

0890-6955(94)00113-8

ANALYSIS OF MINIMUM CLAMPING FORCE

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(Received 11 February 1994; in final form 27 September 1994)

Abstract—Our objective is to determine the minimum clamping force that keep the workpiece stable during the metal cutting process. In analyzing the stability of the workpiece, one usually proves that no instant center of motion can occur on the clamping plane. However, previous search algorithms for the instant center of motion either lack theoretical sufficiency or computational efficiency. This paper presents a new method derived from the correlation between cutting force and clamping moment. This method increases the search efficiency by pruning inadequate search directions. In addition, examples are provided to illustrate minimum clamping force analysis under different fixturing conditions.

NOMENCLATURE

| | |
|----------------|--|
| D | centrode of the distributed clamping friction force |
| F_c | cutting force vector |
| $(F_c)_{\max}$ | maximum allowable cutting force |
| F_N | clamping force (or clamping force in brief) |
| $F_{N,D}$ | total clamping force of a clamping region, applied at its centrode D |
| F_f | (maximum) clamping friction force |
| $F_{f,D}$ | (maximum) clamping friction force due to $F_{N,D}$ |
| G_c | gradient of the cutting moment |
| G_f | gradient of the clamping moment |
| $G_{f,D}$ | gradient of the clamping moment according to $F_{f,D}$ |
| $G_{f,D}^P$ | $G_{f,D}$ at point P |
| ICM | instant center of motion |
| IML | iso-moment lines for cutting force |
| IMML | iso-maximum-moment lines for maximum clamping friction force F_f |
| M_c | cutting moment due to F_c |
| M_f | clamping moment |
| r | position vector |
| r_D | position vector from any point to centrode D |
| r_u | half-length of the major axis of elliptic clamping region |
| ρ_N | intensity of the distributed clamping force |
| Π | the clamping plane |
| μ | friction coefficient |

INTRODUCTION

Fixtures can exert force, referred to as the clamping force, to keep a workpiece stable, or constrain the unwanted motion of the workpiece using a set of contacts. However, the excessive clamping force can cause the workpiece to deform and therefore reduce the machining precision. Our objective is to determine the minimum clamping force that guarantees the workpiece won't slip during the machining process. The difficulty in the minimum clamping force analysis is that both the magnitude and the direction of the static friction force are indeterminate. The direction of the friction force can be determined only when the location of the instant center of motion (ICM) is assumed.

In analyzing the clamping force, Nguyen [1, 2] has proposed a force-closure method to check for the existence of an equilibrium condition for the stability of the general fixturing. Linder and Cipra [3] have proposed a graphical method to analyze the stability of a workpiece under a particular type of three-point frictional constraint. Mason [5] has stated that any pressure distribution that satisfies the equilibrium conditions can be approximated by a tripod. Based on the approximation, Lee and

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Cutkosky [6] proposed the limit surface method, derived from the instant center of motion (ICM) properties, in force/moment space as a formalism to verify the stability of the workpiece. Cogun [7] has investigated mathematically and experimentally the effects of the application sequence of clamping forces on the mounting accuracy of a workpiece. Carlyle [8] and Chou [9] have described a prototype automated fixturing system that has been implemented for prismatic workpieces.

Existing methods [5, 6] for clamping force analysis require either a search in the infinite clamping plane, which is inefficient, or, since the position of the tripod contact points is a function of the position of the instant center of motion, the assumption of a fixed tripod location is incorrect. In this paper, according to the property of instant center of motion, we develop a minimum clamping force analysis that needs neither the tripod simplification nor a search in the infinite domain. In the beginning, we introduce a new stability condition, referred to as the ICM property, and then proceed to the definition of the cutting moment, clamping moment, and their gradients. Based on these definitions, we examine several typical fixturing cases with single clamping area and derive their stability conditions individually. Moreover, the clamping force analysis is carried out in the general multiple clamping area cases.

INSTANT CENTER OF MOTION (ICM) AND CORRESPONDING FRICTION FORCE

It is well known that any planar motion of two rigid bodies is equivalent to rolling moving centred on the instant center of motion (ICM). The product of the static friction coefficient μ and the clamping force F_N is known as the maximum clamping friction force F_f . In Fig. 1, the maximum clamping friction force $F_{f,1}$ is equal to $\mu F_{N,1}$. The direction of the friction force depends on the assumed position of the ICM. Note that the ICM does not actually exist for a stable clamping. It is well known that any plane motion can be transformed into pure rotation about an ICM; therefore, it is sufficient to consider only the moment balance, and the force balance can be ignored.

