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銅製導波管在厘米波段中之表面導電率

Surface Conductivities of Copper Waveguides at Millimeter-Wave Frequencies

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Part I. Experimental Observations

ABSTRACT — Surface conductivities of waveguides having different wall conditions were examined via the insertion loss technique and the Q-technique. It is found that ordinary copper plated waveguide samples are usually about 12% to 19% above the calculated value based upon a wall conductivity 5.8 10^7 mho/m in the frequency range of 40 to 110 GHz.

When the copper waveguides described above are subjected to additional surface treatment (usually imparted to promote dielectric bonding), the losses may or may not change depending on the nature of the surface treatment. For example, a chromated (probably Cr_2O_3) layer up to 300 A will not cause significant added losses in the mm wave frequency band. On the other hand, a surface treatment consisting of a grown copper oxide film (Cu_2 and CuO) will cause a significant increase in loss relative to plated but untreated copper. This additional loss is approximately 20% at 96 GHz.

I. Introduction

To the designers of a communication system it is essential to have the specific informations about the losses in the transmission medium and in the terminal devices. In both cases one needs to know the loss characteristics of the guiding structures which are usually made of smooth copper surface. Theoretically the loss characteristics of a guiding structure can be computed based on the ideally smooth wall assumption and the d.c conductivity σ d.c of single crystal copper at 20°C. In practice the surface may be coated with an oxide layer either due to the exposure to the atomosphere [1] or due to some special considerations [2], for example, to protect the surface from natural oxidation and to introduce a dielectric layer for impedance transformation purpose, etc. Hence one may expect that the surface conductivity of copper guiding surface differs to some degree from the d.c value of the copper conductivity. Further examining the guiding surface under high multiple microscope one noted that the surface appears highly irregular. The smoothness of a guiding surface in the microwave transmission media and the terminal devices is conveniently measured with the so-called root-mean-square of the surface irregularities over a specific length. The additional microwave attenuation due to that smoothness was estimated theoretically [3]. However as is well known the method in literature can only give a qualitative pictures, it deviates from the measured results by large margin [4]. In the recent years some progresses have been made [5] along this line but in this paper we have no attempt to make an in depth studies of the effects of the surface roughness although the measurement data presented in this paper are no doubt including the effects of the surface irregularities.

The primary consideration of this paper is the surface conductivities in frequency range, 40 GHz to 100 GHz. Only limited amount of informations concerning the R.F conductivities in the above-mentioned frequency band are available in literature [6-8]. Moreover except the works reported in ref. 7 the other references concerning the R.F. conductivities need specially built samples.

In the present paper we consider the losses in the commercially available circular waveguides with and without surface treatment. In section II we discuss the methods of experiments, in section III we examine the losses in the cold drawn copper waveguide, in the copper plated waveguide, and in the silver plated waveguide. A thin layer of chromium oxide can be employed to protect the surface from further oxidation. We study the loss effect of this layer. In this section we also examine the loss effect of an copper oxide layer, a few tenth of a micrometer in thickness, introduced by either chemical or electrolytical methods. In section IV we summarize the findings of this paper and discuss some of its implications.

II. Experimental Set-ups

In this section we shall briefly state the methods of measurements and discuss the salient features of the particular test sets.

In Fig. 1 one notes the sketch of a conventional insertion loss measurement set. The method is well known and can be found from standard text book [9].

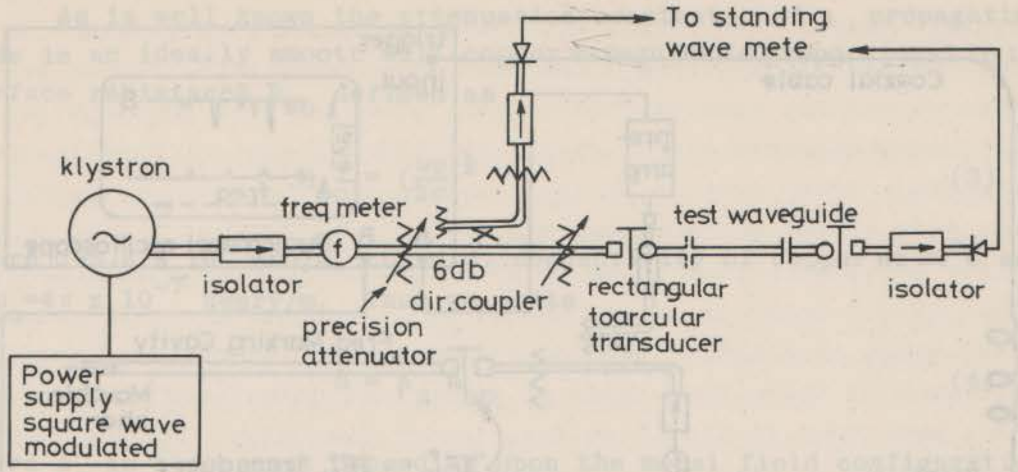

Fig. 1 Circuit diagram for insertion loss measurement, transducer can be either TE_{10}^{o} to TE_{01}^{o} or TE_{10} to TE_{11}^{o}.

In Fig. 2 we illustrate a particular Q-set which is capable to measure the differential frequency with high degree of precision. This Q-set differs from the conventional one in that it contains two cavities, one comprising the testing waveguides and the other one comprising a single end reflected cavity. The R.F. Signals, electronically swept, are introduced into those cavities. The resonance traces from the two cavities can be seen in a dual channel oscilloscope. The trace from the single-ended cavity comprises many spikes, all due to the same waveguide mode resonating at different frequencies. The differential frequency between two consecutive spikes is obtained in Appendix I and is given as

$$\Delta f_1 = \frac{2L}{C} \left[1-\left(\frac{f_c}{f_o}\right)^2\right]^{\frac{1}{2}} \tag{1}$$

where

$$C = \frac{1}{\sqrt{u_o \varepsilon_o}}$$

L = length of the single-ended cavities

f_c = cut off frequency of the resonant mode in the single-ended cavities

f_o = center frequency of the swept range

The trace from the testing cavity, comprising the waveguides to be tested and two coupling irises, also contains many resonant spikes. Those spikes can be identified as the resonant traces of the waveguide modes [10].

