Design of aspheric lens to collimate and uniform irradiance of a light source with Lambertian angular distribution

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ABSTRACT

A design of aspheric lens is proposed to collimate a light source of Lambertian angular distribution and generate homogenous irradiance. Two numerical solutions were calculated. One used even orders of general aspheres to describe the aspheric surfaces and the performance was evaluated by commercial optical software package Tracepro. Both of high numerical aperture (NA) of 0.79 and good RMS uniformity within 95% of designed area in the second surface of lens could be obtained. The reverse of this lens would be used in focusing and the simulated point spread function (PSF) is almost diffraction-limited. The other fitted both of aspheric surface with radial spline surface and the performance was evaluated by commercial optical software package OSLO. With the radial spline surface representation, the aspheric lens has a higher NA of 0.9; meanwhile, the uniformity was still good enough and PSF is also near diffraction-limited.

Keywords: Lens design; geometrical optics, optical design; aspherics

1. INTRODUCTION

Recently the applications of light emitting diode (LED) has attracted much attention, examples can be found in the fields of visualization, lighting, and display technology and so on. In these applications uniform illumination over an extended area at the desired location is generally required. Because the angular distribution of LED light source, which is Lambertian in many cases, is generally not fit to the need, secondary optics is required for redistributing energy. In the literature, reflective optical design to generate uniform illumination with a Lambertian light source has been realized [1-3]. The reflective optical design used in uniform illumination has more collection of flux and results in better optical efficiency. However, these reflectors are usually used off axis to avoid obstruction of the source. Therefore, an overload of alignment is generally inevitable. A half-sphere concentrator to collimate a radially emitted light has been presented [3]. Although this device has a high collecting throughput, the irradiance is not uniform yet.

In the literature, refractive aspheric lens used to generate uniform irradiance is presented for a Lambertian LED light source [4]. The optical efficiency with refractive aspheric lens is not as good as that with reflectors but the advantage of compact system and less alignment issues make a consideration of refractive optics attractive. When the fitting of solved aspheric surface includes the conic constant, a high numerical aperture of 0.866 could be obtained and the uniformity is still good enough. In our design, a refractive aspheric lens with high numerical aperture (NA) of 0.79 has been demonstrated to collimate the light source and to generate homogeneous irradiance. With the help of radial spline surface representation, a higher NA of 0.9 could be obtained without decrease of uniformity at the exit surface of lens and the PSF is still near diffraction-limited.

The goal of this paper is to show how to determine the surface profile of singlet aspheric lens, which generates homogenous irradiance and collimates the uniform bean with a light source of Lambertian angular distribution. This paper was organized as follows: in section 2, the basic formulism is revisited. The numerical results and simulation demonstration is presented in section 3. Section 4 is the conclusion.

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2. BASIC FORMULISM

The principle of energy balance states that the energy of any cross-section is conserved when a bundle of rays transmitted through a lossless optical system. In the literature, the energy balance principle incorporated with ray tracing and the constant condition of optical path length has been utilized in the field of laser beam shaping for the generation of

a uniform irradiance distribution with a well-collimated laser light source [1]. Moreover, the energy balance principle incorporated with ray tracing can be used to design a reflective surface, which converts a Lambertian light source to a uniform irradiance [6]. Tai and Schwarte [4] have used the energy balance principle to design a plano-aspheric lens to generate a homogeneous irradiation for a Lambertian LED light source at the desired location. The energy balance principle plays a role in solving the direction of ray respect to the emitted angle or pupil height in generating homogeneous irradiance. After the direction of ray is determined, the aspheric surface sag can be calculated by solving a differential equation, which is derived from exact ray tracing.

The schematic layout of optical system is shown in Fig. 1, where a bi-aspheric lens is used to collect the light radiated from a finitely extended, but small, source to collimate and to generate a homogeneous irradiance within a circular area of radius R_{max}. The formulae which is used to generate uniform irradiance at image plane is derived form energy balance equation [4], the relation of ray height at image plane between aperture angle can been written as

$$y = \frac{R_{\max}}{\sin\theta_{\max}}\sin\theta \tag{1}$$

where θ_{max} the maximum aperture angle and ray height in image plane y is equal to y_2.

