Quadrilevel Periodic Sequences for Fast Start-Up Equalization and Channel Estimation

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Abstract—**A class of four-level periodic sequences derived from the binary m-sequences are presented. These sequences can be used as the training sequences in adaptive channel estimation and equalization with fast start-up for multilevel PAM or QAM transmission. The sequence generator can be implemented by simple hardware.**

I. Introduction

ADAPTIVE signal processing has been widely applied to channel equalization/estimation of band-limited channels. A training process with a predetermined reference pattern known also by the receiver during the start-up period is often required. Mueller and Spaulding [2] have proposed a fast cyclic training algorithm by using periodic sequences for fast start-up adaptive equalization. The problem can be considered from the viewpoint of optimization of the training sequences [1]–[6], where several candidates have been designed to achieve rapid convergence. In this letter, our attention is concentrated on the construction of the four-level $\{\pm 1, \pm 3\}$ periodic training sequences with desirable correlation property.

II. Four-Level Sequences for Fast Start-Up

To speed up the convergence rate, the eigenvalue spread of the periodic training pattern $\alpha = [a_0\, a_1\, a_2 \cdots a_{N-1}]$ of N symbols must be small. That is, the pattern with a flat spectrum (or equivalently, an impulse-like autocorrelation function) is a good choice. Based on this criterion, self-orthogonal sequences, constant-amplitude and zero-autocorrelated (CAZAC) sequences and PN sequences are usually considered as candidates. If a sequence which is orthogonal to any cyclically shifted version $\alpha^{(j)}$ of itself, i.e., satisfying $\alpha \cdot \alpha^{(j)^*} = \lambda\delta_j$ (λ: sequence energy), then it is called a self-orthogonal sequence [4]. A CAZAC [5], [7] sequence provides constant spectrum amplitude and its correlation satisfies $\alpha \cdot \alpha^{(j)} = \delta_j$. The CAZAC sequence can be used to reduce the variance of the estimate such that the convergence is accelerated. However, the CAZAC sequences with $N > 4$ cannot be constructed by the symbols in $\{\pm 1, \pm 3, \cdots, \pm 2M - 1\}$ [7]. Another solution is to use a short pseudo-random binary sequence of length $N = 2^{m-1}$ for cyclic training [1], [2], [6]. They are usually generated by an m-bit ($m = \log_2(N+1)$) maximum-length sequence (or referred to as m-sequence) generator. Although they are not self-orthogonal and zero-autocorrelation, they are often used in practice as the approximated ones. For high speed transmission, the multilevel signaling method has been widely adopted to achieve bandwidth efficiency. Thus, multilevel cyclic sequences with better correlation properties become useful in channel equalization and/or channel estimation in these cases.

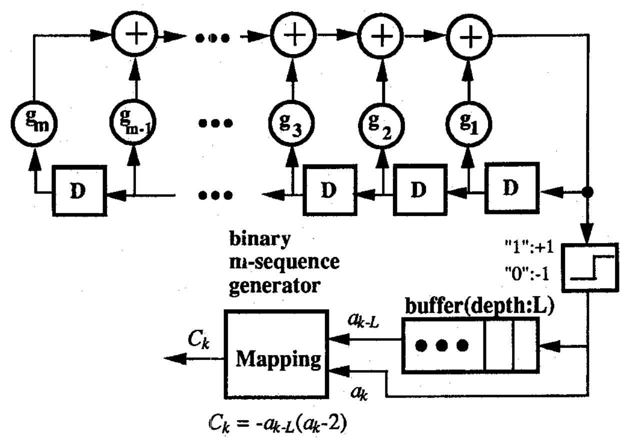

Fig. 1. Proposed sequence generator.

