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Combined Product and Tool Disturbance Estimator for the Mix-Product Process and Its Application to the Removal Rate Estimation in CMP Process

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In this paper, we proposed a control strategy: Combined Product and Tool Disturbance Estimator (CPTDE) which combines threaded double EWMA with the drift compensation scheme, to adaptively estimate the disturbance for a mix-product situation in semiconductor processes. This approach considers the disturbances are related to the combination of the specific product and the tool, and further separates the error into an intercept term and a drift term, where the former is related to the variation of products, whereas the latter is related to the interaction between the tools and the products. The proposed method continuously updates the intercept and drift terms to obtain the recipe for the next run. The simulation case studies, i.e., the fixed schedule process, the random schedule process, and the periodical schedule process, are conducted and the results show that CPTDE control scheme has best control performance when compared with three recently published control schemes. The method is also applied to the estimation of removal rate in mix-product CMP process. The results show that the proposed method has improvements over product-based EWMA control, CF-EWMA control and threaded PCC control by 9.52%, 2523.43% and 11.71% on average, respectively, for estimating removal rate of historical data in mix-product CMP process.

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1. Introduction

Advanced process control (APC) has been recognized as a proper tool for maximizing profitability of semiconductor manufacturing facilities by improving efficiency and product quality. Currently, run-to-run (RtR) process control methods with good quality and reliability performance for APC application are most applicable. Nowadays, many different products enter one tool to process in a high-mix fab. In this high-mix situation, it is hard to find out an optimal recipe for some specified products which have not processed in the tool for a long time, and this frequently leads the process outputs locating outside of the specification limit and decreasing the yield.

Many theorems and control algorithms which related to RtR control have been proposed by several authors. Ingolfsson and Sachs[1] provided an analysis about the stability and sensitivity of EWMA control. They found the EWMA weight and model mismatch would affect the stability region of the system. Sachs et al.[2] presented a framework which combines SPC and feedback control for controlling processes affected by disturbances such as shift and drift. The process was assumed to have no dynamics, and a linear controller based on the EWMA was used. The performance of this approach is highly dependent on the choice of the EWMA weights. Several authors[3-7] pointed out this problem in their studies by proposing a self-tuning EWMA controller. Del Castillo[8] investigated the stability of a predictor collector control (PCC), and analyzed the long run and transient performances. He also established a constrained formula for finding out the optimal weights. In the MIMO case, Tseng et al.[9] proposed a multivariate EWMA controller for a linear MIMO model. Castillo and Rajagopal[10] proposed a MIMO PCC feedback controller for drifting processes. Recently, Lee et al.[11] presented a unified framework called the output disturbance observer (ODOB) structure for the EWMA controller, the double EWMA controller and the PCC controller. The work enhances insight into the well-known established algorithms, and contributes to better understanding of how these algorithms operate and why they can be used successfully in practical application. Other methods in RtR controllers have also been proposed and applied in semiconductor manufacturing.[12-15,26]

Only a few studies have addressed the RtR control of a mixed product process plant. Edger et al.[16] reviewed the problems of mixed product run-to-run control in a high-mix fab. Firth et al.[17] suggested the method Just-in-time Adaptive Disturbance Estimation (JADE) to deal with the variation of products in mix semiconductor processes. They broke down the disturbances into contributions of current product and current tool. In high-mix situation, they could find out the fit recipe by the specific product error in process. Zheng[18] brought up a mix-product process by EWMA control theory and separated into tool-based EWMA (TB-EWMA) control and product-based EWMA (PB-EWMA) control by its process property. In TB-EWMA control, the tool is unconcerned about the variation of products, and only one controller is applied to all products. Contrary to tool-based, PB-EWMA control is depended on the variation of products. A specific controller is used for a particular product individually. According to Zheng's work, PB-EWMA control is a better control scheme to deal with the problems in mix-product situation. Ma et al.[19] studied the effect of production frequency on optimal weights tuning of threaded EWMA (t-EWMA) controller in a high-mixed production. The simulation shows that the optimal weights of t-EWMA controller decrease as the production frequency decrease for stationary disturbance, and the weights of t-EWMA controller increase as the production frequency decrease for non-stationary disturbance. Ai et al.[20] proposed a drift compensatory approach which based on threaded PCC (t-PCC) controller to deal with large deviations at the beginning runs of specific cycle process in a mixed production. For the step and ramp faults in a mixed product processes, the authors also offer a fault-tolerant approach that it could compensate the deviation of process output when the faults had been detected. In a periodic procedure mixed production, Ai et al.[21] proposed a cycle forecasting EWMA (CF-EWMA) approach to deal with the large deviations in the first few runs of each cycle under drift disturbance. The CF-EWMA approach utilized the slop of estimations of disturbance and the length of break products to compensate the deviations for the first run of campaign product in next cycle. The simulation results show that the CF-EWMA approach had better control performance than PB-EWMA[18] under cycle mixed product processes with drift disturbance. Furthermore, several authors proposed the non-threaded state estimation methods[22,23] which share information among different contexts. One of the chief difficulties in those methods was the unobservability in the context matrix which needed to be inverted at every step. Each method utilized a different approach to handing this problem and making the system observable.

This research proposed an RtR mix-product control scheme, Combined Product and Tool Disturbance Estimator (CPTDE) which combined threaded double EWMA with drift compensation scheme. From historical data observation, we separate the disturbances of products in one tool into two parts. One is the intercept term which concerns with the variation of products, and the other is the drift term which is related to the interaction of the tool and the product. That means every product owns its particular drift size in one tool. The CPTDE is tested under three disturbance models, deterministic trend (DT), random walk with drift (RWD), and IMA (1,1) with drift and shows the ability to estimate the drift disturbance, and the final value of this control scheme is on target. Moreover, the method is also applied to the estimation of removal rate in mix-product CMP process.

The rest of the paper is organized as follow. In Section 2, this paper proposes a new mix-product control scheme, CPTDE control scheme. The output performances and stabilities of the CPTDE control schemes are addressed in Section 3. In Section 4, the disturbance model simulations are provided to illustrate the control abilities by PB-EWMA,[18] t-PCC,[20] CF-EWMA[21] and CPTDE control methods in fixed, random and periodic schedule processes, respectively. In Section 5, this paper collects some removal rate historical data in CMP process and compares with the estimated removal rate by these four control methods, in mix-product CMP process. Section 6 concludes this paper.

2. Run-to-run controller for semiconductor manufacturing processes

Most published research and application of run-to-run control have been concerned with the control of a single product type going through one process. Here this paper deliberates on multi-product entering one tool, and the process can be described as a simple linear model of the form:

$$Y_{i,k} = \alpha_i + \beta_i u_{i,k} + \eta_{i,k} \tag{1}$$

where i denotes i'th product, k denotes the number of the runs from the beginning of the process. $Y_{i,k}$ is the process output of product i, α_i is the initial intercept on product i, β_i is the process gain on product i, which may be obtained by Design of Experiment (DOE), $\eta_{i,k}$ is the process output disturbance of product i, and $u_{i,k}$ is the process input recipe of product i. The corresponding control schemes are shown in the following sections.