ICM Property: If \forall points $P \in$ clamping plane Π such that $M_f^P \geq M_c^P$, then the clamping

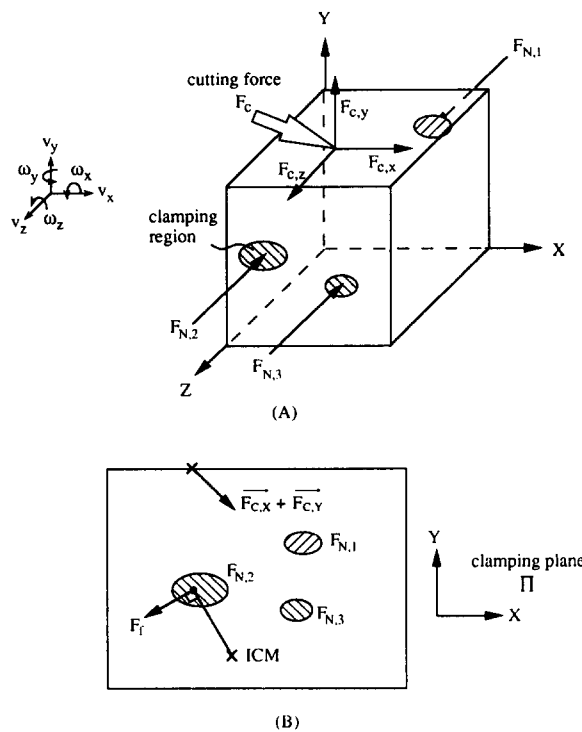


Fig. 1. (a) Cutting force and clamping forces. (b) Clamping plane.

is stable. Equivalently, if \exists a point $P \in \Pi$ for which $M_f^P < M_c^P$, then the clamping is unstable.

GRADIENT OF THE CUTTING MOMENT

As shown in Fig. 2(a), the cutting moment about an arbitrary point P can be written as

$$M_c^P = |\mathbf{PK} \times \mathbf{F}_c| = |\mathbf{PK}| |\mathbf{F}_c| \sin\theta = F_c d \tag{1}$$

where point K is the cutting position and θ is the intersection angle.

The iso-moment lines (IMLs) are the lines parallel to the direction of the cutting force, as shown in Fig. 2(a). The gradient of the cutting moment family is defined as follows.

$$G_c = \|\nabla M_c\|, \tag{2}$$

SINGLE CONCENTRATED-FORCE CLAMPING (SCFC)

In Fig. 2(b), the clamping force F_N is applied to the clamping plane Π at point D . If P is an ICM then point D must tend to move in the direction perpendicular to \mathbf{r} . The clamping moment M_f about point P is as follows

$$M_f^P = \mathbf{r} \times \mathbf{F}_f = rF_f = \mu rF_N. \tag{3}$$

It can be shown that the clamping moment about D is zero. Therefore, any non-

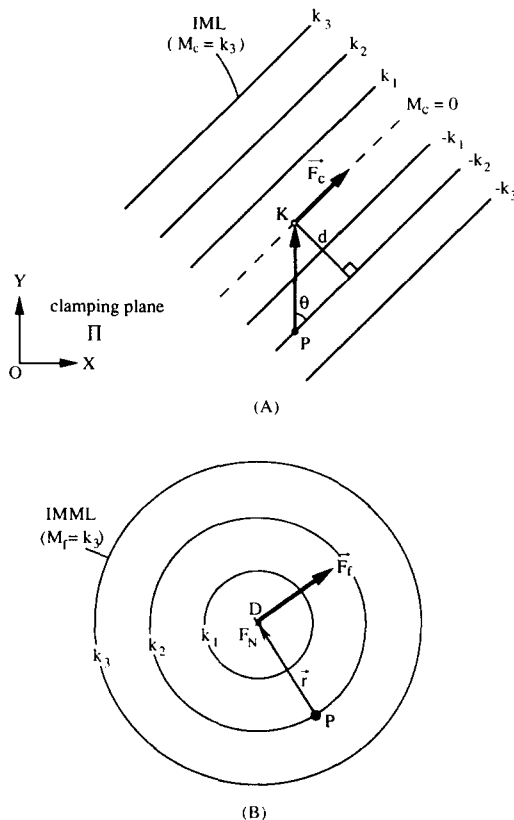


Fig. 2. (a) The iso-moment lines (IML) for the cutting force. (b) The iso-maximum-moment lines (IMML) for the single clamping normal force.

