The Journal of National Chiao Tung University
Vol. 2, December 1976
pp. 221-228

非線性半導體雷射之運算

Self-Consistent Calculation of Nonlinear Laser Theory for Semiconductor Laser

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(Received October 19, 1976)

ABSTRACT — The conventional linear optical model, which was used to describe the lasing behavior, neglects the effects of spontaneous emission and defines the threshold condition intuitively. The defficiency of this model is unable to explain the experimental results. Here, a nonlinear model which takes the effect of spontaneous emission into account was solved by self-consistent method. It has been found that this model is able to illustrate several lasing characteristics which cannot be explained by linear optical model.

I. Introduction

Several papers [1,2,3,4], describing the optical gain of semiconductor laser in Fabry-Perot cavities, have been published in recent years. In these papers, efforts are put on the electronic band structure [1,2,3,4] and transition probability [5,6] while the stimulated emission caused by spontaneous emission before lasing is neglected (linear optical model). The primary inadequacy of such a treatment is that the relationship between the injection current and the output power cannot be derived.

An alternate approach will be discussed in this paper. Stimulated emission caused by spontaneous emission trapped in the cavity before lasing will be included in the power calculation. For simplicity, parabolic electronic energy bands and constant transition probabilities are assumed.

II. Theory

The rate equations for electrons in the active region and photon density can be written as [8]

$$\frac{dN_c}{dt} = G_{en} - \frac{N_c}{\tau_{sp}} - \frac{N_c}{\tau_{non-rad}} - r_{st.} \cdot N_p$$

$$\frac{dN_p}{dt} = \frac{N_c}{\tau_{sp}} \cdot \frac{1}{\Delta E_{sp}} \frac{d\Omega}{4\pi} - \frac{N_p}{\tau_c} + r_{st.} \cdot N_p$$

where

N_c=density of electrons in the conduction band.

G_{en}=electron generation rate due to the injection current.

τ_{sp}=spontaneous emission lifetime.

$\tau_{non\text{-}rad.}$=non-radiative lifetime.

$r_{st.}$=stimulated emission rate.

N_p=photon density for a particular mode.

$d\Omega$=effective solid angle for trapping spontaneous emission.

ΔE_{sp}=linewidth of spontaneous emission.

τ_c=photon lifetime in the cavity

At steady-state $\frac{dN_c}{dt}=0$, $\frac{dN_p}{dt}=0$ and we get

$$N_c=\frac{G_{en}\cdot\tau_{sn}}{1+\frac{\tau_{sn}}{\tau_{sp}}\frac{1}{\Delta E_{sp}}\frac{d\Omega}{4\pi}\frac{r_{st}\cdot\tau_c}{1-r_{st}\cdot\tau_c}} \tag{1}$$

$$N_p=\frac{\tau_c}{1-r_{st}\cdot\tau_c}\frac{N_c}{\tau_{sp}}\frac{1}{\Delta E_{sp}}\frac{d\Omega}{4\pi} \tag{2}$$

where

$$\frac{1}{\tau_{sn}}=\frac{1}{\tau_{sp}}+\frac{1}{\tau_{non\text{-}rad}}$$

Also, the quantum efficiency η is defined as

$$\eta=\frac{\tau_{sn}}{\tau_{sp}}$$

These lumped parameters are related to the band and geometrical factors by the following equations

$$N_c=\int_c \rho_c(E)f_c(E-E_{Fn})dE$$

$$P=\int_v\rho_v(E)[1-f_v(E-E_{Fp})dE]$$

$$r_{st}=\frac{\pi^2c^3h^3}{n^2E^2}\int_c B\rho_c(E')\rho_v(E'-E)[f_c(E'-E_{Fn})-f_v(E'-E-E_{Fp})]dE'$$

$$G_{en}=\frac{j}{e\cdot d}$$

$$d\Omega=2\times\frac{d\cdot w}{\ell^2}\ \text{(for homojunction)}$$

$$\tau_c = \frac{c}{n}\left(\alpha - \frac{1}{\ell}\ln R_1 R_2\right)$$

$$R_1 = R_2 = \left|\frac{n_2 - 1}{n_2 + 1}\right|^2$$

where

n=index of refraction

E=photon energy=$\hbar w$

B=transition probability between conduction band and valence band

$f_c(E)$=Occupation probability of electrons in the conduction band

$$= \frac{1}{1 + e^{(E - E_{Fn})/KT}}$$

$f_v(E)$=Occupation probability of electrons in the valence band

$$= \frac{1}{1 + e^{(E - E_{Fv})/KT}}$$

j=injection current

d=diffusion length for electron

E_{Fn}, E_{Fp}=quasi Fermi-level for electrons and holes

w=width of active layer

ℓ=length of the diode

R_1, R_2=reflectivity of end faces of cavity

α=absorption loss per unit length

Also, the output power density is given by

$$P_{out} = \frac{c}{n} N_p \cdot E$$

III. Self-Consistent Calculation of the Rate Equations

Equations (1) and (2) are solved by self-consistent method which are described in reference 2. r_{st} is approximated by $r_{st \cdot max.}$, this is justified by the extremely narrow linewidth of lasing mode. Various parameters are obtained from reference 9. The algorithm, coded in Fortran IV, is outlined as follows

(1) For a given injection current, a $r_{st.max}$ is estimated.

