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Adaptive synchronization of chaotic systems with unknown parameters via new backstepping strategy

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Abstract In this paper, a new effective approach—backstepping with Ge–Yao–Chen (GYC) partial region stability theory (called BGYC in this article) is proposed to applied to adaptive synchronization. Backstepping design is a recursive procedure that combines the choice of a Lyapunov function with the design of a controller, and it presents a systematic procedure for selecting a proper controller in chaos synchronization. We further combine the systematic backstepping design and GYC partial region stability theory in this article, Lyapunov function can be chosen as a simple linear homogeneous function of states, and the controllers and the update laws of parameters shall be much simpler. Further, it also introduces less simulation error—the numerical simulation results show that the states errors and parametric errors approach to zero much more exactly and efficiently, which are compared with the original one. Two cases are presented in the simulation results to show the effectiveness and feasibility of our new strategy.

Keywords BGYC · Synchronization · Chaotic system

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1 Introduction

A synchronized mechanism that enables a system to maintain a desired dynamical behavior (the goal or target) even when intrinsically chaotic have many applications ranging from biology to engineering [1–4]. Thus, it is of considerable interest and potential utility, to devise control techniques capable of achieving the desired type of behavior in nonlinear and chaotic systems. The control of chaos and bifurcation is concerned with using some designed control input(s) to modify the characteristics of a parameterized nonlinear system.

The control can be static or dynamic feedback control, or open-loop control. The objective can be the stabilization and reduction of the amplitude of bifurcation orbital solutions, optimization of a performance index near bifurcation, reshaping of the bifurcation diagram, or a combination of these. Many approaches and techniques have been proposed for the synchronization of chaos such as the OGY method [5], bang-bang control [6], optimal control [7, 8], active control [9–11], feedback linearization [12–20], differential geometric method [21], adaptive control [22–34], H_∞ control method [35–42], and sliding mode control (SMC) [25, 43–45].

A pragmatical asymptotically stability theorem is proposed to achieve adaptive synchronization in this paper. In current scheme of adaptive synchronization, the traditional Lyapunov stability theorem and Barbalat lemma are used to prove that the error vector approaches zero as time approaches infinity, but the question as to why those estimated parameters also approach the uncertain values has no answer [46–48]. In this article, the pragmatical asymptotically stability theorem and an assumption of equal probability for ergodic initial conditions [49, 50] are used to prove strictly that those estimated parameters approach the uncertain values.

In this paper, a new adaptive synchronizing strategy—backstepping with the Ge, Yao, and Chen partial region stability theory [51, 52] (which is called BGYC) is proposed. Via using this effective approach, the control Lyapunov function can be designed as a simple linear homogeneous function of states, the corresponding controllers and parametric update laws are much simpler, and introduce less simulation error.

The layout of the rest of this paper is as follows. In Sect. 2, the adaptive synchronization with BGYC scheme is presented. In Sect. 3, the simulation results are given. In Sect. 4, the traditional backstepping control and the new approach are presented for comparison. In Sect. 5, conclusions are given.

2 Adaptive synchronization scheme

There are two identical nonlinear dynamical systems, and the master system controls the slave system. The master system is given by

$$\dot{x} = Ax + f(x, B) \tag{2.1}$$

where $x = [x_1, x_2, \dots, x_n]^T \in R^n$ denotes a state vector, A is an $n \times n$ uncertain constant coefficient matrix, f is a nonlinear vector function, and B is a vector of uncertain constant coefficients in f.

The slave system is given by

$$\dot{y} = \hat{A}y + f(y, \hat{B}) + u(t) \tag{2.2}$$

where $y = [y_1, y_2, \dots, y_n]^T \in R^n$ denotes a state vector, $\hat{A}$ is an $n \times n$ estimated coefficient matrix, $\hat{B}$ is a vector of estimated coefficients in f, and $u(t) = [u_1(t), u_2(t), \dots, u_n(t)]^T \in R^n$ is a control input vector.

Our goal is to design a controller $u(t)$ via BGYC so that the state vector of the chaotic system (2.1) asymptotically approaches the state vector of the master system (2.2).

The chaos synchronization can be accomplished in the sense that the limit of the error vector $e(t) = [e_1, e_2, \dots, e_n]^T$ approaches zero:

$$\lim_{t\to\infty} e = 0 \tag{2.3}$$

where

$$e = x - y + K \tag{2.4}$$

where K is a positive constant by which the error dynamics occurs in the first quadrant of state space of e [23].

From Eq. (2.4) we have

$$\dot{e} = \dot{x} - \dot{y} \tag{2.5}$$

$$\dot{e} = Ax - \hat{A}y + f(x, B) - f(y, \hat{B}) - u(t) \tag{2.6}$$

A Lyapnuov function $V(e, \tilde{A}, \tilde{B})$ is chosen as a positive definite function in the first quadrant of state space of e, $\tilde{A}$, $\tilde{B}$.