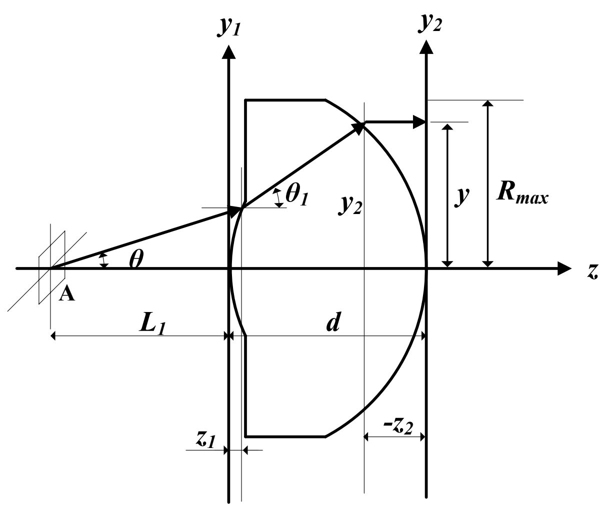

Fig. 1. Schematic diagram of the optical system for collimating a small extended source *A* through a bi-aspheric lens and for generating uniform irradiance.

The slope of first and second surface which we want to design for collimating the ray in the image plane can be written as [7]

$$\frac{dy_1}{dz_1} = \frac{-n\cos\theta_1 + \cos\theta}{n\sin\theta_1 - \sin\theta} \tag{2}$$

$$\frac{dy_2}{dz_2} = \frac{-n\cos\theta_1 + 1}{n\sin\theta_1} \tag{3}$$

where n is the refractive index of the aspheric lens.

From the geometrical relation shown in Fig. 1,

$$y_1 = L_1 + z_1 \tan\theta \tag{4}$$

$$y_2 = (d - z_1 + z_2)\tan\theta_1 + (L_1 + z_1)\tan\theta \tag{5}$$

where L_1 is the distance from source to vertex of lens, d is the thickness of lens.

Combing Eq. (1) into Eq. (4) and after some mathematical manipulations, the angle θ_1 can be written as

$$\tan\theta_1 = \frac{\frac{R_{\max}}{\sin\theta_{\max}}\sin\theta - (L_1 + z_1)\tan\theta}{(d - z_1 + z_2)} \tag{6}$$

Differentiating Eq. (4) and (1) with respect to θ, we find

$$\frac{dy_1}{d\theta} = \frac{dz_1}{d\theta}\tan\theta + \frac{(L_1 + z_1)}{\cos^2\theta} \tag{7}$$

$$\frac{dy_2}{d\theta} = \frac{R_{\max}}{\sin\theta_{\max}}\cos\theta \tag{8}$$

Substituting Eq. (2) and (3) into Eq. (7) and (8) respectively, and using $dy_1/d\theta = (dy_1/dz_1)(dz_1/d\theta)$ and $dy_2/d\theta = (dy_2/dz_2)(dz_2/d\theta)$, we obtain a differential equation that describe the shape of the aspheric surface

$$\frac{dz_1}{d\theta} = \frac{\frac{L_1 + z_1}{\cos^2\theta}}{\frac{-n\cos\theta_1 + \cos\theta}{n\sin\theta_1 - \sin\theta} - \tan\theta} \tag{9}$$

$$\frac{dz_2}{d\theta} = \frac{n\sin\theta_1}{-n\cos\theta_1 + 1} \times \frac{R_{\max}}{\sin\theta_{\max}}\cos\theta \tag{10}$$

where θ_1 is given by Eq. (6).

3. NUMERICAL RESULTS AND SIMULATION DEMONSTRATIONS

A numerical result was calculated with the following conditions: L_1 = 2mm, n=1.517for glass B270 at wavelength λ= 0.78μm, d=30mm, θ_{max}=52°, $R_{\max}$=12mm. In order to evaluate the irradiance distribution of solved biconvex aspheric lens with commercial optical software package, the surface data were fitted to an even aspheric surface as

$$z = \frac{cv\, r^2}{1 + \sqrt{1 - cv^2(cc+1)r^2}} + \sum_{i=1}^{15} asi \times r^{2i} \tag{11}$$

A Monte Carlo ray-tracing package, TracePro (version 3.3.1) [8], was used to evaluate the distribution of irradiation at desired plane. A square-source of 500μm ×500μm with 1W total flux and numbers of 10^6 rays were used in simulations.

The irradiance distribution at a target plane of 10mm to right of lens is shown in Fig. 2(a). In order to evaluate the performance of lens, the RMS uniformity is used as

$$U_{RMS} = 1 - \frac{1}{\frac{1}{N}\sum_{i=1}^{N} E(x_i, y_i)} \sqrt{\frac{\sum_{i=1}^{N}\left[E(x_i, y_i) - \frac{1}{N}\sum_{i=1}^{N} E(x_i, y_i)\right]^2}{N}} \tag{12}$$

in which (x_i, y_i) denotes the location of the sampling pixel on the detection plane and N is the total number of sampling points.