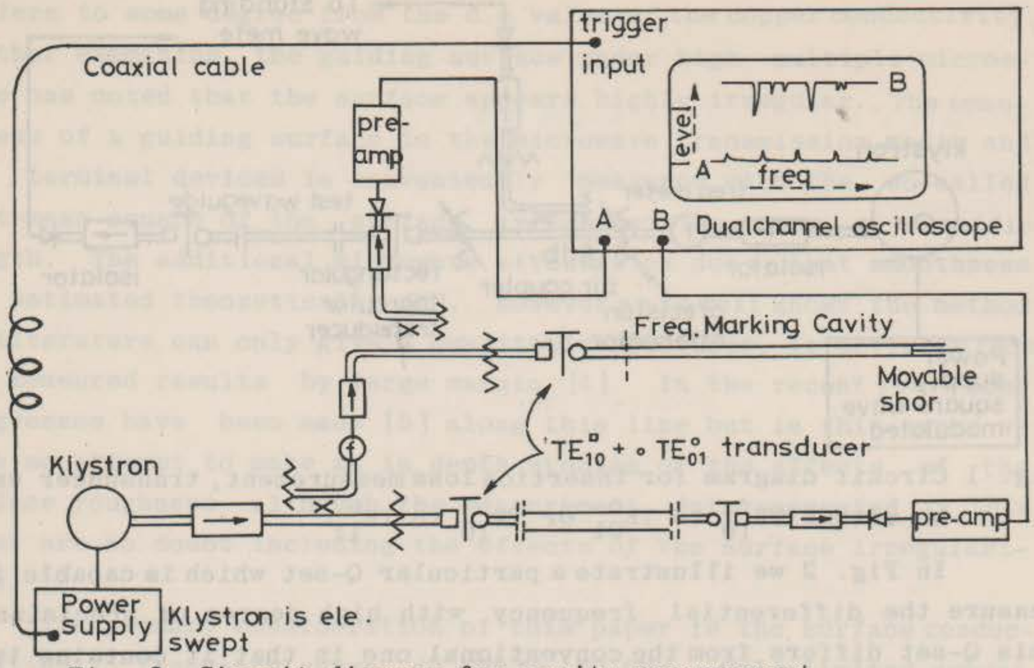

Fig. 2 Circuit diagram for cavity measurement.

The measurement procedures include the selection of a particular mode for examination and the determination of the Q-values of that mode. The crux in obtaining the Q-value of a resonant mode is the measurement of the half power frequency bandwidth Δf_2 which can be accomplished by measuring the distance ℓ_1, between two half power points in the resonant trace of the selected mode and can be computed from

$$\Delta f_2 = \frac{\ell_1}{\ell_2} \Delta f_1 \quad (2)$$

where ℓ_2 is the distance of two consecutive resonant spikes in the trace due to the single-ended cavity. For an improved accuracy it is advised to move the center of Δf_1 approximately coincident with the center of Δf_2.

The measurement accuracy depends upon a number of factors, for example, the swept frequency linearity, the scope linearity, the resolutions of ℓ_1 and ℓ_2, etc. Based upon the bodies of data we estimate the accuracy to be about ±3%. The accuracy can be greatly improved with an automated computer controlled system.

III. Surface Resistances at Millimeter Wave Frequencies

In this section we present the measurement results for a group of circular waveguide of various surface treatments.

As is well known the attenuation constant α of a propagating mode in an ideally smooth wall copper waveguide is proportional to the surface resistance R_{so} defined as

$$R_{so} = (\frac{\omega\mu}{2\sigma})^{\frac{1}{2}} \tag{3}$$

where σ=5.8 x 10^7 mho/m, the d.c. conductivity of copper at 20°C and $\mu=\mu_o=4\pi \times 10^{-7}$ henry/m. Thus we write

$$\alpha = A_\alpha R_{so} \tag{4}$$

where A_α is a constant depending upon the modal field configuration.

Similarly the unloaded quality factor of a waveguide cavity Q_o is also proportional to R_{so} as

$$Q_o = A_Q R_{so} \tag{5}$$

where A_Q again depends upon the resonant mode. The above concept may be extended to the practical waveguide by defining a quantity called effective surface resistance $(R_s)_{eff}$ in the following manner (for simplicity of notation we use R_s instead of $(R_s)_{eff}$ thereafter.)

$$\alpha = A_\alpha R_s \tag{6}$$

$$Q = A_Q R_s \tag{7}$$

It is seen from (4) to (7) that the ratio R_s/R_{so} is independent of the method of measurement. In this section we shall present the measured data, in most cases as R_s/R_{so} vs. frequencies.

In subsection 1 we discuss the normalized surface resistance R_s/R_{so} as a function of frequencies for cold drawn OFHC waveguide, copper plated waveguide and silver plated waveguide. In subsection 2 we study the effects of the layer of chromium oxide grown on the surface of the conducting wall. In subsection 3 we examine the effects of the copper oxide layer grown on the conducting wall.

1. Effect Of The Surface Material

In studying the effect of the surface conductivity we started with three groups of cold drawn Oxygen-Free-High-Conductivity (OFHC) circular copper waveguide, 0.5" in diameter, 2' in length. Each group contains four of the above-mentioned waveguides. Those waveguides were first cleaned with sodium hydroxide and sodium cyanide solution and measured electrically, thereby defining our baseline measurements. After completing the baseline measurement, one group of tubes were copper plated using the so-called alkaline cyanide bathe. Another group was plated using proprietary acid copper bathe, "Udelyte Bright Acid Copper" or simply UBAC. Still another group of tubes was plated using an alkaline cyanide silver bath. The measured surface resistance of those four types of waveguides normalized with respect to R_{so} are shown in Fig. 3 as function of frequency. For the purpose of comparison we plot $1+A_e(f/100)^{0.5}$ in the same chart where A is a parameter and f is the frequency in GHz. We notice that the copper plated tubes have less surface resistance as compared with the cold drawn OFHC copper waveguide.