We have

$$V(e, \tilde{A}, \tilde{B}) = e + \tilde{A} + \tilde{B} \tag{2.7}$$

where $\tilde{A} = A - \hat{A}$, $\tilde{B} = B - \hat{B}$, $\tilde{A}$ and $\tilde{B}$ are two column matrices whose elements are all the elements of matrix $\hat{A}$ and of matrix $\hat{B}$, respectively.

Its derivative along any solution of the differential equation system consisting of Eq. (2.6) and update parameter differential equations for $\tilde{A}$ and $\tilde{B}$ is

$$\dot{V}(e, \tilde{A}, \tilde{B}) = \left[Ax - \hat{A}y + Bf(x) - \hat{B}f(y) - u(t)\right] + \dot{\tilde{A}} + \dot{\tilde{B}} \tag{2.8}$$

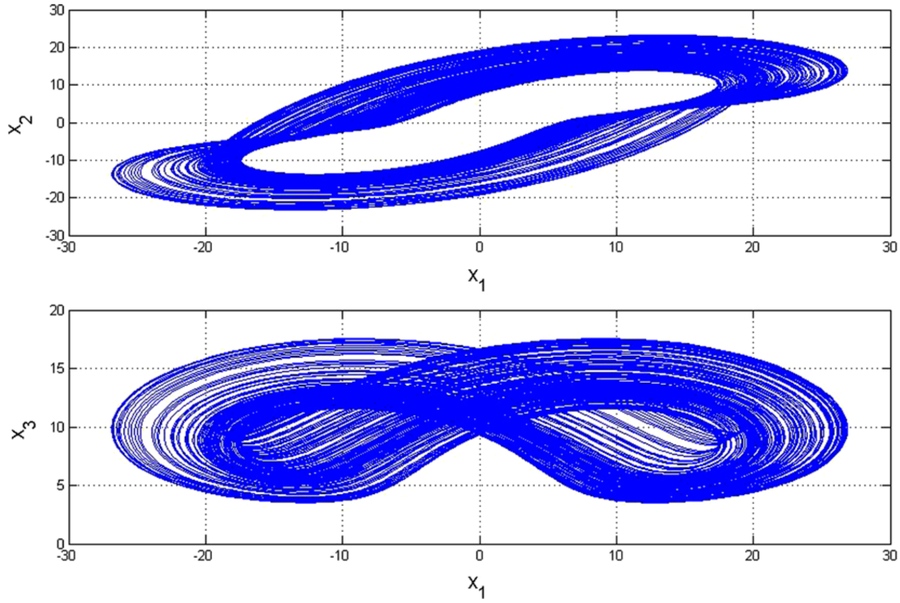


Fig. 1 Projections of phase portrait of chaotic Chen–Lee system with $a_1 = 5$, $b_1 = -10$ and $c_1 = -3.8$

where $u(t)$, $\dot{\hat{A}}$, and $\dot{\hat{B}}$ are chosen so that $\dot{V} = Ce$, C is a diagonal negative definite matrix, and $\dot{V}$ is a negative semidefinite function of e and parameter differences $\tilde{A}$ and $\tilde{B}$. In the current scheme of adaptive control of chaotic motion [18–20], the traditional Lyapunov stability theorem and Babalat lemma are used to prove that the error vector approaches zero, as time approaches infinity. But the question, why the estimated or given parameters also approach to the uncertain or goal parameters, remains no answer. By the pragmatical asymptotical stability theorem [21, 22], the question can be strictly answered.

3 Adaptive synchronization of chaotic systems via BGYC

In this section, Chen–Lee and Newton–Leipnik systems are illustrated for examples to show the effectiveness and flexibility of BGYC in simulation results. In Case I, synchronization of the master and slave Chen–Lee systems is achieved via the controllers designed by BGYC. In Case II, the slave Newton–Leipink system is chosen to trace the master Newton–Leipink system through BGYC design.

Case I Adaptive synchronization of master and slave Chen–Lee systems Chen and Lee reported a new chaotic system [16] in 2004, which is now called the Chen–Lee system [17]. The master and slave systems are described by the following nonlinear differential equations, which are denoted as master Chen–Lee system (3.1) and slave Chen–Lee system (3.2).

$$\begin{cases} \dot{x}_1 = -x_2x_3 + a_1x_1 \\ \dot{x}_2 = x_1x_3 + b_1x_2 \\ \dot{x}_3 = x_1x_2/3 + c_1x_3 \end{cases} \tag{3.1}$$

$$\begin{cases} \dot{y}_1 = -y_2y_3 + \hat{a}_1y_1 + u_1 \\ \dot{y}_2 = y_1y_3 + \hat{b}_1y_2 + u_2 \\ \dot{y}_3 = y_1y_2/3 + \hat{c}_1y_3 + u_3 \end{cases} \tag{3.2}$$

where x_1, x_2, x_3, y_1, y_2, and y_3 are state variables, and a_1, b_1, and c_1 are three system parameters, $\hat{a}_1$, $\hat{b}_1$, and $\hat{c}_1$ are estimated parameters. u_1, u_2, and u_3 are controllers, which shall be designed via BGYC to synchronize the slave Lorenz system to master one. When $(a_1, b_1, c_1) = (5, -10, -3.8)$, initial conditions are chosen as $(x_1, x_2, x_3) = (0.2, 0.2, 0.2)$ systems (3.1) are chaotic attractors, which are demonstrated in Fig. 1. In this case, the initial conditions of the slave system (3.2) are chosen as $(y_1, y_2, y_3) = (20, 10, 15)$ and the estimated parameters are $(\hat{a}_1, \hat{b}_1, \hat{c}_1) = (1, -20, -5.2)$.