(2) A new N_c is obtained from Eq. 1.

(3) For this N_c, a new $r_{st \cdot max}$ is obtained and go back to Step 1.

This process is continued till

$$\frac{r_{st\cdot 2max} - r_{st1\cdot max}}{r_{st2\cdot max}} < 10^{-5}$$

IV. Numerical Example

Numerical calculations were performed for a GaAs injection laser (homojunction) at various temperatures. The result is compared with the existing data qualitatively.

The input data is given as

$N_a = 6\times10^{18}$/c.c.

$N_d = 3\times10^{18}$/c.c.

$\tau_{sn} = 0.28\times10^{-9}$

$\tau_{sn} = 4\times10^{-10}$

$d = 1$ µm

$W = 10$ µm

$\ell = 250$ µm

$n = 3.54$

$m_c^* = 0.072\ m_0$

$m_v^* = 0.5\ m_0$

$m_0 = 0.911\times10^{-27}$ gm.

$T(^oK)$	$\alpha(cm^{-1})$	ΔE_{sp} (eV)
4.2	13	19.8×10^{-3}
77	15	30×10^{-3}
195	16	80×10^{-3}
300	30	0.123

The computed numerical result is given as follows

1. Power v.s. Injection Current

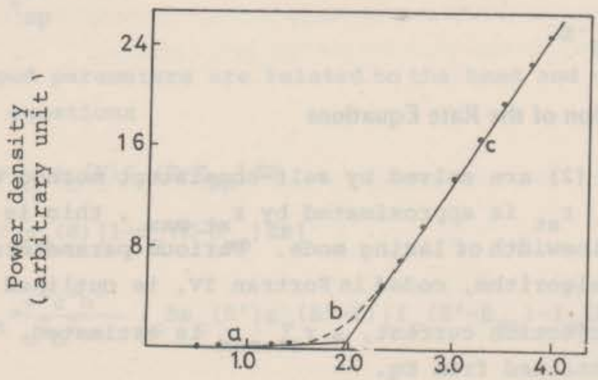

Figure. 1 Relationship between output power density and injection current density calculated by self-consist method with non-linear optical model at 4.2^0k. Three regions are observed: a.

linear region. b. superlinear region. c. linear region.

The computed result is plotted in Fig. 1 and Fig. 2 which clearly indicated three regions similar to the existing experimental data[10]:

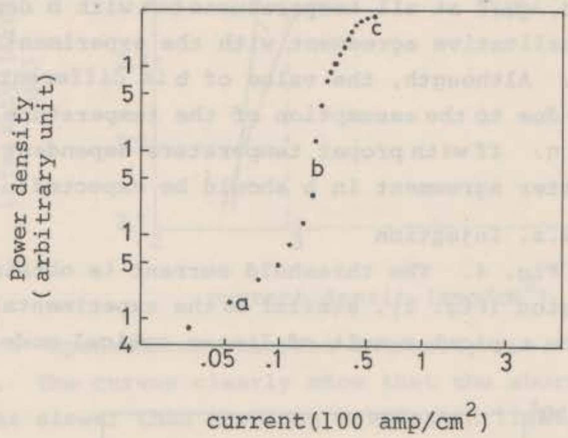

Figure. 2 Logarithm plot of the output power v.s. injection current. It clearly shows three distinct regions: a. linear region. b. superlinear region. c. linear region.

a. linear region: output power dominated by spontaneous emission.

b. superlinear region: population inversion starts to pile up and the spontaneous emission is amplified and the output power increases exponentially

c. linear region: the photon field is so large such that the stimulated emission dominate the whole situation. The population and hence the gain keep almost constant (Fig. 3)

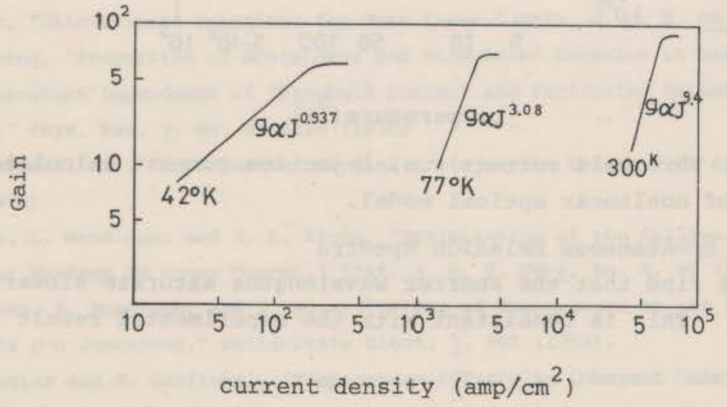

Figure. 3 Gain-current relationship calculated by using of non-linear optical model. The gain approaches assympotatically to the total loss of the cavity.