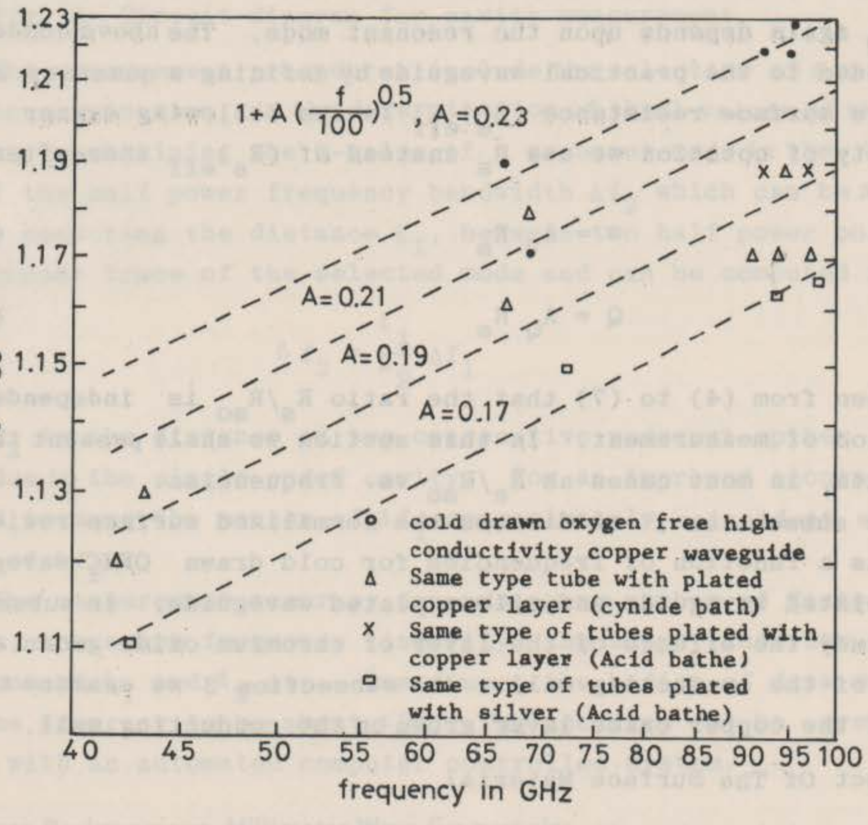

Fig. 3 Normolized surface resistances of cold drawn oxygen free high

conductivity (OFHC) copper waveguide of circular cross-section, 12.7mm in diameter. R_{so} is the surface resistance calculated from (3).

We believe that the lower surface resistance of the plated copper compared to that of the cold drawn copper is due to the smaller grain size and higher density of the lattice defects in the latter type surface.

Referring to Fig.3, it is noted that the cyanide and UBAC copper plated surfaces have comparable loss characteristics. The cyanide copper has larger grain size but rougher surface characteristics compared to UBAC copper. Therefore we feel that those two factors even up and it gives rise the similar conducting ratio R_s/R_{so}.

One may also notice that the silver plated tubes have the lowest R_s/R_{so} ratio. This seems due to the higher intrinsic conductivity of the silver furface (σ_{Ag}/σ_{Cu}=1.06). It is worth mentioning that in order to prevent surface tarnish in the silver plated waveguide a thin layer chromate was electrolytically introduced on top of the silver surface.

It should be mentioned that the thickness of the plated layers, either copper or silver, are less than $5x10^{-6}$m, (typically 2-3 um) one may consider that the plated tubes have the similar grade of surface roughness. Hence the effects of roughness in (6) and (7) are approximately unchanged due to the plating process. Further no foreign layers are introduced to cover the conducting surface. Hence in Fig. 3 the ordinance can also be considered as the measure of the conducting ratio.

2. Chromate Treatment

In this subsection and the subsequent one we will consider the effects of a layer of foreign material grown on the conducting surface. It functions as the protecting layer or as an agent for applying the other dielectric layer to the conducting surface. We began with the chromate treatment.

As before we start with 0.5" diameter, OFHC copper waveguide of circular cross-section. Those tubes were cleaned as described in subsection 3 and were chromated in three different processes:

(i) Thin layer electrolytic process: thickness of chromate is less than 100 Å.

(ii) Thick layer, electrolytic process: thickness of the chromate film is approximately 300 Å.

(iii) Chemical process.

The normalized surface resistances for tubes treated with the above-mentioned three processes were obtained based upon the Q measurement described in Section II. As in subsection 2 we plot these normalized surface resistances as a function of frequency in Fig. 4. It is noted that in the case of the thick and thin electrolytically deposited chromate layers, the losses are approximately the same.

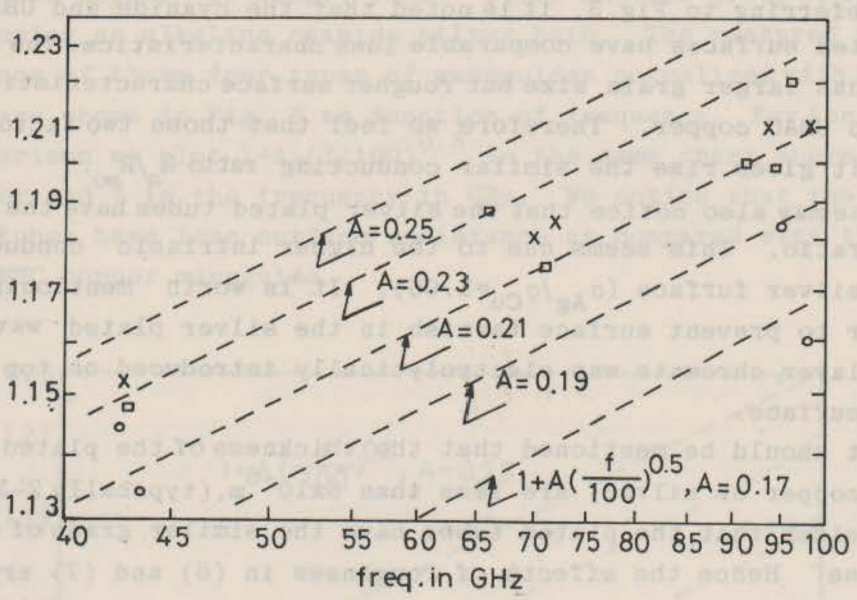

Fig. 4 Normalized surface resistance R_s/R_{so} where R_{so} is the surface resistance of an ideal smooth copper surface with $\delta=5.8\times10^7$ mho/m, waveguide diameter 12.7 mm.