2.1 Description of the proposed Combined Product and Tool Disturbance Estimator (CPTDE)

Most disturbance distributions of each product in mix-product process involve drift trend. To take an example, Fig. 1 shows the historical data of disturbance in mix-product lithography process from semiconductor company. From this figure one observes that the different products (product A and product B) own their particular intercept terms (caused by the product variation) and individual drift sizes which are caused by the interaction of particular product with the tool. Based on this observation, one designs a control method which is called CPTDE control scheme to estimate the intercept and drift disturbance for each product. The following cases describe the updating procedure of CPTDE control:

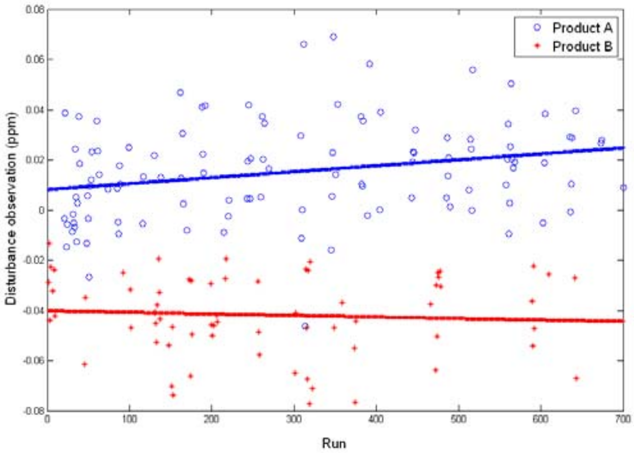


Fig. 1 The historical data of disturbance in mix-product lithography process

Case 1: For the first run of product i

$$u_{i,k}=\frac{T_i-\hat{A}_{i,0}-\hat{P}_{i,0}}{b_i} \tag{2}$$

$$\hat{A}_{i,k}=\left(\hat{A}_{i,0}+\hat{P}_{i,0}\right)+\lambda_{i,1}\left(Y_{i,k}-b_i u_{i,k}-\hat{A}_{i,0}-\hat{P}_{i,0}\right) \tag{3}$$

$$\hat{P}_{i,k}=\hat{P}_{i,0}+\lambda_{i,2}\left(Y_{i,k}-b_i u_{i,k}-\hat{A}_{i,0}-\hat{P}_{i,0}\right) \tag{4}$$

Case 2: For product i keeping on processing

$$u_{i,k}=\frac{T_i-\hat{A}_{i,k-1}-\hat{P}_{i,k-1}}{b_i} \tag{5}$$

$$\hat{A}_{i,k}=\left(\hat{A}_{i,k-1}+\hat{P}_{i,k-1}\right)+\lambda_{i,1}\left(Y_{i,k}-b_i u_{i,k}-\hat{A}_{i,k-1}-\hat{P}_{i,k-1}\right) \tag{6}$$

$$\hat{P}_{i,k}=\hat{P}_{i,k-1}+\lambda_{i,2}\left(Y_{i,k}-b_i u_{i,k}-\hat{A}_{i,k-1}-\hat{P}_{i,k-1}\right) \tag{7}$$

Case 3: For the break product (product j)

$$\hat{A}_{j,k}=\hat{A}_{j,k-1}+\hat{P}_{j,k-1} \tag{8}$$

$$\hat{P}_{j,k}=\hat{P}_{j,k-1} \tag{9}$$

Case 4: Product exchange (product i changes to product j)

$$u_{j,k}=\frac{T_j-\hat{A}_{j,k-1}-\hat{P}_{j,k-1}}{b_j}=\frac{T_j-\hat{A}_{j,k-n}-n\hat{P}_{j,k-n}}{b_j} \tag{10}$$

$$\begin{aligned}\hat{A}_{j,k}&=\left(\hat{A}_{j,k-1}+\hat{P}_{j,k-1}\right)+\lambda_{j,1}\left(Y_{j,k}-b_j u_{j,k}-\hat{A}_{j,k-1}-\hat{P}_{j,k-1}\right)\\&=\left(\hat{A}_{j,k-n}+n\hat{P}_{j,k-n}\right)+\lambda_{j,1}\left(Y_{j,k}-b_j u_{j,k}-\hat{A}_{j,k-n}-n\hat{P}_{j,k-n}\right)\end{aligned} \tag{11}$$

$$\begin{aligned}\hat{P}_{j,k}&=\hat{P}_{j,k-1}+\lambda_{j,2}\left(Y_{j,k}-b_j u_{j,k}-\hat{A}_{j,k-1}-\hat{P}_{j,k-1}\right)\\&=\hat{P}_{j,k-n}+\lambda_{j,2}\left(Y_{j,k}-b_j u_{j,k}-\hat{A}_{j,k-n}-n\hat{P}_{j,k-n}\right)\end{aligned} \tag{12}$$

Case 1 presents the CPTDE updating procedure at the first run of product i (i = 1, 2, 3…), where $\hat{A}_{i,0}$ and $\hat{P}_{i,0}$ denote the initial estimations of intercept term and drift term of product i, respectively, T_i and b_i denote the target and model gain of product i, respectively; $\lambda_{i,1}$ and $\lambda_{i,2}$ are the weights of product i for CPTDE controller. In Eq. (3), $\hat{A}_{i,k}$ denotes the intercept term estimation of product i on run k, and in Eq. (4), $\hat{P}_{i,k}$ denotes the drift term estimation of product i on run k, which is caused by the interaction of tool and product i. When product i keeps on processing in the tool (Case 2), the intercept estimation is updated by Eq. (6), and the drift estimation is updated by Eq. (7), based on the measurement $Y_{i,k}$ on run k and the estimations ($\hat{A}_{i,k-1}$ and $\hat{P}_{i,k-1}$) on run $k-1$. However, the estimations of the other break products (product j, j = 1, 2, 3…, $j \neq i$) cannot be updated by their measurement $Y_{j,k}$, because they did not enter the tool. For this reason, those estimations of break product j are only updated by their individual information on the last run (Case 3). The intercept term of each product is updated individually by the sum of intercept and drift estimations on the last run by Eq. (8). Similarly, the drift term of each product is equal to its drift estimation on the last run by Eq. (9). Combining Eq. (8)-(9), we have

$$\begin{aligned}\hat{A}_{j,k}&=\hat{A}_{j,k-1}+\hat{P}_{j,k-1}\\&=\hat{A}_{j,k-2}+\hat{P}_{j,k-2}+\hat{P}_{j,k-1}\\&\;\;\vdots\\&=\hat{A}_{j,k-n}+\hat{P}_{j,k-n}+\cdots+\hat{P}_{j,k-3}+\hat{P}_{j,k-2}+\hat{P}_{j,k-1}\\&=\hat{A}_{j,k-n}+n\hat{P}_{j,k-n}\end{aligned} \tag{13}$$

where $\hat{A}_{j,k-n}$ and $\hat{P}_{j,k-n}$ denote the estimated intercept term and drift term of product j on run $k-n$, respectively, by assuming product i has been processed for $n-1$ runs; furthermore, $\hat{A}_{j,k-n}$ and $\hat{P}_{j,k-n}$ are used to compensate the intercept of product j on run k. Eq. (13) also shows that if product j did not enter the tool for n runs, the intercept term is updated n times by Eq. (8), but the drift term is still equal to the same value which is estimated from the last run before n runs by Eq. (9). If product i changed into product j on run k (Case 4) by assuming product i has been processed for $n-1$ runs, the recipe of product j is given by Eq. (10), and the estimations of $\hat{A}_{j,k}$ and $\hat{P}_{j,k}$ are updated by the pervious estimations of product j on run $k-n$ and the current measurement $Y_{j,k}$ in Eq. (11)-(12).

