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Moiré Fringes by Two Spiral Gratings and Its Applications on Collimation Tests

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The characteristics of the moire fringes produced by the superposition of various kinds of spiral gratings are derived. According to the shapes of the resultant moire fringes, the moire fringes can be used for checking the collimation of a light beam. The resolution of each condition is discussed.

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I. INTRODUCTION

In the previous Letter [1], a new method that uses spiral gratings and Talbot interferometry for checking the quality of a collimated light is proposed. According to the shape of the moiré fringes produced by the superposition of the two special spiral gratings, it is very easy to judge the quality of the collimation of light. Moire fringes produced by linear gratings [2-7] or circular gratings [7-9] are widely investigated and can be applied to optical and mechanical measurements. There are few papers [10,11] related to the moiré fringes produced by spiral gratings. In this paper, the characteristics of moiré fringes produced by various kinds of spiral gratings will be investigated entirely. In addition, the resolution of each condition as it is used to judge the vergence of the light is discussed.

II. MOIRE FRINGES PRODUCED BY SPIRAL GRATINGS

The optical system is shown in Fig. 1. As shown in the figure, a monochromatic, nearly collimated light of wavelength λ passes through two special spiral gratings SG1 and SG2, and the moire fringes appear in the observation plane OP that is placed just behind SG2. SG1 and SG2 have the same radial period p. SG2 is located at one of the Talbot images of SG1, i.e., the distance z_T between SG1 and SG2 equals $2mp^2/\lambda$, where m is a positive integral and p is the radial period of SG1.

For simplicity, let the amplitude transmission of the gratings be sinusoidal distributions, then the amplitude transmission of SG1 is written as

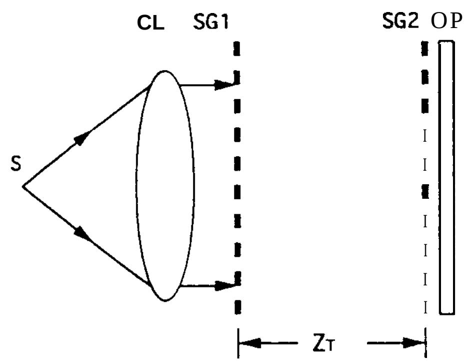


FIG. 1. The optical arrangement for measuring the collimated light. CL: collimating lens; SG: special spiral grating; OP: observation plane.

$$t_1(r,\theta) = \frac{1}{2}\left[1 + \cos\left(\frac{2\pi r}{p} - N_1\theta\right)\right]. \tag{1}$$

This means that it has counterclockwise spiral distribution with a radial period of p and has N_1 repetitions when θ is from 0° to 360° in a constant radius r_0. For example, the shape for $N_1 = 4$ is shown in Fig. 2.

Since SG2 is positioned at the Talbot image of SG1, the electric field distribution just before SG2 can be represented as

$$E_1'(r,\theta) \sim \frac{1}{2}\left[1 + \cos\left(\frac{2\pi r}{p'} - N_1\theta\right)\right], \tag{2}$$

where p' is the projected radial period of SG1 on SG2. Because SG1 is a spiral grating and

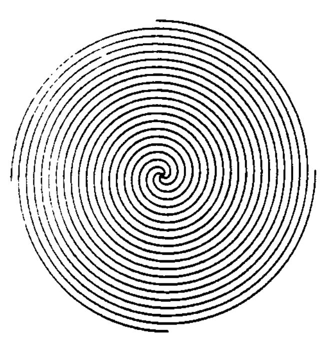


FIG. 2. The diagram of SG1 when $N_1 = 4$.