- □ Cold drawn oxygen free high conductivity copper waveguide of circular cross-section coated with thin chromate layer.
- × Same type of waveguides with thick chromate layer (appr. 300 Å).
- o Chemically chromated.

The chemically chromated tubes appear to have lower losses than the electrolytically prepared samples at high frequencies. This result seems due to the experimental error since the chemically chromated samples are believed to have the same characteristics of the thin electrolytically chromated samples. In this part of experiment only two samples were measured.

3. Copper Oxide Treatment

Another method of surface treatment to be considered is to oxidize the copper surface either electrically or chemically. surface treatment of this kind has an unique loss characteristic which is of interest by itself. To advance an investigation into that loss characteristic several sets of experiments were performed as described below:

The first experiment involve three pieces of electroformed copper waveguide 4.5 mm inner diameter and approximately 1" outside diameter. Those waveguides were initially cleaned with sodium cyanide solution and were measured via the insertion loss technique to establish the base line for the TE_{01} mode as shown in Fig. 5, curve number 1. Those tubes were then oxidized by dipping them into a solution bearing the trade name "Ebonol C", a proprietrary solution designed to produce a black oxide layer on copper for decorative purpose. A layer of copper oxide approximately 12×10^3 Å in thickness is grown over the copper surface. After we rinse the tube with water and dry it in the oven of $100^{\circ}C$ for 30 min we measured the insertion loss as before. The results are plotted in Fig. 5, curve No.2. We see a significant increase in loss. The oxide layer was then stripped using a dilute HCl solution 5%. After rinsing with water and drying, the electric losses were remeasured. To one's surprise the loss, shown in Fig. 5 curve No. 3, is less than curve No.1 in same figure. This seems due to the fact that we stripped the top layer containing crystal defects introduced in fabricating the electroformed waveguide [6].

These same waveguides were then oxidized to a different oxide thickness. After we determined the electric losses, the oxide layer was stripped and we remeasured the losses. Repeating this process a few more times, we found that the losses in stripped waveguides were in one range and the losses in oxidized waveguides are in another range as illustrated in Fig. 6. From Fig. 6 one may notice that the losses in oxidized waveguide are weakly dependent upon the thickness of the oxide layer.

To extend the frequency band of observation we decide to excite TE_{11} mode instead of TE_{01} mode in the electroformed circular waveguide since the cutoff frequencies of TE_{11} mode and TE_{01} mode in a 4.5 mm inner diameter waveguide are 39 GHz and 81.2 GHz respectively. The measurement results for the frequency range 50 to 80 GHz were recorded in Fig. 7. As before the waveguides oxidized in the Ebonol C solution to form an oxide layer 6~7000 Å thick were considered.

To further extend our frequency range of observation we employed 18 pieces of copper plated steel tube with inner diameters of 13.22 mm. We measured the attenuation constant of TE_{01} mode via the insertion loss technique in the frequency band 37 to 50 HGs. An oxide layer was then introduced into these waveguide by the electrolytic technique. The TE_{01} mode losses in those oxidized waveguide were determined in the same frequency range. The losses of these samples with and without the oxide laver are shown in Fig. 8.

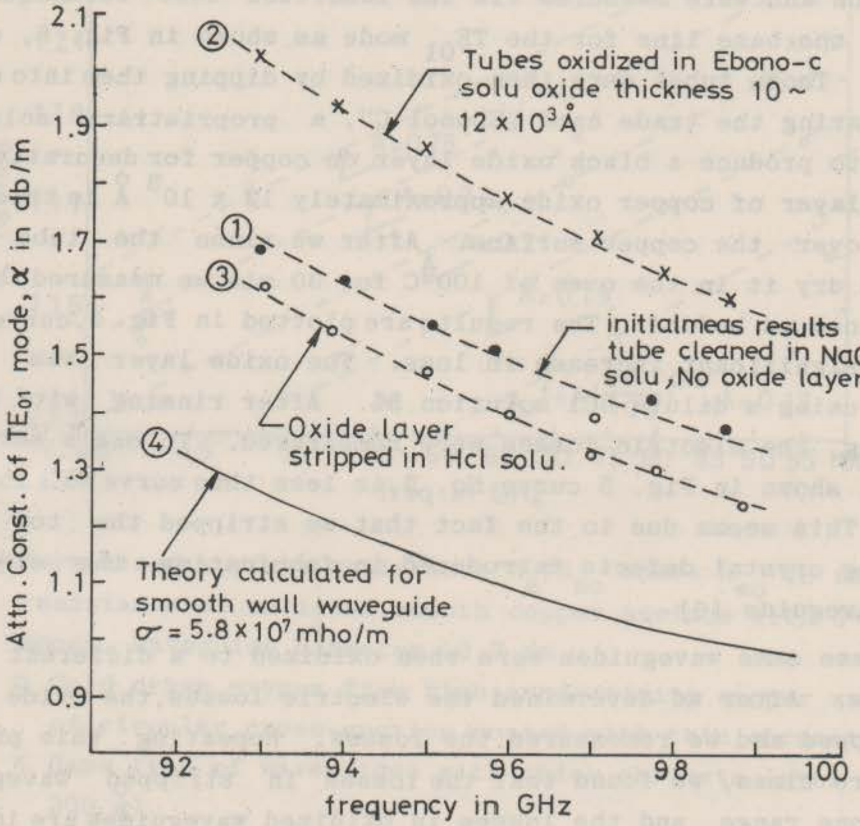

Fig. 5 Attenuation of TE_{01} mode in circular waveguide, I.D.=4.5 mm.

