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Equilibrium pricing and lead time decisions in a competitive industry

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ABSTRACT

Pricing and lead time are two crucial decisions to a success in today's competitive markets. This paper examines the equilibrium pricing and lead time decisions in a duopoly industry consisting of two *large* and several *smaller* firms with competition. We solve the sufficient Karush–Kuhn–Tucker (KKT) optimality conditions for the Nash equilibrium solution. We characterize the existence and uniqueness of the Nash equilibrium solution of pricing and lead time decisions for both homogenous and heterogeneous firms. Our case study provides important managerial insights about firms' behavior under price and lead time competition in a semiconductor manufacturing industry.

1. Introduction

Observation of time-based marketplaces indicates that firms that provide more products and services in shorter lead times can charge a higher price and thus capture greater market share (Stalk, 1988; Stalk and Hout, 1990). In this paper we examine the equilibrium decision about the price and lead time in a duopoly consisting of two *large* firms and several *smaller* firms in a competitive market, where only the two large firms have dominant control over the market.

Capital-intensive industries, such as semiconductor manufacturing and its related industries, are typical examples of a duopoly in the marketplace. For instance, Intel and AMD are the leading global producers of microprocessor chips. Taiwan Semiconductor Manufacturing Company (TSMC) and United Microelectronics Corporation (UMC) dominate the semiconductor foundry industry, accounting for more than 80% market share (The Register, 2003). Samsung Electronics and LG Philips in Korea and AUO and CMO in Taiwan dominate the thin-film transistor liquid crystal display (TFT-LCD) industry (Chang, 2005). In such oligopolistic markets, each individual firm has its own profit function and often is unwilling to reveal information. Decisions made by competing firms can be influenced by other firms' behavior, especially in consumer product markets characterized by shortened product life-cycles.

A literature survey reveals that some researchers have examined individual firms' decisions about the equilibrium price or lead time (Bertrand and van Ooijen, 2000; Das and Abdel-Malek, 2003; Kunnumkal and Topagloglu, 2008; Cai et al., 2011; Glock, 2012), while others (Atamer et al., 2011; Qian, 2011; Maihami and Kamalabadi, 2012) have focused on optimization within a single firm and neglected the competition among firms. The underlying concept is that pricing and lead time are trade-offs—a short lead time typically results in a high price.

Palaka et al. (1998) examine the lead time setting, capacity utilization, and pricing decisions facing a firm serving customers sensitive to quoted lead times. Hatoum and Chang (1997) present a model to determine the optimal demand level using a mechanism of quoted lead time and price. Ray and Jewkes (2004) present an analytical approach for a firm to maximize its profit by optimal selection of a lead time. ElHafsi (2000) develops a model that includes enough detail so that realistic price and day-to-day lead time can be achieved and quoted to the customer. So and Song (1998) study the impact of using delivery time guarantees as a competitive strategy in service industries where demands are sensitive to both price and delivery time. Ray (2005) develops analytical models that can assist a firm in deciding on its optimal pricing, stocking and investment values in varying operating environments. Pekgun et al. (2008) study a firm serving customers sensitive to quoted price and lead time and analyze the inefficiencies created by the decentralization of the decisions, where pricing decisions are made by the marketing department and lead time decisions by the production department.

A growing body of literature on competitive models relies upon economic theory to analyze the behavior of independent firms in a market where no firm is better off by a unilateral change in its decision (see Gibbons, 1992). Several papers examine competitive supply, particularly in a duopoly industry, of goods or services to time-sensitive customers. For example, Kalai et al. (1992) study competition in service rates without consideration of pricing competition. Chen and Wan (2003) consider the

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duopoly price competition of make-to-order (MTO) firms. The results in Chen and Wan (2003) show that whenever market equilibrium exists and is unique, the firm with greater capacity, a higher service value, or a lower waiting cost can enjoy a price premium and a large market share. Li and Lee (1994) present a model of market competition in which a customer values cost, quality, and speed of delivery. Lederer and Li (1997) study the competition between firms that produce goods or services for customers sensitive to time delay where firms compete by setting prices and production rates for each type of customer and by choosing scheduling policies. So (2000) studies a similar issue of delivery time guarantees and pricing for service delivery. In reality, industries seldom consist of only two firms (a duopoly). Most research appears to ignore the effect of other smaller firms on pricing and lead time decisions in a duopoly industry. Thus our paper fills an important research void.

Our objective is to predict the equilibrium decision of the prices and lead times of the goods provided by all of the firms where none will be benefited by unilaterally deviating from its current decision. The remainder of this paper is organized as follows. The model and its equilibrium outcomes are described in Section 2. In Section 3 we conduct a case study in a semiconductor foundry industry and produce several managerial insights about price and lead time decisions. Section 4 presents our concluding remarks and suggestions for further research.

2. The model

2.1. The duopoly market model

We model a duopoly market consisting of two large firms and several smaller firms in a duopoly market. Regardless of firm size, all customers are served on a first-come, first-served basis. The large firms compete non-cooperatively to provide a type of goods in an MTO fashion. Both are independent entities and are modeled as queues with exponential service times with a common source of potential customer arrivals. Often, the decision variables for each firm will be influenced by the other's behavior. We also consider the effects of the smaller firms' decisions.

To begin, we denote the set of the two large firms by $N=\{X,Y\}$ and the group of smaller firms by M. We assume that the customer arrival rate of firm $i\in N$, λ_i, depends on firm i's decision of the price, p_i, and lead time, t_i. In a competitive market, λ_i is also influenced by the decisions of its major competitor, firm $j\in N, j\neq i$, and of the smaller firms. In other words, λ_i is also a function of p_j, t_j, p_M, and t_M in addition to p_i and t_i. In our hypothetical model, customers prefer lower prices and shorter lead times compared to the decisions offered by the other firms as shown in (1), where λ_i is proportional to the differences of prices and lead times between the other firms and firm $i\in N$.

$$\begin{aligned}&\lambda_i \propto (p_j-p_i)\\&\lambda_i \propto (p_M-p_i)\\&\lambda_i \propto (t_j-t_i)\\&\lambda_i \propto (t_M-t_i)\end{aligned} \tag{1}$$

Next, we elaborate on the precise relationships between prices and lead times in the market as shown in (2). The customer arrival rate λ_i of firm i is a function of the difference between firm i's decisions and the decisions of its competitors. Let α_M and α_C denote the preference factors accounting for the effect of the decision differences by the smaller firms, M, and the competitor, firm j. Similarly, β_t and β_p represent the preference factors for explaining the effect of lead times and prices on the arrival rate. We assume that the competition effect is a convex combination between i's competitor, firm j, and the smaller firms ($\alpha_M+\alpha_C=1$, α_M, $\alpha_C\geq 0$), and the decision effect is a convex combination of the price and lead time ($\beta_t+\beta_p=1$, β_t, $\beta_p\geq 0$). We now assume the arrival rate is

$$\begin{aligned}\lambda_i = \lambda_0 - m_t\beta_t[\alpha_M(t_i-t_M)+\alpha_C(t_i-t_j)]\\ -m_p\beta_p[\alpha_M(p_i-p_M)+\alpha_C(p_i-p_j)]\end{aligned} \tag{2}$$

where λ_0 denotes the arrival rate when both prices and lead times of all firms in the market are identical, and m_t and m_p represent the lead time sensitivity and price sensitivity of the arrival rate, respectively (λ_0, m_t, $m_p\geq 0$). A linear form of the arrival rate helps us obtain qualitative insights without much analytical complexity. It also has the desirable properties for approaching the equilibrium decisions of prices and lead times of the firms in the market. For illustrative purposes, we write λ_i as $\lambda_i(p_i,t_i|p_j,t_j,p_M,t_M)$; p_i and t_i are decisions of firm i, and p_j, t_j, p_M, t_M are decisions of other firms. We note the arrival rate shown in (2) does not limit itself to the situation where there are only two large firms and a group of smaller firms in a competitive industry. The form of (2) allows a decision maker to take into account the other major competitor and the group of all firms. A similar model appears in (Hatoum and Chang, 1997; Ray and Jewkes, 2004).