not a linear grating, so the approximation symbol "$\sim$" is used here instead of "=". Let the amplitude transmission of SG2 be written as

$$t_2(r,\theta) = \frac{1}{2}\left[1 + \cos\left(\frac{2\pi r}{p} \mp N_2\theta\right)\right], \tag{3}$$

where the upper part "$-$" means that SG2 has the same spiral direction with SG1, and the lower part "+" means opposite. Consequently, the intensity distribution in OP is given by

$$\begin{aligned} I(r,\theta) &= |E_1'(r,\theta)t_2(r,\theta)|^2 \\ &\sim \frac{1}{16}\left\{1 + 2\cos\left[\frac{2\pi r}{p'} + \frac{2\pi r}{p} - (N \pm N_2)\theta\right]\right. \\ &\quad +2\cos\left[\frac{2\pi r}{p'} - \frac{2\pi r}{p} - (N_1 \mp N_2)\theta\right] \\ &\quad + \cos^2\left(\frac{2\pi r}{p'} - N_1\theta\right)\cos^2\left(\frac{2\pi r}{p} \mp N_2\theta\right) \\ &\quad +2\cos\left(\frac{2\pi r}{p'} - N_1\theta\right) + 2\cos\left(\frac{2\pi r}{p} \mp N_2\theta\right) \\ &\quad + \cos^2\left(\frac{2\pi r}{p'} - N_1\theta\right) + \cos^2\left(\frac{2\pi r}{p} \mp N_2\theta\right) \\ &\quad +2\cos\left(\frac{2\pi r}{p'} - N_1\theta\right)\cos^2\left(\frac{2\pi r}{p} \mp N_2\theta\right) \\ &\quad +2\cos\left(\frac{2\pi r}{p'} - N_1\theta\right)\cos^2\left(\frac{2\pi r}{p} \mp N_2\theta\right) \\ &\quad \left.+2\cos\left(\frac{2\pi r}{p} \mp N_2\theta\right)\cos^2\left(\frac{2\pi r}{p'} - N_1\theta\right)\right\}. \end{aligned} \tag{4}$$

It is obvious that the intensity of the moire fringes is given in the form of

$$I_M(r,\theta) \sim \frac{1}{8}\cos\left[\frac{2\pi r}{p'} - \frac{2\pi r}{p} - (N_1 \mp N_2)\theta\right]. \tag{5}$$

Then according to the spiral directions of SG2, we derive the following two cases.

II-l. SG2 has the same direction as SG1

The intensity of the moire fringes is

$$I_M(r,\theta) \sim \frac{1}{8}\cos\left[\frac{2\pi r}{p'} - \frac{2\pi r}{p} - (N_1 - N_2)\theta\right]. \tag{6}$$

(i) For a perfectly collimated light, we have $p' = p$, and

$$I_M(r,\theta) \sim \frac{1}{8}\cos(N_1 - N_2)\theta. \tag{7}$$

It is obvious that there are $|N_1 - N_2|$ sets of moire fringes. In addition, if

(a) $N_1 = N_2$, then
$I_M(r, \theta) \sim$ constant.
No moire fringes will appear.

(b) $N_1 \neq N_2$,
the shapes of these $|N_1 - N_2|$ sets of moire fringes are determined by the following equation:

$$|N_1 - N_2|\theta = 2k\pi \; ; \qquad k = 0, 1, \cdots |N_1 - N_2| - 1. \tag{8}$$

It is seen that the moire fringes are radial with an angular period of $\frac{2\pi}{|N_1 - N_2|}$.

(ii) For an imperfectly collimated light, we have $p' \neq p$, and if

(a) $N_1 = N_2$, then

$$I_M(r, \theta) \sim \frac{1}{8} \cos\left(\frac{2\pi r}{p'} - \frac{2\pi r}{p}\right). \tag{9}$$

So the shapes of the moire fringes are concentrical with a radial period of $|\frac{pp'}{p'-p}|$.

(b) $N_1 \neq N_2$, then from Eq. (6), we know that there are $|N_1 - N_2|$ sets of moire fringes. And the shapes of these fringes are determined by the equation:

$$\left(\frac{1}{p'} - \frac{1}{p}\right)2\pi r - (N_1 - N_2)\theta = 2k\pi \; ; \qquad k = 0, 1, \cdots |N_1 - N_2| - 1. \tag{10}$$

It is clear that they are spiral with the same radial period as ii(a).

