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ORIGINAL PAPER

Generating tri-chaos attractors with three positive Lyapunov exponents in new four order system via linear coupling

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Abstract This paper presents a new hyperchaotic system with three positive Lyapunov exponents (called Tri-Chaos). Via linear coupling, Mathieu, and van der Pol systems are coupled with each other and then become a new four order system—Mathieu–van der Pol autonomous system. As we know, two positive Lyapunov exponents confirm hyperchaotic nature of its dynamics and it means that the system can present more complicated behavior than ordinary chaos. We further generate three positive Lyapunov exponents in a new coupled nonlinear system and anticipate the advanced application in secure communication. Not only a new chaotic system with three Lyapunov exponents is proposed, but also its implementation of an electronic circuit is put into practice in this article. The phase portrait, electronic circuit, power spectrum, Lyapunov exponents, and 2-D and 3-D parameter diagram of tri-chaos with three positive Lyapunov exponents of the new system will be shown in this paper.

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Keywords Tri-Chaos · Mathieu–van der Pol system · Lyapunov exponent · 3-D parametric diagram

1 Introduction

Nonlinear systems are capable of exhibiting a variety of behaviors, ranging from fixed points via limit cycles and tori to the more complex chaotic phenomenon, especially hyperchaotic attractors. Chaos and hyperchaos in a nonlinear system can occur in various kinds of nature and man-made systems [1–10]; those systems are characterized by great sensitivity to initial conditions [11]. In order to appear chaotic motion, continuous time systems of integer order must be at least third order. On the other hand, to obtain hyperchaos, the system must be at least fourth order.

Chaotic systems are characterized by one positive Lyapunov exponent (PLE) in the Lyapunov spectrum [12–19]. The one PLE just indicates that the dynamics of the underlying chaotic attractor expands only in one direction. If a chaotic attractor is characterized by more than one positive Lyapunov exponent, it is termed hyperchaos. In this case, the dynamics of the chaotic attractor expands in more than one direction giving rise to a "thick" chaotic attractor [20–24]. There are both theoretical and practical interests in hyperchaos. Hyperchaos was first reported from computer simulations of hypothetical ordinary differential equations in [25–27]. The first observation of hyperchaos from a real physical system, a fourth-order electrical circuit, was later reported in [28]. Very few hyperchaos generators have been reported since then [29–32].

Hyperchaotic Chua's circuit [33, 34] and Rossler system [35] are two well-known examples of hyperchaotic models. In recent years, different methods and techniques have been proposed for chaotification chaos, in which the linear feedback control method is simple, but effective to chaotify the chaotic system. Li and Chen [36] introduced a hyperchaotic Chen system via state feedback control. Chen et al. [37] constructed a new hyperchaotic system based on Lü system by using a state feedback. More recent studies by Wang and Liu [38] realized hyperchaos based on the chaotic dynamical system that was introduced in [39] using an additional state input. Hu et al. [40] presented a new hyperchaotic system, which was obtained by adding an approximate time-delay state feedback to the second equation of the three-dimensional Lorenz chaotic system. Jia [41] also presented a hyperchaotic system by adding a nonlinear quadratic controller to the first equation. Wang [42] constructed a hyperchaotic Lorenz system by adding a simple linear controller to the second equation. Hyperchaotic attractors with two positive Lyapunov exponents can be generated through all these methods.

As the numerical example, recently developed new Mathieu–van der Pol autonomous oscillator with four state variables is used. For this new system, four Lyapunov exponents are not zero. Although by traditional theory [43], for four-dimensional continuous-time systems, there must be a zero Lyapunov exponent; however, on the history of science, as mentioned by Kuhn in his book, "*The Structure of Scientific Revolution*," the unexpected discovery or anomality (counterinstance) is not simply factual in its import and the scientist's world is qualitatively transformed as well as quantitatively enriched by fundamental novelties of either fact or theory. "Conversion as a feature of revolutions in science" is the conclusion of the book "*Revolution in Science*" written by Cohen [44]. One of the patterns of the evolution of science is: current paradigm → normal science → anomality (counterinstance) → crisis → emergence of scientific theories → new paradigm.

Contributing editor—Professor Evan Ratliff wrote a comment about Barack Obama's technology strategy which is published in the issue 17.02 of Science [45] as: "Extraordinary claims require extraordinary evidence." Recently, Ott and Yorke [46] show that the existence of Lyapunov exponents is a subtle question for systems that are not conservative. They describe a simple continuous-time flow such that Lyapunov exponents fail to exist at nearly every point in the phase space. Ge and Yang [47] firstly find out the simulation results of 3PLES in Quantum Cellular Neuro Network autonomous system with four state variables. Thus, we call the chaotic motions with three Lyapunov exponents "tri-chaos," which means there exist three positive Lyapunov exponents in a nonlinear system.

In this paper, a new system, Mathieu–van der Pol autonomous system, with four state variables is introduced by linear coupling and generated to show the tri-chaos for 3PLEs. The complex dynamics behaviors are going to be investigated by phase portrait, power spectrum, Lyapunov exponents, and parameter diagram in the following simulation results.

2 Differential equations for New Mathieu–van der Pol system and its basics properties

2.1 The New Mathieu–van der Pol system

Mathieu equation and van der Pol equation are two typical nonlinear nonautonomous systems:

$$\begin{cases} \dot{x} = y \\ \dot{y} = -(a + b\sin\omega t)x - (a + b\sin\omega t)x^3 \\ \qquad - cy + d\sin\omega t \end{cases} \tag{2.1}$$

$$\begin{cases} \dot{z} = w \\ \dot{w} = -ez + f(1 - z^2)w + g\sin\omega t \end{cases} \tag{2.2}$$

In order to generate tri-chaos in a four order nonlinear system, we exchange $\sin\omega t$ in (2.1) with z and $\sin\omega t$ in (2.2) with x for linear coupling, and then we obtain the autonomous new Mathieu–van der Pol system.