zero cutting force will cause the ICM to occur at D . Hence, SCFC can never be a stable clamping. The iso-maximum-moment lines (IMMLs) for the clamping moment are a family of circles. The gradient of the clamping moment is defined as follows

$$G_f = \|\nabla M_f\|. \tag{4}$$

CLAMPING BY DISTRIBUTED-FORCE ON CIRCULAR AREA (CDFC)

Refer to Fig. 3(a), the intensity ρ_N is uniform. The clamping moment is the integration of the elementary force dF_N , as follows

$$M_f^P = \mu \rho_N \int_A r dA \tag{5}$$

where r denotes the distance between point P and the position of the elementary force.

As shown in Fig. 3(b), the clamping force is distributed in a circular area with a total clamping force $F_{N,D}$ applied at the centre D . A point K is arbitrarily chosen so that $F_c \perp K'D$, where F_c denotes the cutting force.

CDFC Stability Criterion: If \exists point $R \in K'D$, where $G_f^R \geq G_c^R$, such that \forall point $Q \in \overline{DR}$, where $M_f^Q > M_c^Q$, then the clamping is stable.

The above criterion can be proven in three steps, as follows.

(1) Proof that no ICM can be within or on the circle C_2 , as shown in Fig. 3(b):
At an arbitrary circle C_1 within the circle C_2

$$M_c^{Q\theta} = M_c^D + G_c^{D*} * |\overline{DQ}| \cos\theta \tag{6}$$

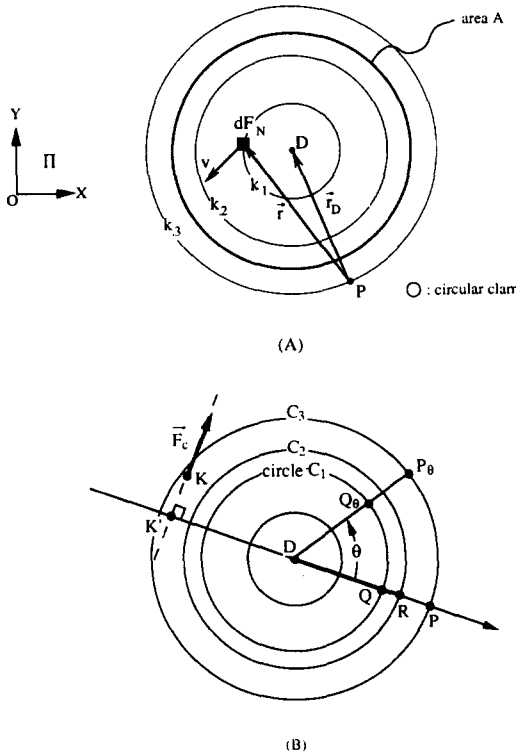


Fig. 3. (a) IMML for CDFC. (b) CDFC stability analysis.

point Q is at $\theta = 0$,

$$M_c^Q - M_c^{Q_0} = (1 - \cos\theta) * G_c^D * |\overline{DQ}| \geq 0 . \tag{7}$$

Since the IMML are symmetric about D , we have

$$M_f^Q = M_f^{Q_0} . \tag{8}$$

Since it is given that $M_f^Q > M_c^Q$, equations (7) and (8) yield

$$M_f^{Q_0} > M_c^{Q_0} . \tag{9}$$

According to the ICM property, the ICM of the workpiece will not fall within the circle C_2 .

(2) Proof that no ICM can be on **RP**:

Since $F_c \perp DR$, equation (5) yields

$$M_c^P = M_c^R + G_c^R * |\overline{RP}| \tag{10}$$

$$M_f^P = M_f^R + \left(G_f^R + \Delta G_f^{R \rightarrow P} \right) * |\overline{RP}| . \tag{11}$$

It can be verified that $\Delta G_f^{R \rightarrow P} > 0$. From the conditions $M_f^R > M_c^R$ and $G_f^R \geq G_c^R$, we obtain $M_f^P > M_c^P$ based on equations (10) and (11). By the ICM property, the ICM of the workpiece should not fall within the semi-infinite line segment **RP**.