Another method for measuring the losses is to employ the Q-set described in section II. We selected a four oxidized 13.22 inner bore copper plated steel tubes and inserted them in the Q-set and measured the Q-value in the frequency range 90-98 GHz. before and after the tubes were oxidized. The Q-values of the results were shown in Fig. 9. For stripping the oxide layer we have used an NaCN oxide stripping system as well as an HCl stripping system, and have found no difference in electrical behavior of the stripped copper in either case, thereby supporting our assumption that the morphology of the underlying copper is not significantly modified by the HCl stripping system used in most of these studies.

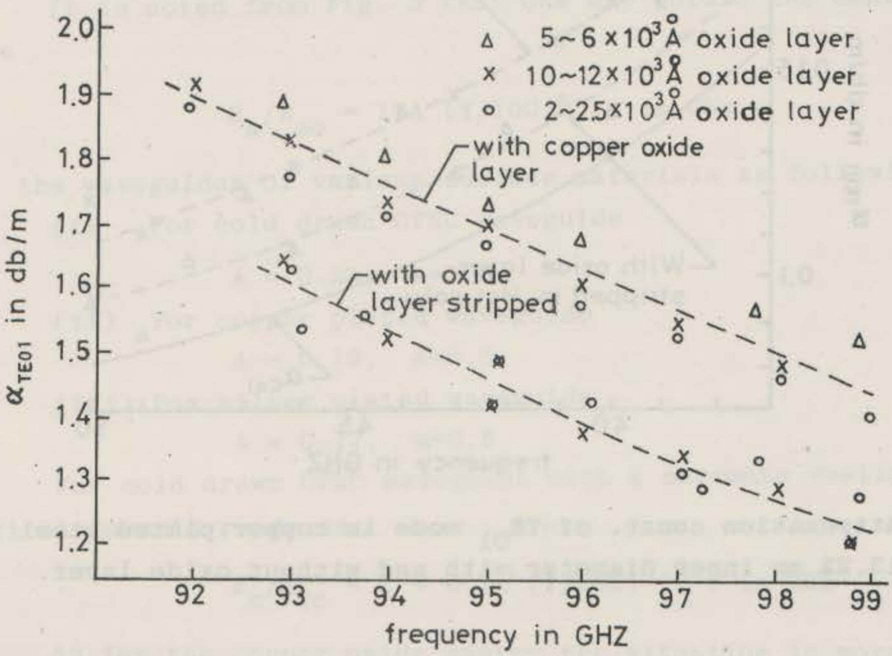

Fig. 6 Attenuation constant of TE_{01} mode in electroformed copper waveguide of circular cross-section 4.5 mm I.D.

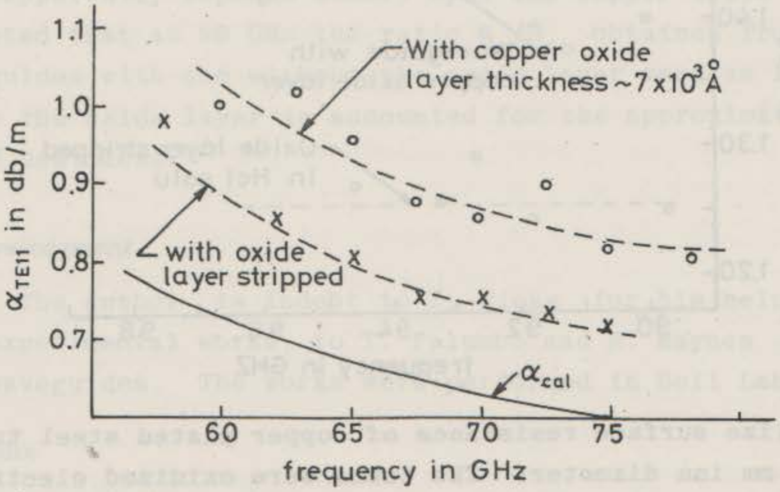

Fig. 7 Attenuation constant of TE_{11} mode in 4.5 mm I.D. electroformed copper waveguide.

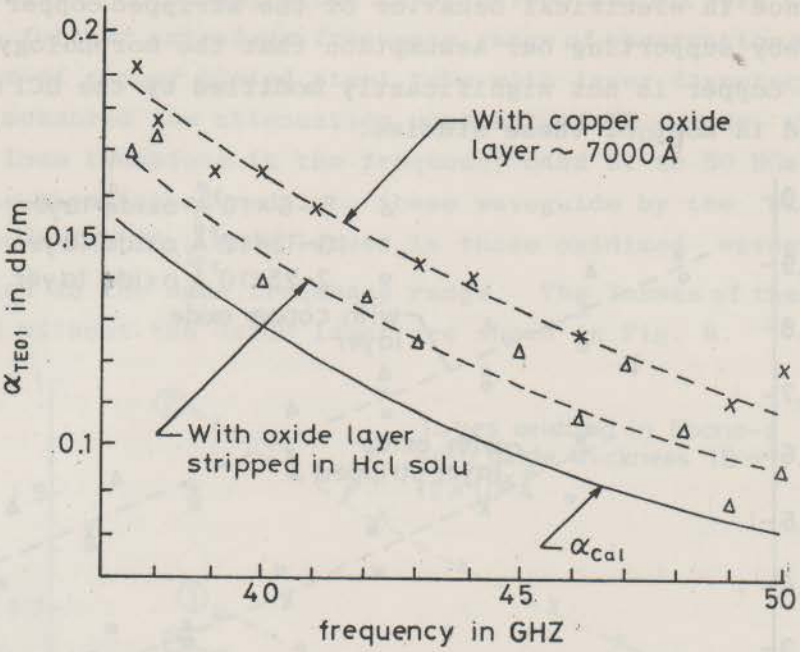

Fig. 8 Attenuation const. of TE_{01} mode in copper plated steel tube, 13.22 mm inner diameter with and without oxide layer.