For the Mathieu–van der Pol system, the following differential equations are obtained:

$$\begin{cases} \dot{x} = y \\ \dot{y} = -(a + bz)x - (a + bz)x^3 - cy + dz \\ \dot{z} = w \\ \dot{w} = -ez + f(1 - z^2)w + gx \end{cases} \tag{2.3}$$

where x, y, z, and w are four stats of the system, a, b, c, d, e, f, and g are parameters of the Mathieu–van der Pol system.

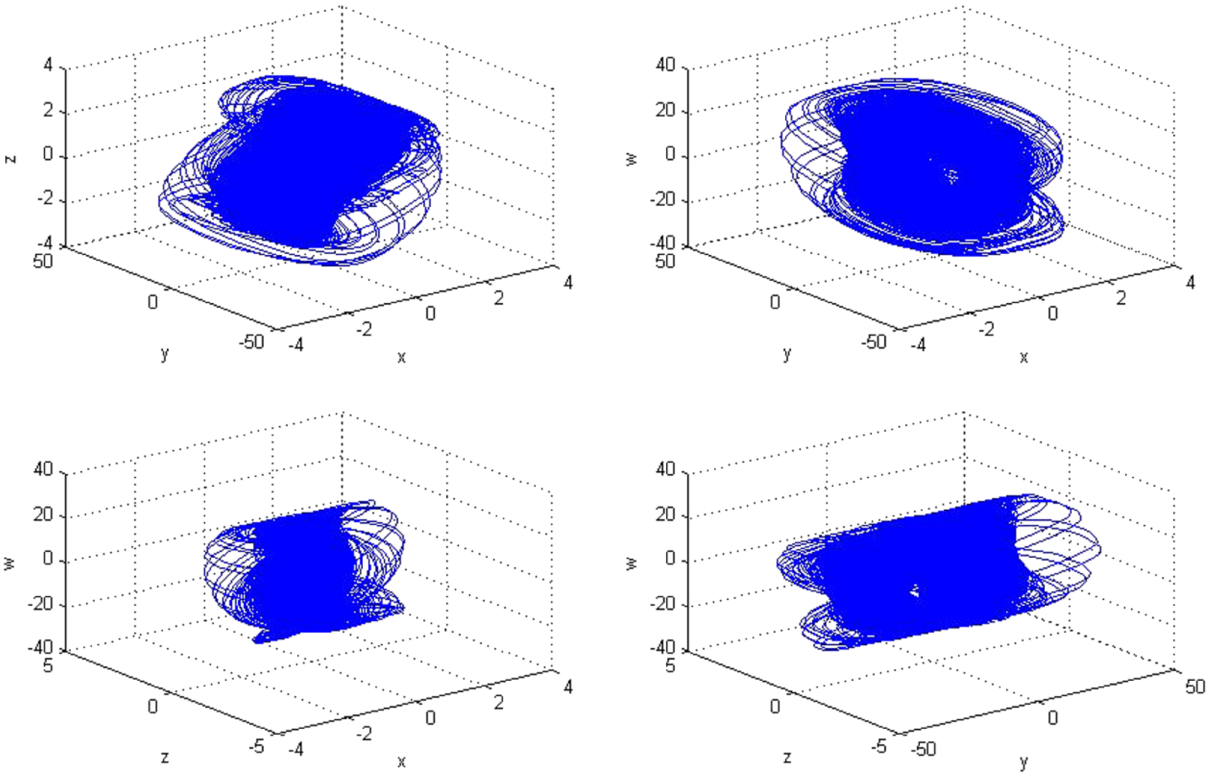


Fig. 1 Phase portrait of Mathieu–van der Pol system with $a = 91.17$, $b = 5.023$, $c = -0.001$, $d = 91$, $e = 87.001$, $f = 0.0180$, and $g = 9.5072$

2.2 Equilibria analysis

The equilibria of the new system can be found by solving the following equations simultaneously:

$$\begin{cases} y = 0 \\ -(a+bz)x - (a+bz)x^3 - cy + dz = 0 \\ w = 0 \\ -ez + f(1-z^2)w + gx = 0 \end{cases} \tag{2.4}$$

Here, we use the Lyapunov's linearization method to investigate the local stability of the new nonlinear system in (2.3). Thus, the linearization of (2.3) for equilibrium point $E_1(0,\ 0,\ 0,\ 0)$ is defined as:

$$\dot{X} = J_1 X \tag{2.5}$$

where

$$J_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ a & -c & d & 0 \\ 0 & 0 & 0 & 1 \\ g & 0 & -e & f \end{bmatrix}, \tag{2.6}$$

$X = [x_1, x_2, x_3, x_4]^T$

In order to gain the eigenvalues for E_1, we take:

$$|J_1 - \lambda I| = \begin{vmatrix} -\lambda & 1 & 0 & 0 \\ a & -c-\lambda & d & 0 \\ 0 & 0 & -\lambda & 1 \\ g & 0 & -e & f-\lambda \end{vmatrix} = 0 \tag{2.7}$$

and the characteristic equation is obtained:

$$\begin{aligned} \det|J_1 - \lambda I| &= \lambda^4 + (c-f)\lambda^3 + (e-a-cf)\lambda^2 \\ &\quad + (ce+af)\lambda + (-ae-dg) = 0 \end{aligned} \tag{2.8}$$

Substituting the constants $a = 91.7$, $b = 5.023$, $c = 0.01$, $d = 91$, $e = 87.001$, $f = 0.0180$, and $g = 9.5072$, then the eigenvalues which are corresponding to the equilibrium point $E_1(0, 0, 0, 0)$ are obtained as

$\lambda_1 = -9.8239, \qquad \lambda_2 = 9.8146,$

$\lambda_3 = 0.0086 + 9.5769\text{i}, \qquad \lambda_4 = 0.0086 - 9.5769\text{i}$

where λ_2 are positive real numbers and the real parts of λ_3, λ_4 are positive numbers as well.