(3) Proof that no ICM can be outside of C_2 :

Since

$$M_c^P > M_c^{P_0} \tag{12}$$

and similar to equation (8)

$$M_f^P = M_f^{P_0} \tag{13}$$

then from the above proof, we can deduce that

$$M_f^{P_0} > M_c^{P_0} \tag{14}$$

for any point P_0 on circle C_3 .

CLAMPING BY DISTRIBUTED-FORCE ON ELLIPTICAL AREA (EDFC)

The IMML of the elliptic distributed clamping force is shown in Fig. 4(a). Let r_a and r_b denote the half-length of the major and minor axes of the elliptical clamping area, respectively. The numerical analysis, based on equation (5), shows that when $0.02 < r_b/r_a < 50$ and $r_D > 2 r_a$, the variation in the clamping moment M_f on the same circle is within 5%. Hence the IMMLs are nearly circles for $r_D > 2 r_a$.

In Fig. 4(b), L_1 denotes the major axis of the elliptical clamping region when the centre of the distributed clamping force is at point D . The intersection points between the circles C_1, C_2, C_3 and the line L_1 are denoted by Q_0, P_0 and R_0 , respectively. Again, a point K' is arbitrarily chosen so that $F_c \perp K'D$, where F_c denotes the cutting force.

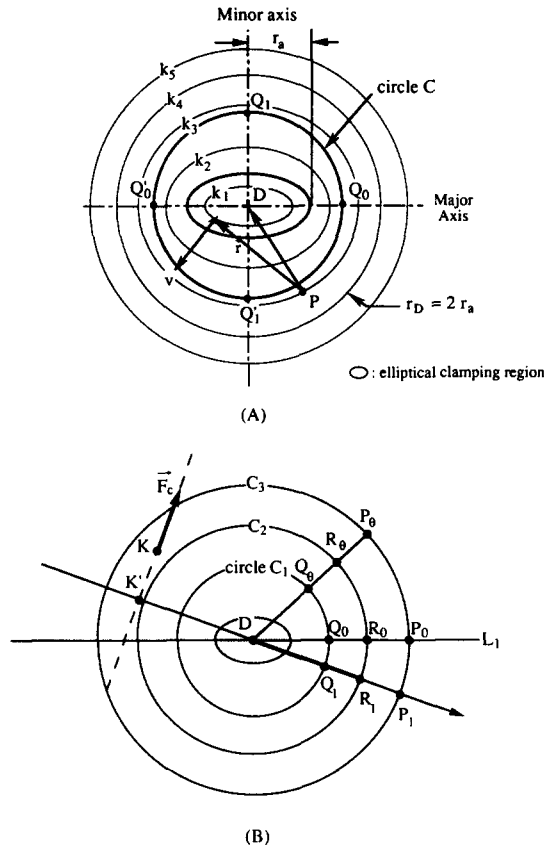


Fig. 4. (a) IMML for EDFC. (b) EDFC stability analysis.

EDFC Stability Criterion: If \exists point $R_1 \in K'D$, where $G_f^{R_0} \geq G_c^{R_1}$ such that \forall points $Q_1 \in DR_1$, where $M_f^{Q_0} > M_c^{Q_1}$, then the clamping is stable.

The EDFC stability criterion can be proven following a line of reasoning similar to that used to prove the CDFC stability criterion. Since the ratio of the lengths of the major/minor axes of elliptic clamping region is between 0.02 and 50 and $r_D > 2r_a$, the variation in the clamping moment M_f is less than 5%. Therefore, the constraints on EDFC stability criterion are reasonably tight.

CLAMPING BY MULTIPLE CONCENTRATED-FORCE (MCFC)

In Fig. 5(a), let $M_{f,i}^P, i = 1, 2, \dots, n$, denote the clamping moment due to the i th concentrated clamping force $F_{N,i}$ applied at (x_i, y_i) . Let D denote the centroid of all concentrated forces. Let $M_{f,D}^P$ denote the clamping moment due to the total concentrated force $F_{N,D}$ applied at the centroid D , where

$$F_{N,D} = \sum_{i=1}^n F_{N,i} .$$

MCFC property: $\forall P \in \Pi,$

$$\sum_{i=1}^n M_{f,i}^P \geq M_{f,D}^P .$$

(The sum of the clamping moment due to each individual concentrated force is no less than the clamping moment of the sum of the concentrated forces applied at the centroid.)