The objective of each firm is to maximize its own expected profit. Since the capacity is fixed, maximizing the expected profit is equivalent to maximizing the expected revenue. We assume an M/M/1 queuing system with mean service rate μ_i for firm $i\in N$. To prevent quoting unrealistic lead times, we assume that all of the firms maintain a certain minimum service level, s, which can be set by each firm in response to competitiveness or to the industry in general. The probability that the total sojourn time in firm i is less than the quoted lead time is $1-e^{-(\mu_i-\lambda_i)t_i}$ for an M/M/1 system (Kleinrock, 1975). Therefore, the requirement that the probability of meeting the quoted lead time for firm i must be at least s (e.g., 95%) can be represented in the following constraint as

$$1-e^{-(\mu_i-\lambda_i)t_i}\geq s$$

or equivalently,

$$-(\mu_i-\lambda_i)t_i\leq \ln(1-s)$$

Since firm $i\in N$ is assumed to maximize its own profit per unit time, the maximization model for firm i can be written as

$$\operatorname*{Max}_{p_i,t_i}\quad \pi_i(p_i,t_i|p_j,t_j,p_M,t_M)=p_i\lambda_i(p_i,t_i|p_j,t_j,p_M,t_M) \tag{3}$$

$$\text{s.t.}-(\mu_i-\lambda_i)t_i\leq \ln(1-s) \tag{4}$$

$$\begin{aligned}&\lambda_i(p_i,t_i|p_j,t_j,p_M,t_M)\\&\quad=\lambda_0-m_t\beta_t[\alpha_M(t_i-t_M)+\alpha_C(t_i-t_j)]\\&\qquad-m_p\beta_p[\alpha_M(p_i-p_M)+\alpha_C(p_i-p_j)]\end{aligned} \tag{5}$$

$$p_i,t_i>0. \tag{6}$$

Firm i maximizes its profit function π_i by quoting the price, p_i and lead time, t_i. Clearly, firm i's profit function, π_i, is a function of p_i and t_i, but it also depends on p_j, t_j, p_M, and t_M. Constraint (4) ensures that firm i maintains a minimum service level, and (5) and (6) are the arrival rate definition and bounds for prices and lead times.

2.2. Establishing equilibrium price and lead time

In this section we elaborate upon our algorithm. In equilibrium, the firms have no incentive to deviate from the current quotation of their pricing and lead time decisions, given the other firms' decisions.

2.2.1. Preliminary

Consider a real single-valued scalar function $f(x)$ defined on some nonempty closed set $\mathbf{X}$ in the n-dimensional Euclidean space. Function $f(x)$ is assumed to be twice continuously differentiable on $\mathbf{X}$. Let $\nabla f(x)$ and $\nabla^2 f(x)$, respectively, denote the gradient and the Hessian matrix of f evaluated at x. A sufficient condition for function f to be pseudo-convex is given in Definition 1. Let $M(X,\beta)$ be the $n \times n$ matrix and let T denote the transpose operator.

$$M(X,\beta) = \nabla^2 f(x) + \beta \nabla f(x) \times \nabla f(x)^T, \tag{7}$$

where β is a nonnegative real number.

Definition 1. (see Mereau and Paquet, 1974) *A sufficient condition for $f(x)$ to be pseudo-convex on the convex set $\mathbf{X}$ is that there exists a real number β, $0 \le \beta < +\infty$, such that $M(X,\beta)$ is positive semi-definite.*

2.2.2. The solution algorithm

Our goal is to solve for the equilibrium solution of the competing firms in our model. We demonstrate the Karush–Kuhn–Tucker (KKT) approach to find the Nash equilibrium solution, defining the equilibrium as a set of decisions that satisfy each firm's first-order conditions (KKT) for profit maximization. The solution satisfying those conditions possesses the property that no firm wants to alter its decision unilaterally and is known as the Nash equilibrium solution (Hobbs, 2001). However, we note that KKT conditions are necessary optimality conditions for the local optimum in general, not sufficient conditions for the optimum. Therefore, to satisfy the properties of the Nash equilibrium, we need to solve the *globally sufficient* KKT conditions simultaneously for the equilibrium instead of solving the general *locally necessary* KKT conditions; otherwise, we need to examine all possible KKT points for the equilibrium solution. This leads to Lemmas 1 and 2 for the solution algorithm derivation.

Lemma 1. *Profit function* (3) *of firm $i \in N$ is pseudo-concave function.*

Proof. See Appendix.

Lemma 2. *The feasible region of constraints* (4)–(6) *is a convex set.*

Proof. See Appendix.

From Lemmas 1 and 2, we can conclude that the KKT optimality conditions to the problem (3)–(6) of firm $i \in N$ are both necessary and sufficient (Bazaraa et al., 1993). The KKT optimality conditions of firm $i \in N$ are stated as

$$\Omega_i = p_i \lambda_i - a_1(-(\mu_i - \lambda_i)t_i - \ln(1-s)) + a_2 p_i + a_3 t_i \tag{8}$$

$$\frac{\partial \Omega_i}{\partial p_i} = 0 \tag{9}$$

$$\frac{\partial \Omega_i}{\partial t_i} = 0 \tag{10}$$

$$a_1(-(\mu_i - \lambda_i)t_i - \ln(1-s)) = 0 \tag{11}$$

$$a_2 p_i = 0 \tag{12}$$

$$a_3 t_i = 0 \tag{13}$$

$$-(\mu_i - \lambda_i)t_i \le \ln(1-s) \tag{14}$$

$$p_i > 0 \tag{15}$$

$$t_i > 0 \tag{16}$$

$$a_1,\ a_2,\ a_3 \ge 0 \tag{17}$$

where a_1, a_2, and a_3 are dual variables to constraints (4) and (6). Constraint (8) is the Lagrangian function definition for the purpose of notational simplicity. Constraints (9), (10) and (17) are corresponded to dual feasibility equalities, (11)–(13) are complementary slackness conditions, and (14)–(16) are primal feasibility equalities. Similarly, we also derive the KKT conditions of the other competing firm. The equilibrium solution of prices and lead times can be obtained by simultaneously solving the combined KKT conditions. Since the KKT optimality conditions of the model presented in this paper are sufficient, any solution simultaneously satisfying the combined KKT optimality conditions is optimal to each firm. In other words, this solution follows the definition of the Nash equilibrium where no firm wishes to alter its decision unilaterally.

Observation 1 The dual variables a_2 and a_3 are equal to zero in the KKT optimality conditions.

Proof. Since we only focus on a nontrivial solution, we assume that the equilibrium solution of prices and lead times is a positive value. This allows us to simplify the KKT conditions by letting a_2 and a_3 be zero in (12) and (13). □

Observation 2 The dual variable a_1 is non-zero in the KKT optimality conditions.