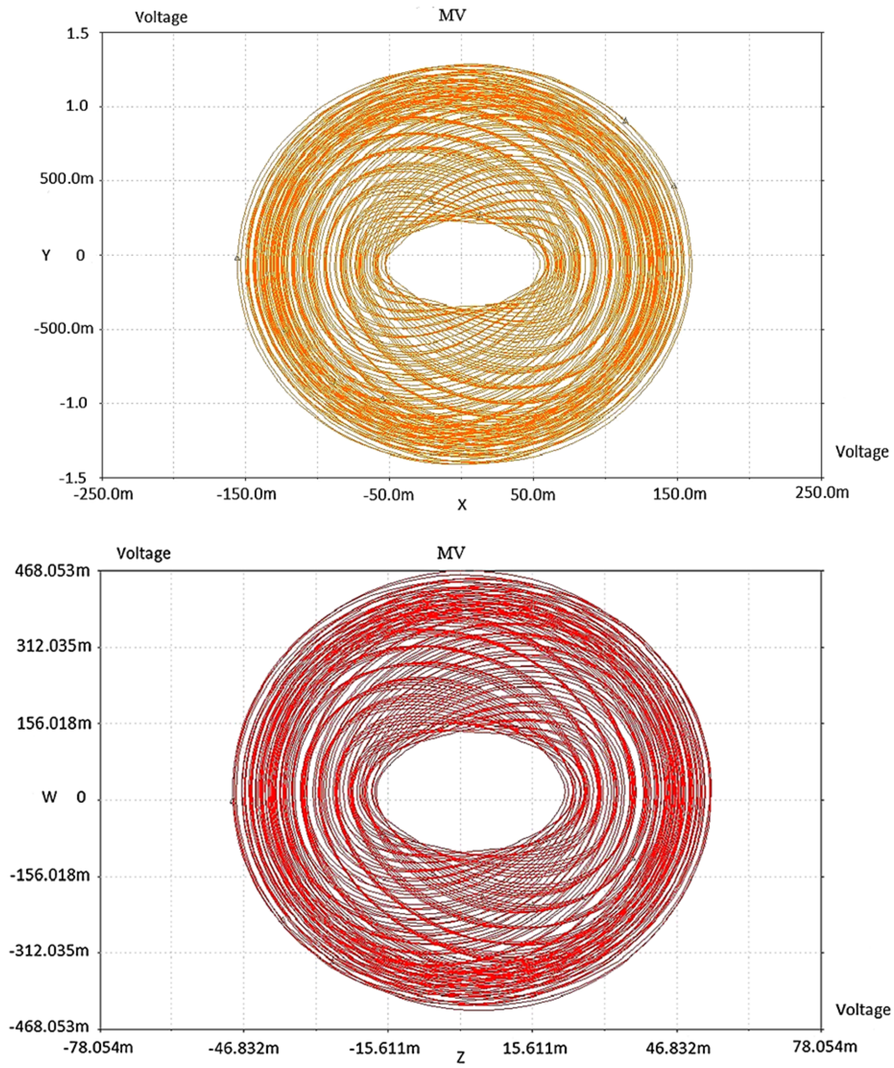


Fig. 2 Projection of phase portraits outputs in electronic circuit for Mathieu–van der Pol system

Theorem 1 [48, 49] *Lyapunov's linearization method.*

According to Lyapunov's direct method, the following results make precise the relationship between the stability of the linear system and that of the original nonlinear system.

1. If the linearized system is strictly stable (i.e., if all eigenvalues of J are strictly in the left-half complex plane), then the equilibrium point is asymptotically stable (for the actual nonlinear system).
2. If the linearized system is unstable (i.e., if at least one eigenvalue of J is strictly in the right-half complex plane), then the equilibrium point is unstable (for the nonlinear system).
3. If the linearized system is marginally stable (i.e., all eigenvalues of J are in the left-half complex plane, but at least one of them is on the jw axis), then one cannot conclude anything from the linear approximation (the equilibrium point may be stable, asymptotically stable, or unstable for the nonlinear system.

Therefore, based on Theorem 1, the equilibrium point $E_1(0, 0, 0, 0)$ is a saddle point, i.e., unstable. For those nonzero equilibria, it is not a necessary work to numerically evaluating their stabilities since one unstable equilibrium point has been found at zero.