11-2. SG2 has the opposite direction to **SG1**

The intensity of the moire fringes is

$$I_M(r, \theta) \sim \frac{1}{8} \cos\left[\frac{2\pi r}{p'} - \frac{2\pi r}{p} - (N_1 + N_2)\theta\right]. \tag{11}$$

It is obvious that there are $(N_1 + N_2)$ sets of moire fringes.

(i) For a perfectly collimated light

We have $p' = p$, and the shapes of moire fringes are determined by the following equation:

$$(N_1 + N_2)\theta = 2k\pi \; ; \qquad k = 0, 1, \cdots (N_1 + N_2 - 1). \tag{12}$$

They are radial with an angular period of $\frac{2\pi}{(N_1+N_2)}$.

(ii) For an imperfectly collimated light

We have $p' \neq p$, and the shapes of these (N_1+N_2) sets of moire fringes are determined by the next equation:

$$\left(\frac{1}{p'}-\frac{1}{p}\right)2\pi r-(N_1+N_2)\theta=2k\pi\ ; \qquad k=0,1,\cdots(N_1+N_2-1). \tag{13}$$

They are spiral with a radial period of $\left|\frac{pp'}{p'-p}\right|$.

III. RESOLUTION FOR COLLIMATION METHOD

Here, resolution is defined as the smallest detectable divergent or convergent angle of the tested beam, as shown in Fig. 3. Let the radius of curvature of the tested light incident on SG2 be r' where it is positive for divergent, and negative for convergent. From the geometrical relation shown in Fig. 3, we have

$$p'=\frac{r'}{r'-z_T}p. \tag{14}$$

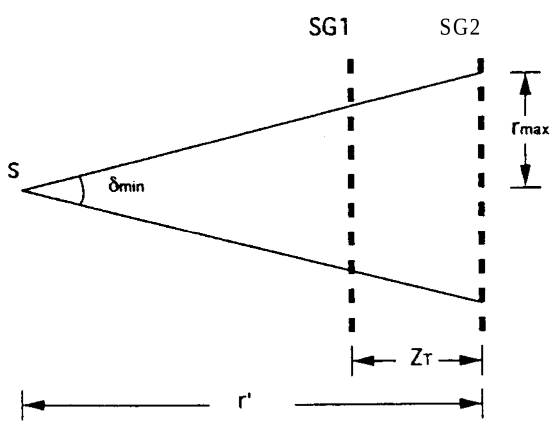


FIG. 3. The geometrical diagram related to Fig. 1.

III-l. SG2 has the same direction as SG1

(i) $N_1 = N_2$

Because the fringes are concentrical, so it needs at least one fringe to judge the quality of the vergence. Subsequently, from Eq. (9), we have

$$2\pi r_{\max}\left|\frac{1}{p'} - \frac{1}{p}\right| = \pi\,; \tag{15}$$

where $r_{\max}$ **is** the radius of the tested light incident on SG2. Combining Eqs. (14) and (15), we get

$$|r'| = \frac{2r_{\max}}{p} z_T\,. \tag{16}$$

Therefore, the resolution $\delta_{\min}$ is

$$\delta_{\min} = \frac{2r_{\max}}{|r'|} = \frac{p}{z_T} = \frac{\lambda}{2mp}\,. \tag{17}$$

(ii) $N_1 \neq N_2$,

The fringes are radial for perfectly collimated light, and spiral for imperfectly collimated light. So, as the collimating lens is changed from focus to defocus, the end part of the radial fringes will be curved continuously. As a result, the fringes become spiral. Let $\Delta\theta = |\theta(r \cong r_{\max}) - \theta(r \cong 0)|$, and $\theta_{\min}$ be the least distinguishable angle with naked eyes. That is, $\Delta\theta = \theta_{\min}$. Then, from Eq. (10), we get

$$\left|\left(\frac{1}{p'} - \frac{1}{p}\right)2\pi r_{\max}\right| = |N_1 - N_2|\theta_{\min}\,. \tag{18}$$