Proof. From (10), we have $\frac{\partial \Omega_i}{\partial t_i} = -m_t \beta_t p_i - a_1(-\mu_i + \lambda_i - m_t \beta_t t_i) + a_3 = 0$. Obviously, m_t, β_t, and p_i are non-zero, and $a_3 = 0$ due to Observation 1. Thus, $a_1(-\mu_i + \lambda_i - m_t \beta_t t_i)$ is non-zero as well and it completes the proof. □

2.3. Demonstration

To illustrate the use of the KKT approach, we construct a simple and symmetric example of two leading firms, X and Y, and a group of smaller firms, M. We assume that the price and lead time of M are given: that is, $p_M = 10$ and $t_M = 5$. The customer arrival rate, λ_0, is 3 when X and Y have the same prices and lead times. The mean service rates of X and Y are $\mu_X = \mu_Y = 5$. The minimum service levels, s, for both firms are set at 95%. Other required parameters are $\alpha_M = .2$, $\alpha_C = .8$, $\beta_t = .5$, $\beta_p = .5$, and $m_t = 1$, $m_p = .5$. The KKT optimality conditions of firm X can be stated as

$$(4 - .5p_X + .2p_Y - .5t_X + .4t_Y) + .25a_1 t_X + a_2 = 0$$

$$-.5p_X - a_1(-1 - .25p_X + .2p_Y - t_X + .4t_Y) + a_3 = 0$$

$$a_1[(-1 - .25p_X + .2p_Y - .5t_X + .4t_Y)t_X + 3] = 0$$

$$a_2 p_X = 0$$

$$a_3 t_X = 0$$

$$(-1 - .25p_X + .2p_Y - .5t_X + .4t_Y)t_X \le -3$$

$$p_X, t_X > 0$$

$$a_1,\ a_2,\ a_3 \ge 0.$$

The KKT optimality conditions of firm Y can be stated as

$$(4 - .5p_Y + .2p_X - .5t_Y + .4t_X) + .25a_4 t_Y + a_5 = 0$$

$$-.5p_Y - a_4(-1 - .25p_Y + .2p_X - t_Y + .4t_X) + a_6 = 0$$

$$a_4[(-1 - .25p_Y + .2p_X - .5t_Y + .4t_X)t_Y + 3] = 0$$

$$a_5 p_Y = 0$$

$$a_6 t_Y = 0$$

$$(-1 - .25p_Y + .2p_X - .5t_Y + .4t_X)t_Y \le -3$$

$p_Y, t_Y > 0$

$a_4, a_5, a_6 \geq 0.$

Observation 1 allows us to simplify the combined KKT conditions by allowing a_2, a_3, a_5, and a_6 to be zero. We use the commercial package, Mathematica 7 (Wolfram, 2009), to solve the combined KKT conditions of X and Y for the equilibrium price and lead time; the equilibrium solution is

$$(p_X, p_Y, t_X, t_Y) = (16.67, 16.67, 1.51, 1.51),$$

and $a_1 = a_4 = 3.09$, $a_i = 0$, $i = 2, 3, 5$, and 6. The corresponding profits of X and Y are 50.24, respectively. Intuitively, the solution and profits are identical for the two firms in this symmetrical example.

2.4. Existence and uniqueness of equilibrium

Analyzing the required conditions for the existence and uniqueness of the equilibrium price and lead time of X and Y paves the way to solving the nonlinear system of KKT optimality conditions for the equilibrium solution. We first show a trivial observation for derivation purposes followed by two cases: homogeneous firms and heterogeneous firms.

Observation 3 Firm i determines the equilibrium price and lead time such that the equality of constraint (4) holds.

Proof. This result follows from Observation 2 and (11). □

2.4.1. Homogeneous firms

In the case of homogeneous firms, two competing firms have identical capacity, production technology, cost structure, quality, etc; in other words, $\mu = \mu_X = \mu_Y$. In addition, it is reasonable assuming that these two firms return the identical equilibrium decision of prices and lead times. For notational simplicity, let p and t denote the equilibrium price and lead time of the two homogeneous firms without the subscript index. From Observations 1 and 2, the variables of the combined KKT conditions are reduced to only (t, p, a_1) and the combined KKT conditions of the firms can be further reduced using (43), simplified as

$$u_0' - u_1 \alpha_M t - u_2(1+\alpha_M)p + a_1 u_2 t = 0 \tag{18}$$

$$-u_1 p - a_1(u_0' - u_1(1+\alpha_M)t - u_2 \alpha_M p - \mu) = 0 \tag{19}$$

$$(u_0' - u_1 \alpha_M t - u_2 \alpha_M p - \mu)t + u_5 = 0 \tag{20}$$

$$p, t, a_1 > 0 \tag{21}$$

Lemma 3. A unique positive solution exists to the equation $ax^3 + bx + c = 0$ when $ac < 0$ and b is a real number.

Proof. See Appendix. □

Proposition 1. If the two competing firms are homogenous, there exists a unique equilibrium solution of the price and lead time.

Proof. There are only three unknowns in (18)–(21) and we only focus on nontrivial positive solutions of prices and lead times. Rearranging (20), we have

$$p = \tfrac{1}{u_2 \alpha_M}\left[(u_0' - u_1 \alpha_M t - \mu) + \frac{u_5}{t}\right] \tag{22}$$

From (18) and (19), it is easy to have (23) without a_1

$$u_1 u_2 t p = [u_0' - u_1 \alpha_M t - u_2(1+\alpha_M)p]$$
$$[u_0' - u_1(1+\alpha_M)t - u_2 \alpha_M p - \mu] \tag{23}$$

Substituting (22) into (23), we have

$$-u_1 \mu t + u_5\left[\tfrac{u_0'}{\alpha_M} - (1+\tfrac{1}{\alpha_M})\mu\right]\tfrac{1}{t} + \left(1+\tfrac{1}{\alpha_M}\right)\tfrac{u_5^2}{t^2} = 0. \tag{24}$$

Multiplying t^2 on both sides, (24) can be represented as

$$-At^3 - Bt + C = 0 \tag{25}$$

where

$$A = u_1 \mu$$
$$B = -u_5\left[\tfrac{u_0'}{\alpha_M} - \left(1+\tfrac{1}{\alpha_M}\right)\mu\right]$$
$$C = \left(1+\tfrac{1}{\alpha_M}\right)u_5{}^2$$
$$A, C > 0.$$

Since $(-A)C < 0$ and from Lemma 3, a unique positive t exists such that (25) holds. In addition, (22) shows the one-to-one correspondence between t and p. As a result, a unique equilibrium price exists as well. □

2.4.2. Heterogeneous firms

In the case of heterogeneous firms, two competing firms do not have identical capacity, production technology, cost structure, quality, etc; in other words, $\mu_X \neq \mu_Y$ in this paper. With Observations 1, 2, and 3, the combined KKT optimality conditions for X and Y can be rewritten as

$$\frac{\partial \Omega_X}{\partial p_X} = u_0' - u_1 t_X - 2u_2 p_X + u_1 \alpha_C t_Y + u_2 \alpha_C p_Y + a_1 u_2 t_X = 0 \tag{26}$$

$$\frac{\partial \Omega_X}{\partial t_X} = -u_1 p_X - a_1(u_0' - \mu_X - 2u_1 t_X - u_2 p_X + u_1 \alpha_C t_Y + u_2 \alpha_C p_Y) = 0 \tag{27}$$

$$(u_0' - \mu_X - u_1 t_X - u_2 p_X + u_1 \alpha_C t_Y + u_2 \alpha_C p_Y)t_X + u_5 = 0 \tag{28}$$

$$\frac{\partial \Omega_Y}{\partial p_Y} = u_0' - u_1 t_Y - 2u_2 p_Y + u_1 \alpha_C t_X + u_2 \alpha_C p_X + a_4 u_2 t_Y = 0 \tag{29}$$

$$\frac{\partial \Omega_Y}{\partial t_Y} = -u_1 p_Y - a_4(u_0' - \mu_Y - 2u_1 t_Y - u_2 p_Y + u_1 \alpha_C t_X + u_2 \alpha_C p_X) = 0 \tag{30}$$

$$(u_0' - \mu_Y - u_1 t_Y - u_2 p_Y + u_1 \alpha_C t_X + u_2 \alpha_C p_X)t_Y + u_5 = 0 \tag{31}$$

$$a_1,\ a_4,\ t_X,\ t_Y,\ p_X,\ p_Y > 0 \tag{32}$$

Rearranging (28) and (31), the prices can be represented by the functions of lead times

$$p_X = \tfrac{1}{u_2(1-\alpha_C^2)}\left[u_0'(1+\alpha_C) - \mu_X - \alpha_C \mu_Y\right] - \tfrac{u_1 t_X}{u_2} + \tfrac{u_5}{u_2(1-\alpha_C^2)t_X} + \tfrac{\alpha_C u_5}{u_2(1-\alpha_C^2)t_Y} \tag{33}$$

$$p_Y = \tfrac{1}{u_2(1-\alpha_C^2)}\left[u_0'(1+\alpha_C) - \mu_Y - \alpha_C \mu_X\right] - \tfrac{u_1 t_Y}{u_2} + \tfrac{u_5}{u_2(1-\alpha_C^2)t_Y} + \tfrac{\alpha_C u_5}{u_2(1-\alpha_C^2)t_X} \tag{34}$$