2. Optical Gain v.s. Injection Current

The optical gain and r_{st}. is related by the equation $g=\frac{n}{c}r_{stim}$. Optical gain v.s. injection current is plotted in Fig. 3 at three different temperatures, two points are worth of notice:

a. the gain approach asympotatically to the loss of the cavity and never exceeds the loss, i.e. $g \lesssim \alpha$.

b. below threshold, $g \alpha J^b$ at all temperatures but with b depending on temperature. This is in qualitative agreement with the experimental results presented on reference 11. Althougth, the value of b is different quantitively, this is believed to be due to the assumption of the temperature independent of the quantum efficiency η. If with proper temperature dependent of the quantum efficiency assumed, better agreement in b should be expected.

3. Threshold Current v.s. Injection

This is plotted in Fig. 4. The threshold current is obtained by extrapolation of the upper region (Fig. 1), similar to the experimental process. The result is similar to the typical result of linear optical model.

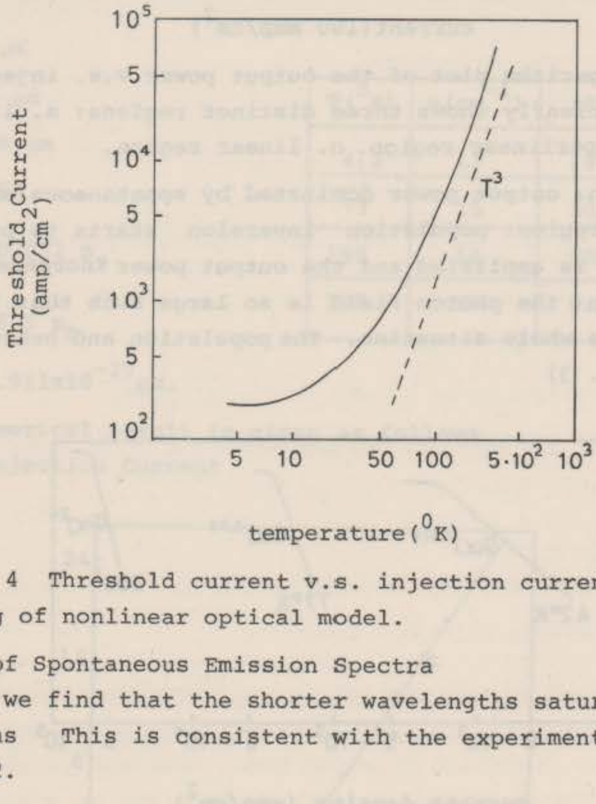

Figure. 4 Threshold current v.s. injection current calculated by using of nonlinear optical model.

4. Phenomena of Spontaneous Emission Spectra

In Fig. 5, we find that the shorter wavelengths saturate slower than the long wavelengths This is consistent with the experimental result presented on reference 12.

V. Conclusion

If has been found that although the nonlinear optical model is relatively simple, it is quite powerful. Several phenomena which cannot be explained by the linear optical model seems quite obvious in this nonlinear optical model.

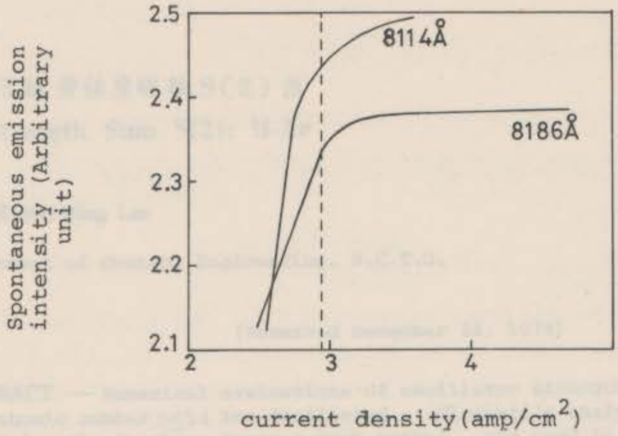

Figure. 5 Spontaneous emission intensity v.s. injection current density. The curves clearly show that the short wavelength light saturates slower than the long wavelength light.

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