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貝提連續抽樣計畫之最佳性

An Optimality of Beattie's Continuous Sampling Plan

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Abstract––Beattie's continuous (acceptance) sampling plan (CSP) is very easy to operate once the parameters (k,h,h*) are chosen for given an Acceptable Quality Level (AQL) p_1, a Rejectable Quality Level (RQL) p_2, the consumer's risk α and producer's risk β . In this paper, a method for determining the parameters of Beattie's CSP is proposed. Given an AQL p_1, a RQL p_2, and α , β, this method gives a faster way to get the appropriate parameters, so that $P(a \mid p_1) \geqslant 1-\alpha$ and $P(a \mid p_2) \leqslant \beta$, which is difficult to do by means of previously published methods. We begin by reviewing the existing methods, then we describe our method. Finally, a selection of plans under the binomial assumption are provided.

I. Introduction

Consider the situation that product is being made continuously and it is desired to tell one whether to continue accepting or to cease accepting the product based on the number of defectives observed, where the sampling test may be destructive, the number of items available for the test is relatively small and it is required to keep a constant sampling rate. It is desired to discriminate between the prescribed AQL p_1 and RQL p_2 such that $P(a \mid p_1) \geqslant 1-\alpha$ and $P(a \mid p_2) \leqslant \beta$. Where α , β is the consumer's risk and producer's risk respectively.

The conventional continuous sampling plans are not appropriate in the above situation. For instance, Dodge's CSP-l and CSP-2 are designed with rectify property, also they entail a variable sampling rate which are not possible when testing is destructive. However, a continuous acceptance sampling plan does appear to be well suited and appliable to the above situation is one proposed by Beattie (1962) which is based on cumulative sums.

Beattie's CSP is to set up a cumulative sum chart which has an accept zone and a reject zone and to accept or reject the product appropriately. A small sample of size n is selected at regular intervals of time from the process and the number of defectives y is recorded. A reference value k is chosen such that the cusum $S_m = \Sigma\ (y_i - k)$ decreases if $p=p_1$ and increases if $p=p_2$, hence the decision lines (S_m=h and S_m=h+h*) on the accept and reject zones can be horizontal. S_m is computed and plotted according to the rules below :

(1) Start the cumulation at zero.

(2) Accept product as long as $S_m < h$. When $S_m < 0$, return the cumulation to zero.

(3) Reject product when $S_m \geqslant h$, restart cumulation at S_m=h+h* and continue rejecting product until $S_m < h$. When $S_m >$ h+h*, return cumulation to h+h*.

(4) When h is crossed or reached from above, accept product and restart cumulation at zero.

In the above description, values of n, k, h and h* can be found such that product will be accepted 100(1- α)% of the time when $p=p_1$, i.e. $P(a|p_1)=1-\alpha$ and rejected 100(1- β)% of the time when $p=p_2$, i.e. $P(a|p_2)=\beta$.

The properties of such a plan are defined by an OC curve which is determined by the Average Run Length (ARL) in the accept zone or reject zone. By ARL we mean the average number of samples taken for a given quality level p before the cusum reaches or acrosses the decision line. An OC curve gives the probability of acceptance of an item as a function of incoming quality. The proportion of items accepted (P(a | p)) is given by the ratio

$$P(a \mid p) = \frac{L(0, p)}{L(0,p)+L^*(h+h^*,p)},$$

where L(0,p) is the ARL in the accept zone, L*(h+h*,p) is the ARL in the reject zone and the sampling rate is the same in both the accept and reject zone.

Prairie and Zimmer (1973) presented brief underlying mathematics to Beattie's CSP and provided graphs and tables for determining a plan under the binomial assumption and the Poisson approximation. However, there are many shortcomings of their approach to select the parameters discriminating between the prescribed p_1 and p_2 values, satisfying the required α, β values. For instance, suppose we would like to select the parameters (k,h,h*) of Beattie's CSP for p_1=0.01, p_2=0.12, α =0.05 and β = 0.10, n=5, the existing approach need to look for tables and graphs provided to find an appropriate parameters satisfying the requirements. Even when the graphs are provided, obtaining the desired plan through use of the figures is difficult. It may be easy to find a plan with the desired P(a | p_1), but a lot of trial and error is needed to find a plan satisfying both P(a | p_1) $\approx$ 1- α and P(a | p_2) $\approx\beta$. Also in order to meet such needs, too many tables and graphs need to be made for various values. Their method of constructing a graph is as follows. Fixed the sample size n first, choose k, h, h* arbitrary, put those values into the following two equations derived by Ewan and Kemp (1960), which may be solved iteratively or by standard matrix inversion methods, to get L(0,p) and L*(h+h*,p) over a suitable range of p by using computer.