For a four-level m-sequence, it can be generated directly from Galois field GF(4), but they are not widely accepted due to the complexity problem [8]. A four-level sequence generator based on a binary m-sequence generator with generating polynomial $g(x)$ is presented and shown in Fig. 1. The incoming binary m-sequence is stored in a buffer with depth of L. The four-level sequence $\{C_k\}$ is related to the binary symbols a_{k-L} and a_k by $C_k = -a_{k-L}(a_k - 2)$, where $a_{k-L},\ a_k \in \{+1, -1\}$. The symbol C_k is represented equivalently by a pair of binary digits (a_{k-L}, a_k). This operation of mapping and buffering can be further simplified by Boolean algebra and realized by combinational logic gates. By using the computer-aided search method, several generators with sequences having an impulse-like autocorrelation function are tabulated in Table I. These sequence generators are uniquely characterized by $g(x)$, denoted by an octal representation, and the memory depth L. There are two choices of memory depth L for each case. Due to the hardware complexity consideration, the time span (N) of the equalizer or channel estimator usually considered is less than 128. Thus, those four-level sequence generators with $N < 128$ are listed. Two other longer

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TABLE I
FOUR-LEVEL SEQUENCE GENERATORS

Degree m	Length N	Generator Polynomial $g(x)$(in octal)	Buffer Length L	
3	7	13*	4	5
4	15	23*	11	12
5	31	45*	13	14
		75	11	12
		67	12	13
6	63	103*	57	58
		147	38	39
		155	55	56
7	127	211*	96	97
		217	40	41
		235	9	10
		367	54	55
		277	108	109
		325	20	21
		203*	120	121
		313	88	89
		345	113	114
8	255	435	230	231
9	511	1021*	381	382

*: only two feedback connections

sequences ($N > 128$) are also supplemented. In general, the autocorrelation function $\{\gamma_j\}$ for a periodic sequence can be defined by $\gamma_j \equiv \frac{1}{N}\alpha \cdot \alpha^{(j)},\ j = 0, 1, 2, \cdots, N-1$. Based on this definition, the autocorrelation of the four-level sequences obtained from Table I satisfies

$$\gamma_j = \begin{cases} 5 - \frac{4}{N}, & j = 0 \pmod N \\ 2 - \frac{7}{N}, & j = 1 \pmod N \\ -\frac{9}{N}, & \text{otherwise} \end{cases} . \tag{1}$$

As the sequence length N increases, its correlation approaches that of a four-level CAZAC sequence but with a correlation time confined among two symbol periods. Although there is no CAZAC sequence on $\{+3, +1, -1, -3\}$ with length $N > 4$, we actually find the approximated ones. Since the sequence generator can be realized simply by several storage elements and logic gates, it is very efficient in the hardware implementation.

III. CONCLUSION

A class of four-level periodic sequences with special correlation properties are presented. They can be used for fast start-up channel equalization or estimation. Several sequence generators with useful length are obtained by means of a computer search.

REFERENCES

[1] R. W. Chang and E. Y. Ho, "On fast start-up data communication systems using pseudo-random training sequences," *Bell Syst. Tech. J.*, vol. 51, pp. 2013–2027, Nov. 1972.

[2] K. H. Mueller and D. A. Spaulding, "Cyclic equalization—A new rapidly converging equalization technique for synchronous data communication," *Bell Syst. Tech. J.*, vol. 54, pp. 369–406, Feb. 1975.

[3] S. U. H. Qureshi, "Fast start-up equalization with periodic training sequences," *IEEE Trans. Inform. Theory*, vol. IT-23, pp. 553–563, Sept. 1977.

[4] A. P. Clark, Z. C. Zhu, and J. K. Joshi, "Fast start-up channel estimation," *IEE Proc.*, Pt. F, pp. 375–381, July 1984.

[5] A. Milewski, "Periodic sequences with optimal properties for channel estimation and fast start-up equalization," *IBM J. Res. Develop.*, vol. 27, pp. 425–528, Sept. 1983.

[6] G. D. Foney, "Training adaptive linear filters," U.S. Patent 372 391, Mar. 1973.

[7] J. A. C. Bingham, *The Theory and Practice of Modem Design*. New York: Wiley, 1989, ch. 8.

[8] V. N. Yarmolik and S. N. Demidenko, *Generation and Application of Pseudorandom Sequences for Random Testing*. New York: Wiley, 1988, ch. 1.