Therefore, combined with Eq. (14), the resolution $\delta_{\min}$ is

$$\delta_{\min} = \frac{|N_1 - N_2|\theta_{\min}}{2m\pi}\frac{\lambda}{p}\,. \tag{19}$$

111-2. SG2 has the opposite direction to SG1

Similarly, from Eqs. (13) and (14), we get

$$\delta_{\min} = \frac{|N_1 + N_2|\theta_{\min}}{2m\pi}\frac{\lambda}{p}\,. \tag{20}$$

For clearness, we have summarized the above results in Table I.

On the other hand, if SG1 has a clockwise spiral direction, then the amplitude transmission of SG1 can be written as

TABLE I. The behaviors of the moiré fringes for different combinations of SG1 and SG2 as SG1 has a counterclockwise direction.

SG1	SG2	condition of N_1, N_2	vergence of tested light	number sets of moiré fringes	shapes of moiré fringes	the resolution δ_{min}
↺	↺	$N_1=N_2$	collimated	none	none	
↺	↺	$N_1=N_2$	noncollimated	$\left\lvert\frac{(p'-p)}{pp'}\right\rvert r_{max}$	concentrical-circular	$\frac{\lambda}{2mp}$
↺	↺	$N_1>N_2$	collimated	N_1-N_2	radial	
↺	↺	$N_1>N_2$	divergent	N_1-N_2	spiral ↻	$\frac{(N_1-N_2)\theta_{min}}{2mn}\frac{\lambda}{p}$
↺	↺	$N_1>N_2$	convergent	N_1-N_2	spiral ↺	
↺	↺	$N_1<N_2$	collimated	N_2-N_1	radial	
↺	↺	$N_1<N_2$	divergent	N_2-N_1	spiral ↺	$\frac{(N_2-N_1)\theta_{min}}{2mx}\frac{\lambda}{p}$
↺	↺	$N_1<N_2$	convergent	N_2-N_1	spiral ↻	
↺	↻	any	collimated	N_1+N_2	radial	
↺	↻	any	divergent	N_1+N_2	spiral ↻	$\frac{(N_1+N_2)\theta_{min}}{2mx}\frac{\lambda}{p}$
↺	↻	any	convergent	N_1+N_2	spiral ↺	

↺ : counterclockwise direction

↻ : clockwise direction

TABLE II. The behaviors of the moiré fringes for different combinations of SG1 and SG2 as SG1 has a clockwise direction.

SG1	SG2	condition of N_1, N_2	vergence of tested light	number sets of moire fringes	shapes of moire fringes	the resolution δ_{min}
↻	↻	$N_1=N_2$	collimated	none	none	$\frac{\lambda}{2mp}$
↻	↻	$N_1=N_2$	noncollimated	$\left\lvert \frac{(p'-p)}{pp'} r_{max} \right\rvert$	concentrical -circular	
↻	↻	$N_1>N_2$	collimated	N_1-N_2	radial	$\frac{(N_1-N_2)\theta_{min}}{2m\pi}\frac{\lambda}{p}$
↻	↻	$N_1>N_2$	divergent	N_1-N_2	spiral ↺	
↻	↻	$N_1>N_2$	convergent	N_1-N_2	spiral ↻	
↻	↻	$N_1<N_2$	collimated	N_2-N_1	radial	$\frac{(N_2-N_1)\theta_{min}}{2m\pi}\frac{\lambda}{p}$
↻	↻	$N_1<N_2$	divergent	N_2-N_1	spiral ↻	
↻	↻	$N_1<N_2$	convergent	N_2-N_1	spiral ↺	
↻	↺	any	collimated	N_1+N_2	radial	$\frac{(N_1+N_2)\theta_{min}}{2m\pi}\frac{\lambda}{p}$
↻	↺	any	divergent	N_1+N_2	spiral ↺	
↻	↺	any	convergent	N_1+N_2	spiral ↻	