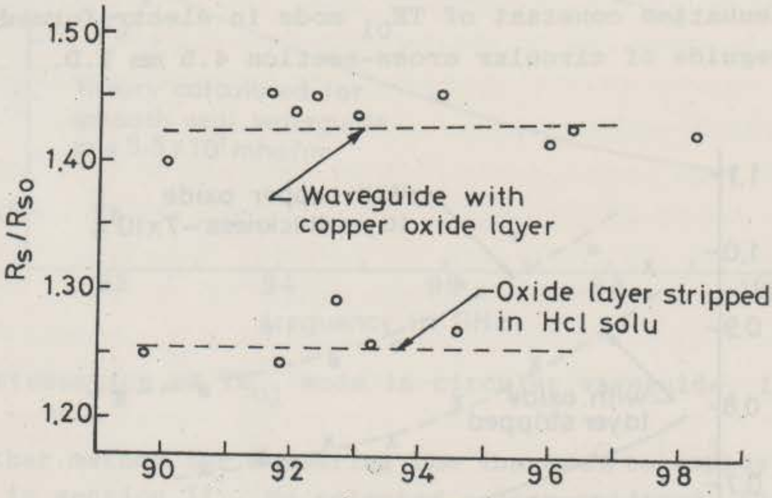

Fig. 9 Normalize surface resistance of copper plated steel tubes, 13.22 mm inn diameter. The tubes were oxidized electrolytically. $R_{so} = \sqrt{\frac{\omega\mu}{2\sigma}}$ where $\sigma = 5.8 \times 10^7$ mho/m.

IV. Concluding Remarks

In this section we attempt to draw some conclusions from the earlier observations.

It is noted from Fig. 3 that one may obtain the empirical formula

$$R_s/R_{so} = 1+A\ (f/100)^a \quad ,f \text{ in GHz} \tag{8}$$

for the waveguides of various surface materials as following

(i) For cold drawn OFHC waveguide

$$A = 0.23, \quad a=0.5 \tag{9}$$

(ii) For copper plated waveguide

$$A = 0.19, \quad a=0.5 \tag{10}$$

(iii) For silver plated waveguide

$$A = 0.17, \quad a=0.5 \tag{11}$$

For cold drawn OFHC waveguide with a chromate coating the empirical formula reads as

$$R_s/R_{so} = 1 + 0.21\ (f/100)^{0.5} \quad f \text{ in GHz} \tag{12}$$

As for the copper oxide system the situation is more complicated. We are not able to obtain a simple empirical formula as (8) and (12). But from Fig. 5 through Fig. 9 one clearly notice an anomalous heat loss associated with copper oxide layer. That added loss apparently depends weakly upon the copper oxide thickness. It is noted that at 96 GHz the ratio R_s/R_{so} obtained from Fig. 9 for waveguides with and without the oxide layer read as 1.44 and 1.25. Hence the oxide layer is accounted for the approximately 20% gam of added heat loss.

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Appendix

Derivation of (1)

Let the length of the single end cavity be L and is in resonant condition at a fequency f_1 we have

$$\frac{2L}{c} f_1 \sqrt{1 - (f_c/f_1)^2} = n \qquad (A-1)$$

where f_c be the cutoff frequency of the resonant mode. n is the number of half wavelength. At a fequency slightly higher than f_1 the cavity is again in resonant condition with the number of half wavelenghth increased by 1, i.e.,

$$\frac{2L}{c} (f_1+\Delta f_1) \sqrt{1-[f_c/(f_1+\Delta f)]^2} = n+1 \qquad (A-2)$$

Subtracting (A-1) from (A-2) and after some algebraic manipulation we have

$$\Delta f_1 = \frac{2L}{c} \sqrt{1 - (f_c/f_1)^2} \qquad (A-3)$$

References

1. Beck A. C. and Dawson, R. W.: "Conductivity measurement at microwave frequencies",Pro. Inst. Radio Eng., 38, p. 1181 (1950); Paghis I.: "Surface losses in electromagnetic cavity reserrators", Conadian J. Phy. 30, p. 174 (1952)
2. Unger, H. G.: "Waveguides with anisotropic impedance walls" in Electromagnetic Theory and Antennas Ed. by E. C. Jordan, pp. 919-939, Macmillan, 1962.
3. S. P. Morgan "Effect of surface roughness on eddy-current losses at microwave frequencies" J. Appl. Phys. 20, pp. 352-362 (1949).
4. Benson, F. A. and Allison, T.: "Surface roughness and attenuation of precision drawn waveguides",Proc. Inst. Elec. Eng. 102B, pp. 251-258 (1955); Benson, F. A., "Surface properties of waveguides" 17th Int. Scientific Radio Union General Assembly,Warsaw, Poland Aug. 1972.
5. Sanderson, A. E.: "Effect of surface raughness on propagation of the TEM mode", Advances of Microwaves 5, pp. 1-57 (1971).
6. Thorp, J. S. : "R. F. Conductivity in copper at 8 mm wavelengths",Proc. Inst. of Elec. Eng. Vol 101, Part III, No. 74, pp. 357-359, Nov. 1954.
7. Benson, F. A., and Steven, D. H.: "Rectangular waveguide attenuation at millimeter wavelengths",Proc. Inst. Elect. Eng. Vol 110, pp. 1008-1014, 1963.
8. Tischer,F.J.: "R. F. Conductivity in copper at 8 mm wavelengths",Proc. Inst. of Elec. Eng. 44, pp. 411-420 (1974).
9. Ginston, E. L.:Microwave measurements, McGraw-Hill, pp. 466-476, 1957.
10. Klinger, Y.: "The measurement of spurious modes in over moded waveguides"Proc. IEE,Vol 106 B Suppl. (13), pp. 89-93, 1959.