The error can be described as

$$\begin{aligned} e &= \begin{bmatrix} e_1(t) & e_2(t) & e_3(t) \end{bmatrix} \\ &= [x_1 - y_1 + K \quad x_2 - y_2 + K \quad x_3 - y_3 + K] \\ &> 0 \end{aligned} \tag{3.3}$$

where $K = 100$, the addition of $K = 100$ makes the error dynamics always happen in the first quadrant.

From Eq. (3.3), we have the following error dynamics:

$$\begin{aligned} \dot{e}_1 &= -x_2 e_3 - y_3 e_2 + a_1 e_1 + \tilde{a}_1 y_1 \\ &\quad + (x_2 + y_3)K - u_1 \\ \dot{e}_2 &= x_1 e_3 + y_3 e_1 + b_1 e_2 + \tilde{b}_1 y_2 \\ &\quad - (x_1 + y_3)K - u_2 \\ \dot{e}_3 &= x_1 e_2/3 + y_2 e_1/3 + c_1 e_3 + \tilde{c}_1 y_3 \\ &\quad - \big((x_1 + y_2)/3\big)K - u_3 \end{aligned} \tag{3.4}$$

where $\tilde{a}_1 = a_1 - \hat{a}_1 > 0$, $\tilde{b}_1 = b_1 - \hat{b}_1 > 0$ and $\tilde{c}_1 = c_1 - \hat{c}_1 > 0$ are the error of parameters, which are positive numbers.

Step 1 For the first equation of Eq. (3.6), we choose the Lyapunov function as

$$V_1 = e_1 + \tilde{a}_1 \tag{3.5}$$

Its time derivative is

$$\begin{aligned} \dot{V}_1 &= \dot{e}_1 + \dot{\tilde{a}}_1 \\ &= -x_2 e_3 - y_3 e_2 + a_1 e_1 + \tilde{a}_1 y_1 \\ &\quad + (x_2 + y_3 - a_1)K - u_1 + \dot{\tilde{a}}_1 \end{aligned} \tag{3.6}$$

We assume e_2 as the virtual controller, and choose the update laws of parameters and controller u_1 as

$$\begin{cases} e_2 = \alpha e_1 = 0 \quad (\alpha = 0) \\ \dot{\tilde{a}}_1 = -\dot{\hat{a}}_1 = -y_1 \tilde{a}_1 \\ u_1 = -x_2 e_3 + 2a_1 e_1 + (x_2 + y_3 - a_1)K \end{cases} \tag{3.7}$$

Then we can obtain

$$\dot{V}_1 = -a_1 e_1 < 0 \tag{3.8}$$

This means that $e_1 = 0$ is asymptotically stable.

Step 2 For studying the (e_1, w_1) system:

According to $e_2 = \alpha e_1 = 0$, we have

$$w_1 = e_2 - \alpha e_1 = e_2 \tag{3.9}$$

then the $(e1, w2)$ system (3.10) can be described as follows:

$$\begin{aligned} \dot{e}_1 &= a_1 e_1 + \tilde{a}_1 y_1 \\ \dot{w}_1 &= x_1 e_3 + y_3 e_1 + b_1 w_1 + \tilde{b}_1 y_2 - u_2 \end{aligned} \tag{3.10}$$

Choose the Lyapunov function as

$$V_2 = V_1 + w_1 + \tilde{b}_1 \tag{3.11}$$

Its time derivative is

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + \dot{w}_1 + \dot{\tilde{b}}_1 \\ &= \dot{V}_1 + x_1 e_3 + y_3 e_1 + b_1 w_1 + \tilde{b}_1 y_2 \\ &\quad - (x_1 + y_3 + b_1)K - u_2 + \dot{\tilde{b}}_1 \end{aligned} \tag{3.12}$$

We assume e_3 as the virtual controller, and choose the update laws of parameters and controller u_2 as

$$\begin{cases} e_3 = \beta e_2 = 0 \\ \dot{\tilde{b}}_1 = -\dot{\hat{b}}_1 = -\tilde{b}_1 y_2 \\ u_2 = y_3 e_1 - (x_1 + y_3 + b_1)K \end{cases} \tag{3.13}$$

Then we can obtain

$$\dot{V}_2 = -a_1 e_1 + 2b_1 w_1 < 0, \quad \text{where } b_1 = -10 \tag{3.14}$$

This means that $e_2 = 0$ is asymptotically stable.