↺ : counterclockwise direction

↻ : clockwise direction

$$t_1(r,\theta) = \frac{1}{2}\left[1 + \cos\left(\frac{2\pi r}{p} + N_1\theta\right)\right]. \tag{21}$$

Similarly, the results can be obtained as above and may be summarized in Table II.

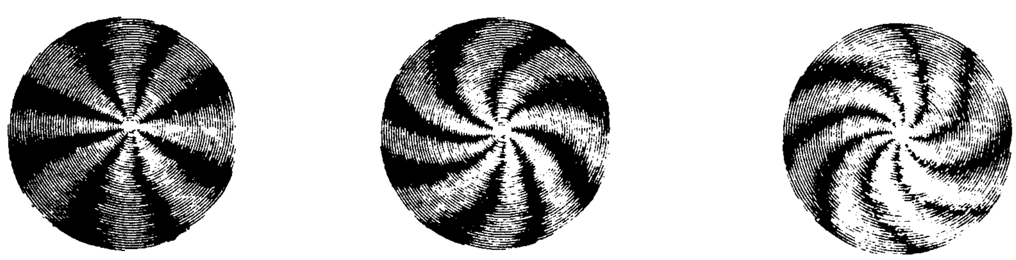
IV. EXPERIMENT WORK

At first, an IBM 386-compatible personal computer and a Roland DXY880A plotter are used to plot the desired spiral figures on a transparent thin film. Secondly, these spiral figures are exposed on an Agfa 10E75 photographic plate by contact printing. After developing, a spiral grating is obtained.

In our experiment, we use a He-Ne laser of wavelength 632.8 nm and two self-made spiral gratings with a radial period of 0.4 mm/line and $N_1 = N_2 = 4$. SG2 is positioned at the first Talbot image of SG1, that is, m = 1 and $z_T \cong 0.51m$. We set SG1 in the counterclockwise direction and SG2 in the opposite direction, the resultant moire fringes are shown in Fig. 4 for (a) quasi-collimated light, (b) divergent light, and (c) convergent light. Then, we get SG1 and SG2 in the same direction, the moire fringes are shown in Fig. 5 for noncollimated light, and no fringes for collimated light.

V. DISCUSSION

From Table I and Table II, the resolution $\delta_{\min}$ is proportional to $\frac{\lambda}{2mp}$, which equals $\frac{p}{z_T}$. Therefore, the resolution is proportional to p when z_T is remained fixed. And the minimal radial period p depends on the technique of the manufacture. In practical performance, the resolution, the length of the overall optical setup, the cost for spiral gratings and the



(a) (b) (c)

FIG. 4. The moiré fringes for (a) quasi-collimated light, (b) divergent light, and (c) convergent light, as SG1 and SG2 being in the opposite direction.

FIG. 5. The moird fringes for noncollimated light as SG1 and SG2 being in the same direction

easiness for observation should be considered. In our experiment, the conditions SG1 and **SG2** are chosen in opposite directions and $N_1 = N_2 = 4$ are used, thus there are eight sets of moire fringes which are symmetrical with respect to the origin in the observation plane. The angular interval between the adjacent moire fringes is $\frac{360°}{8} = 45°$, hence it is very easy for observation. If a very loose condition $\theta_{min} = 10''$ is chosen, then at least we have the resolution $\delta_{min} = 0.35$ mrad.

VI. CONCLUSION

In this paper, the characteristics of the moire fringes produced by the superposition of various kinds of spiral gratings are derived. According to the shapes of the resultant moire fringes, they can be used for checking the collimation of a light beam. And the resolution of each condition is discussed.

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