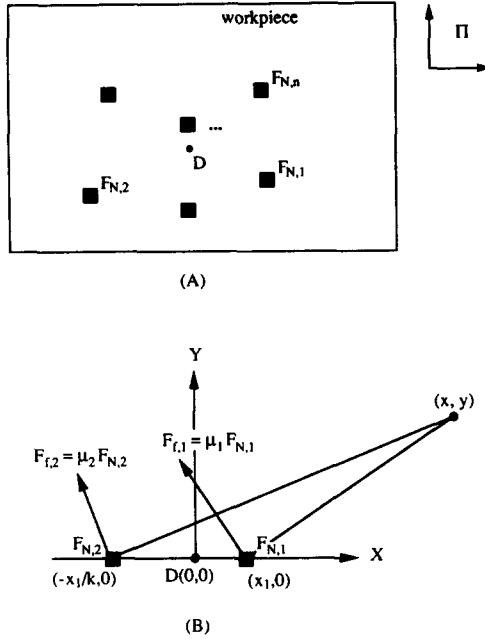


Fig. 5. MCFC property.

In the following, we show that the combination of any two concentrated forces at the centre of the two concentrated forces will introduce a smaller clamping moment. As shown in Fig. 5(b), when $n = 2$, the first clamping force $F_{N,1}$ applied at $(x_1, 0)$ and the second clamping force $F_{N,2}$, equal to $kF_{N,1}$, must be located on the x -axis $(-x_1/k, 0)$. The centre is at the origin $(0, 0)$, where k is an arbitrary positive real number. The stationary condition of the clamping moment for an arbitrary point P with coordinate (x, y) is written as

$$\frac{\partial \sum_{i=1}^2 M_{f,i}^P}{\partial x_1} = \frac{\partial \left\{ \sqrt{(x-x_1)^2 + y^2} + k \sqrt{\left(x + \frac{x_1}{k}\right)^2 + y^2} \right\}}{\partial x_1} F_{f,1} = 0. \tag{15}$$

Since $F_{f,1} = \mu F_{N,1} \neq 0$,

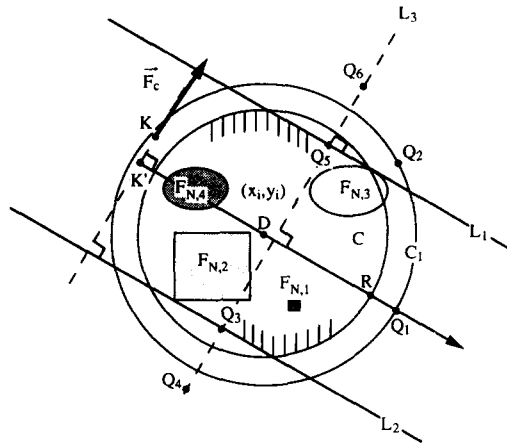
$$\left\{ (x-x_1)^2 - \left(x + \frac{x_1}{k}\right)^2 \right\} y^2 = 0. \tag{16}$$

The above equation yields $x_1 = x_1/k$, which implies that $x_1 = 0$. The second derivative of equation (15) is semi-positive definite at $x_1 = 0$, hence the concentrated force provides a minimum clamping moment. Therefore, the combination of two concentrated forces at the centre of the two concentrated forces, i.e. to cause $x_1 = 0$, will introduce a smaller clamping moment.

For the case of $n > 2$, the proof can be continued by deduction, i.e. the combination of any two of the n concentrated forces at the centre of the two concentrated forces will introduce a smaller clamping moment. Therefore, the MCFC property holds.