Step 3 For studying the (e_1, w_1, w_2) system:

According to $e_3 = \beta e_2 = 0$, we have

$$w_2 = e_3 - \beta e_2 = e_3 \tag{3.15}$$

then the $(e1, w2)$ system (3.10) can be described as follows:

$$\begin{aligned} \dot{e}_1 &= -a_1 e_1 + \tilde{a}_1 y_1 \\ \dot{w}_1 &= b_1 w_1 + \tilde{b}_1 y_2 \\ \dot{w}_2 &= x_1 e_2/3 + y_2 e_1/3 + c_1 w_2 + \tilde{c}_1 y_3 \\ &\quad - \big((x_1 + y_2)/3 + c_1\big)K - u_3 \end{aligned} \tag{3.16}$$

Choose the Lyapunov function as

$$V_3 = V_1 + V_2 + w_1 + \tilde{c}_1 \tag{3.17}$$

Its time derivative is

$$\begin{aligned} \dot{V}_3 &= \dot{V}_1 + \dot{V}_2 + \dot{w}_2 + \dot{\tilde{c}}_1 \\ &= \dot{V}_1 + \dot{V}_2 + x_1 e_2/3 + y_2 e_1/3 + c_1 w_2 + \tilde{c}_1 y_3 \\ &\quad - \big((x_1 + y_2)/3 + c_1\big)K - u_3 + \dot{\tilde{c}}_1 \end{aligned} \tag{3.18}$$

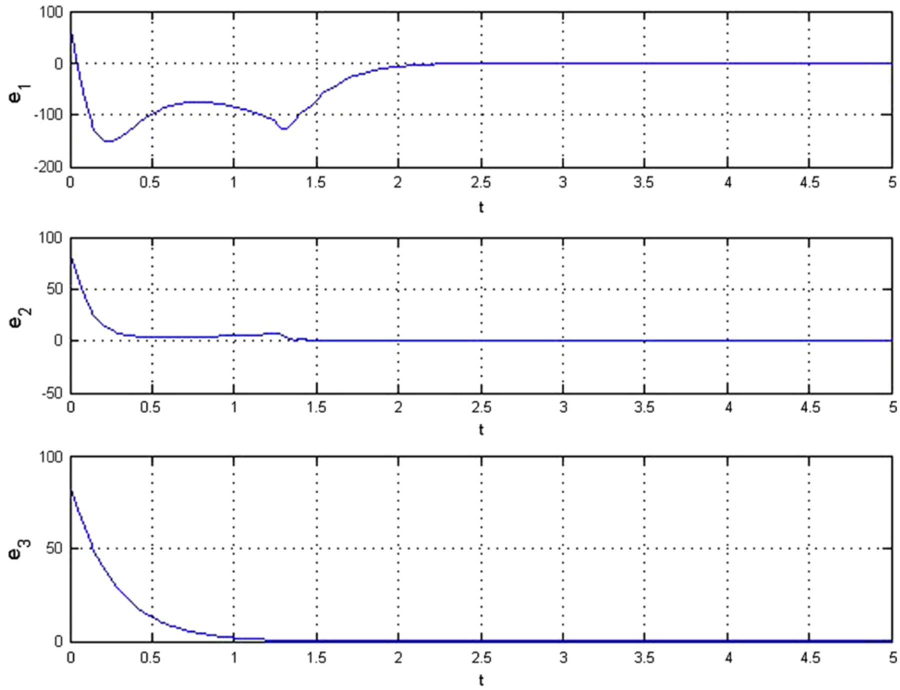


Fig. 2 Time histories of errors for Case I

We choose the update laws of parameters and controller u_3 as

$$\begin{cases} \dot{\tilde{c}}_1 = -\dot{\hat{c}}_1 = -\tilde{c}_1 y_3 \\ u_3 = x_1 e_2/3 + y_2 e_1/3 \\ \qquad - \big((x_1 + y_2)/3 + c_1\big) K \end{cases} \tag{3.19}$$

Then we can obtain

$$\dot{V}_3 = -a_1 e_1^2 + b_1 w_1^2 + c_1 w_2^2 < 0, \quad \text{where } a_1 = 5,\ b_1 = -10 \text{ and } c_1 = -3.8 \tag{3.20}$$

This means that $e_3 = 0$ is asymptotically stable.
The Lyapunov asymptotical stability theorem is not satisfied here. We cannot obtain that the common origin of error dynamics and parameter dynamics is asymptotically stable. By the pragmatical asymptotically stability theorem [49, 50], D is a 6-manifold, $n = 6$, and the number of error state variables $p = 3$. When $e_1 = e_2 = e_3 = 0$ and $\hat{a}, \hat{b}, \hat{c}$ take arbitrary values, $\dot{V} = 0$, so X is of 3 dimensions, $m = n - p = 6 - 3 = 3$, $m + 1 < n$ is satisfied. According to the pragmatical asymptotically stability theorem, error vector e approaches zero and the estimated parameters also approach the uncertain parameters. The equilibrium point is pragmatically asymptotically stable. Under the assumption of equal probability, it is actually asymptotically stable. The simulation results are shown in Figs. 2 and 3.
However, our goal is to synchronize the slave system (y_1, y_2, y_3) to trace the master system (x_1, x_2, x_3). As a result, all we have to do is shift the simulation results from $(y_1 + K, y_2 + K, y_3 + K)$ to (y_1, y_2, y_3), where K is constant designed via BGYC.