In the accept zone, let L(z,p) denote the ARL of a plan of the procedure for which the first cumulation starts at a point z units above the 0 boundary.

$$L(z,p)=1+\sum_{y=1}^{h-1} L(y,p)f(y+k-z)+L(0,p)\sum_{x=0}^{k-z} f(x),\quad z=0,1,2,\ldots\ldots,h-1. \tag{1}$$

In the reject zone, L*(z,p) denote the ARL of the plan of the procedure for which the first cumulation starts at a point z units below the h+h* boundary.

$$L^*(z,p)=1+\sum_{y=1}^{h^*-1} L^*(y,p)f(z-y+k)+L^*(h+h^*,p)\sum_{x=z}^{\infty} f(k+x),\quad z=h+1,\ h+2,\ldots,h+h^*. \tag{2}$$

Where f(x) is the probability of having exactly x defectives in the sample; f(x) may be taken as the binomial or Poisson density functions.

Next compute the probability of acceptance P(a | p), then pick the p_1 and p_2 satisfying P(a | p_1) $\approx$ 1- α and P(a | p_2) $\approx\beta$. The method is really backward.

In the next section, we will describe the new approach we proposed, i.e., define the cusum test first, prove cusum test has an optimality property, determine the parameters of Beattie's CSP from the cusum test, and provide some tables under the binomial assumption and Poisson approximation. Finally, we will indicate strengths and weakness of our approach and what future research is needed.

II. The Cusum Test

In the acceptance sampling inspection of a continuously manufactured products a small sample of size n is selected at regular intervals of time from the production process. Letting the random variable Y denote the number of defectives in a sample, we shall denote the successive observations on Y by $y_1, y_2, \ldots\ldots$, etc., and the distribution of Y at p_i (i=1, 2) by $f(y \mid p_i)=f_i(y)$.

For any positive integer m, let $f_{im}=f_i(y_1)f_i(y_2)\ldots\ldots f_i(y_m)$, (i=1, 2. m=1,2,3,....) Given any two real numbers $A > 1$ and $B < 1$, the cusum test for the simple hypothesis $H_1: p=p_1$ versus the alternative $H_2: p=p_2$ is defined as follows:

Letting $z_i=\log\frac{f_2(y_i)}{f_1(y_i)}$, the cusum $Z_m = \sum_1^m z_i$ is computed at each stage m. In the accept (reject) zone, Z_m is started at zero, and the sampling procedure is continued as long as $Z_m < \log A (Z_m > \log B)$. If $Z_m \geqslant \log A$ ($Z_m \leqslant \log B$) the procedure shift (reject) to the reject (accept) zone.

Under the binomial assumption, let $f_i(y)=\binom{n}{y}p_i^y(1-p_i)^{n-y}$, i=1,2, then we have

$$Z_m = \sum_{i=1}^m z_i = \sum_i^m \log\frac{f_2(y_i)}{f_1(y_i)} = \sum_{i=1}^m \log\frac{\binom{n}{y_i}\, p_2^{\,y_i}\,(1-p_2)^{n-y_i}}{\binom{n}{y_i}\, p_1^{\,y_i}\,(1-p_1)^{n-y_i}}$$

$$= \sum_{i=1}^m y_i \log\frac{p_2(1-p_1)}{p_1(1-p_2)} + nm\log\frac{1-p_2}{1-p_1}$$

In the accept zone, we reject H_1 if

$$\sum_{i=1}^m y_i \log\frac{p_2(1-p_1)}{p_1(1-p_2)} + nm\log\frac{1-p_2}{1-p_1} \geqslant \log A \tag{3}$$

We continue the acceptance sampling by taking an additional observation if

$$\sum_{i=1}^m y_i \log\frac{p_2(1-p_1)}{p_1(1-p_2)} + nm\log\frac{1-p_2}{1-p_1} < \log A \tag{4}$$

In the reject zone, we accept H_1 if

$$\sum_{i=1}^m y_i \log\frac{p_2(1-p_1)}{p_1(1-p_2)} + nm\log\frac{1-p_2}{1-p_1} \leqslant \log B \tag{5}$$

and continue sampling by taking an additional observation if

$$\sum_{i=1}^m y_i \log\frac{p_2(1-p_1)}{p_1(1-p_2)} + nm\log\frac{1-p_2}{1-p_1} > \log B \tag{6}$$

The usual measures of the performance of such a procedure are the error probabilities, α and β, and the expected number of observations. We will prove an optimality theorem for the cusum test in the next section.