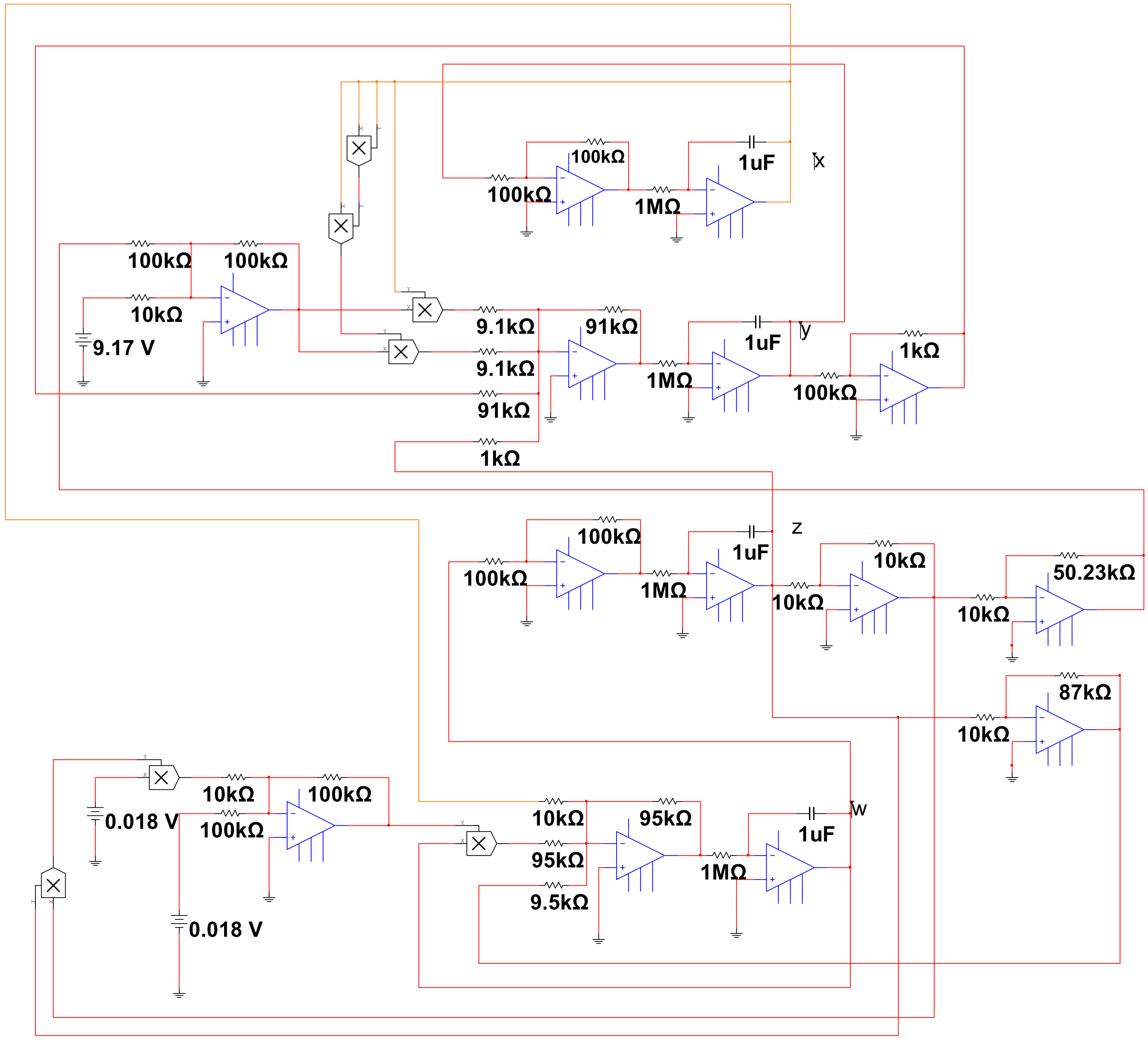


Fig. 3 The configuration of electronic circuit for Procedo-chaotic Mathieu–van der Pol system

3 Phase portraits and its implementation of electronic circuits

It is well known that the phase space can present the evolution of a set of trajectories emanating from various initial conditions. When the solution becomes stable, the asymptotic behaviors of the phase trajectories are particularly interested and the transient behaviors in the system are neglected. As a result, the phase portrait of the Mathieu–van der Pol system, (2.3), is plotted in Fig. 1. In these numerical studies, the parametric values are taken to be $a = 91.7$, $b = 5.023$, $c = 0.01$, $d = 91$, $e = 87.001$, $f = 0.0180$, and $g = 9.5072$ for plotting the tri-chaotic phase portrait. The new system can be represented as an electronic oscillator circuit shown in Fig. 2. We have implemented it using an electronics simulation package Multisim (previously called Electronic Workbench, EWB) and the approximated nonlinear electronic circuits are presented to realize the disordered behavior in the new chaotic system. The voltage outputs have been normalized to 1 V and the operational amplifiers are considered to be ideal. The phase diagrams are plotted within the time interval 500 s. The time step is 0.001 s. Due to the limit

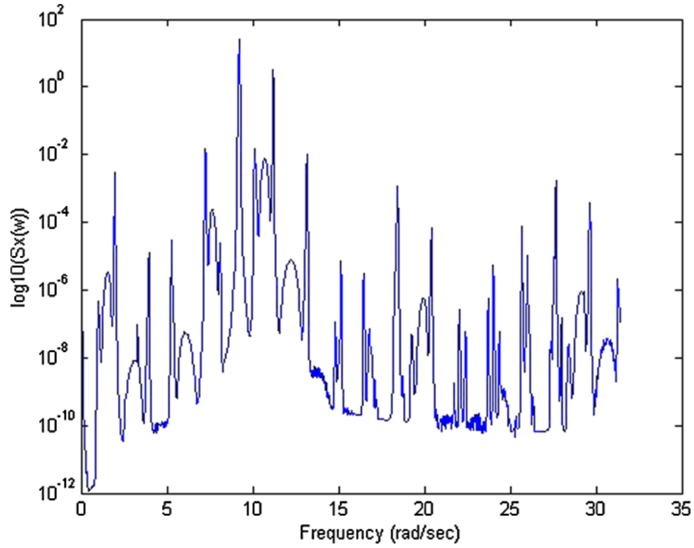


Fig. 4 Power spectrum of x for Mathieu–van der Pol system with $a = 91.17$, $b = 5.023$, $c = -0.001$, $d = 91$, $e = 87.001$, $f = 0.018$, and $g = 9.5072$