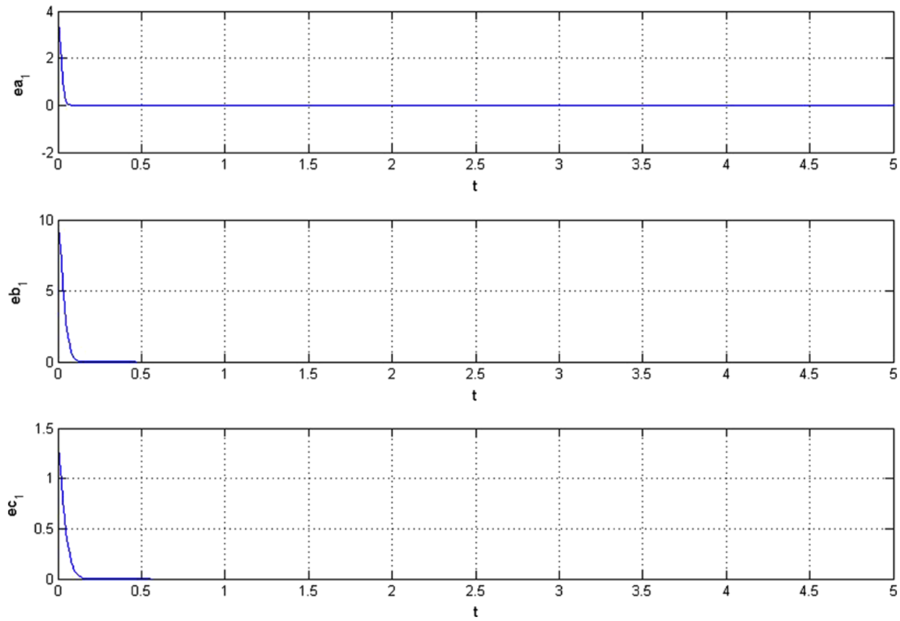
Case II Adaptive synchronization of the master and slave Newton–Leipink systems The master and slave Newton–Leipnik system is described by

$$\begin{cases} \dot{x}_1 = -a x_1 + x_2 + 10 x_2 x_3 \\ \dot{x}_2 = -x_1 - 0.4 x_2 + 5 x_1 x_3 \\ \dot{x}_3 = b x_3 - 5 x_1 x_2 \end{cases} \tag{3.21}$$

$$\begin{cases} \dot{y}_1 = -\hat{a} y_1 + y_2 + 10 y_2 y_3 + u_1 \\ \dot{y}_2 = -y_1 - 0.4 y_2 + 5 y_1 y_3 + u_2 \\ \dot{y}_3 = \hat{b} y_3 - 5 y_1 y_2 + u_3 \end{cases} \tag{3.22}$$

where a, b are positive parameters. The Newton–Leipnik system in Eq. (3.21) is a chaotic system with two strange attractors, which is shown in Fig. 4. For the two system parameter $a = 0.4$, $b = 0.175$, and ini-

Fig. 3 Time histories of parametric errors for Case I



tial states (0.349, 0, −0.160) and (20, 10, 15).

$$\begin{aligned} e &= \begin{bmatrix} e_1(t) & e_2(t) & e_3(t) \end{bmatrix} \\ &= [x_1 - y_1 + K \quad x_2 - y_2 + K \quad x_3 - y_3 + K] > 0 \end{aligned} \tag{3.23}$$

where $K = 20$, the addition of $K = 20$ makes the error dynamics always happen in the first quadrant.

From Eq. (3.23), we have the following error dynamics:

$$\begin{aligned} \dot{e}_1 &= -ax_1 + x_2 + 10x_2x_3 + \hat{a}y_1 - y_2 - 10y_2y_3 - u_1 \\ \dot{e}_2 &= -x_1 - 0.4x_2 + 5x_1x_3 + y_1 + 0.4y_2 \\ &\quad - 5y_1y_3 - u_2 \\ \dot{e}_3 &= bx_3 - 5x_1x_2 - \hat{b}y_3 + 5y_1y_2 - u_3 \end{aligned} \tag{3.24}$$

Equation (3.24) can be rearranged as follows:

$$\begin{aligned} \dot{e}_1 &= -ae_1 - \tilde{a}y_1 + e_2 + 10x_2e_3 + 10y_3e_2 \\ &\quad + (a - 10x_2 - 10y_3 - 1)K - u_1 \\ \dot{e}_2 &= -e_1 - 0.4e_2 + 5x_1e_3 + 5y_3e_1 \\ &\quad + (1.4 - 5x_1 - 5y_3)K - u_2 \\ \dot{e}_3 &= be_3 + \tilde{b}y_3 - 5x_1e_2 - 5y_2e_1 \\ &\quad + (5x_1 + 5y_2 - b)K - u_3 \end{aligned} \tag{3.25}$$

where $\tilde{a} = a - \hat{a}$ and $\tilde{b} = b - \hat{b}$ are the error of parameters.