By algebraic manipulations, X's KKT optimality conditions, (26)–(28), can be represented as

$$\frac{u_5^2(2-\alpha_C^2)}{t_X^2(1-\alpha_C^2)} - u_1 \mu_X t_X + \tfrac{u_5}{t_X t_Y(1-\alpha_C^2)} u_5 \alpha_C$$
$$+ \tfrac{u_5 t_Y}{t_X t_Y(1-\alpha_C^2)}\left[u_0'(1+\alpha_C) - \mu_X(2-\alpha_C^2) - \alpha_C \mu_Y\right] = 0 \tag{35}$$

Similarly, Y's KKT optimality conditions, (29)–(31), can be represented as

$$\frac{u_5^2(2-\alpha_C^2)}{t_Y^2(1-\alpha_C^2)} - u_1 \mu_Y t_Y + \tfrac{u_5}{t_X t_Y(1-\alpha_C^2)} u_5 \alpha_C$$
$$+ \tfrac{u_5 t_X}{t_X t_Y(1-\alpha_C^2)}\left[u_0'(1+\alpha_C) - \mu_Y(2-\alpha_C^2) - \alpha_C \mu_X\right] = 0. \tag{36}$$

For notational simplicity, (35) and (36) can be rewritten as

$$E - F_1 t_X^3 + G\tfrac{t_X}{t_Y} + H_1 t_X = 0 \tag{37}$$

$$E - F_2 t_Y^3 + G\frac{t_Y}{t_X} + H_2 t_Y = 0. \tag{38}$$

where

$E = \frac{u_5^2(2-\alpha_C^2)}{1-\alpha_C^2}$

$F_1 = u_1 \mu_X$

$F_2 = u_1 \mu_Y$

$G = \frac{u_5^2 \alpha_C}{1-\alpha_C^2}$

$H_1 = \frac{u_5}{1-\alpha_C^2}\left[u_0'(1+\alpha_C) - \mu_X(2-\alpha_C^2) - \alpha_C \mu_Y\right]$

$H_2 = \frac{u_5}{1-\alpha_C^2}\left[u_0'(1+\alpha_C) - \mu_Y(2-\alpha_C^2) - \alpha_C \mu_X\right].$

Up to this point, the combined KKT optimality conditions of the two heterogeneous competing firms are represented as two equations with two unknowns, t_X and t_Y, as shown in (37) and (38). We can now analyze the equations plotted in a $t_X t_Y$-plane composed of horizontal axis t_X and vertical axis t_Y. Rearranging (37), we have

$$t_Y = \frac{G t_X}{F_1 t_X^3 - H_1 t_X - E} \tag{39}$$

From Lemma 3, there is a unique positive solution, t_X^A, to $F_1 t_X^3 - H_1 t_X - E = 0$ since $F_1(-E) < 0$. As t_X approaches t_X^A, the denominator of (39) approaches zero. Therefore, we have $\lim_{t_X \to t_X^A} t_Y = \lim_{t_X \to t_X^A} (G t_X / F_1 t_X^3 - H_1 t_X - E) = \infty$. As t_X approaches infinity, t_Y approaches zero by L'Hospital Rule (Salas et al., 2003, pp. 615–616). Thus line $t_X = t_X^A$ and axis t_X are asymptotic to (37) on $t_X t_Y$-plane. Similarly, line $t_Y = t_Y^A$ and axis t_Y are asymptotic to (38) on $t_X t_Y$-plane where t_Y^A is the solution to $F_2 t_Y^3 - H_2 t_Y - E = 0$.

In addition, the first derivative of (39) with respect to t_X is $-G(2F_1 t_X^3 + E)/\left(F_1 t_X^3 - H_1 t_X - E\right)^2$, where G, F_1, E, and t_X are positive. As a result, $(\partial t_Y / \partial t_X) < 0$ and it implies that (39) decreases in t_X. We next examine concavity or convexity of (39) on $t_X t_Y$-plane. Taking the second derivative of (39) with respect to t_X, we have

$$\frac{\partial^2 t_Y}{\partial t_X^2} = \frac{2G[E(6F_1 t_X^2 - H_1) + F_1 t_X^3 (3F_1 t_X^2 + H_1)]}{(F_1 t_X^3 - H_1 t_X - E)^3} \tag{40}$$

Observation 4 $F_1 t_X^3 - H_1 t_X - E > 0$ and $F_2 t_Y^3 - H_2 t_Y - E > 0$ when $t_X > 0$ and $t_Y > 0$.

Proof. See Appendix.

From Observation 4, the denominator of (40) is positive. With $G > 0$, the sign of $\frac{\partial^2 t_Y}{\partial t_X^2}$ can be determined by inspecting the sign of $E(6F_1 t_X^2 - H_1) + F_1 t_X^3 (3F_1 t_X^2 + H_1)$. For notational simplicity, we let $T(t_X) = E(6F_1 t_X^2 - H_1) + F_1 t_X^3 (3F_1 t_X^2 + H_1)$. We discuss the sign of $T(t_X)$ in the following two disjunctive cases: $H_1 \geq 0$ and $H_1 < 0$.

Proposition 2. As $H_1 \geq 0$, $T(t_X)$ is positive for all $t_X > 0$.

Proof. See Appendix.

Proposition 3. $H_1 < 0$, $T(t_X)$ is positive for all $t_X > 0$ when $1/6H_1\sqrt{(-H_1/6F_1)} + 4E > 0$, but $T(t_X)$ is negative for some $t_X > 0$ when $1/6H_1\sqrt{(-H_1/6F_1)} + 4E < 0$.

Proof. See Appendix.

From Propositions 2 and 3, $\partial^2 t_Y / \partial t_X^2$ is positive for all $t_X > 0$ when $H_1 \geq 0$, or when $H_1 < 0$ and $1/6H_1\sqrt{(-H_1/6F_1)} + 4E > 0$. Thus, (37) on $t_X t_Y$-plane is convex when $H_1 \geq 0$, or when $H_1 < 0$ and $1/6H_1\sqrt{(-H_1/6F_1)} + 4E > 0$. Similarly, (38) on $t_X t_Y$-plane is convex when $H_2 \geq 0$, or when $H_2 < 0$ and $1/6H_2\sqrt{(-H_2/6F_2)} + 4E > 0$. We let SS denote

$$\left\{ \begin{pmatrix} E, F_1, F_2, \\ G, H_1, H_2 \end{pmatrix} \middle| \begin{array}{l} E > 0, F_1 > 0, F_2 > 0, G > 0, \\ H_1 > 0 \cup \left\{ H_1 < 0,\ \frac{1}{6} H_1 \sqrt{\frac{-H_1}{6F_1}} + 4E > 0 \right\}, \\ H_2 > 0 \cup \left\{ H_2 < 0,\ \frac{1}{6} H_2 \sqrt{\frac{-H_2}{6F_2}} + 4E > 0 \right\} \end{array} \right\} \tag{41}$$

Proposition 4. There exists a unique equilibrium solution of prices and lead time if the parameters, E, F_1, F_2, G, H_1, and H_2, of two competing firms satisfy (41).