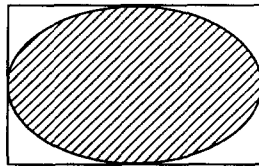
CLAMPING BY DISTRIBUTED-FORCE ON MULTIPLE AREA (MDFC)

In Fig. 7, ρ_i^N denotes the intensity of the i th distributed clamping regions R_i , A_i denotes the area of clamping regions R_i , and μ_i denotes the friction coefficient of

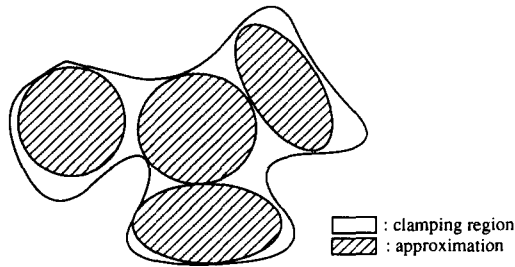


▤ : the vertical lines indicates the region for possible ICM.

Fig. 6. MDFC stability analysis.



(A)



(B)

Fig. 7. ADFC. (a) Rectangular clamping area. (b) Arbitrary clamping area

clamping regions R_i . The friction force, due to a total normal force, applied at the centroide D_i is defined as

$$F_{f,D} = \int \mu_i \rho_{N,D} dA_i \tag{17}$$

$M_{f,D}^i$ denotes the clamping moment due to the above friction forces and M_f^i denotes the actual clamping moment due to the distributed clamping region R_i .

In Fig. 6, D denotes the centroide of all clamping forces. The line L_3 is parallel to F_c . The lines L_1 and L_2 are perpendicular to F_c and chosen so as to bound a minimum area which includes all forces. Circles C and C_1 are centered at centroide point D . The radius of circle C is $|\overline{DR}|$. A point K' is arbitrarily chosen so that $F_c \perp K'D$, where F_c denotes the cutting force.

MDFC Stability Criterion: If \exists a point $R \in K'D$, where $F_{f,D} |\overline{DR}| = F_c |\overline{K'R}|$ such that \forall points P with the region bounded by the lines L_1 , L_2 and the circle C where $\sum_{i=1}^n M_{f,i}^P > M_c^P$, then the clamping is stable.

The condition for the above criterion can be proven by showing that the ICM must be bounded by the circle C and bounded by the lines L_1 and L_2 . To shorten the paper, the proof is reserved for exercises.

CLAMPING BY DISTRIBUTED-FORCE ON AN ARBITRARY AREA (ADFC)

As shown in Fig. 7(a), a rectangular clamping area can be replaced by an inscribed elliptical clamping region with the same intensity clamping force. Since the area of the elliptical region is always smaller than that of the rectangular one, the EDFC conversion increases the efficiency at the cost of a reduction in accuracy. As shown in Fig. 7(b), the arbitrary clamping area, which should not be inscribed by one ellipse, can always be separated into many disconnected clamping areas. Consequently, the MDFC stability criterion can then be applied.

CLAMPING FORCE ANALYSIS

Two types of clamping force analysis problem, with different input conditions, are examined in the following sections. For a given cutting path (or contour), the cutting forces can be derived from the cutting conditions, such as feedrate, material, tool blade type, and material removal rate [12].

As shown in Fig. 8, once the fixture types and the fixture locations are known, then the clamping region is identified. If the fixture is as simple as a wrench with two identical clamping areas on each side of the workpiece, such as in the layout shown in Fig. 8(a), then it can be equivalent to a single clamping region. If there are multiple fixtures, such as in the layout shown in Fig. 8(b), then we have to calculate the

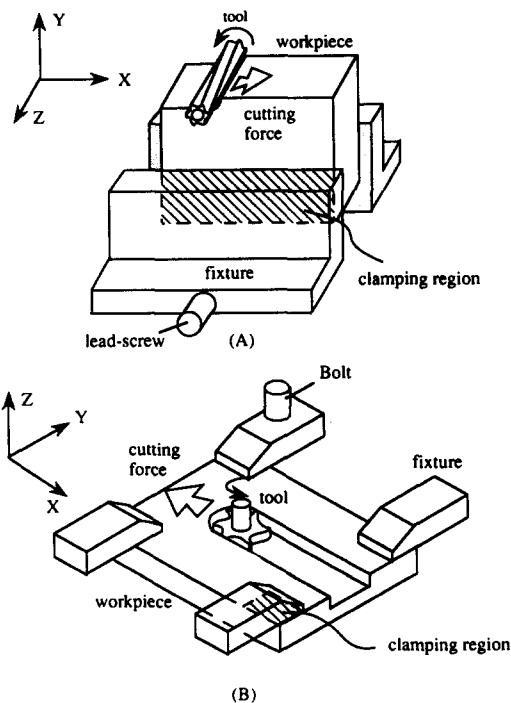


Fig. 8. Two fixturing examples.

centres for each of the fixtures in order to convert the problem into one with the MCFC property.