Step 1 For the first equation of Eq. (3.25), we choose the Lyapunov function as

$$V_1 = e_1 + \tilde{a} \tag{3.26}$$

Its time derivative is

$$\begin{aligned} \dot{V}_1 &= \dot{e}_1 + \dot{\tilde{a}} \\ &= \big(-ae_1 - \tilde{a}y_1 + e_2 + 10x_2e_3 + 10y_3e_2 \\ &\quad + (a - 10x_2 - 10y_3 - 1)K - u_1\big) + \dot{\tilde{a}} \end{aligned} \tag{3.27}$$

We assume e_2 as the virtual controller, and choose the update laws of parameters and controller u_1 as

$$\begin{cases} e_2 = \alpha e_1 = 0 \quad (\alpha = 0) \\ \dot{\tilde{a}}_1 = -\dot{\hat{a}}_1 = \tilde{a}y_1 \\ u_1 = 10x_2e_3 + (a - 10x_2 - 10y_3 - 1)K \end{cases} \tag{3.28}$$

Then we can obtain

$$\dot{V}_1 = -ae_1 < 0 \tag{3.29}$$

This means that $e_1 = 0$ is asymptotically stable.

Step 2 For studying the (e_1, w_1) system:

According to $e_2 = \alpha e_1 = 0$, we have

$$w_1 = e_2 - \alpha e_1 = e_2 \tag{3.30}$$

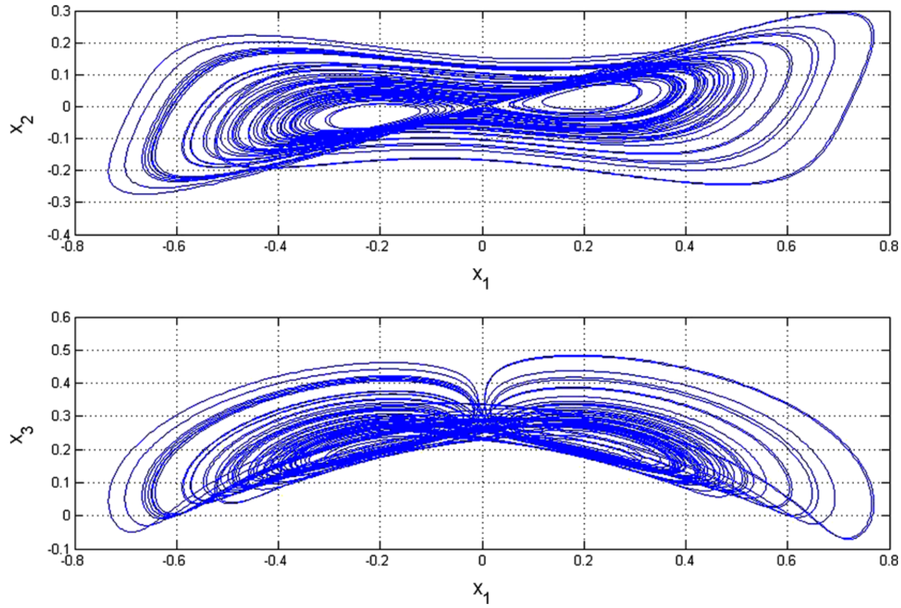


Fig. 4 Projections of phase portrait of chaotic Newton–Leipink system with $a = 0.4$, $b = 0.175$

then the $(e1, w2)$ system (3.31) can be described as follows:

$$\dot{e}_1 = -ae_1 - \tilde{a}y_1$$
$$\dot{w}_1 = -e_1 - 0.4w_1 + 5x_1e_3 + 5y_3e_1 + (1.4 - 5x_1 - 5y_3)K - u_2 \quad (3.31)$$

Choose the Lyapunov function as

$$V_2 = V_1 + w_1 \quad (3.32)$$

Its time derivative is

$$\begin{aligned}\dot{V}_2 &= \dot{V}_1 + w_1\dot{w}_1 \\ &= \dot{V}_1 + -e_1 - 0.4w_1 + 5x_1e_3 + 5y_3e_1 \\ &\quad + (1.4 - 5x_1 - 5y_3)K - u_2\end{aligned} \quad (3.33)$$

We assume e_3 as the virtual controller, and choose the update laws of parameters and controller u_2 as

$$\begin{cases} e_3 = \beta e_2 = 0 \quad (\beta = 0) \\ u_2 = -e_1 + 5y_3e_1 + (1.4 - 5x_1 - 5y_3)K \end{cases} \quad (3.34)$$

Then we can obtain

$$\dot{V}_2 = -ae_1 - 0.4w_1 < 0 \quad (3.35)$$

This means that $e_2 = 0$ is asymptotically stable.