III. An Optimality Theorem

Let P_i and E_i denote the probability and expectation on the space of infinite sequences $(y_1, y_2, \ldots\ldots)$ determined by f_i (i=1,2).

Theorem: Let (t_1, t_2) denote the stopping rules of the cusum test in the accept zone and reject zone respectively, and α, β denote the error probabilities of the cusum test. Define the set of stopping rules: $\tau = \{(t_1^*, t_2^*): P_1(t_1^* < \infty)$

$< P_1(t_1 < \infty), P_2(t_2^* < \infty) < P_2(t_2 < \infty) \}$ and $\tau_{\alpha\beta} = \{(t_1^*, t_2^*): \alpha^* \leq \alpha, \ \beta^* \leq \beta\} \cap \tau$. Then $(t_1, t_2) \in \tau_{\alpha\beta}$ and is optimal in the sense that $E_2 t_1$ and $E_1 t_2$ are minimum over $\tau_{\alpha\beta}$.

The main part of the proof of the theorem is contained in the solution of the following auxiliary problem. Consider the simple hypothesis testing problem $H_1: p=p_1$ against $H_2: p=p_2$. In the accept zone of the cusum sampling plan, there is a constant cost c per observation if $p=p_2$, however, if $p=p_1$, then there is no sampling cost, but we incur a cost of c amount if we ever reject H_1. Hence it appears that if we are sampling from $f_2(y)$, we would like to shift to the reject zone as soon as possible, whereas we would like to continue sampling (accepting) indefinitely as long as it appears that the distribution of Y is $f_1(y)$. In the reject zone, each observation cost c amount if $p=p_1$; however, if $p=p_2$, there is no sampling cost until we incur a cost of c amount for rejecting H_2.

Suppose that the prior distribution of quality p is of the following type: $P(p=p_1)= \pi$ and $P(p=p_2)= \bar{\pi}$, $0 < \pi < 1$, such that $\pi + \bar{\pi} = 1$. In the accept zone, corresponding to the losses given above, we have the risk function:

$$R(t) = c\,\pi\, P_1(t < \infty) + c\,\bar{\pi}\, E_2 t \tag{7}$$

We would like to find an extended stopping rule minimizing this risk. For any stopping rule t for which $P_1(t < \infty, f_{2t}=0)=0$ and $E_2 t < \infty$, we have:

$$P_1(t < \infty) = E_1 \chi_{(t=\infty)} = \sum_1^\infty \int_{(t=n)} f_{1n} = \sum_1^\infty \int \frac{f_{1n}}{f_{2n}} \cdot f_{2n} = E_2 \left(\frac{f_{1n}}{f_{2n}}\right)$$

Putting $-x_n = c\,\pi \frac{f_{1n}}{f_{2n}} + c\,\bar{\pi}\, n$, $\mathcal{F}_n = \beta(y_1, y_2, \ldots, y_n), (n \geq 1)$, we see that to minimize (7), it suffices to maximize $E_2 x_t$, i.e., to solve the optimal stopping problem for the stochastic sequence $(x_n, \mathcal{F}_n)$ under the probability P_2.

Letting $u_n = \frac{f_{1n}}{f_{2n}} \cdot u$, $u \geq 0$, we can express x_n as a function ϕ_n of u_n. i.e., $x_n = \phi_n(u_n) = -c\,\pi u_n - c\,\bar{\pi} n$. Chow et. al. (1971) define a stopping rule σ_1 for this situation given by

$$\sigma_1 = \text{first } n \geq 1 \text{ such that } x_n \geq E_2(\mathcal{F}_{n+1} \mid \mathcal{F}_n)$$

$$= \infty \text{ if no such n exists}$$

where $\mathcal{F}_i = \max(x_i, E_2(\mathcal{F}_{i+1} \mid \mathcal{F}_i))$

Note that $\{u_n\}$ forms a stationary Markov sequence with initial point u.