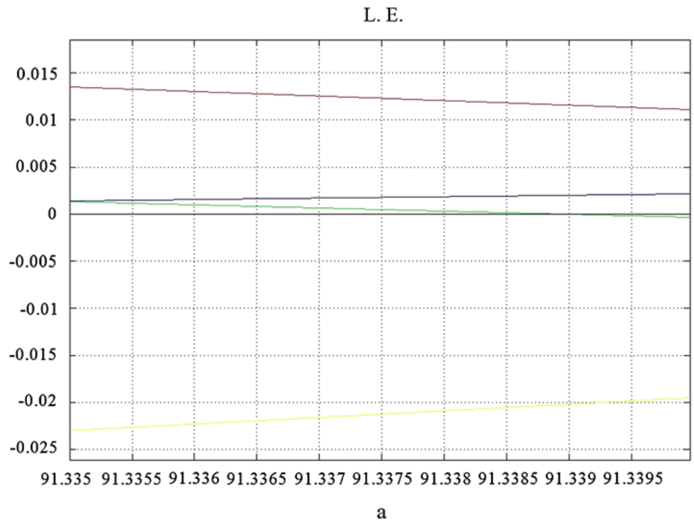


Fig. 5 Lyapunov exponents of Mathieu–van der Pol system with $b = 5.023$, $c = -0.001$, $d = 91$, $e = 87.001$, $f = 0.018$, and $g = 9.5072$

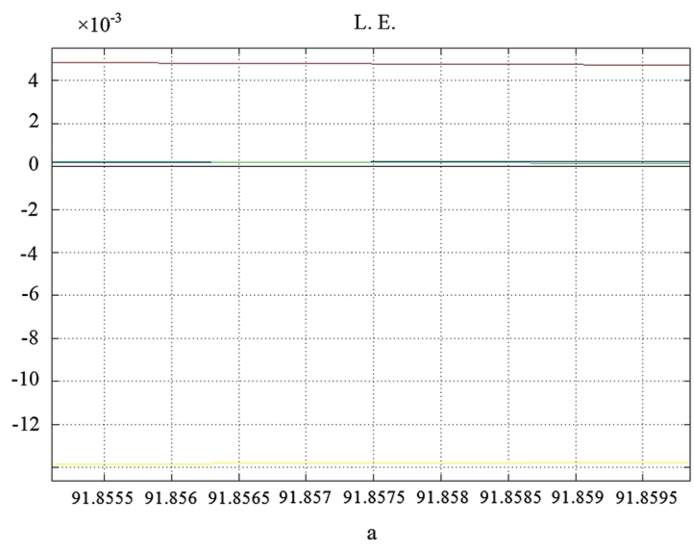


Fig. 6 Lyapunov exponents of Mathieu–van der Pol system with $b = 5.023$, $c = -0.001$, $d = 25$, $e = 87.001$, $f = 0.018$, and $g = 9.5072$

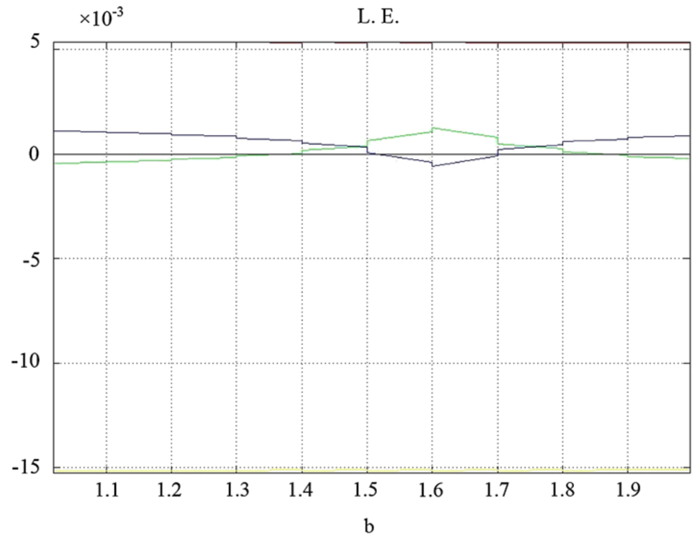


Fig. 7 Lyapunov exponents of Mathieu–van der Pol system with $a = 96.326680$, $c = -0.001$, $d = 25$, $e = 87.001$, $f = 0.018$, and $g = 9.5072$

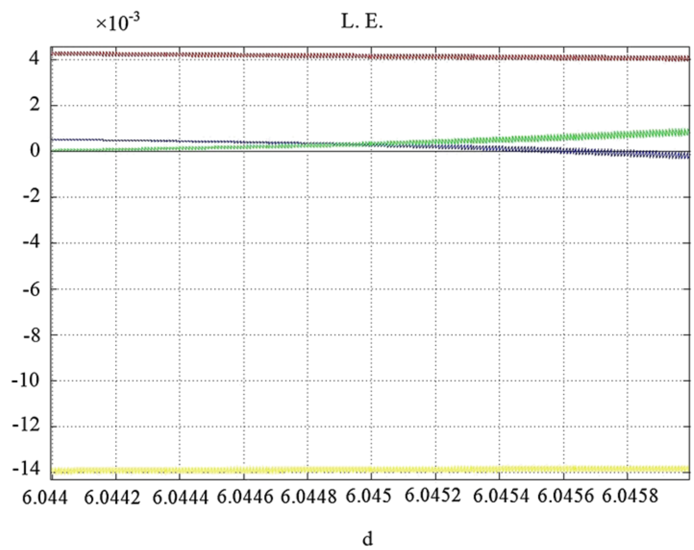


Fig. 8 Lyapunov exponents of Mathieu–van der Pol system with $a = 96.326680$, $b = 5.023$, $c = -0.001$, $e = 87.001$, $f = 0.018$, and $g = 9.5072$

