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—— 短文 *Short Notes*

無鏡固體雷射
Solid-State Laser without External Mirrors

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Abstract — We report the successful operation of solid-state laser using 90 degree cone reflectors to provide total internal reflection, with no any other external components required. Also, the stability condition of resonators of this type are discussed in terms of ray tracing technique. The result shows a stable resonator can be found only when the sum of the deviation from 90 degree is smaller than zero.

Total internal reflecting surfaces have been proposed as a regenerative structures for solid-state lasers and the laser power is extracted from the resonator through frustrated total reflection by using of a coupling prism [1,2], instead of using frustrated total reflection to abstract the power, we found that the energy can be extracted through a "hole" on the tip of the end of the total reflecting surface. In this paper we examine the stability of the proposed structure as a solid state laser and report the construction and tested results of laser built according to the proposal.

If an optical ray incident from optical dense material with index of refraction n into air, then as the incident angle larger than the cricital angle ($\sin^{-1}\frac{1}{n}$), then total internal reflection occurs.

Consider the Nd: Yag material which has an index of refraction approximately equal to 1.8 at the lasing wavelength 1.06 μm and this corresponds to a critical angle of 32.4 degree. Thus if we polish the two ends of the laser rod into 90 degree cones then we obtain a laser resonator.

However, manutacturing tolerance are always exist in fabricating a 90 degree cone, hence it seems instructive to study how the manufacturing tolerance affect the stability of such a resonator. This problem is approached by ray tracing technique and the ray transfer matrix for nearly 90 degree cone is reshown in table 1 [3].

Consider Fig. 1, where γ_s and γ_s' are the height and slope of the ray after S round trips. γ_{s+1} and γ_{s+1}' are the height and slope of the ray after (s+1) round trips. Also, we assume the rays hit the upper surface when s=0. Then γ_{s+1}, γ_{s+1}' are related to γ_s, γ_s' by Eq. (4) [4].

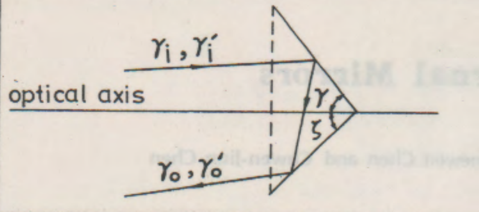
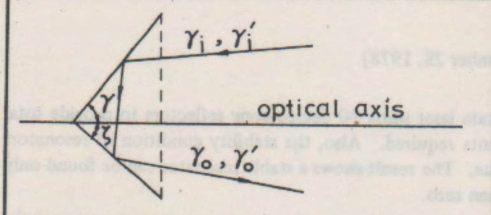
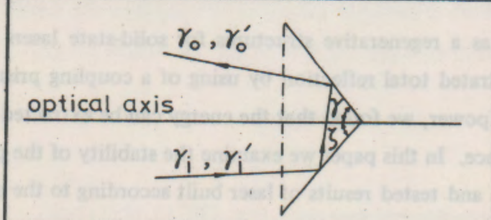
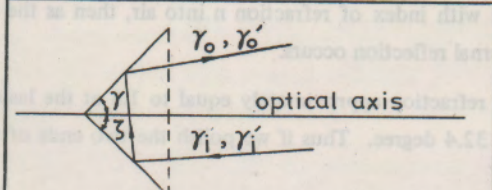
$$\begin{bmatrix}\gamma_{s+1}\\ \gamma_{s+1}'\end{bmatrix} = \begin{bmatrix}1 & d\\ 0 & 1\end{bmatrix}\left\{\begin{bmatrix}-1 & -2a\\ 0 & -1\end{bmatrix}\begin{bmatrix}1 & d\\ 0 & 1\end{bmatrix}\left(\begin{bmatrix}-1 & -2a\\ 0 & -1\end{bmatrix}\begin{bmatrix}\gamma_s\\ \gamma_s'\end{bmatrix} - \begin{bmatrix}2a(\delta+\phi)\\ 2(\delta+\phi)\end{bmatrix}\right) + \begin{bmatrix}2a(\alpha+\eta)\\ 2(\alpha+\eta)\end{bmatrix}\right\}$$

$$= \begin{bmatrix}1 & 4a+2d\\ 0 & 1\end{bmatrix}\begin{bmatrix}\gamma_s\\ \gamma_s'\end{bmatrix} + \begin{bmatrix}(6a+4d)(\delta+\phi)+(2a+2d)(\alpha+\eta)\\ 2(\delta+\phi+\alpha+\eta)\end{bmatrix}$$

$$= \begin{bmatrix}A & B\\ C & D\end{bmatrix}\begin{bmatrix}\gamma_s\\ \gamma_s'\end{bmatrix} + \begin{bmatrix}E\\ F\end{bmatrix} \qquad (1)$$

Table 1: Ray matrices for possible combination of nearly 90 degree roof-prisms and incoming rays.