Theorem 5.2 of Chow et. al. (1972) says that in the stationary Markov case, there is a version of (γ_n) such that for n=1,2,......

$$V_n(\cdot) = E_2(\gamma_{n+1}(u_{n+1}) \mid u_n = \cdot) \quad \text{where} \quad \gamma_n(\cdot) = \max(\phi_n(\cdot), V_n(\cdot))$$

Thus, our stopping rule σ_1 becomes

$$\sigma_1 = \text{first } n \geq 1 \text{ such that } \phi_n(u_n) \geq V_n(u_n)$$

$$= \infty \text{ if no such n exists.}$$

Under the assumptions of Theorem 5.2, suppose that $\phi_n = \phi - c\,\bar{\pi} n$ (n=1,2,.....) and let $S = \{ u: \phi(u) \geq V_0(u) \}$, we have $V_n = V_0 - c\bar{\pi} n$, where $V_0(u) = \sup_{n \epsilon C} E_2\, \phi_n(u_n)$. $C = \{\, t : E_2(\phi_{t+k}(u_{t+k}) \mid u_t = u) \} < \infty$.

Since $\lim x_n = -\infty$, we have $P(\sigma_1 < \infty) = 1$ and σ_1 is optimal by Theorem 4.5 in the same book. Thus, the optimal stopping rule for this sequential decision problem is

$$\sigma_1 = \text{first } n \geqslant 1 \quad \text{such that} \quad \phi(u_n) \geqslant V_0(u_n)$$

where $\phi(u_n) = -c\,\pi\, u_n$.

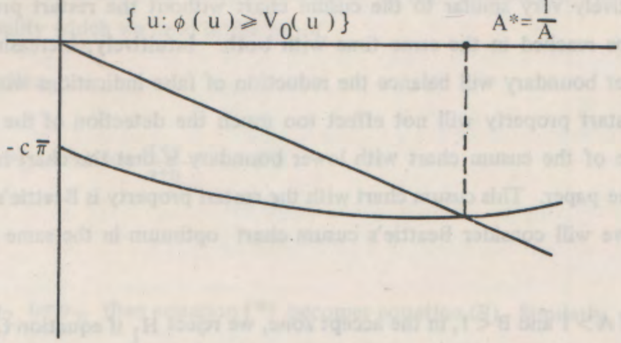

Figure 1

It is easy to see that $V_0(0) = -c\bar{\pi}$ and that $V_0(\cdot)$ is convex. It follows that for some constant $A^* = \frac{1}{A}$

$$\sigma_1 = \text{first } n \geqslant 1 \quad \text{such that } u_n \geqslant \frac{1}{A} \text{ or } \frac{f_{2n}}{f_{1n}} \geqslant Au \tag{8}$$

Now return to our problem where u=1. Let $Z_n = \Sigma \log \frac{f_{2n}}{f_{1n}}$, we note that (8) becomes

$$\sigma_1 = \text{first } n \geqslant 1 \quad \text{such that } Z_n \geqslant \log A.$$

Thus, the Bayes risk (7) is minimized by a cusum test with boundary A as shown in Figure 1. In the reject zone, we can show in the same manner that the cusum test with boundary B minimize the Bayes risk.

The required connection between the auxiliary problem and the original one is established by the following lemma.

Lemma: Given A, B and π , with $0 < B < 1 < A < \infty$ and $0 < \pi < 1$, there exists a sequential decision problem (i.e., a choice of $c > 0$), such that the cusum test is Bayes with respect to π. (that is, such that the relation shown in Fig. 1 holds in the accept zone, and a similar relation holds in the reject zone).

Proof: For any given $A > 1$ and $0 < \pi < 1$, there is a choice of cost $c > 0$, such that Fig. 1 holds i.e., the cusum test is Bayes with respect to π , it follows by the similar argument for the reject zone.

Proof of optimality theorem

Let (T_1, T_2) be any other stopping rules in τ. The risk function is $R(T_1) = c\pi P_1(T_1 < \infty) + c\bar{\pi} E_2 T_1$ in the accept zone. Since t_1 is a Bayes rule, we have

$$0 \leq R(T_1)-R(t_1)= c\pi\ [P_1(T_1<\infty)-P_1(t_1<\infty)]+ c\bar{\pi}(E_2T_1-E_2t_1) \leq c\bar{\pi}(E_2T_1-E_2t_1)$$

which implies that $E_2T_1 \geq E_2t_1$.

In the reject zone, we can show that $E_1T_2 \geq E_1t_2$ in the same manner.

Thus, we have shown that E_2t_1 and E_1t_2 are minimum over τ and $\tau_{\alpha\beta}$ since $\tau_{\alpha\beta} \subset \tau$.