3. The application of run-to-run controllers in fixed schedule

In order to obtain analytic solutions for investigating characteristics of CPTDE control under some disturbance models, we set a special sequence of process schedule as shown in Fig. 2, where each product periodically enters the tool and n denotes the run number of one cycle in the schedule.

In Zheng's work,[18] they considered the disturbance types of IMA (1,1) with drift and white noise, and calculated the output asymptotic variance and asymptotic mean square error (AMSE). Here, we analyze CPTDE control scheme under three disturbance

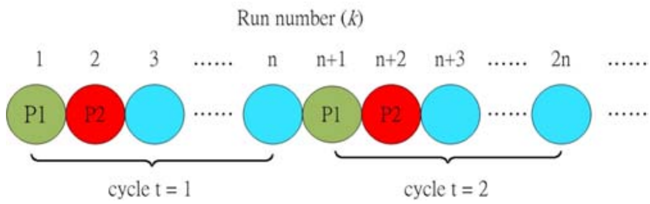


Fig. 2 A special case of mix-product schedule

models, i.e., DT, RWD, and IMA (1,1) with drift, where the white noise term is satisfied with $\varepsilon_k \sim N(0,\sigma^2)$ and calculate the asymptotic mean, asymptotic variance, and AMSE, respectively, under the above-mentioned mix-product schedule. These disturbance models can be defined as follows:

DT:

$$\eta_k = \eta_{k-1} + \varepsilon_k - \varepsilon_{k-1} + \delta = \varepsilon_k + k\delta \tag{14}$$

RWD:

$$\eta_k = \eta_{k-1} + \varepsilon_k + \delta \tag{15}$$

IMA (1,1) with drift:

$$\eta_k = \eta_{k-1} + \varepsilon_k - \theta\varepsilon_{k-1} + \delta \tag{16}$$

where δ denotes the drift size per run, ε_k denotes the white noise, and θ denotes the moving average term.

3.1 The stability of CPTDE control scheme

Lemma 1: If the targets of product i are zero, the output response of Y_i under fixed schedule is given by

$$Y_i(z) = \frac{z^{2n} - 2z^n + 1}{z^{2n} + \left[\xi_i\left(\lambda_{i,1} + n\lambda_{i,2}\right) - 2\right]z^n + \left(1 - \xi_i\lambda_{i,1}\right)}\eta_i(z), \quad i = 1\cdots n \tag{17}$$

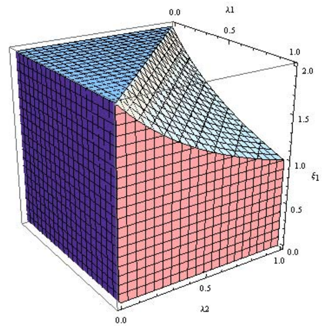
where ξ_i denotes the model mismatch $(\xi_i = \beta_i / \mathrm{b}_i)$ of product *i*.

Proof: See Appendix for algebraic details, and Eq. (17) is adopted from Eq. (A-6) in Appendix.

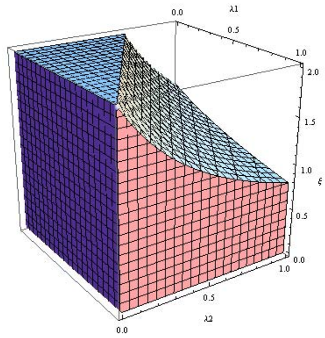
The stability of CPTDE control scheme in the fixed general schedule (as shown in Fig. 2) is investigated using Jury's stability test. The following equation dictates the characteristic equation of CPTDE control in the fixed general schedule,

$$\Delta_i = z^{2n} + \left[\xi_i\left(\lambda_{i,1} + n\lambda_{i,2}\right) - 2\right]z^n + \left(1 - \xi_i\lambda_{i,1}\right), \ i = 1\cdots n \tag{18}$$

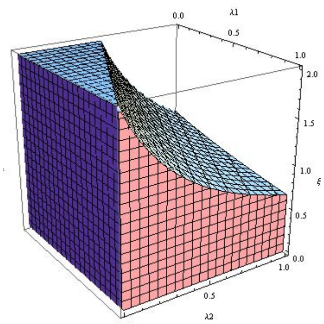
Fig. 3 shows the results for product 1 as an example where the model mismatch is changed from zero to two and the value of n from two to six. From Fig. 3, it is observed that the stable region of CPTDE (enclosure of the shaded surface) is getting smaller when n and λ_2 increase. For the RtR control in semiconductor process, the weight λ_2 related to the estimate of drift term is usually small, which means the particular drift size is no need to be updated rapidly. Fig. 4 shows that the stable range of CPTDE control scheme when n is equal to 6 and the weight λ_2 is set 0.01. The result shows the stable region is close to that of PB-EWMA.[18]



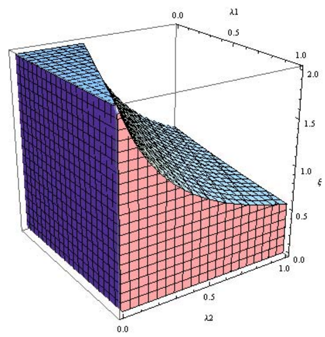
(a) n = 2



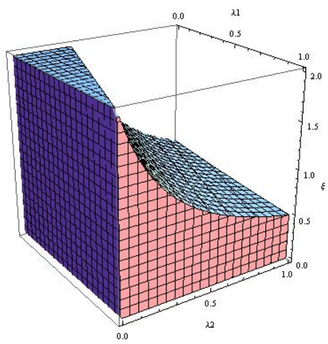
(b) n = 3



(c) n = 4



(d) n = 5



(e) n = 6

Fig. 3 The stable range of λ_1, λ_2, and ξ by CPTDE control scheme

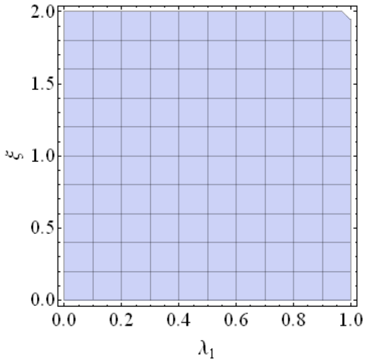


Fig. 4 The stable range of λ_1 and ξ by CPTDE control scheme ($\lambda_2 = 0.01$ and $n = 6$)

3.2 The output performances of CPTDE control scheme

By substituting the DT, RWD and IMA (1,1) with drift for $\eta_i(z)$ in Eq. (17), one can obtain the final values of Y_i :

Lemma 2: If the targets of product i are zero and all poles of $Y_i(z)$ are inside the unit circle, the final values of process output of Y_i under fixed schedule with three disturbance models are given by

$$E(Y_i) = \lim_{z \to 1} \frac{z-1}{z} Y_i(z) = 0, \quad i = 1 \cdots n \tag{19}$$

Proof: See Appendix for algebraic details, and Eq. (19) is adopted from Eq. (A-10) in Appendix.