Proof. From Propositions 2 and 3, (37) and (38) on $t_X t_Y$-plane are convex when $(E, F_1, F_2, G, H_1, H_2) \in SS$. It is trivial to argue that the slopes of (37) and (38) are different by inspecting the first derivative. With the asymptotic lines of (37) and (38), the two equations can only cross once and the result follows. □

3. Sensitivity analysis: A case study

Our case study is designed to examine the behavior of the equilibrium price and lead time as predicted by our model when different firms compete under varying market conditions. We predict the required parameters in our model based on available public data for two leading semiconductor foundry manufacturers in Taiwan. Without implying identity, X and Y denote the two large firms.

3.1. Case study data

Our case study is based upon timely representative data for the semiconductor foundry manufacturing industry. We note that the data will differ for other industry sectors, geographic regions, and/or time epochs and the case study results may alter depending on different data sets.

The average quarterly shipment for 200-mm wafer products per firm in 2007 is 722.5 thousand (K) (Science & Technology Policy Research and Information Center (STPI), 2008). This ballpark number allows us to predict the customer arrival rate, λ_0, when both prices and lead times of all firms in the market are identical, since the demand of a duopolist is roughly close to the market average per firm due to the nearly half of weighting in the average for duopoly firms. Therefore, we let λ_0 equal 722.5K pieces per quarter in the case study. From public financial information, we know that the quarterly shipments of X and Y in 2007 are about 2.001 million (M) and 1.077M pieces, respectively; this data allow us to estimate the ballpark number of firms' service rates. We intentionally set an identical service rate of the duopoly firms to leave room for conducting the sensitivity analysis of the service rates. The service rate of the firms is quoted at the average quarterly shipments of X and Y, i.e., $\mu_X = \mu_Y = 1.54$M pieces per quarter. The total market share of X and Y in Taiwan is 68% with smaller firms accounting for approximately 32% (IDC, 2008). We refer to the total market share of X and Y as the preference factor, i.e., $\alpha_C = .68$ and consequently, $\alpha_M = .32$, since a high market share firm potentially has a larger impact on the decision difference for the demand change and vice versa for the smaller firms. We assume an equal preference factor of lead times and prices; in other words, $\beta_t = \beta_p = .5$. In addition, we assume that the two firms' minimum service levels s_X and s_Y are both .95. The average selling price (ASP) of finished wafer products is US \$1000 (GS, 2008). The production lead time in the foundry industry ranges from 30 to 50 day and the number, 36-day, is chosen for prediction of m_t and m_p. We note that different products may have different production lead times. The purpose of the data set presented in this section is to predict a reasonable data set of m_t and m_p from the available public information for our later use in the sensitivity analysis. Our data set allows us to roughly predict and round m_t=180,000 and m_p=15,000 such that the available data satisfy the KKT optimality conditions of (8)–(17) along with Observations 1 and 2.

3.2. Impact of firm characteristics

To study the effects of each firm's characteristics of service rate and service level on the equilibrium prices, lead times, and firm profits, we consider X and Y with the parameters proposed in Section 3.1, but only vary X's service rate (μ_X) and service level (s_X), respectively. We compute the ratios of their corresponding equilibrium prices, lead times, and profits to observe how the equilibrium solution changes when only X adjusts its parameters. The results are given in Fig. 1 and the ballpark number of profits can be found in Fig. 2(b) and Fig. 3(b).

In Fig. 1(a), X's service rate (μ_X) varies from 924, 1232, 1540, 1848, to 2156K pieces per quarter and Y's service rate (μ_Y) remains the same (μ_Y=1540). In a similar setting in Fig. 1(b), X's service level (s_X) varies from 0.9, 0.925, 0.95, 0.975, to 0.99 while firm Y's service level (s_Y) is 0.95. We summarize the major observations as follows:

(i) The ratios of the equilibrium prices remain almost the same; the ratios of the equilibrium lead times decrease; the ratios of the corresponding profits increase as the ratios of the service rate increase. The trend of the ratios of the corresponding profits is not surprising, but it is worth further examination of the relationship between predicted equilibrium prices and lead times. An increase in the service rates implies an increase in capacity. This observation allows us to infer that a firm appears not to raise its price due to competitiveness as its capacity increases; otherwise, the firm may lose market share. Meanwhile, an increase in capacity enables a firm to reduce its lead time so that it can attract more demand to increase its corresponding profit.

(ii) The ratios of the corresponding profits and equilibrium prices remain almost the same; the ratios of the equilibrium lead times increase as the ratios of the service levels increase. An increase in service levels implies a more conservative perspective in quoted lead times. In other words, management tends to quote a long lead time so the firm can easily satisfy the requirement of a high service level (defined as the probability that the total production cycle time is less than or equal to the quoted lead time). From the numerical results, an increase in lead times results in a decrease in the selling price. However, due to competitiveness, the other firm tends to reduce its selling price as well at the equilibrium. As a result, the ratios of both firms' corresponding profits and equilibrium prices remain almost the same even though their profits and selling prices decrease as one firm's service level increases. This shows that the ability to offer a higher service level does not benefit the firm with the higher service level.

3.3. Impact of preference factors of firms

We next study how the preference factors affect the equilibrium prices, lead times and profits. We consider different values of α_M, α_C, β_t, and β_p such that $\alpha_M+\alpha_C=1$ and $\beta_t+\beta_p=1$. Again,

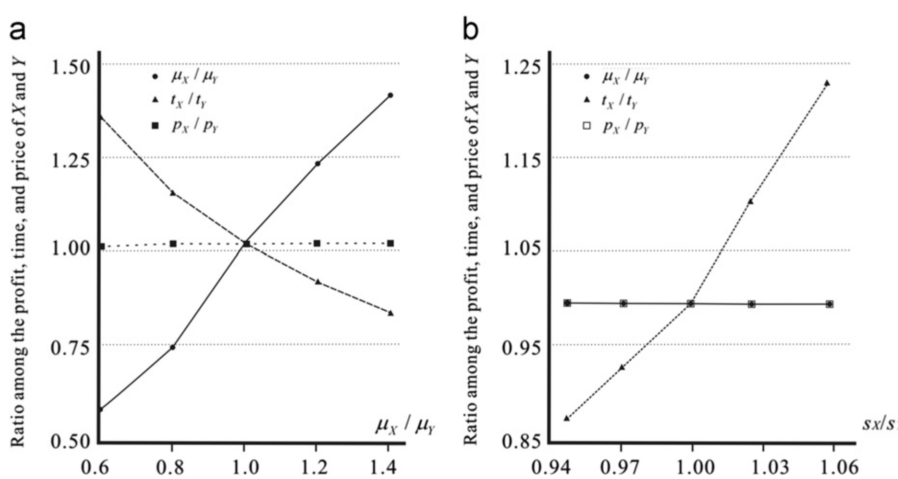

Fig. 1. Impact of service rate (μ_X) and service level (s_X).