The RMS uniformity of irradiance distribution within 95% of desired target area at second surface of lens is 97% and degrades slightly to 96% at target 200mm to right of lens although the irradiance has decreased a considerable quantity.

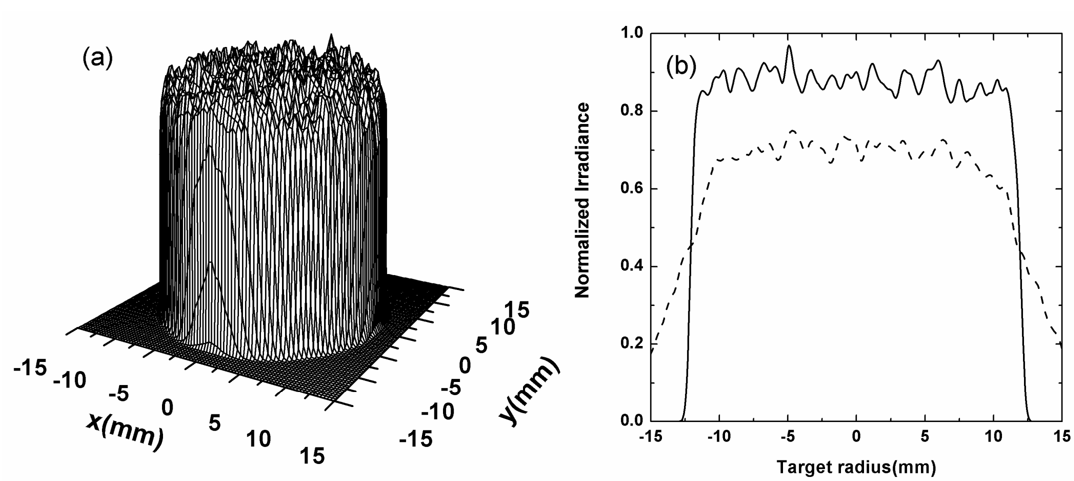

Fig. 2. Simulated irradiance distribution with a square source of size 500μm ×500μm. (a) Three-dimensional relief plot of irradiance distribution on target of second surface lens. (b) Irradiance of radial slice through the center of target. The solid curve is on target of second surface of lens and dashed curve is on target of 200mm to right of lens

The Point Spread Function (PSF) by reversing this aspheric is evaluated with commercial optical software package OSLO. The simulated result is shown in Fig. 3; the left is the 2 dimensional plane view of this aspheric lens and the right is PSF calculated by commercial optical software package OSLO [9]. The calculated PSF has a good performance and is almost approach to diffraction limited. Therefore, this lens could be one example of initial design for a pick-up head lens with high NA of 0.79.

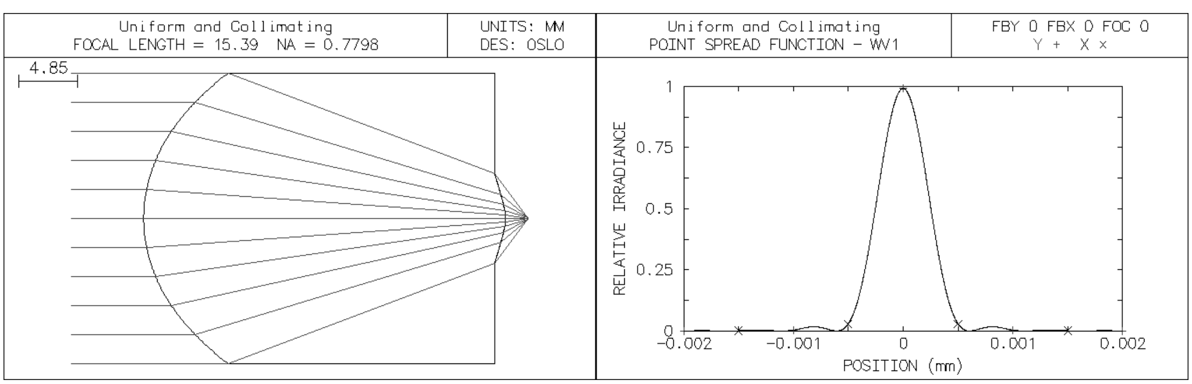

Fig. 3. The simulation with OSLO for reverse of aspheric lens describe above for using in focusing. (a) 2D plane view of this aspheric lens. (b) The PSF of this aspheric lens.