Åström and Wittenmark[24] presented a theory to calculate the output asymptotic variance. Contrary to the state-space theory or time series analysis used by most of the scholars,[1,7,18,25] this computing skill is more simple and rapid. Using Eq. (A-7)-(A-9), one can calculate the asymptotic variances of Y_i :

Lemma 3: For product i, the asymptotic variances of the process outputs under fixed schedule with the three disturbance models are given by

$$AVAR,\ AMSE_{DT} = \frac{2\left(2\lambda_{i,1} + n\lambda_{i,2}\right)}{\lambda_{i,1}\left[4 - \xi_i\left(2\lambda_{i,1} + n\lambda_{i,2}\right)\right]}\sigma_i^2,\ i = 1 \cdots n, \tag{20}$$

$$AVAR,\ AMSE_{RWD} = \frac{2n}{\xi_i \lambda_{i,1}\left[4 - \xi_i\left(2\lambda_{i,1} + n\lambda_{i,2}\right)\right]}\sigma_i^2,\ i = 1 \cdots n, \tag{21}$$

$$AVAR,\ AMSE_{IMA(1,1)\text{ with drift}} = \frac{2n(1-\theta)^2 + 2\theta\xi_i\left(2\lambda_{i,1} + n\lambda_{i,2}\right)}{\xi_i \lambda_{i,1}\left[4 - \xi_i\left(2\lambda_{i,1} + n\lambda_{i,2}\right)\right]}\sigma_i^2, \quad i = 1 \cdots n. \tag{22}$$

Proof: See Appendix for details.

Because the final value is on the target, the asymptotic variances and the AMSE are the same. According to Eq. (19), the proposed method, CPTDE control can maintain the asymptotic mean of process output on target.

4. Simulations under disturbance models

Some simulations are conducted to compare the performance of these four control methods: PB-EWMA[18] control, t-PCC[20] control, CF-EWMA[21] control and CPTDE control, under three disturbance models mentioned in the previous section with performance index, mean square error (MSE), defined as:

$$MSE = \frac{\sum_{i=1}^{n} (y_i - T)^2}{n} \tag{23}$$

The optimum weights λ_1 and λ_2 are obtained by sweep the value from 0 to 0.99 with 0.01 increment.

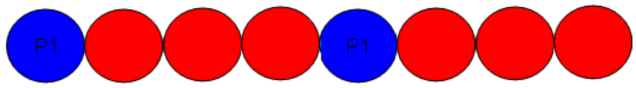


Fig. 5 The fixed product schedule

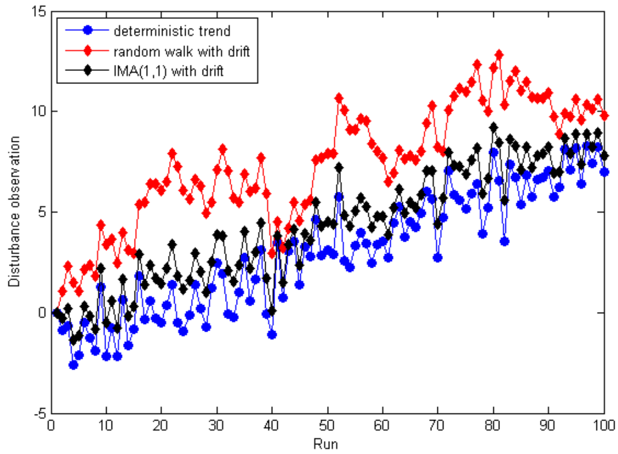


Fig. 6 The realization of the three disturbance models

Table 1 The optimum weight for each controller

Weight	DT		RWD		IMA(1,1) with drift	
	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
PB-EWMA	0.66		0.99		0.75	
CF-EWMA	0.16		0.26		0.19	
t-PCC	0.1	0.09	0.99	0.01	0.55	0.03
CPTDE	0.12	0.003	0.99	0.001	0.49	0.001

Table 2 The comparison of performances under each disturbance

		DT	RWD	IMA(1,1) with drift
MSE	PB-EWMA	1.8599	4.2032	1.8117
	CF-EWMA	1.8607	4.1945	1.8117
	t-PCC	1.1410	4.0429	1.4532
	CPTDE	1.1322	4.0399	1.4345
Variance	PB-EWMA	1.4922	4.0393	1.5266
	CF-EWMA	1.4697	4.0459	1.5341
	t-PCC	1.1407	4.0423	1.4531
	CPTDE	1.1324	4.0373	1.4324

4.1 Simulations under fixed schedule process

The fixed product schedule is shown in Fig. 5, where three other products (red ones) are inserted between the concerned products (blue ones). In this case, the process gain (β) and the model gain (b) are equal to 1 and 10000 steps are simulated under three disturbance models, i.e., DT, RWD, and IMA (1,1) with drift, where $\theta = 0.7$ and $\delta = 0.1$. Fig. 6 shows the realization of each disturbance. The optimum weights for each controller by the sweep method are shown in Table 1. The MSE and variance of the process outputs for the four control schemes are shown in Table 2 and it clearly shows that CPTDE control has superior control capability in comparison with other three controls in fixed general schedule under these three typical disturbance models. Furthermore, Table 2 also shows that both PB-EWMA and CF-EWMA based on single EWMA algorithm cannot completely compensate the disturbance with drift as double EWMA dose.

4.2 Simulations under random schedule process

If the production schedule is random, the system becomes too complicated to investigate analytically. Therefore, according to Zheng's work[18] a simple random schedule simulation with two tools and five products is provided to illustrate their control capabilities. The parameters of the linear plant for each product are $\beta = [1, 2, 3, 1.5, 2.5]$, and the parameters of nominal models are $b = [0.8, 1.4, 2.4, 1.5, 3]$. Hence, the model mismatches in process gains are $\xi = [0.8, 0.7, 0.8, 1, 1.2]$. The disturbances of the two tools are both IMA (1,1) with drift. The drift and moving averaging parameters are $\delta = [0.1, 0.2]$ and $\theta = [0.3, 0.7]$, while the white noise are sequences with zero mean and unit variance, $\varepsilon \sim N(0,1)$. Some common mix-product control schemes, PB-EWMA control, CF-EWMA control, t-PCC control and CPTDE control, are applied to this simulation for comparison. The system outputs are shown in Figs. 7-10. Figs. 7-10 show the outputs of the methods, PB-EWMA control, CF-EWMA control, t-PCC control and CPTDE control respectively, and Table 3 indicates their MSE and variance of system outputs under the random schedule process. Inspection of the results presented in Table 3 shows that CPTDE control scheme provides a more-accurate estimation. Note that the CF-EWMA control scheme gets worse control performance because the disturbance estimator of CF-EWMA control scheme is in the transient state always during random schedule production. Among these methods, this random schedule simulation indicates that CPTDE control has the best control capability.

4.3 Simulations under periodic schedule process

In this section, the PB-EWMA, t-PCC, CF-EWMA and CPTDE control schemes are compared by using periodic schedule production which was provided by Ai et al.[21] simulation case. There are one tool, two products and four cycles processes under IMA (1,1) with drift disturbance. The drift and moving average parameters are $\delta = 0.1$ and $\theta = 0.7$, while the white noise are sequences with zero mean and 0.01 variance, $\varepsilon \sim N(0, 0.1^2)$. The numbers of product 1 and product 2 for each cycle are [100, 150], [150, 100], [50, 100] and [100, 50]. The model mismatches for product 1 and product 2 are [ξ_1, ξ_2] = [2, 0.5]; the true parameters of the processes and the initial values of these four control schemes are [$\alpha_{1,0}$, $\alpha_{2,0}$] = [2, 1], [$\hat{A}_{1,0}$, $\hat{A}_{2,0}$] = [2, 1] and [$\hat{P}_{1,0}$, $\hat{P}_{2,0}$] = [0, 0]. The output targets for products are [T_1, T_2] = [0, 5], and the optimum weights for each controller by the sweep method are shown in Table 4. The system outputs are shown in Figs. 11-14.