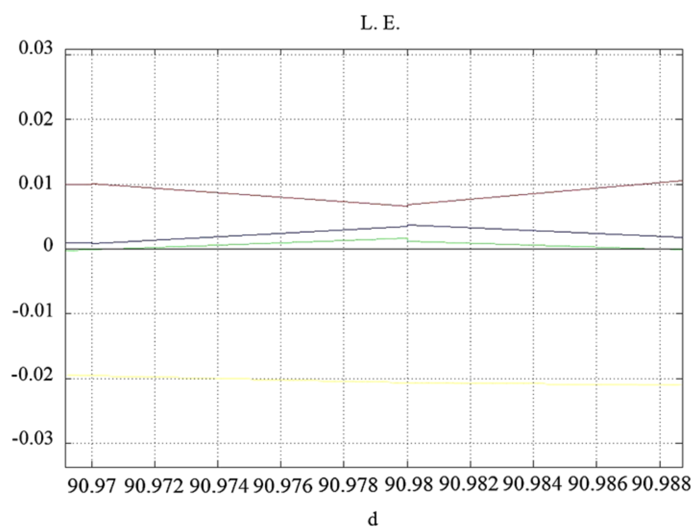


Fig. 9 Lyapunov exponents of Mathieu–van der Pol system with $a = 96.326680$, $b = 5.023$, $c = -0.001$, $e = 87.001$, $f = 0.018$, and $g = 9.5072$

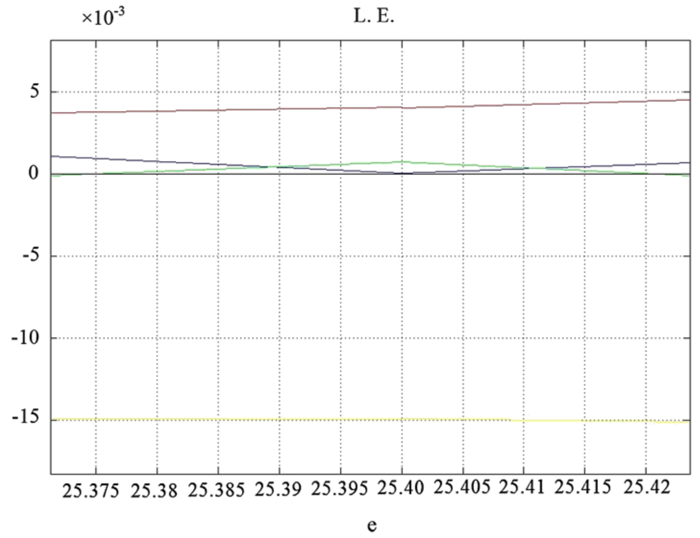


Fig. 10 Lyapunov exponents of Mathieu–van der Pol system with $a=96.326680$, $b=5.023$, $c=-0.001$, $d=25$, $f=0.018$, and $g=9.5072$

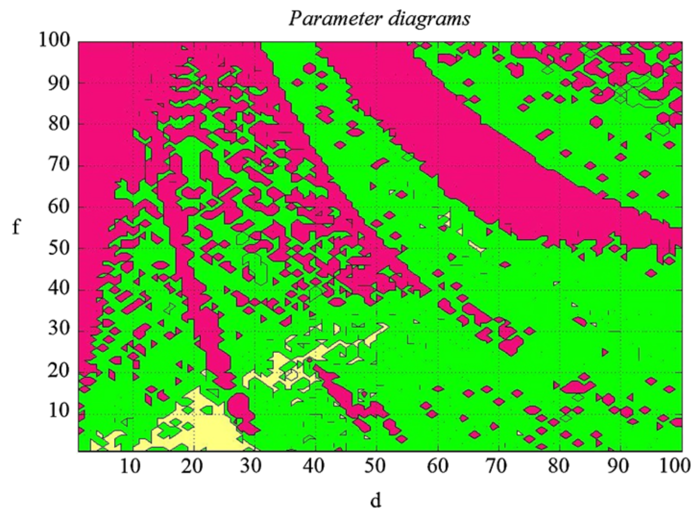


Fig. 11 Parameter diagrams of Mathieu–van der Pol system with $a=96.326680$, $b=5.023$, $c=-0.001$, $e=87.001$, and $f=0.018$

of the scope of implementation of electronic circuits, the phase portraits can be only shown in two dimensions. In Fig. 3, the configuration of electronic circuits is also given.

4 Power spectrum

The power spectrum analysis of the nonlinear dynamical system, (2.3) with $a=91.7$, $b=5.023$, $c=0.01$, $d=91$, $e=87.001$, $f=0.0180$, and $g=9.5072$, is shown in Fig. 4. The noise-like spectrum is one of the characteristics of the chaotic dynamical system.