In order to achieve a higher NA, we used a higher refractive index of glass material and the spline surface to describe the aspheric surface [10]. Another numerical result was calculated with the following conditions: L_1 = 2mm, n=1.57 at wavelength λ= 0.78μm, d=40mm, θ_{max}=66°, R_{max}=20mm. The numerical aperture now is about 0.9 and all of the aspheric surfaces are expressed with radial spline surface. We used OSLO for evaluation of the aspheric lens performance because it has already built with radial spline surface representation. Both of aspheric surfaces have used 500 spline surface zones in OSLO and the half angle of light source has been limited within 66°. A square light source with 500μm ×500μm and numbers of 10^7 rays were used to simulate the irradiance distribution in target planes. Fig. 4 shows the normalized irradiance in different distances from the second surface of lens. Fig. 4(a) indicates a good uniformity of irradiance on the second surface of lens and Fig. 4(b) shows the irradiance simulated at the location on target of 200mm to right of lens. There is a decrease of normalized irradiance at central uniform region and spread of window at this location but the uniformity at center region is still not bad. In this simulation, this lens has a higher NA of 0.9 but the uniform volume is smaller than the above lens with NA of 0.79. A higher NA is more crucial to has larger uniform volume.

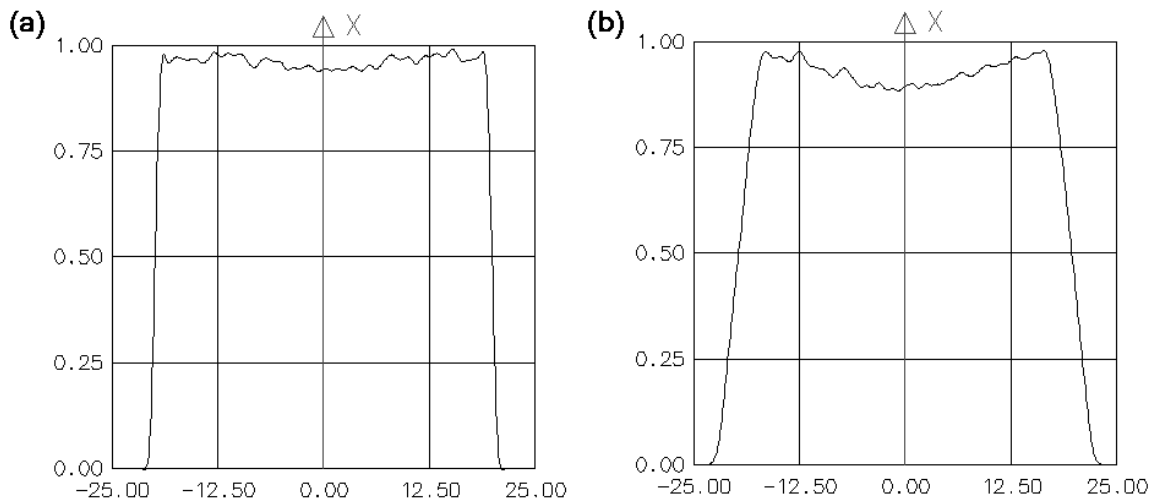

Fig. 4. The irradiance at target plane simulation with OSLO. (a) Normalized irradiance on target of second surface of lens. (b) Normalized irradiance on target of 200mm to right of lens.

The reverse of lens could also been used in focusing, the simulated result is shown in Fig. 5. The left is the 2 dimensional plane view of this lens and right is the calculated PSF. The PSF is still approach diffraction-limited and it indicates the ability of this lens in pick-up head lens.