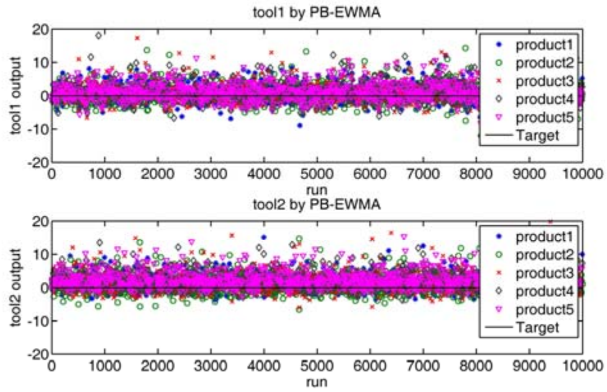


Fig. 7 The system outputs using PB-EWMA control approach

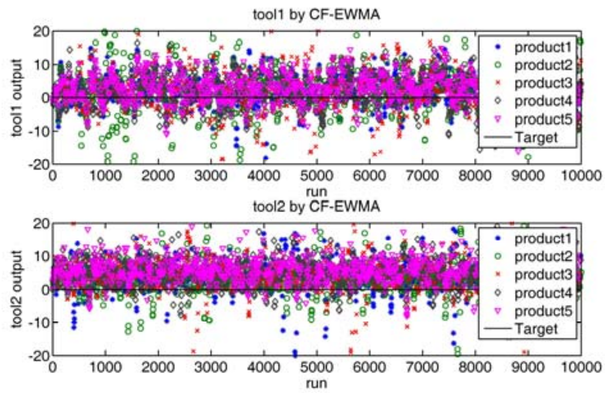


Fig. 8 The system outputs using CF-EWMA control approach

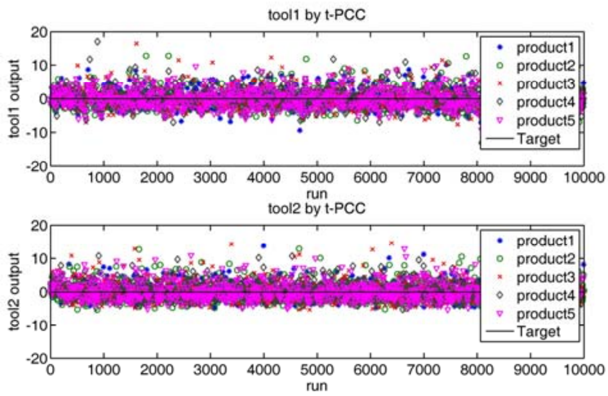


Fig. 9 The system outputs using t-PCC control approach

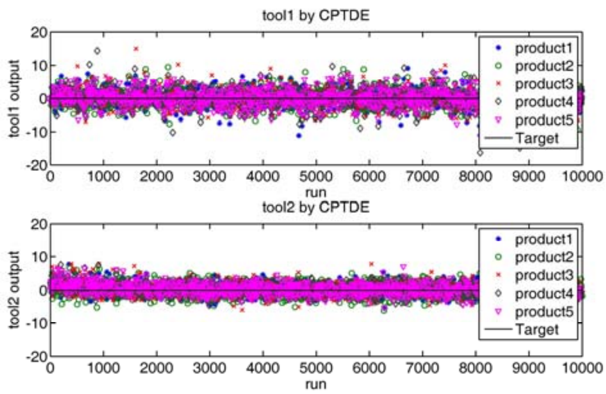


Fig. 10 The system outputs using CPTDE control approach

Table 3 The MSE and variance of these four controllers in each tool

		PB-EWMA	CF-EWMA	t-PCC	CPTDE
MSE	Tool 1	8.5311	32.62	7.3381	6.4928
	Tool 2	10.2458	42.8336	5.7361	2.6379
Variance	Tool 1	7.5372	31.6691	7.3685	6.7895
	Tool 2	6.4536	17.4816	6.0724	2.9256

Table 4 The optimum weight for each controller

	PB-EWMA	CF-EWMA	t-PCC		CPTDE	
Weight	λ_1	λ_1	λ_1	λ_2	λ_1	λ_2
Product 1	0.51	0.63	0.49	0.01	0.35	0.01
Product 2	0.99	0.99	0.99	0.01	0.99	0.01

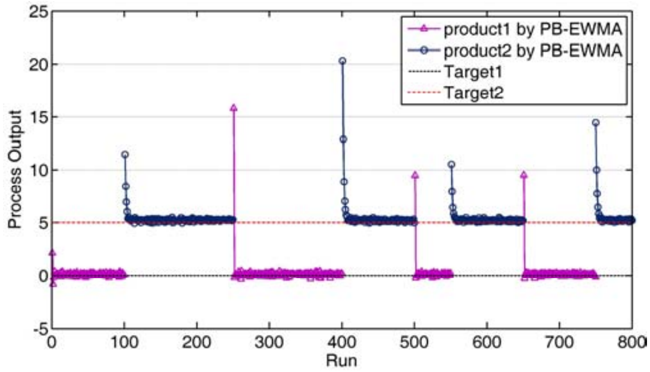


Fig. 11 The system outputs using PB-EWMA control approach

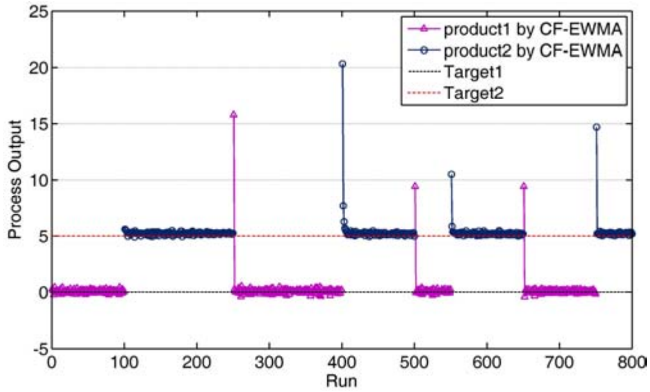


Fig. 12 The system outputs using CF-EWMA control approach

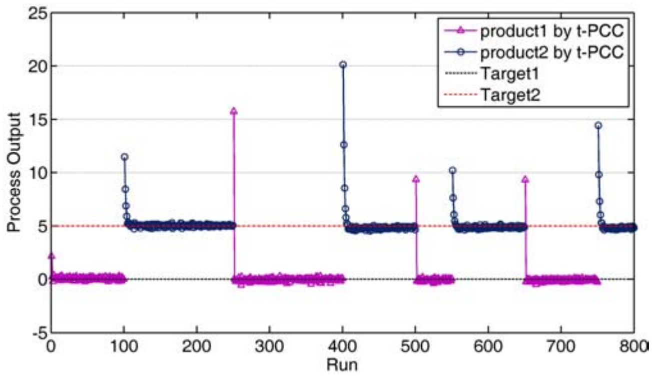


Fig. 13 The system outputs using t-PCC control approach

Figs. 13-14 show that t-PCC and CPTDE control schemes drive the system output on target except for the PB-EWMA and CF-EWMA control methods.