$\gamma = \frac{\pi}{4} + \delta\ ,\quad \zeta = \frac{\pi}{4} + \phi,\quad \delta \ll 1,\quad \phi \ll 1.$

	$\begin{bmatrix}\gamma_o\\ \gamma_o'\end{bmatrix} = \begin{bmatrix}-1 & -2a\\ 0 & -1\end{bmatrix}\begin{bmatrix}\gamma_i\\ \gamma_i'\end{bmatrix} - \begin{bmatrix}2a(\delta+\phi)\\ 2(\delta+\phi)\end{bmatrix}$
	$\begin{bmatrix}\gamma_o\\ \gamma_o'\end{bmatrix} = \begin{bmatrix}-1 & -2a\\ 0 & -1\end{bmatrix}\begin{bmatrix}\gamma_i\\ \gamma_i'\end{bmatrix} - \begin{bmatrix}2a(\delta+\phi)\\ 2(\delta+\phi)\end{bmatrix}$
	$\begin{bmatrix}\gamma_o\\ \gamma_o'\end{bmatrix} = \begin{bmatrix}-1 & -2a\\ 0 & -1\end{bmatrix}\begin{bmatrix}\gamma_i\\ \gamma_i'\end{bmatrix} + \begin{bmatrix}2a(\delta+\phi)\\ 2(\delta+\phi)\end{bmatrix}$
	$\begin{bmatrix}\gamma_o\\ \gamma_o'\end{bmatrix} = \begin{bmatrix}-1 & -2a\\ 0 & -1\end{bmatrix}\begin{bmatrix}\gamma_i\\ \gamma_i'\end{bmatrix} + \begin{bmatrix}2a(\delta+\phi)\\ 2(\delta+\phi)\end{bmatrix}$

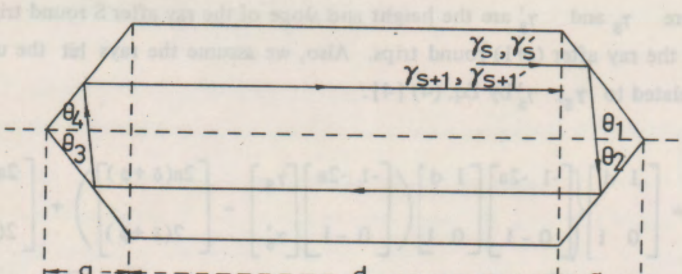


Fig. 1. $\theta_1 = \frac{\pi}{4} + \delta\ ,\quad \theta_2 = \frac{\pi}{4} + \phi,\quad \theta_3 = \frac{\pi}{4} + \eta,\quad \theta_4 = \frac{\pi}{4} + \alpha\ .$

Schematic diagram used to derive the stability condition of a quasi-90 degree resonator. Where the deviation from 90 degree of the cone angle is due to manufacturing tolerance.

where

$$A = 1$$
$$B = 4a + 2d$$
$$C = 0$$
$$D = 1$$
$$E = (6a + 2d)(\delta + \phi) + (2a + 2d)(\alpha + \eta)$$
$$F = 2(\delta + \phi + \alpha + \eta)$$

Hence

$$\gamma_{s+1} = A\gamma_s + B\gamma'_s + E$$

$$\gamma'_{s+1} = C\gamma_s + D\gamma'_s + F$$

From the above two equation, we will obtain the difference equation shown below [4]

$$\gamma_{s+2} - (A+D)\gamma_{s+1} + (AD-BC) = BF + E - DE$$

or

$$\gamma_{s+2} - 2\gamma_{s+1} + \gamma_s = (8a+4d)(\delta + \phi + \alpha + \mu) \qquad (2)$$

The solution of this difference equation with the boundary condition

$$\gamma_o = y_o$$

$$\gamma_1 = y_o + (4a+2d)\mu_o + (6a+4d)(\delta + \phi) + (2a+2d)(\alpha + \eta)$$

is given by

$$\gamma_s = y_o + (4a+2d)\mu_o S + [(2a+2d)(\delta + \phi) - 2a(\alpha + \eta)]S + (4a+2d)(\delta + \phi + \alpha + \eta)S^2$$

$$= y_o + [(4a+2d)\mu_o + (2a+2d)(\delta + \phi) - 2a(\alpha + \eta) + (4a+2d)(\delta + \phi + \alpha + \eta)S]S \qquad (3)$$

Where y_o is the height of the ray and μ_o is its slope when S=0 and γ_1 is obtained from Eq. (1) by setting S=0. Since we are considering paraxial rays only, then when the ray hits the right, upper cone surface, we will have $y_o > 0$.

The term in the bracket will have negative quantity if $(\delta + \phi + \alpha + \eta) < 0$ for sufficiently large value of S. (Note S is a positive number). In other words, the ray tends to move closer and closer to the optical axis if $(\delta + \phi + \alpha + \eta) < 0$ and for sufficiently large number of S.

If $(\delta + \phi + \alpha + \eta) > 0$, then γ_s increases as S increases and eventually the ray will run away from the optical resonator.