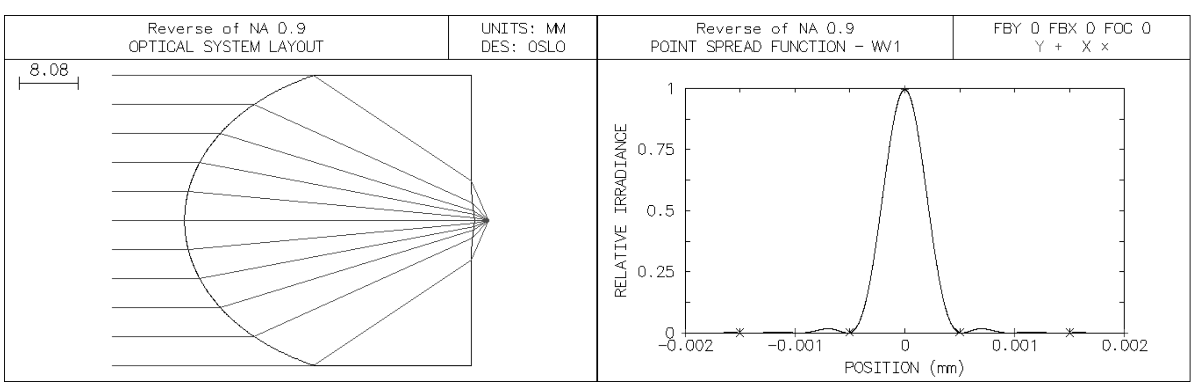

Fig. 5. The simulation with OSLO for reverse of NA 0.9 aspheric lens with spline surface representation for focusing. (a) 2D plane view of this aspheric lens. (b) The PSF of this aspheric lens.

For the reverse of aspheric lens, the candela plot with uniform and collimated rays would indicate the source distribution which we used above. The Lambertian light source is a cosine function with angular distribution. Therefore, we simulated this situation with grid ray trace in TracePro and the simulated result is shown in Fig. 6. The normalized intensity is plotted with angular degree in which the solid line is a Lambertian distribution (cosine) and the dotted line is the simulated candela plot. The inset of Fig. 6 is the same plot but only within 42°. This simple diagram seeks to capture the fact that the reverse of this aspheric lens incident with uniform and collimated rays would generate the original source distribution in image plane. A discrepancy between these two curves above 42° in Fig. 6 is due to the transmission loss, Fresnel loss and a larger fitting error in higher aperture radius.

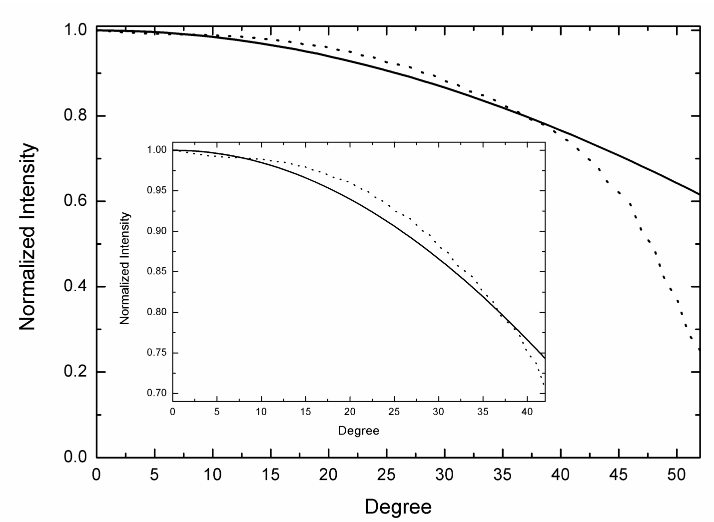

Fig. 6. The candela plot by grid ray trace with reverse of first aspheric lens in TracePro. The candela plot is represented with normalized intensity verse angle in image plane. The solid curve is cosine distribution for reference and the dotted line is simulated result. The inset is the same plot within 42° and it indicates a Lambertian-like distribution in image plane within 42°. A discrepancy between these two curves above 42° is due to the Fresnel loss and a larger fitting error

4. CONCLUSION

We have solved the design of collimating and of generating homogeneous irradiance with an single aspheric lens. A large numerical aperture of 0.79 is obtained in first example and thus the throughput collected by this lens is 62% moreover a glass of low refractive index has been used. The RMS uniformity of 97% within 95% of designed area at the location of second surface of lens reveals its good performance and 96% at target 200mm to right of lens is still good even the irradiance has a amount of decrease. The reverse of this aspheric lens with uniform and collimated beam incidence indicates a good performance of a near diffraction-limited PSF and the source distribution of original design. If a higher refractive index of glass has been used, a larger numerical aperture could be obtained and therefore more throughputs could be collected. Consequently, in second example, a lens with numerical aperture of 0.9 is obtained and the uniformity at the second surface is still preserved, although the uniform volume has been reduced. The radial spline surface fit helps the representation of this high numerical aperture lens and hence the simulation of a near diffraction-limited PSF in second example. Furthermore, a source Lambertian angular distribution is used in the present design, for nonLambertian angular distribution light source it can also been extended easily. Moreover, generating the specified spatial irradiance such as Gaussian distribution by an aspheric lens would also been designed with this technique.

5. ACKNOWLEDGEMENTS

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