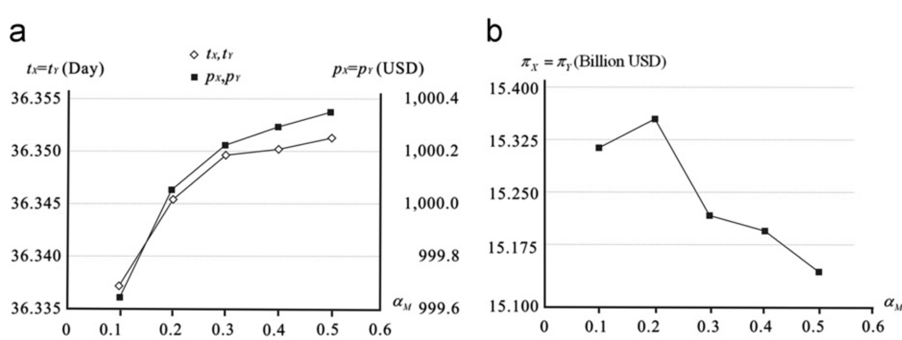

Fig. 2. Impact of preference factor (α_M).

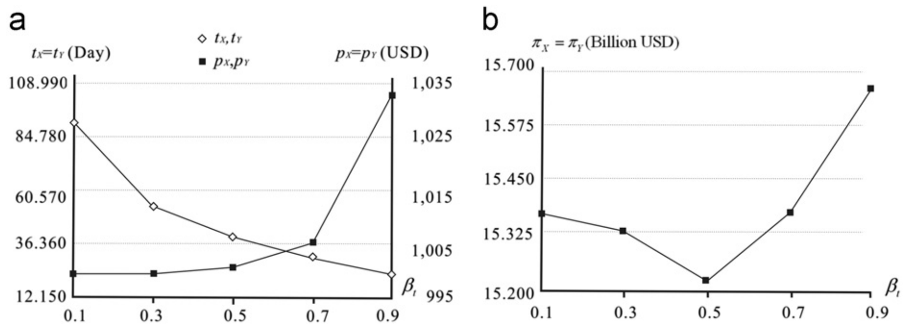

Fig. 3. Impact of preference factor (β_t).

other parameters remain the same as proposed in Section 3.1; the results are shown in Figs. 2 and 3.

The factor, α_M, can be interpreted as the preference measure accounting for the effect of the decision difference by the smaller firms. As α_M increases, the impact of the decision difference between the large firm and the smaller firms on the customer arrival rate increases. Meanwhile, as α_M increases, the impact of the decision difference between the two large firms on the customer arrival rate decreases. We note this is a symmetric case where both X and Y share the same parameters; therefore, X and Y have identical equilibrium prices, lead times, and resulting profits. Fig. 2 presents the trend of the equilibrium price, lead time, and profit as α_M increases. Only several discrete data sets have been investigated in the case study so that the trend may behave in a non-smooth manner. Clearly, both firms' equilibrium prices and lead times increase in α_M and their corresponding profits decline after α_M=.2. A large value of α_M indicates a small value of α_C, which represents a smaller impact of the decision difference between the two large firms on the customer arrival rate. Firms X and Y would not pay much attention to the competition between them; hence, both the equilibrium price and lead time increase as α_M increase. However, the results shown in Fig. 2 indicate that both an increase in the price and lead time do not necessarily reduce the profitability of both firms depending on the level of α_M.

The factors, β_t and β_p, can be interpreted as the preference measures accounting for the effect of the lead times and prices on the arrival rate, respectively. As β_t increases, the impact of lead time decisions on the customer arrival rate increases; meanwhile, the impact of price decisions on the customer arrival rate decreases. Similarly, X and Y have identical equilibrium prices, lead times, and resulting profits in the symmetric case. Fig. 3 presents the trend of the equilibrium price, lead time, and profit as β_t increases. Again, only several discrete data sets have been investigated in the case study so that the trend may behave in a non-smooth manner. It is clear that the equilibrium lead times of X and Y decrease, but the equilibrium prices increase in β_t. Thus, if firms pay more attention to the lead time decision, but less attention to the price decision, the results are low equilibrium lead times and high prices. The corresponding profits increase after β_t=.5; hence, the trend of profitability is not determined based on the value of β_t or β_p.

4. Conclusions

This paper has examined the equilibrium pricing and lead time decisions in a duopoly industry consisting of two leading firms and a group of smaller firms all competing to provide goods or services to customers. We consider each firm as a system that behaves as an M/M/1 queue. The objective function of each firm depends on its own decision variables as well as on the decision variables of the competition. All firms attempt to maximize their profits by making decisions about the price and lead time subject to the constraints needed to satisfy the minimum required service level which is defined as the probability of meeting the promised lead time quotation.

We solve the combined KKT conditions for the equilibrium decision of the price and the lead time. In equilibrium, no firm wishes to deviate from its current decision given the others' decisions. The equilibrium solution is obtained by simultaneously solving the sufficient KKT optimality conditions instead of the necessary conditions. Our model shows the sufficiency of the KKT optimality conditions so that the solution to the KKT conditions is indeed an equilibrium decision rather than an examination of all possible KKT points for the equilibrium. Thus, we characterize the analytical condition of the existence and uniqueness of the Nash equilibrium of price and lead time decisions for both homogenous and heterogeneous cases.

The case study of two leading semiconductor foundry manufacturers and several smaller firms examines the behavior of the equilibrium price and lead time as the firms compete under varying market conditions. The results produce some helpful managerial insights. A unilateral increase in one firm's capacity does not appear to raise its price due to competitiveness, but instead tends to reduce the lead time to attract more demands. The ability to offer a higher service level does not automatically guarantee a benefit to the firm with the higher service level. The equilibrium prices and lead times of the two large firms increase when they pay less attention to the competition between them, but it does not necessarily imply reduced profitability for either firm. Likewise, high prices and low lead times result when firms pay more attention to their lead time decisions, and less to the price decisions.

We suggest that three possible extensions to our prototypical duopoly model are worth investigation. First, since we base our model (and results) on a linear structure of the customer arrival function, it would be useful to extend the model to other structures of the customer arrival rate. Second, the service level and the preference measures are assumed as given and fixed in our model. Additional modeling should consider the service level or preference measures as decision variables that can also affect the customer arrival rate. Third, noting that unsatisfied orders may cause a significant loss in profits due to potential penalties, both penalty design and policy analysis are fruitful topics for further consideration.

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Appendix

Proof of Lemma 1. For notational simplicity, we rewrite the arrival rate λ_i as

$$\begin{aligned}&\lambda_i(p_i,t_i|p_j,t_j,p_M,t_M)\\&=\lambda_0-m_t\beta_t[\alpha_M(t_i-t_M)+\alpha_C(t_i-t_j)]-m_p\beta_p[\alpha_M(p_i-p_M)+\alpha_C(p_i-p_j)]\\&=u'_0-u_1t_i-u_2p_i+u_3t_j+u_4p_j \end{aligned}\tag{42}$$

where

$$\begin{aligned}&u'_0=\lambda_0+\alpha_M\beta_tm_tt_M+\alpha_M\beta_pm_pp_M\\&u_1=\beta_tm_t\\&u_2=\beta_pm_p\\&u_3=\alpha_C\beta_tm_t\\&u_4=\alpha_C\beta_pm_p\\&u_5=-\ln(1-s)\\&u_1,\ u_2,\ u_3,\ u_4,u_5>0.\end{aligned}\tag{43}$$

By inserting (42) into (4) and rearranging it, we have

$$(u_0-u_1t_i-u_2p_i)t_i\le -u_5 \tag{44}$$

where

$$u_0=u'_0+u_3t_j+u_4p_j-\mu_i.$$

Proving that the profit function of firm i, π_i, is pseudo-concave is equivalent to showing that $\pi'_i=-\pi_i$ is pseudo-convex. Since π'_i is twice continuously differentiable, the gradient, $\nabla(\pi'_i)$, and Hessian matrix, $\nabla^2(\pi'_i)$, of π'_i can be computed as

$$\nabla(\pi'_i)^T=\begin{bmatrix}\frac{\partial\pi'_i}{\partial p_i} & \frac{\partial\pi'_i}{\partial t_i}\end{bmatrix}=\begin{bmatrix}-\lambda_i+u_2p_i & u_1p_i\end{bmatrix},$$

and

$$\nabla^2(\pi'_i)=\begin{bmatrix}\frac{\partial^2\pi'_i}{\partial p_i^2} & \frac{\partial^2\pi'_i}{\partial p_i\partial t_i}\\ \frac{\partial^2\pi'_i}{\partial t_i\partial p_i} & \frac{\partial^2\pi'_i}{\partial t_i^2}\end{bmatrix}=\begin{bmatrix}2u_2 & u_1\\ u_1 & 0\end{bmatrix}.$$