IV. Determination of the Parameters of Beattie's CSP

Ewan and Kemp (1960) claimed that a cusum chart with lower boundary zero and upper boundary h using a reference value k is effectively very similar to the cusum chart without the restart property, and a decision that quality has changed can be reached in the same time with both. Intuitively, increasing the decision intervals of the cusum chart with lower boundary will balance the reduction of false indications with increased time to correct detections, so that the restart property will not effect too much the detection of the change of quality. Besides that, a practical advantage of the cusum chart with lower boundary is that the chart is bounded in the sense that cusum does not run off the paper. This cusum chart with the restart property is Beattie's sampling plan. As a result of the above discussion, we will consider Beattie's cusum chart optimum in the same sense as that of the regular cusum chart.

For any given constants $A > 1$ and $B < 1$, in the accept zone, we reject H_1 if equation (3) is true. i.e.,

$$\sum_{i=1}^{m} y_i \log \frac{p_2(1-p_1)}{p_1(1-p_2)} + nm \log \frac{1-p_2}{1-p_1} \geq \log A$$

or

$$\sum_{i=1}^{m} \left(y_i - n\frac{\log \frac{1-p_1}{1-p_2}}{\log \frac{p_2(1-p_1)}{p_1(1-p_2)}}\right) \geq \frac{\log A}{\log \frac{p_2(1-p_1)}{p_1(1-p_2)}},$$

compare it with $\sum_{i}^{m} (y_i - k) \geq h$,

We have $h = \dfrac{\log A}{\log \frac{p_2(1-p_1)}{p_1(1-p_2)}}$, $k = \dfrac{n \log \frac{1-p_1}{1-p_2}}{\log \frac{p_2(1-p_1)}{p_1(1-p_2)}}$

Similarly, in the reject zone, we accept H_2 if equation (5) is ture.

$$\sum_{i=1}^{m} y_i \log \frac{p_2(1-p_1)}{p_1(1-p_2)} + nm \log \frac{1-p_2}{1-p_1} \leq \log B,$$

$$\sum_{i=1}^{m} \left(y_i - n\frac{\log \frac{1-p_1}{1-p_2}}{\log \frac{p_2(1-p_1)}{p_1(1-p_1)}}\right) \leq \frac{\log B}{\log \frac{p_2(1-p_1)}{p_1(1-p_2)}},$$

compare it with $\sum_{i}^{m} (y_i - k) \lessgtr h^*$, we get $h^* = \dfrac{\log B}{\log \frac{p_2(1-p_1)}{p_1(1-p_2)}}$

V. The Optimal Choice of k

Ewan and Kemp (1960) indicated that the optimal choice k is np_o where p_o is a midpoint between p_1 and p_2 but from the preceding section, we have $p_0 = \dfrac{\log \frac{1-p_1}{1-p_2}}{\log \frac{p_2(1-p_1)}{p_1(1-p_2)}}$. Our choice of k for any given AQL p_1, RQL p_2

and specified α , β is optimal in the sense that it is dictated by the fact that the cusum test has an optimality property.

Now we would like to verify that indeed $p_1 < p_0 < p_2$. Let us show that $p_1 < \frac{\log \frac{1-p_1}{1-p_2}}{\log \frac{p_2(1-p_1)}{p_1(1-p_2)}}$ first, which after some algebraic manipulation is equivalent to

$$p_1 \log \frac{p_2}{p_1} < (1-p_1) \log \frac{1-p_1}{1-p_2} \tag{9}$$

Hence if we can prove that the equation (9) is true, we are done. In the "Inequalities" by Hardy, G. G. et. al. (1952) we have the following inequality which we need in the proof.

If all the numbers are positive, then

$$x \log \frac{x}{a} + y \log \frac{y}{b} > (x+y) \log \frac{x+y}{a+b} \qquad (*)$$

unless $\frac{x}{a} = \frac{y}{b}$

Let $x=1-p_1$, $y=p_1$, $a=1-p_2$ $b=p_2$, then equation (*) becomes equation (9). Similarly, one can show that

$$\frac{\log \frac{1-p_1}{1-p_2}}{\log \frac{p_2(1-p_1)}{p_1(1-p_2)}} < p_2$$ by the same algebraic steps.