Two stability criteria are used in the analysis. The EDFC stability criterion can consequently be converted into a line search with the bounds corresponding to the gradients G_c and G_f . The line search algorithm can be as simple as an optimization algorithm for finding the maximum of the M_c to M_f ratio. If the maximum is greater than one, then the clamping is unstable. The MDFC stability criterion can also be verified through an area search algorithm with the bounds corresponding to the MCFC properties and gradients G_c and G_f .

MINIMUM CLAMPING FRICTION FORCE

The fixturing layout and cutting conditions are shown in Fig. 9. As shown in Fig. 10, for the straight-line cutting paths, e.g. path Nos 1–3, the MDFC stability criterion is applied to each 2 mm interval of cutting path. For the circular paths, e.g. path Nos 4–6, the stability criterion is applied to each 20° interval. Here, θ_t denotes the intersection angle between the line from cutter center to contact point and the horizontal line. We obtain the intersection angle between the horizontal line and the direction of any cutting force as follows

$$\theta_c = \theta_t + 90^\circ - \theta_\gamma$$

and

$$\begin{aligned} x_c &= x_r + 5.0 \cos(-\theta_t) \\ y_c &= y_r + 5.0 \sin(-\theta_t) . \end{aligned} \tag{18}$$

Substituting $\theta_t = 30^\circ$, $\theta_\gamma = 5^\circ$ and $F_c = 100 \text{ Nt}$, an arbitrary selection, into the above equation, we calculate the minimum total clamping friction forces corresponding to different fixturing cases as shown in Table 1. Table 1 shows that the maximum M_c/M_f occurs when the tool is just about to move away from the workpiece (at the end of path No. 3). According to Table 1, the fixturing layout case 1 requires the largest clamping friction force for the following reasons.

- (1) In case 1, the centroid D is farthest from the geometric center of the entire

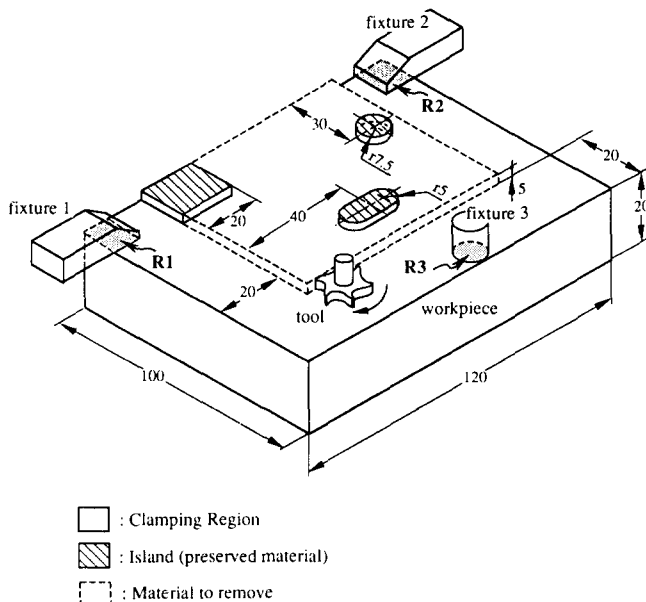


Fig. 9. Example of fixturing layout and cutting conditions for minimum clamping force analysis.