Rearranging (7), we have

$$\begin{aligned}M(X,\beta)&=\begin{bmatrix}2u_2 & u_1\\ u_1 & 0\end{bmatrix}+\beta\begin{bmatrix}(\lambda_i-u_2p_i)^2 & -u_1p_i(\lambda_i-u_2p_i)\\ -u_1p_i(\lambda_i-u_2p_i) & (u_1p_i)^2\end{bmatrix}\\&=\begin{bmatrix}2u_2+\beta(\lambda_i-u_2p_i)^2 & u_1-\beta u_1p_i(\lambda_i-u_2p_i)\\ u_1-\beta u_1p_i(\lambda_i-u_2p_i) & \beta(u_1p_i)^2\end{bmatrix}.\end{aligned}$$

The determinant of $M(X,\beta)$ is $u_1^2(2\beta p_i\lambda_i-1)$. Since we only focus on a nontrivial solution of the equilibrium price and lead time, it is reasonable assuming that lower and upper bounds exist for the prices and lead times of firm $i\in N$. We let $\underline{p_i}$, $\overline{p_i}$, $\underline{t_i}$, and $\overline{t_i}$ denote these lower and upper bounds of prices and lead times, respectively. Let $\beta=1/2\phi$ where $\varphi=\underline{p_i}\underline{\lambda_i}$ and $\underline{\lambda_i}=u_0-u_1\overline{t_i}-u_2\overline{p_i}+u_3t_j+u_4p_j$. It is obvious that ϕ is a lower bound of the profit function of firm i since the customer arrival rate has the highest value for the variable with negative coefficients and the lowest value for the variable with positive coefficients. As a result, the determinant of $M(X,\beta)$ is positive for such β. In addition, the diagonal elements of $M(X,\beta)$ are nonnegative. This gives the result that there exists a real number β, $0\le\beta<+\infty$, such that $M(X,\beta)$ is positive semi-definite. Following Definition 1, π'_i is a pseudo-convex function. □

Proof of Lemma 2. The feasible region C of constraints (4)–(6) can be represented as

$$C=\{(p_i,t_i)|p_i>0,t_i>0,\ \text{and}(u_0-u_1t_i-u_2p_i)t_i\le -u_5\}$$

Let $\vec{z}_1=(p_1,t_1)\in C$ and $\vec{z}_2=(p_2,t_2)\in C$. Consider the point $\vec{z}=(p,t)=\alpha\vec{z}_1+(1-\alpha)\vec{z}_2$, $0\le\alpha\le 1$. Because $p_1>0$ and $p_2>0$, it is obvious that $p=\alpha p_1+(1-\alpha)p_2>0$. Similarly, $t>0$. These show that (6) holds for $\vec{z}=(p,t)$.

Constraint (4) can be represented as (44). For $\vec{z}_1$ and $\vec{z}_2$, we have

$$u_0-u_1t_1-u_2p_1\le -u_5/t_1 \tag{45}$$

and

$$u_0-u_1t_2-u_2p_2\le -u_5/t_2. \tag{46}$$

Considering $\vec{z}=(p,t)$, the left side of (44) can be rewritten as

$$\begin{aligned}(u_0-u_1t-u_2p)t=&[\alpha(u_0-u_1t_1-u_2p_1)\\&+(1-\alpha)(u_0-u_1t_2-u_2p_2)](\alpha t_1+(1-\alpha)t_2).\end{aligned}$$

Because of (45) and (46), and the Cauchy inequality (see Bartle, 1976), we have

$$\begin{aligned}(u_0-u_1t-u_2p)t&\le\left(\alpha\frac{-u_5}{t_1}+(1-\alpha)\frac{-u_5}{t_2}\right)(\alpha t_1+(1-\alpha)t_2)\\&=-u_5\left(\alpha^2+(1-\alpha)^2+\alpha(1-\alpha)\left(\frac{t_2}{t_1}+\frac{t_1}{t_2}\right)\right)\\&\le -u_5(\alpha^2+(1-\alpha)^2+2\alpha(1-\alpha))\\&=-u_5.\end{aligned}$$

Therefore

$$(u_0-u_1t-u_2p)t\le -u_5. \tag{47}$$

Due to (47), $\vec{z}=(p,t)$ also satisfies (4) and (5). In summary, for $\vec{z}_1=(p_1,t_1)\in C$ and $\vec{z}_2=(p_2,t_2)\in C$, we show that the point $\vec{z}=(p,t)=\alpha\vec{z}_1+(1-\alpha)\vec{z}_2$, $0\le\alpha\le 1$ satisfies constraints (4)–(6). It completes the proof.

Proof of Lemma 3. There are two situations such that $ac<0$: $a<0$, $c>0$ and $a>0$, $c<0$. We first examine the former case when $a<0$ and $c>0$. Let function $f(x)$ be ax^3+bx+c, which is obviously continuous as $x\ge 0$. The first and second derivatives of f are $f'(x)=3ax^2+b$ and $f''(x)=6ax$. Function f is a concave function as $x>0$. If $b\le 0$, f is a decreasing function when x is positive. In addition, we have $f(0)=c>0$. As a result, f definitely intersects axis x at a sufficiently large value of x. In other words, there exists a unique positive solution to $ax^3+bx+c=0$ if $b\le 0$.

We then analyze the case when $b>0$. There are two points such that the first derivative of f equals 0; that is, $x=\pm\sqrt{(-b/3a)}$. Again, we have $f(0)=c>0$. Function f increases within the range $\left[0,\sqrt{(-b/3a)}\right]$, but decreases when $x>\sqrt{(-b/3a)}$. With the concavity property, f intersects axis x at a sufficiently large value of x as well. A similar argument follows for the latter case when $a>0$, $c<0$ and it completes the proof. □

Proof of Observation 4. Rearranging (37), we have $t_Y=(Gt_X/F_1t_X^3-H_1t_X-E)>0$. The inequality follows since $G>0$. Similarly, $F_2t_Y^3-H_2t_Y-E>0$ follows. □

Proof of Proposition 2. We discuss the sign of $T(t_X)$ in the following two disjunctive cases: $H_1=0$ and $H_1>0$. When $H_1=0$, it is obvious that $T(t_X)$ is positive since all other terms are positive. We then examine the case when $H_1>0$. Let function $B(t_X)$ be $F_1t_X^3-H_1t_X-E$. One can easily distinguish the critical points (local maximum or minimum) of $B(t_X)$ since $B(t_X)$ is a continuous and twice differentiable function. As a result, $B(t_X)$ increases as $t_X<-\sqrt{(H_1/3F_1)}$, $B(t_X)$ decreases as $-\sqrt{(H_1/3F_1)}\le t_X<$

$\sqrt{(H_1/3F_1)}$, and $B(t_X)$ increases as $t_X \geq \sqrt{\frac{H_1}{3F_1}}$. In addition, we have $B(0)=-E<0$, and $B(t_X)$ needs to be positive from Observation 4. Combining these, we argue that the feasible region of t_X is greater than $\sqrt{(H_1/3F_1)}$; namely, $3F_1t_X^2-H_1>0$, and leading to the result that $6F_1t_X^2-H_1>0$. Under this inequality, we have $T(t_X)>0$ as $H_1>0$. □