If the ray hits the right, lower cone surface first, the solution will be of the form

$$\gamma_s = y''_o + [(4a+2d)\mu''_o - (2a+2d)(\delta + \phi) + 2a(\alpha + \eta) - (4a+2d)(\delta + \phi + \alpha + \eta)S]$$

where y''_0 is the height of the ray when S=0 and μ''_0 is its slope (note $\tan \mu''_0 \approx \mu''_0$ for paraxial ray).

By the same argument as before, we know here y''_0 is a negative quantity. Thus as $(\delta + \phi + \alpha + \eta) < 0$ and for sufficiently large S, the ray will move closer and closer to the optical axis too.

Similar arguments holds for these rays which hits the left cone surface first.

Thus, if any manufacturing tolerance is required, we should keep the tolerance smaller than zero (i.e., $\delta + \phi + \alpha + \eta < 0$). Otherwise the paraxial rays will eventually run away from the cavity and results in unstable resonator.

In this experiment, fabrication of the proposed solid state laser, we polish the two ends of a Nd: Yag laser rod into 90 degree cones and the cone tips are polished into small flat surfaces (1 mm in diameter, no intention has been made to optimize the output energy) as shown in Fig. 2. Where the 90 degree cones form total internal reflecting surfaces and the small flat surfaces are used as energy output port. Intiutively, this structure is more or less similar to the hole coupling used in CO_2 lasers.

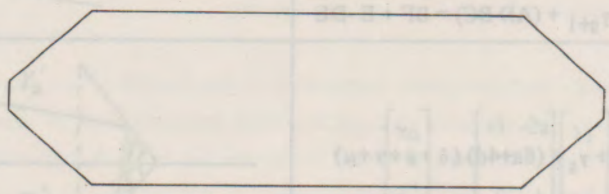


Fig. 2. Schematic diagram of solid-state laser without external mirrors.

This Nd: Yag laser rod, 69 mm in length and 5 mm in diameter with 1.2% concentration, is then put into a gold-coated circular cavity and pumped by a pulsed Xeron flash lamp. The typical discharge condition was a capacity of 50μf and 600V. The output energy detected by a p-i-n photodiods (with RG780, 1.06 μm filter, in front of the detector) is about 25 mj and shown in Fig. 3 and Fig. 4.

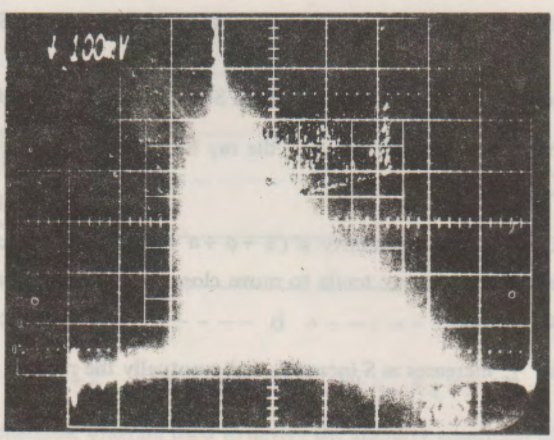


Fig. 3. Pulsed output of Nd: Yag laser with total internal reflecting surfaces using "hole" coupling. This picture shows about a 58 μs delay of laser pulse.

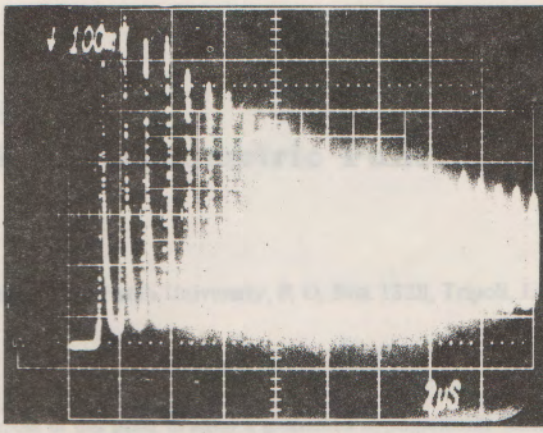


Fig. 4. Laser pulse output of Fig. 2 with smaller time scale.

Where the oscilloscope traces show a sudden rise of output energy in Fig. 3 and relaxation oscillation of Fig. 4 will ensure that laser oscillation does occur in such a structure. No such oscillation has been observed if we did not polish the cone surfaces under otherwise the same pumping condition. This is believed due to large scattering loss.

In conclusion we note:

1. From discussion in section II, it is found that if we try to make a mirroless resonator, we prefer to make the tip angle slightly smaller than 90 degree if any manufacturing tolerance is required.

2. Since total internal reflection has no dielectric loss, those we expect this resonator will minimize the threshold pumping power and hence maximize the output power under optimum coupling condition. Also, this structure is expected to be more useful in field operation where vibration problem may be serious. This is because no external components are required and hence no alignment problem exist after the laser rod is fabricated.

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