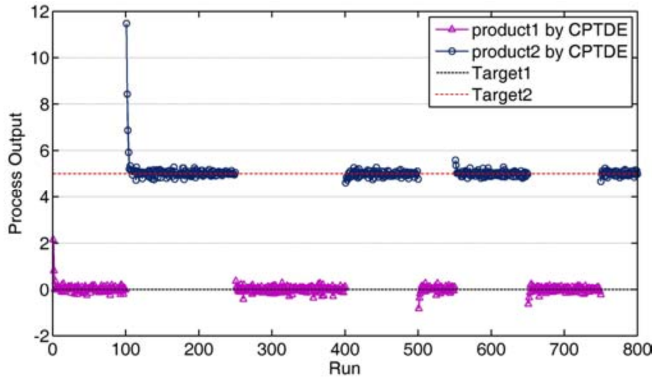


Fig. 14 The system outputs using CPTDE control approach

Table 5 The MSE and variance of these four controllers in each product

		PB-EWMA	CF-EWMA	t-PCC	CPTDE
MSE	Product 1	1.1134	1.0971	1.0889	0.0294
	Product 2	1.1881	0.9771	1.3126	0.1574
Variance	Product 1	1.0871	1.0771	1.0967	0.0290
	Product 2	1.0618	0.8963	1.3170	0.1543

The performance indices, MSE and variance, of system outputs for these four control schemes are shown in Table 5. Table 5 indicates that the CPTDE control not only reduces the MSE and variance significantly but also has superior control capability in comparison with other three control schemes in periodic schedule process.

5. Application to the removal rate estimation in mix-product CMP process

In this section, the PB-EWMA control, CF-EWMA control, t-PCC control and CPTDE control are applied to estimate the removal rate using historical data in mix-product chemical mechanical polishing process.

In CMP process, the polish rate $R_{i,k}$ can be written as a simple model:

$$R_{i,k} = R_{i,0} + \eta_{i,k} \tag{24}$$

where $R_{i,0}$ denotes the initial polish rate of product i, $\eta_{i,k}$ is the polish rate error of product i on run k, and k is the batch number. In CMP process, the polish efficiency becomes worse over time. There are so many factors affect the polish rate, degradation of the pad, changes in the slurry flow, pressure, carrier speed, platen speed, etc. The tool is necessary to continually change the process recipe (polish time) to compensate for the drift nature of the polish rate. Eq. (25) indicates the relationship between polish thickness and polish rate:

$$\text{Polish thickness} = \text{Polish rate} \times \text{Polish time} \tag{25}$$

The study only considers one factor, i.e., degradation of the pad, and the other factors are fixed. As shown in Fig. 15, the historical data contains four pads and three layers in each pad.

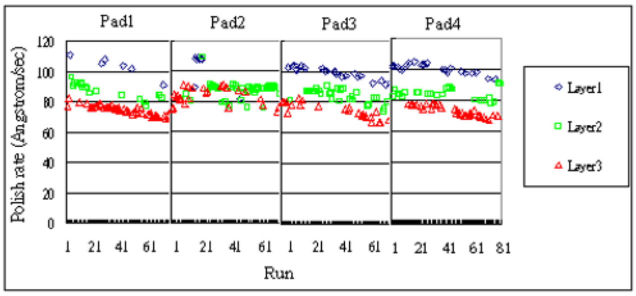


Fig. 15 The measurement of polish rate in each pad

Table 6 The initial values of intercept and drift for each layer

	Intercept (Å/s)	Drift (Å/s · run)
Layer1 (product 1)	111.09	-0.28546
Layer2 (product 2)	96.017	-0.18463
Layer3 (product 3)	77.333	-0.14472

Table 7 The optimum weight for each layer

	PB-EWMA	CF-EWMA	t-PCC		CPTDE	
Layer	λ_1	λ_1	λ_1	λ_2	λ_1	λ_2
1	0.99	0.01	0.55	0.51	0.99	0.01
2	0.93	0.99	0.89	0.01	0.75	0.01
3	0.39	0.36	0.16	0.09	0.22	0.01

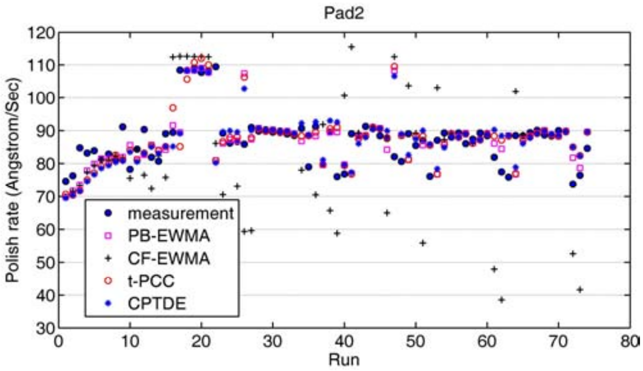


Fig. 16 The comparison of measurements and estimations in pad 2

The historical data of the four pads with constant recipes and equally spaced measurements were acquired from CMP tool, an Applied Material Reflexion, at the Powerchip Semiconductor Corporation of Taiwan. Different layers can be regarded as different products to fit the algorithms. The initial conditions (Table 6) and the optimum weights (Table 7) are found based on the first pad for each layer for use in other pads, i.e., the initial values of intercept are based on the first polish rate, and the initial values of drift are based on the slopes of linear regression in each layer. The optimum weighs are chosen by sweep method when the MSE is the minimum. Note that the PB-EWMA control and CF-EWMA control are just concerned about the initial values of intercept. Applying the initial values and optimum weights to the other pads, i.e., pad 2, pad 3, and pad 4, the measurements and the estimations by the four control schemes are shown in Figs. 16, 17 and 18. The important performance index MSE of the estimation error between the measurement and the estimation by four control schemes are shown in Table 8. Comparing with the MSE of these four control schemes, CPTDE control reduces the MSE significantly. The degradation of

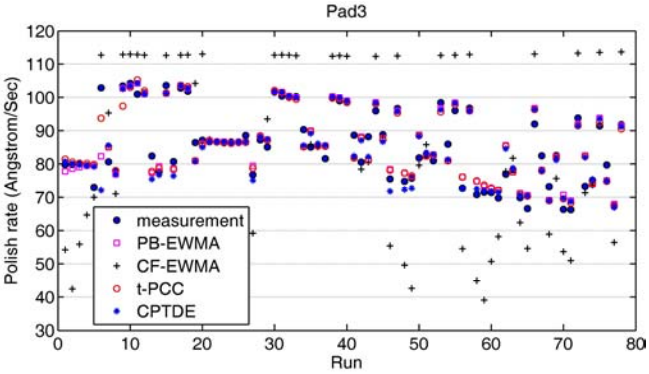


Fig. 17 The comparison of measurements and estimations in pad 3

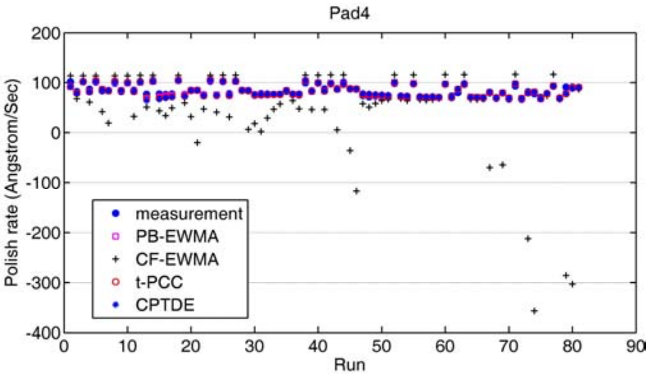


Fig. 18 The comparison of measurements and estimations in pad 4

Table 8 MSE of the estimated polish rate by four control schemes

MSE (Å/s)2	Pad 2	Pad 3	Pad 4	Average Improvement by CPTDE
PB-EWMA	64.159	14.731	9.432	9.52%
CF-EWMA	180.617	68.375	646.697	2523.43%
t-PCC	66.295	14.690	9.718	11.71%
CPTDE	56.594	13.072	9.201	