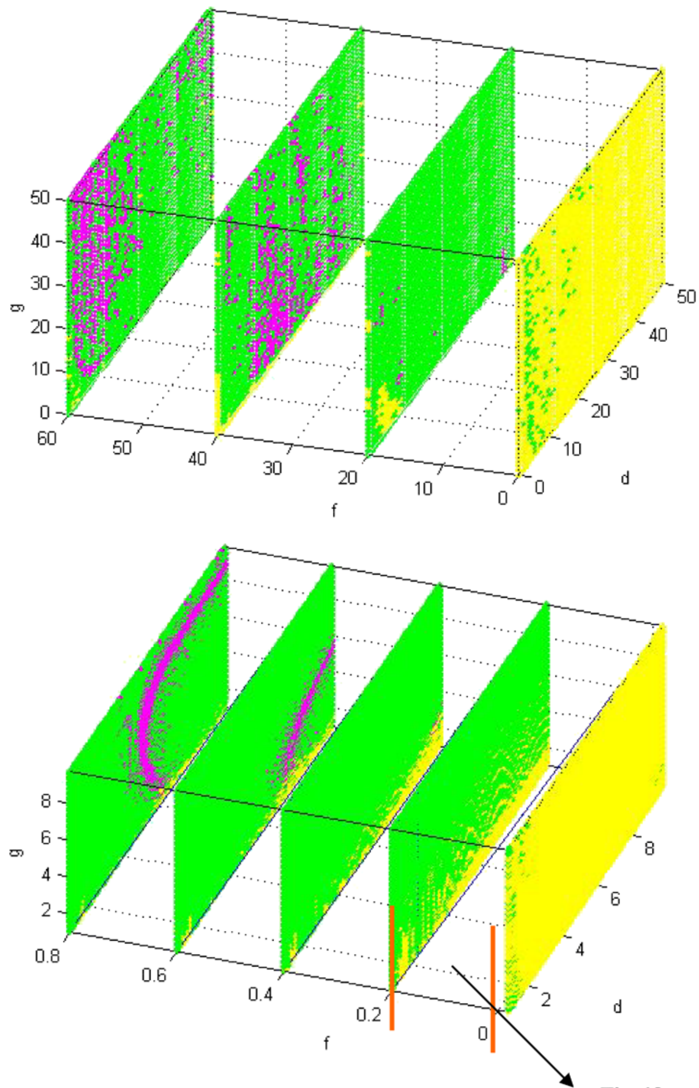
5 Lyapunov exponents

The Lyapunov exponents of Mathieu–van der Pol system with 3PLEs are plotted in Figs. 5–10. These figures show that there exist at least one PLEs in the Lyapunov spectrum for our new system, and which Lyapunov exponents of Mathieu–van der Pol system are varied separately with parameters a, b, d, and e.

6 Parameter diagrams

A system with more than one positive Lyapunov exponent can be classified as a hyperchaotic system. In this

Fig. 12 2D Parameter diagrams varied with f. $a = 96.326680$, $b = 5.023$, $c = -0.001$ and $e = 87.001$. Part A and B are shown in Fig. 7



study, the parameter values, b, d, g, and f, are varied to observe the regions of chaotization of our new system. The enriched information of chaotic behaviors of the system can be obtained from the Figs. 11–16. In order to discover the behavior of such complicated nonlinear systems in detail, in this section, we further develop a series of MATLAB codes to calculate the total number of positive Lyapunov exponents and then plot the 2-D and 3-D parameter diagrams automatically.

In Figs. 11–16, the regions of 3PLEs are presented in yellow, 2PLEs are in green, and 1PLEs are in purple. It can be realized that the Mathieu–van der Pol system is chaotic in several different regions, especially hyperchaos with 3 PLEs are found in many regions between chaos with 2 PLEs and period with 1 PLE. The special 3D plots show that the variations of chaotic and regular regions are smooth, gradual, and continuous.

In Fig. 11, we plot the parameter diagram of Mathieu–Van der Pol system with $0 < d < 100$ and $0 < f < 100$. Large parts of this diagram are 2PLEs and 1PLEs areas, only area about $0 < d < 50$ and $0 < f < 30$ is 3PLE. In Fig. 12, an alternative form of a parameter diagram is given—which is a 3-D diagram and presents the chaotic states of the Mathieu–van der Pol system with $0 < d < 50$, $0 < g < 50$ and $f = 0.05, 20, 40, 60$, and with $0 < d < 10$, $0 < g < 10$ and $f = 0.05, 0.2, 0.4, 0.6$, and 0.8. Figure 13 shows

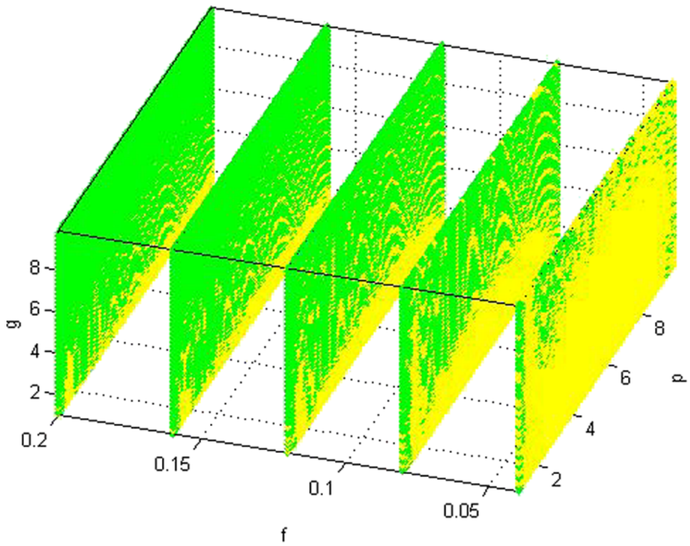


Fig. 13 2D Parameter diagrams varied with f. $a = 96.326680$, $b = 5.023$, $c = -0.001$ and $e = 87.001$. Part C is shown in Fig. 8