Part II. On the Anomalous Heat Loss in Waveguides with Copper Oxide Layer at Millimeter Frequencies

ABSTRACT — Anomalous heat loss in waveguides with copper oxide layer reported in Part I is further considered. Based on the observations made in Part I a model called variable conductivity model is propered. According to that model the percentage increase of heat loss is propertional to the signal frequency,which is qualitatively in line with the expérimental observations.

I. Introduction

In part I of this paper it is reported that a sizable amount of an anomalous heat loss was observed in copper waveguide with copper oxide layer, 0.6 μm to 1.2 μm in thicknesses. For example at 96 GHz the added heat loss is about 20% of the loss in the same waveguide with the copper oxide layer removed, irrespect to its thickness. This unique loss characteristic seems warrant further studies. In this paper we attempt to develope a theory for that anomalous heat loss.

As is well known the copper oxides either cupric or cuprous posses small conductivities, many order of magnitudes smaller than the metals'. Together with the thickness of the copper oxide layer mentioned above it becomes apparent that the copper oxide layer proper can not contribute to the amount of the observed heat loss. On the other hand it was reported in [1] that mechanical works could introduce defects into the crystal structure of the metallic wall of some waveguide cavities and in turn caused higher heat losses in those cavities. The added loss can be removed by removing the defect layer. Returning to the problem of the anomalous heat losses in copper waveguides with copper oxide layers we believe that the oxidation process not only generate the copper oxide layer proper but also introduce defects to the substrate copper in a region near the oxide layer proper. The conductivity of that region is considered to be some what smaller than the substrate copper but is in the same order of magnitude as the substrate copper. Preliminary computations based on bimetal system showed that a lower conductivity layer, a fraction of skin depth thick can cause the amount of the added heat loss comparable to the observed anomalous heat loss mentioned earlier. In section II of this paper we further consider that the defect density gradually diminishes with the depth of the substrate, i.e., the conductivity increases with depth to the bulk value σ_b of the substrate copper. For convenience we shall refer this model as the variable conductivity model. In section III we compare the calculated loss characteristics with the experiments and draw the conclusions.

II. Variable conductivity model

In this section we model the gradually increasing conductivity with the exponential function

$$\sigma(z) = \sigma_b[1-(1-(\sigma_s/\sigma_b)e^{-z/z_o}] \quad (1)$$

for the transition region between the copper oxide proper and the substrate copper. For convenience of analysis we consider an interface which separates the transition region from the oxide layer proper as shown in Fig. 1, σ_s is the assumed conductivity at that plane, σ_b is the bulk conductivity of the copper substrate. We further consider that $\sigma_s \ll \sigma_b$. Zo is a characteristic depth. It is noted from Appendix I that the surface impedance at the interface of the variable conductivity region is given by

$$Z_s=-(1+j)\sqrt{\frac{\omega\mu_o}{2\sigma_b}}\,[J_{2\xi}(-2\xi)\,/J'_{2\xi}(-2\xi)] \quad (2)$$

where $\xi = \tau z_o = \sqrt{j\omega\mu_o\sigma_b}\; z_o$ (3)

$J_{2\xi}(-2\xi)$ is the Bassel junction of order 2ξ and argument -2ξ. The prime indicates the differentication of the Bessel function with respect to the argument. For the case $|\tau z_o| \ll 1$ one may employ the small argument approximation for the Bessel function and arrives at

$$Z_s=(1+j)\sqrt{\frac{\omega\mu_o}{2\sigma_b}}\,(1+\tau z_o-\tau^2 z_o^2) \quad (4)$$

The surface resistance R_s can be readily obtained from (4).

$$R_s = \sqrt{\frac{\omega\mu_o}{2\sigma_b}}\,[1+2(\omega\mu_o\sigma_b)z_o^2] \quad (5)$$

One notes that R_{so} defined in

$$R_{so} = \sqrt{\frac{\omega\mu_o}{2\sigma_b}} \quad (6)$$

is the surface resistance of an ideally smooth copper surface. Further we introduce the skin depth of the copper wall at a reference frequency f_o

$$\delta_o = \sqrt{2/\omega_o \mu \sigma_b}$$

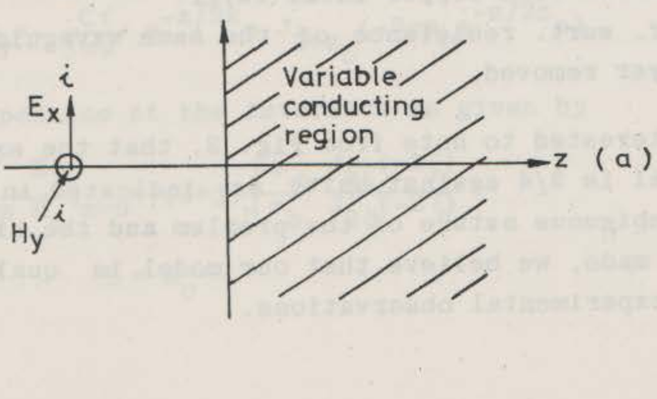
R_s becomes

$$R_s = R_{so}\,[1+2(z_o'/\delta_o)^2(\frac{f}{f_o})] \qquad (7)$$

In the subsequent section we shall compare (7) with the experimental results.

III. Comparison with Experimental Results

In Fig. 6 through Fig. 9 of the part I of this paper the losses of either TE_{01} mode or TE_{11} mode in copper waveguide of circular cross-section with and without copper oxide layer are platted against frequencies. In order to compare the percentage added losses due to the presence of the copper oxide layer it is of interest to plot the ratio of the losses in waveguides with and without the copper oxide layer as a function of frequencies. To do so we drew the best fit average curves for those figures and chose to consider at three different frequencies, the lower end, the middle and the lighter end of the frequency range of a particular figure. The results are shown in Fig. 2. Since (7) is obtained for the purpose of interpretation of the added heat loss due to the oxidation of the copper surface, we enter (7) in Fig. 2 with z_o/δ_o=0.33 for comparison. f_o in (7) is taken to be 100 GHz and δ_o is the skin depth of an ideally smooth copper surface at that frequency. Making use of the tabulated values of the permeability and conductivity of copper it is noted that δ_o at 100 GHz is slightly larger than 0.2μm. The characterisitic depth of our model is 0.07μm.