the pad in CMP process indeed brings large drift size, and it leads a better performance in CPTDE control scheme. In the later three pads, Table 8 indicates that CPTDE control has the best performance to estimate the removal rate in mix-product situation, and improves over PB-EWMA control, CF-EWMA control and t-PCC control by 9.52%, 2523.43% and 11.71% on average, respectively.

6. Conclusions

This research proposed an RtR mix-product control scheme, Combined Product and Tool Disturbance Estimator (CPTDE) which combined threaded double EWMA with drift compensation scheme. The key point of this method is that we separate the disturbances of products in one tool into two parts. One is the intercept term which concerns with the variation of products, and the other is the drift term which is related to the interaction of the tool and the product. That means every product owns its particular drift size in one tool. The CPTDE shows the ability to estimate the drift disturbance, and the mean value of this control scheme is on target under three disturbance models. The simulation case studies as well as a real case study are conducted and the results show that CPTDE control scheme has the best control performance compared with other three mix-product control schemes.

ACKNOWLEDGEMENT

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APPENDIX

In order to obtain analytic solutions in fixed schedule case (as shown in Fig. 2), this paper utilizes the cycle number, t, and the run numbers of one cycle, n, to replace the run number k. For example, the run $n+1$ can be presented as run $nt-(n-1)$ with $t=2$. Since Case 1 does not affect the steady-state property and Case 2 does not apply to this schedule, only Case 3 and Case 4 need to be considered. With the compensation action considered in Eq. (13) for Case 3, Eq. (10)-(12) can be rewritten as following equations according to the Fig. 2 on run $n+1$ for the product 1 for Case 4:

$$u_{1,nt-(n-1)} = \frac{-\hat{A}_{1,n(t-1)-(n-1)} - n\hat{P}_{1,n(t-1)-(n-1)}}{b_1} \tag{A-1}$$

$$\begin{aligned}\hat{A}_{1,nt-(n-1)} = \hat{A}_{1,n(t-1)-(n-1)} + n\hat{P}_{1,n(t-2)+1} + \lambda_1(Y_{1,nt-(n-1)} \\ - b_1 u_{1,nt-(n-1)} - \hat{A}_{1,n(t-1)-(n-1)} - n\hat{P}_{1,n(t-1)-(n-1)})\end{aligned} \tag{A-2}$$

$$\begin{aligned}\hat{P}_{1,nt-(n-1)} = \hat{P}_{1,n(t-2)+1} + \lambda_2(Y_{1,nt-(n-1)} \\ - b_1 u_{1,nt-(n-1)} - \hat{A}_{1,n(t-2)+1} - n\hat{P}_{1,n(t-2)+1})\end{aligned} \tag{A-3}$$

where the target of product 1 is zero, $T_1 = 0$. Let the Eq. (A-2) and (A-3) to be expressed in transfer function form as shown in Eq. (A-4).

$$\begin{aligned}
\begin{bmatrix}\hat{A}_{1,nt-(n-1)}\\ \hat{P}_{1,nt-(n-1)}\end{bmatrix} &= \begin{bmatrix}\lambda_1\\ \lambda_2\end{bmatrix}\left(Y_{1,nt-(n-1)} - b_1 u_{1,nt-(n-1)}\right) \\
&\quad + \begin{bmatrix}1-\lambda_1 & n(1-\lambda_1)\\ -\lambda_2 & (1-n\lambda_2)\end{bmatrix}\begin{bmatrix}\hat{A}_{1,n(t-1)-(n-1)}\\ \hat{P}_{1,n(t-1)-(n-1)}\end{bmatrix} \\
\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}\hat{A}_{1,nt-(n-1)}\\ \hat{P}_{1,nt-(n-1)}\end{bmatrix} &= \begin{bmatrix}\lambda_1\\ \lambda_2\end{bmatrix}\left(Y_{1,nt-(n-1)} - b_1 u_{1,nt-(n-1)}\right) \\
&\quad + \begin{bmatrix}(1-\lambda_1)z^{-n} & n(1-\lambda_1)z^{-n}\\ (-\lambda_2)z^{-n} & (1-n\lambda_2)z^{-n}\end{bmatrix}\begin{bmatrix}\hat{A}_{1,nt-(n-1)}\\ \hat{P}_{1,nt-(n-1)}\end{bmatrix} \\
\begin{bmatrix}1-(1-\lambda_1)z^{-n} & -n(1-\lambda_1)z^{-n}\\ -(-\lambda_2)z^{-n} & 1-(1-n\lambda_2)z^{-n}\end{bmatrix}\begin{bmatrix}\hat{A}_{1,nt-(n-1)}\\ \hat{P}_{1,nt-(n-1)}\end{bmatrix} &= \begin{bmatrix}\lambda_1\\ \lambda_2\end{bmatrix}\left(Y_{1,nt-(n-1)} - b_1 u_{1,nt-(n-1)}\right) \\
\begin{bmatrix}\hat{A}_{1,nt-(n-1)}\\ \hat{P}_{1,nt-(n-1)}\end{bmatrix} &= \begin{bmatrix}1-(1-\lambda_1)z^{-n} & -n(1-\lambda_1)z^{-n}\\ -(-\lambda_2)z^{-n} & 1-(1-n\lambda_2)z^{-n}\end{bmatrix}^{-1}\begin{bmatrix}\lambda_1\\ \lambda_2\end{bmatrix} \\
&\quad \left(Y_{1,nt-(n-1)} - b_1 u_{1,nt-(n-1)}\right) \\
\begin{bmatrix}\hat{A}_{1,nt-(n-1)}\\ \hat{P}_{1,nt-(n-1)}\end{bmatrix} &= \frac{1}{[1-(1-n\lambda_2)z^{-n}][1-(1-\lambda_1)z^{-n}] - \lambda_2 z^{-n}[-n(1-\lambda_1)z^{-n}]} \\
&\quad \begin{bmatrix}1-(1-n\lambda_2)z^{-n} & n(1-\lambda_1)z^{-n}\\ (-\lambda_2)z^{-n} & 1-(1-\lambda_1)z^{-n}\end{bmatrix}\begin{bmatrix}\lambda_1\\ \lambda_2\end{bmatrix} \\
&\quad \left(Y_{1,nt-(n-1)} - b_1 u_{1,nt-(n-1)}\right) \\
\begin{bmatrix}\hat{A}_{1,nt-(n-1)}\\ \hat{P}_{1,nt-(n-1)}\end{bmatrix} &= \frac{z^{2n}}{z^{2n} - (2-n\lambda_2-\lambda_1)z^n + (1-\lambda_1)} \\
&\quad \begin{bmatrix}[1-(1-n\lambda_2)z^{-n}]\lambda_1 + [n(1-\lambda_1)z^{-n}]\lambda_2\\ [(-\lambda_2)z^{-n}]\lambda_1 + [1-(1-\lambda_1)z^{-n}]\lambda_2\end{bmatrix} \\
&\quad \left(Y_{1,nt-(n-1)} - b_1 u_{1,nt-(n-1)}\right)
\end{aligned} \tag{A-4}$$