VI. Computation

From the previous section, we have the parameters h and h* of Beattie's continuous sampling plan as $h = \frac{\log A}{\log \frac{p_2(1-p_1)}{p_1(1-p_2)}}$ and $h^* = \frac{\log B}{\log \frac{p_2(1-p_1)}{p_1(1-p_2)}}$ which are related to the constants A and B. But since we can not find the values of A and B analytically, we determine h and h* directly by numerical method. Given a p_1 and a p_2 we can determine the optimal k. For various fractional value of h and h*, we compute L(0,p), L(Z+Z*,p) and P(a | p) numerically by using computer. We then pick the h and h* which close to the requirements $P(a \mid p_1) \approx 1-\alpha$ and $P(a \mid p_2) \approx \beta$ for the specified α and β . With k selected as optimal, we can get an optimal plan much faster.

In TABLE I, we provide a selection of Beattie's CSP under the binomial assumption, which can be adopted for practical use.

VII. Summary and Conclusions

Given the quadruple (α , p_1, β , p_2) for the continuous manufacturing process, Beattie proposed a continuous acceptance sampling plan based on a cusum chart for the number of defectives with parameters (k,h,h*) having the following desired properties:

(1) The test may be destructive.

(2) The sample size n may be relatively small.

(3) The sampling rate may remain constant.

(4) $P(a \mid p_1) \approx 1-\alpha$ and $P(a \mid p_2) \approx \beta$.

TABLE I

A selection of plans and their characteristics under the binomial assumption

n	k	h	h*	AQL	RQL	L (0) AQL	L*(0) AQL	L (0) RQL	L*(0) RQL	p_1	p_2
5	1/8	2	7/4	0.01	0.05	404.0	19.51	15.39	137.9	0.954	0.100
5	3/20	5/3	5/4	0.01	0.070	247.5	11.35	8.52	77.75	0.956	0.099
10	3/10	4/3	6/5	0.01	0.070	80.34	5.16	3.92	33.31	0.940	0.105
5	4/25	3/2	9/8	0.01	0.075	181.4	9.93	7.08	64.58	0.948	0.099
5	1/5	11/10	11/10	0.01	0.10	146.3	7.13	5.78	54.22	0.954	0.096
5	3/10	1/2	3/10	0.01	0.20	20.40	1.65	1.49	12.36	0.954	0.107
5	1/5	17/5	11/5	0.025	0.075	399.2	20.71	18.50	169.2	0.951	0.099
5	7/25	1	19/10	0.025	0.10	175.7	10.77	8.86	79.34	0.942	0.100
5	7/20	8/5	6/5	0.025	0.15	107.1	5.39	4.37	42.41	0.952	0.093
5	2/5	41/10	51/10	0.05	0.10	584.6	30.78	30.07	288.2	0.950	0.094
5	9/20	12/5	21/10	0.05	0.15	137.8	9.05	8.12	70.62	0.938	0.103
5	11/20	7/4	3/2	0.05	0.20	82.91	4.76	4.39	36.65	0.940	0.107
5	3/5	7/5	9/10	0.05	0.25	42.08	2.91	2.60	21.39	0.935	0.108
5	7/10	1	4/5	0.05	0.30	41.67	2.63	2.07	19.71	0.941	0.095
5	1/2	47/10	63/20	0.075	0.15	314.7	19.56	18.4	192.4	0.942	0.087
5	3/5	16/5	11/5	0.075	0.20	178.3	8.29	8.15	79.71	0.956	0.093
5	7/10	11/5	29/20	0.075	0.25	94.37	4.85	4.50	47.54	0.951	0.090
5	4/5	3/2	1	0.075	0.30	65.45	3.06	3.12	29.00	0.955	0.097
5	7/10	43/10	33/10	0.10	0.20	255.0	13.64	13.75	133.2	0.948	0.094
5	4/5	14/5	11/5	0.10	0.25	116.2	6.36	6.57	54.15	0.948	0.108
5	9/10	11/5	9/5	0.10	0.30	74.78	4.26	4.06	39.53	0.946	0.093

Beattie's continuous sampling plan is very easy to operate once the suitable values of parameters are determined. Given p_1, p_2, α, β and sample size n, our proposed method can get optimal k immediately and find appropriate h and h* much faster than the existing approach. Unfortunately, we did not get the analytic solutions for h and h*. More study is needed to determine A and B analytically, which in turn decide h and h*. Once this is done, our approach will appear to be ideal. For given any p_1, p_2, α, β and sample size n we can relate those values to parameters (k, h, h*) immediately.

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