 1.095$$

and the slope, S, at $t=t_i$, is

$$S = \frac{M}{b}e^{-at_i}\sin bt_i = Me^{-at_i}$$

Again, the ultimate response, B_u, is given by

$$B_u = \lim_{t\to\infty} c(t) = M$$

Then, we can obtain

$$K_p = B_u/M = 1$$

$$T = \frac{B_u}{S} = e^{at_i} = 2.27$$

and $$T_d = t_i - e^{at_i} + 2a = 0.325$$

The Cohen-Coon method recommended the following PID controller setting

$$K_c = \frac{1}{K_p}\frac{T}{T_d}\left(\frac{4}{3} + \frac{T_d}{4T}\right)$$

$$\tau_I = T_d\frac{32+6T_d/T}{13+8T_d/T}$$

$$\tau_D = T_d\frac{4}{11+2T_d/T}$$

Using the values of T_d, T and K_p determined in above, K_c, τ_I and τ_D can be calculated and are given by

$$K_c = 9.56$$

$$\tau_I = 0.755$$

$$\tau_D = 0.1152$$

Therefore, Cohen-Coon method gives the controller as

$$u(t) = -9.56\left\{0.1152\frac{dc}{dt} + c + \frac{1}{0.755}\int_0^t c\,d\tau\right\}$$

or $$u(t) = -\left\{1.10\frac{dc}{dt} + 9.56c + 12.65\int_0^t c\,d\tau\right\}$$

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