Substitute Eq. (A-4) into Eq. (A-1) to yield:

$$\begin{aligned}
\hat{A}_{1,nt-(n-1)} + n\hat{P}_{1,nt-(n-1)} &= \frac{\left[(\lambda_1+n\lambda_2)z^n - \lambda_1\right]z^n}{z^{2n} - (2-n\lambda_2-\lambda_1)z^n + (1-\lambda_1)} \\
&\quad \left(Y_{1,nt-(n-1)} - b_1 u_{1,nt-(n-1)}\right) \\
\hat{A}_{1,nt-(n-1)} + n\hat{P}_{1,nt-(n-1)} &= \frac{(\lambda_1+n\lambda_2)z^n - \lambda_1}{z^{2n} - (2-n\lambda_2-\lambda_1)z^n + (1-\lambda_1)} \\
&\quad \left(Y_{1,n(t+1)-(n-1)} - b_1 u_{1,n(t+1)-(n-1)}\right)
\end{aligned}$$

$$\begin{aligned}
u_{1,nt+1} &= -\frac{1}{b_1}\frac{(\lambda_1+n\lambda_2)z^n - \lambda_1}{z^{2n} - (2-n\lambda_2-\lambda_1)z^n + (1-\lambda_1)} \\
&\quad \times\left(Y_{1,n(t+1)-(n-1)} - b_1 u_{1,n(t+1)-(n-1)}\right) \\
u_{1,nt+1} &= -\frac{1}{b_1}\frac{(\lambda_1+n\lambda_2)z^n - \lambda_1}{z^{2n} - (2-n\lambda_2-\lambda_1)z^n + (1-\lambda_1)}Y_{1,nt+1} \\
&\quad + \frac{(\lambda_1+n\lambda_2)z^n - \lambda_1}{z^{2n} - (2-n\lambda_2-\lambda_1)z^n + (1-\lambda_1)}u_{1,nt+1} \\
u_{1,nt+1} &= -\frac{1}{b_1}\frac{(\lambda_1+n\lambda_2)z^n - \lambda_1}{z^{2n} - 2z^n + 1}Y_{1,nt+1}
\end{aligned} \tag{A-5}$$

Substitute Eq. (A-5) into the linear model of process output Eq. (1), and then we can obtain the final process response as below:

$$\begin{aligned}
Y_{1,nt+1} &= -\xi_1\frac{(\lambda_1+n\lambda_2)z^n - \lambda_1}{z^{2n} - 2z^n + 1}Y_{1,nt+1} + \eta_{1,nt+1} \\
Y_{1,nt+1} &= \frac{z^{2n} - 2z^n + 1}{z^{2n} + \left[\xi_1(\lambda_1+n\lambda_2) - 2\right]z^n + 1 - \xi_1\lambda_1}\eta_{1,nt+1}
\end{aligned} \tag{A-6}$$

where $\xi_1 = \beta_1/b_1$ is defined as model mismatch. Substitute the DT, RWD and IMA (1,1) with drift for $\eta_{1,nt+1}$ in Eq. (A-6), we have

DT:

$$\begin{aligned}Y_1(z) &= \frac{z^{2n}-2z^n+1}{z^{2n}+\left[\xi_1\left(\lambda_1+n\lambda_2\right)-2\right]z^n+1-\xi_1\lambda_1}\frac{z\delta}{\left(z-1\right)^2}\\ &= \frac{\left(\sum_{i=0}^{n-1}z^i\right)^2\left(z-1\right)^2}{z^{2n}+\left[\xi_1\left(\lambda_1+n\lambda_2\right)-2\right]z^n+1-\xi_1\lambda_1}\frac{z\delta}{\left(z-1\right)^2} \qquad \text{(A-7)}\\ &= \frac{\left(\sum_{i=0}^{n-1}z^i\right)^2}{z^{2n}+\left[\xi_1\left(\lambda_1+n\lambda_2\right)-2\right]z^n+1-\xi_1\lambda_1}z\delta\end{aligned}$$

RWD:

$$\begin{aligned}Y_1(z) &= \frac{z^{2n}-2z^n+1}{z^{2n}+\left[\xi_1\left(\lambda_1+n\lambda_2\right)-2\right]z^n+1-\xi_1\lambda}\left[\frac{z}{z-1}\varepsilon(z)+\frac{z\delta}{\left(z-1\right)^2}\right]\\ &= \frac{\left(\sum_{i=0}^{n-1}z^i\right)^2 z}{z^{2n}+\left[\xi_1\left(\lambda_1+n\lambda_2\right)-2\right]z^n+1-\xi_1\lambda_1}\left[\left(z-1\right)\varepsilon\left(z\right)+\delta\right]\end{aligned} \qquad \text{(A-8)}$$

IMA (1,1) with drift

$$\begin{aligned}Y_1(z) &= \frac{z^{2n}-2z^n+1}{z^{2n}+\left[\xi_1\left(\lambda_1+n\lambda_2\right)-2\right]z^n+1-\xi_1\lambda}\left[\frac{z-\theta}{z-1}\varepsilon(z)+\frac{z\delta}{\left(z-1\right)^2}\right]\\ &= \frac{\left(\sum_{i=0}^{n-1}z^i\right)^2}{z^{2n}+\left[\xi_1\left(\lambda_1+n\lambda_2\right)-2\right]z^n+1-\xi_1\lambda_1}\left[\left(z-1\right)\left(z-\theta\right)\varepsilon\left(z\right)+z\delta\right]\end{aligned} \qquad \text{(A-9)}$$

In general schedule case, the asymptotic expectation of process outputs under three disturbance models can be shown by final value theory. The discrete final value theory states $\lim_{k\to\infty} y(k) = y_{ss} = \lim_{z\to 1}(1-z^{-1})Y(z)$ if all of the poles of $(1-z^{-1})Y(z)$ i.e., $Y(z)$, are inside the unit circle. It implies that $Y(z)$ should be bounded. Thus, the asymptotic expectation of process outputs (Eqs. (A-7)-(A-9)) can be obtained by assuming all the poles of $Y_1(z)$ are inside the unit circle, i.e., the characteristic equation in Eq. (18) should be stable. Here, one takes the DT disturbance case as an example:

$$\begin{aligned}E\left(Y_1\right) &= \lim_{z\to 1}\frac{z-1}{z}\frac{\left(\sum_{i=0}^{n-1}z^i\right)^2}{z^{2n}+\left[\xi_1\left(\lambda_1+n\lambda_2\right)-2\right]z^n+1-\xi_1\lambda_1}z\delta \qquad \text{(A-10)}\\ &= 0\end{aligned}$$

Using Eq. (A-7)-(A-9), the asymptotic variance can be calculated by the method proposed from Åström and Wittenmark.[24] The derivation results are shown in Eq. (20)-(22).