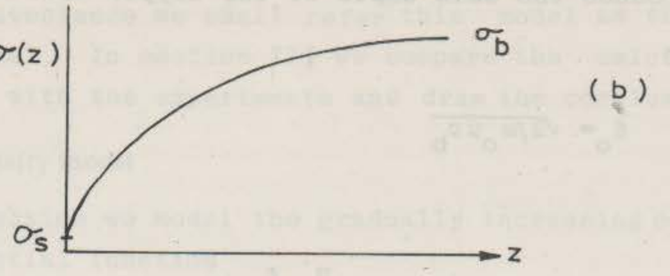


Fig. 1 Variable conductivity modal $\sigma(z)=\sigma_b[1-(1-\sigma_s/\sigma_b)e^{-z/z_o}]$

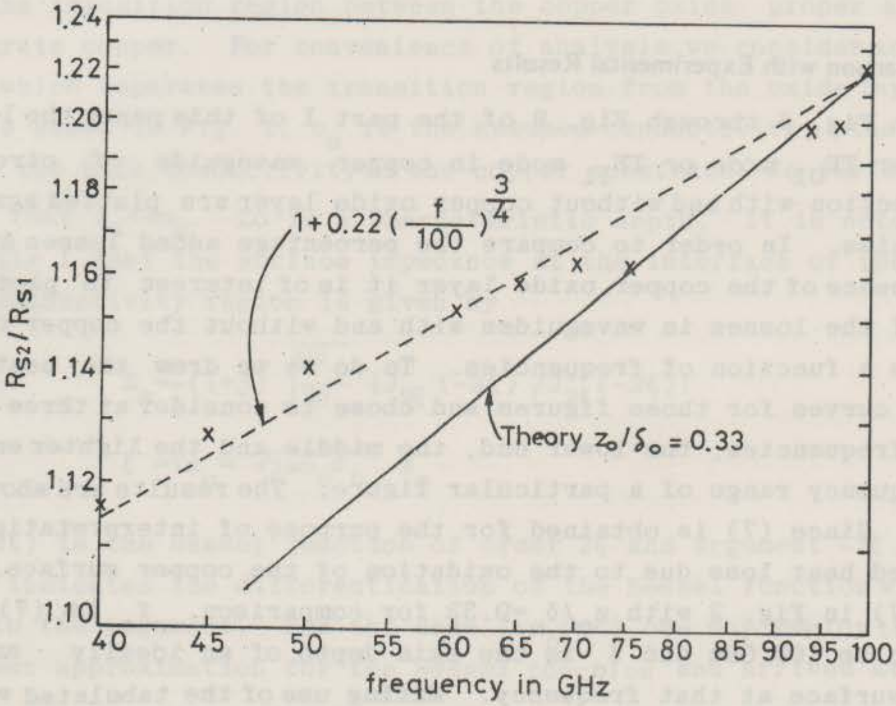


Fig. 2 Comparison of the theory with measurement results

R_{s2} = Eff. surt. resistance of copper waveguide, circular cross-section with copper oxide layer.

R_{s1} = Eff. surt. resistance of the same waveguide with oxide layer removed.

It is interested to note from Fig. 2. that the exponent of the empirical formal is 3/4 against unity as indicated in (7). Considering the ambiguous nature of the problem and the kind approximation we have made, we believe that our model is qualitatively in line with the experimental observations.

Appendix I. Surface Impedance of an Exponentially Varying Conductivity Substrate

In reference to Fig. 1 we consider a plane wave incident from the z<0 half space. For the half space z>0 the wave equation of the electric field E_x can be obtained from Maxwell's Equations [2] as

$$\frac{d^2E_x}{dz^2} - \tau^2 [1-(1-\sigma_s/\sigma_b)e^{-z/z_o}]E_x=0 \qquad (I-1)$$

where $$\tau^2 = j\omega\mu_o\sigma_b \qquad (I-2)$$

In the text we indicated that σ_s is considered to be much smaller than σ_b, the bulk conductivity of the substrate copper, hence (I-1) reduces to

$$\frac{d^2E_x}{dz^2} - \tau^2 [1-e^{-z/z_o}] E_x = 0 \qquad (I-3)$$

Making use of the substitution of variable

$$w = -2z_o\tau e^{-z/2z_o} \qquad (I-4)$$

the solution of (I-3) can be obtained as

$$E_x=C\ J_{2\tau z_o}(-2\tau z_o e^{-z/2z_o}) \qquad (I-5)$$

The magnetic field is given by [2]

$$-j\omega\mu H_y = \frac{dE_x}{dz} \qquad (I-6)$$

Substituting (I-5) in (I-6) yields

$$H_y=-\frac{C\tau}{j\omega\mu} e^{-z/2z_o} J_{2\tau z_o}(-2\tau z_o e^{-z/2z_o}) \qquad (I-7)$$

The surface impedance at the interface is given by

$$Z_s=\frac{E_x}{H_y}\Big|_{z=0}=-(1+j)\sqrt{\frac{\omega\mu_o}{\sigma_b}}\ \frac{J_{2\xi}(-2\xi)}{J'_{2\xi}(-2\xi)} \qquad (I-8)$$

where $$\xi = z_o \qquad (I-9)$$

References

1. Thorp, J. S. "R. F. Conductivity in Copper at 8 mm Wavelength", Pro. IEE, Vol. 101, Part III, No. 74, Nov. 1954.
2. Ramo, S. Whinnery, J. R., Van Duger, T.; Fields and Waves in Communication Electronics John Wiley, New York 1967, pp. 482-483.