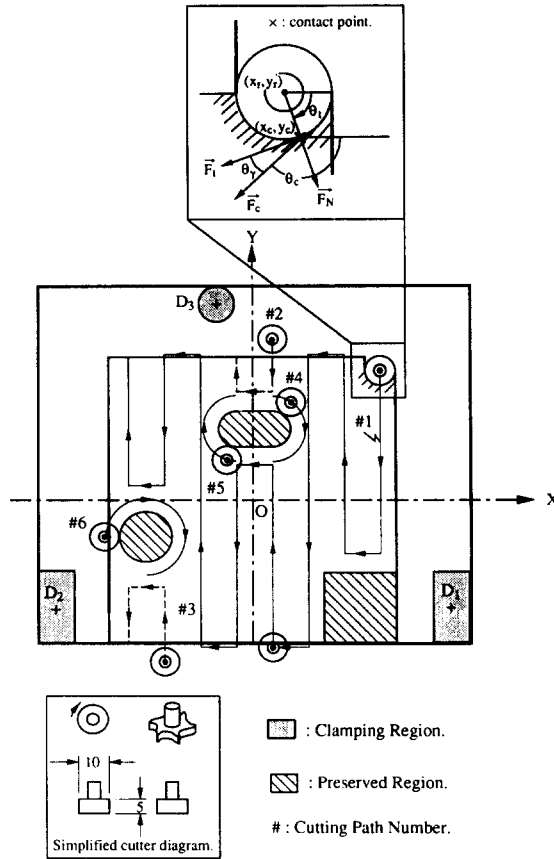


Fig. 10. Cutting paths.

Table 1. Minimum clamping force analysis for different clamping layout cases

| Analysis fixturing layout | Layout case 1 | Layout case 2 | Layout case 3 |
|--|--|---|--|
| Clamping area | R1: 200 mm ² (20*10) rectangular R2: 200 mm ² (20*10) rectangular R3: 25 π mm ² (radius=5) circular | | |
| Clamping force distribution intensities ratio R1 : R2 : R3 | 1.50 : 1.00 : 1.27 | 1.50 : 1.00 : 1.27 | 1.00 : 1.00 : 1.00 |
| Centrode of individual fixture clamping force | D ₁ (55.0,-30.0) D ₂ (-55.0,-30.0) D ₃ (-10.00,55.0) | D ₁ (55.0,-30.0) D ₂ (-55.0,17.5) D ₃ (-55.0,55.0) | D ₁ (55.0,8.5) D ₂ (-55.0,-30.0) D ₃ (0.0,55.0) |
| Centrode of all clamping force | D(7.5,-15.8) | D(0.0,0.0) | D(0.0,-0.1) |
| Minimum total clamping force [†] | 196.60 Nt | 172.80 Nt | 165.00 Nt |
| Cutting position corresponding to the maximum M _c /M _r | (-39.3,40.0) | (-39.3,40.0) | (-39.3,40.0) |

[†]The clamping force for each individual fixture can be calculated through their intensity ratio and area.

cutting region. Based on the IML property, the same cutting force applied at more distant locations makes it easier to induce a larger cutting moment about *D*. Therefore, a larger total clamping friction force is required to keep the workpiece stable.

(2) The ratio of the clamping friction force intensities in different clamping regions is 1.5:1:1.27 for case 1 and 1:1:1 for case 3. Case 3 has a more uniform clamping force distribution than case 1. According to the MCFC property, if the total clamping friction forces for clamping cases 1 and 3 are the same, then the clamping moment of case 1 is smaller than that of case 3. This reason holds for when the minimum total clamping friction force of case 2 is larger than that of case 3.

CONCLUSION

According to the property of instant center of motion, we have examined several typical types of fixturing with single clamping area and derived their stability conditions individually. It is also found that an arbitrary shape of clamping area can be replaced by several elliptical areas. Subsequently, the problem can be analyzed based on the stability conditions for multiple clamping areas. Toward the multiple clamping area analysis, we have first derived the MCFC property that properly bounds the stability clamping moment. According to the bounds, we derived the general stability condition for the clamping by distributed force on multiple areas. Finally, we presented a fixturing design example to demonstrate the minimum force analysis for different fixturing layouts. This new method for clamping force analysis transforms a clamping analysis problem into a simple line search or finite region search, therefore it is highly efficient. In addition, in our model the tripod simplification is unnecessary, so the accuracy of the analysis result is also improved. This new method can directly support on-line fixturing design. Once the cutting paths have been generated by the path generation program and the cutting conditions given, the clamping force analysis can be used to estimate the required clamping force and to evaluate the corresponding fixturing design.

Acknowledgements—This research was supported in part by the R.O.C. National Science Council under grant No. NSC 81-0422-E009-08. The authors would like to express their appreciation to Prof. An-Chen Lee, Director of the Automation Center at NCTU, for his valuable suggestions concerning the cutting force analysis.

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