Proof of Proposition 3. To inspect the locations of critical points of $T(t_X)$, we take the first and second derivatives of $T(t_X)$ as shown below

$$T'(t_X)=3F_1t_X(5F_1t_X^3+H_1t_X+4E)$$

$$T''(t_X)=6F_1(10F_1t_X^3+H_1t_X+2E).$$

For notational simplicity, we let $P(t_X)$ denote $5F_1t_X^3+H_1t_X+4E$. One can easily distinguish the critical points of $P(t_X)$ since $P(t_X)$ is a continuous and twice differentiable function. As a result, $P(t_X)$ increases as $t_X<-\sqrt{(-H_1/15F_1)}$, $P(t_X)$ decreases as $-\sqrt{(-H_1/15F_1)} \leq t_X < \sqrt{(-H_1/15F_1)}$, and $P(t_X)$ increases as $t_X \geq \sqrt{(-H_1/15F_1)}$. We discuss the sign of $T(t_X)$ in the following two disjunctive cases: (i) $(2/3)H_1\sqrt{(-H_1/15F_1)}+4E>0$, and (ii) $(2/3)H_1\sqrt{(-H_1/15F_1)}+4E<0$.

(i) $\frac{2}{3}H_1\sqrt{\frac{-H_1}{15F_1}}+4E>0$:

We have $P(0)=4E>0$. The minimum of $P(t_X)$ for all $t_X>0$ is at $t_X=\sqrt{(-H_1/15F_1)}$ and with the value of $P(\sqrt{(-H_1/15F_1)})$, which is equal to $(2/3)H_1\sqrt{(-H_1/15F_1)}+4E$. Trivially, under condition (i), $P(t_X)$ is positive for all $t_X>0$; namely, $T'(t_X)>0$ for all $t_X>0$. Therefore, $T(t_X)$ increases as $t_X>0$. In addition, we have $T(0)=-EH_1>0$. Thus, $T(t_X)>0$ for all $t_X>0$.

(ii) $\frac{2}{3}H_1\sqrt{\frac{-H_1}{15F_1}}+4E<0$:

Under condition (ii), the local minimum of $P(t_X)$ at $t_X=\sqrt{(-H_1/15F_1)}$ is negative. In addition, $P(0)=4E>0$ and $P(t_X)$ is an increasing and convex function as $t_X>\sqrt{(-H_1/15F_1)}$. One can argue that $P(t_X)=0$ for $t_X>0$ has two roots denoted by t_{X_1} and t_{X_2}, where

$$0<t_{X_1}<\sqrt{\frac{-H_1}{15F_1}} \tag{48}$$

$$t_{X_2}>\sqrt{\frac{-H_1}{15F_1}} \tag{49}$$

At $t_X=t_{X_1}$ and $t_X=t_{X_2}$, $T(t_X)$ is in its local maximum and minimum since $T''(t_{X_1})<0$ and $T''(t_{X_2})>0$. Since $T(t_{X_2})$ is the minimum point when $t_X>0$, thus, $T(t_X)$ is positive for all $t_X>0$ if $T(t_{X_2})$ is positive. This allows us to ignore the case that $t_X=t_{X_1}$. We then examine the sign of $T(t_{X_2})$ in the following by inspecting two disjunctive cases: (ii-a) $\frac{H_1}{6}\sqrt{(-H_1/6F_1)}+4E<0$ and (ii-b) $(H_1/6)\sqrt{(-H_1/6F_1)}+4E>0$.

(ii-a) $\frac{H_1}{6}\sqrt{\frac{-H_1}{6F_1}}+4E<0$:

Under case (ii-a), $P(\sqrt{(-H_1/6F_1)})=(H_1/6)\sqrt{(-H_1/6F_1)}+4E<0$. In addition, since $P(t_{X_2})=0$, we have $P(\sqrt{(-H_1/6F_1)})$ $P(t_{X_2})$. As mentioned earlier, $P(t_X)$ increases as $t_X>\sqrt{(-H_1/15F_1)}$. From (49) and the obvious inequality $\sqrt{(-H_1/6F_1)}>\sqrt{(-H_1/15F_1)}$, both t_{X_2} and $\sqrt{(-H_1/6F_1)}$ are greater than $\sqrt{(-H_1/15F_1)}$. Within this range, $P(t_X)$ is increasing in t_X. Therefore

$$t_{X_2}>\sqrt{\frac{-H_1}{6F_1}} \tag{50}$$

Again, since $P(t_{X_2})=0$, we have

$$5F_1t_{X_2}^3+H_1t_{X_2}+4E=0 \tag{51}$$

Substituting (51) into $T(t_{X_2})$, we have

$$T(t_{X_2})=\frac{t_{X_2}}{4}(H_1+6F_1t_{X_2}^2)(H_1-3F_1t_{X_2}^2). \tag{52}$$

Since $H_1<0, H_1-3F_1t_{X_2}^2<0$. Since $t_{X_2}>\sqrt{(-H_1/6F_1)}$ and F_1 is positive, we have $H_1+6F_1t_{X_2}^2>0$. Combining the above inequalities, therefore, $\mathrm{T}(t_{X_2})<0$. Thus, $T(t_X)$ is negative for some $t_X>0$ when $(2/3)H_1\sqrt{(-H_1/15F_1)}+4E<0$ and $(1/6)H_1\sqrt{(-H_1/6F_1)}+4E<0$.

(ii-b) $\frac{H_1}{6}\sqrt{\frac{-H_1}{6F_1}}+4E>0$:

Under case (ii-b), $P(\sqrt{(-H_1/6F_1)})=\frac{H_1}{6}\sqrt{(-H_1/6F_1)}+4E>0$. In addition, since $P(t_{X_2})=0$, we have $P(\sqrt{(-H_1/6F_1)})>P(t_{X_2})$. As mentioned earlier, $P(t_X)$ increases as $t_X>\sqrt{(-H_1/15F_1)}$. It is similar to case (a), where both t_{X_2} and $\sqrt{(-H_1/6F_1)}$ are greater than $\sqrt{(-H_1/15F_1)}$. Within this range, $P(t_X)$ is increasing in t_X. Therefore

$$t_{X_2}<\sqrt{\frac{-H_1}{6F_1}} \tag{53}$$

Since $t_{X_2}<\sqrt{(-H_1/6F_1)}$ and F_1 is positive, we have $H_1+6F_1t_{X_2}^2<0$. Again, $H_1-3F_1t_{X_2}^2<0$ since $H_1<0$. Therefore, the sign of $T(t_{X_2})$ is positive. Thus, $T(t_X)$ is positive for all $t_X>0$ when $(2/3)H_1\sqrt{(-H_1/15F_1)}+4E<0$ and $(1/6)H_1\sqrt{(-H_1/6F_1)}+4E>0$.

To summarize the above proof for clarification purposes, from (i), we have shown that $T(t_X)>0$ for all $t_X>0$ when $(2/3)H_1\sqrt{(-H_1/15F_1)}+4E>0$. There are two disjunctive cases in (ii) in which we state that $T(t_X)$ is negative for some $t_X>0$ when $(2/3)H_1\sqrt{(-H_1/15F_1)}+4E<0$ and $(1/6)H_1\sqrt{(-H_1/6F_1)}+4E<0$, but $T(t_X)$ is positive for all $t_X>0$ when $(2/3)H_1\sqrt{(-H_1/15F_1)}+4E<0$ and $(1/6)H_1\sqrt{(-H_1/6F_1)}+4E>0$. In addition, the range of $(2/3)H_1\sqrt{(-H_1/15F_1)}+4E>0$ is contained in the range of $(1/6)H_1\sqrt{(-H_1/6F_1)}+4E>0$. Hence, it completes the proof.

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