Step 3 For studying the (e_1, w_1, w_2) system: According to $e_3 = \beta e_2 = 0$, we have

$$w_2 = e_3 - \beta e_2 = e_3 \quad (3.36)$$

then the $(e1, w2)$ system (3.37) can be described as follows:

$$\begin{aligned}\dot{e}_1 &= -a_1e_1 - \tilde{a}y_1 \\ \dot{w}_1 &= -0.4w_1 \\ \dot{w}_2 &= bw_2 + \tilde{b}y_3 - 5x_1e_2 - 5y_2e_1 \\ &\quad + (5x_1 + 5y_2 - b)K - u_3\end{aligned} \quad (3.37)$$

Choose the Lyapunov function as

$$V_3 = V_1 + V_2 + w_2 + \tilde{b} \quad (3.38)$$

Its time derivative is

$$\begin{aligned}\dot{V}_3 &= \dot{V}_1 + \dot{V}_2 + \dot{w}_2 + \dot{\tilde{b}} \\ &= \dot{V}_1 + \dot{V}_2 + \big(bw_2 + \tilde{b}y_3 - 5x_1e_2 - 5y_2e_1 \\ &\quad + (5x_1 + 5y_2 - b)K - u_3\big) + \dot{\tilde{b}}\end{aligned} \quad (3.39)$$

We choose the update laws of parameters and controller u_3 as

$$\begin{cases} \dot{\tilde{b}} = -\dot{\hat{b}} = -\tilde{b}y_3 \\ u_3 = (b + 1)w_2 - 5x_1e_2 - 5y_2e_1 \\ \quad\quad + (5x_1 + 5y_2 - b)K \end{cases} \quad (3.40)$$

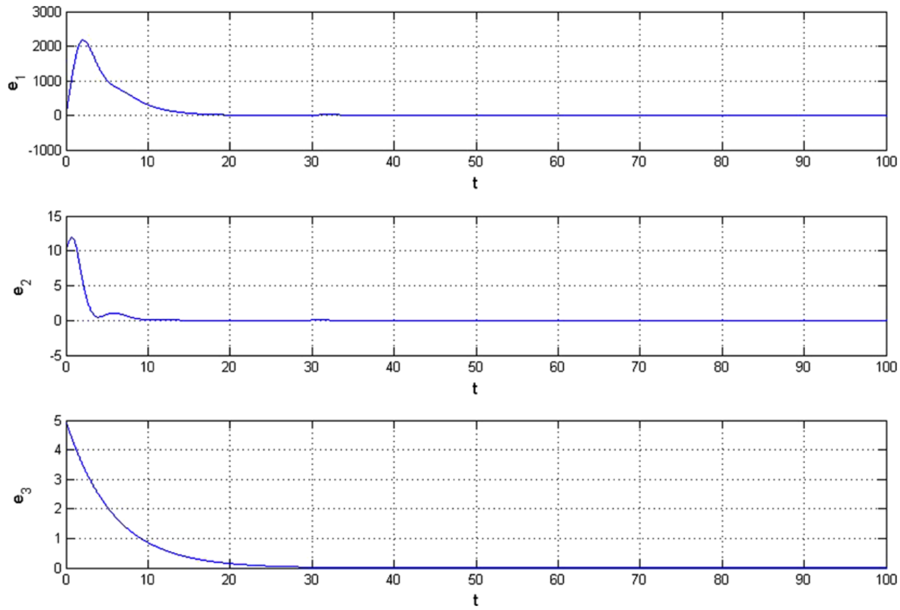


Fig. 5 Time histories of errors for Case II

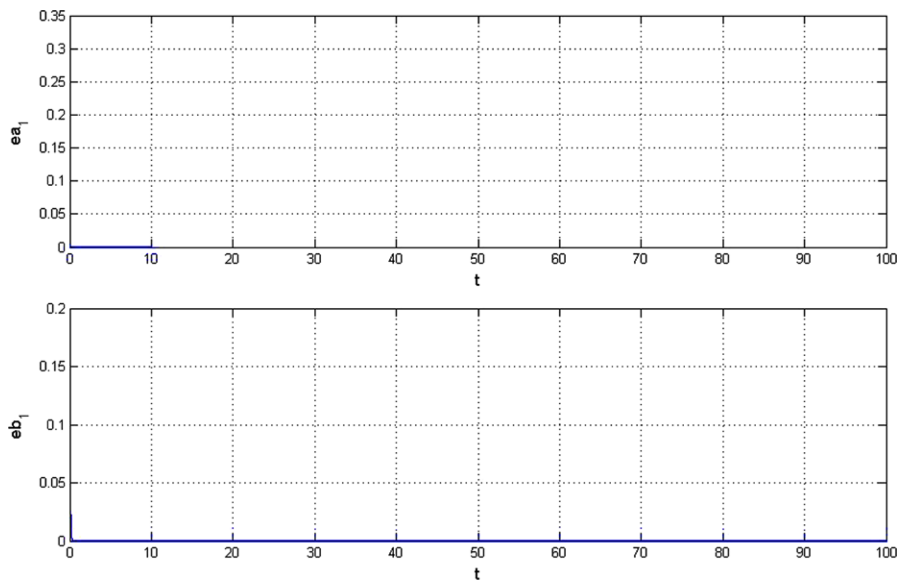


Fig. 6 Time histories of parametric errors for Case II

Then we can obtain

$$\dot{V}_3 = -a_1 e_1 - 0.4 w_1 - w_2 < 0,$$
$$\text{where } a = 0.4 \text{ and } b = 0.175 \quad (3.41)$$

This means that $e_3 = 0$ is asymptotically stable.