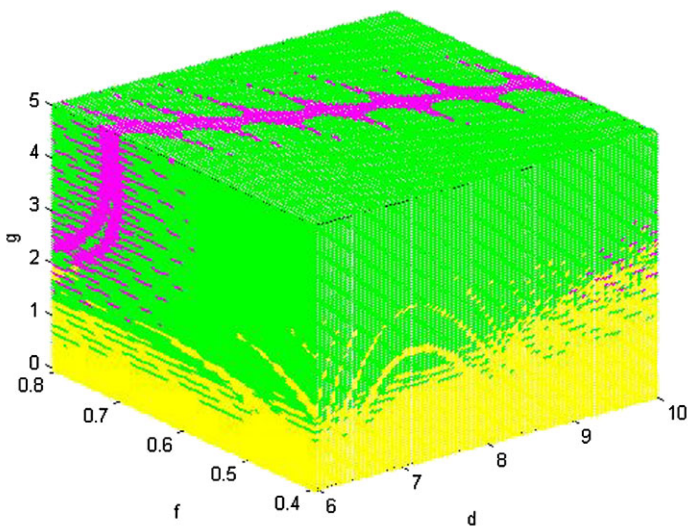


Fig. 14 3D Parameter diagrams of Mathieu–van der Pol system with $a = 96.326680$, $b = 5.023$, $c = -0.001$, and $e = 87.001$

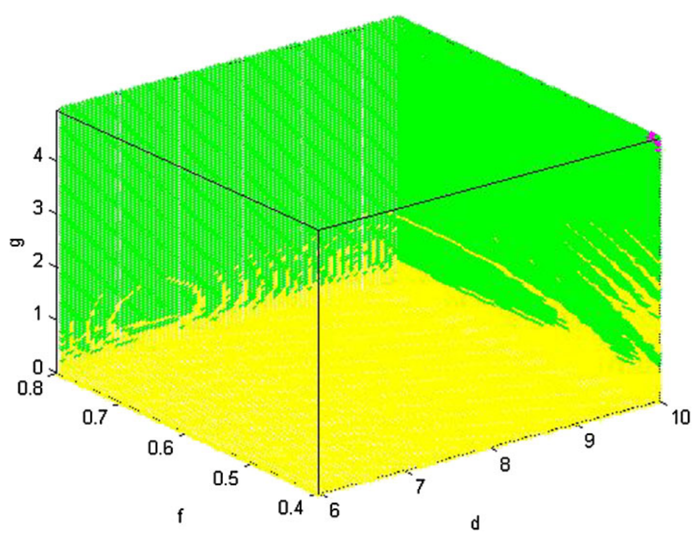


Fig. 15 3D Parameter diagrams of Mathieu–van der Pol system with $a = 96.326680$, $b = 5.023$, $c = -0.001$, and $e = 87.001$

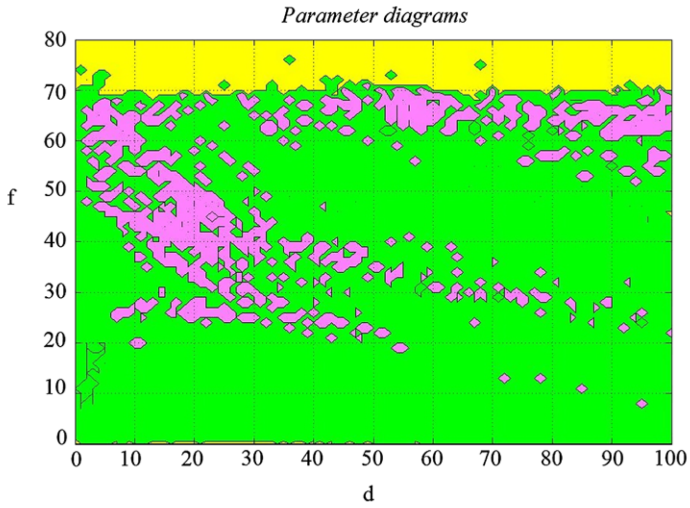


Fig. 16 Parameter diagrams of Mathieu–van der Pol system with $a = 96.326680$, $b = 5.023$, $c = -0.001$, $e = 87.001$, and $g = 9.5072$

that when $f \to +0.05$, the area of chaotic state with 3PLEs is increasing in the parameter diagram. We can use this strategy (from Figs. 12 and 13) to find out such complicated behavior (3PLEs) of a nonlinear system step by step. In Figs. 14 and 15, 3-D parameter diagrams with $6 < d < 10$, $0.4 < f < 0.8$, and $0 < g < 5$ are provided for investigation. It is clear to find out that the area of 3PLEs is within $6 < d < 10$, $0.4 < f < 0.8$, and $0 < g < 1$.

7 Conclusions

"Extraordinary claims require extraordinary evidence" is quoted by the contributing editor of Science, Professor Evan Ratliff. As a result, in this paper, we have shown that the autonomous continuous-time Mathieu–van der Pol autonomous system with four state variables as described by (2.1) can exhibit tri-chaos with three positive Lyapunov exponents. The simulation results have been investigated in phase portrait, power spectrum, parameter diagram, and the Lyapunov spectrum. An implementation of electronic circuit for the new chaotic system is further given to show the complicated and disordered behavior as well.

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