The Lyapunov asymptotical stability theorem is not satisfied here. We cannot obtain that common origin of error dynamics and parameter dynamics is asymptotically stable. By the pragmatical asymptotically stability theorem [49, 50], D is a 5-manifold, $n = 5$ and the number of error state variables $p = 3$. When $e_1 = e_2 = e_3 = 0$ and $\hat{a}, \hat{b}, \hat{c}$ take arbitrary values, $\dot{V} = 0$, so X is of 3 dimensions, $m = n - p = 5 - 3 = 2$, $m + 1 < n$ is satisfied. According to the pragmatical asymptotically stability theorem, error vector e approaches zero and the estimated parameters also approach the uncertain parameters. The equilibrium point is pragmatically asymptotically stable. Under the assumption of equal probability, it is actually asymptotically stable. The simulation results are shown in Figs. 5, 6, and 7.

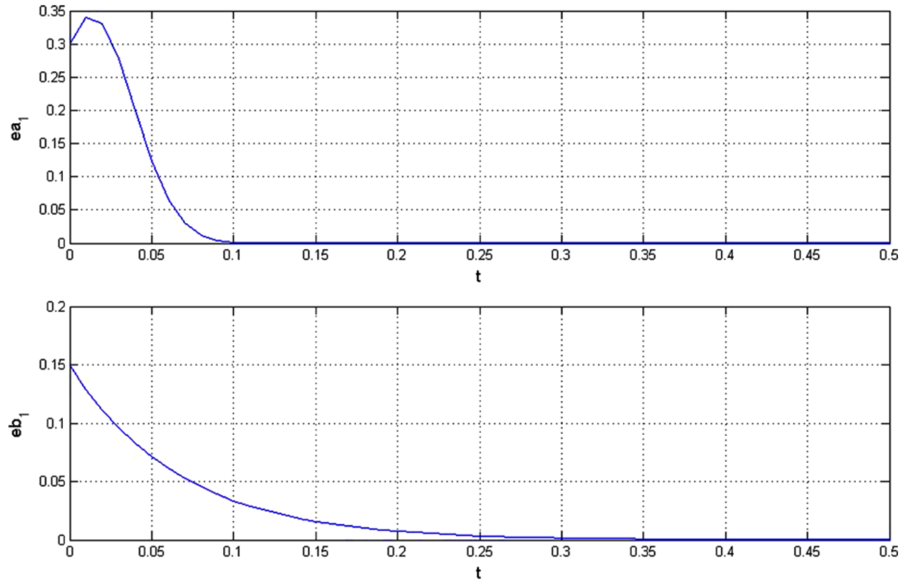


Fig. 7 Time histories of parametric errors for Case II-2

Again, our goal is to synchronize the slave system (y_1, y_2, y_3) to trace the master system (x_1, x_2, x_3). As a result, all we have to do is shift the simulation results from $(y_1 + K, y_2 + K, y_3 + K)$ to (y_1, y_2, y_3), where K is constant designed via BGYC.

4 Comparison

In this section, the simulation results of adaptive synchronizations with the traditional backstepping method for the two cases discussed in Sect. 3 are further given to demonstrate the effectiveness and the power of BGYC. The simulation results are investigated by the time history of errors and the time history of parametric errors which are shown in the figures.

In *Case I*, the controllers and update laws designed via the traditional method can be concluded as follows:

$$\begin{cases} u_1 = -x_2e_3 + 2a_1e_1 \\ u_2 = y_3e_1 \\ u_3 = x_1e_2/3 + y_2e_1/3 \\ \dot{\tilde{a}}_1 = -\dot{\hat{a}}_1 = -y_1e_1 \\ \dot{\tilde{b}}_1 = -\dot{\hat{b}}_1 = -y_2w_1 \\ \dot{\tilde{c}}_1 = -\dot{\hat{c}}_1 = -y_3w_2 \end{cases} \tag{4.1}$$

then, through simulation via the MATLAB/Simulink, we have the simulation results of time history of the errors and time history of parametric errors which are given in Figs. 8 and 9. The red dash lines refer to the simulation results of BGYC and the blue lines present the simulation results of the traditional one. It is obvious that the performance of adaptive synchronization is hugely raised up, especially in the parameters adapting, which are reaching the goal of parameters in 0.5 sec.

In *Case II*, the controllers and update laws designed via traditional method can be derived as follows:

$$\begin{cases} \dot{\tilde{a}} = -\dot{\hat{a}} = y_1e_1 + \tilde{a}e_1 \\ \dot{\tilde{b}} = -\dot{\hat{b}} = -y_3w_2 + \tilde{b}w_2 \\ u_1 = 10x_2e_3 + \tilde{a}^2 \\ u_2 = -e_1 + 5y_3e_1 \\ u_3 = (b+1)w_2 - 5x_1e_2 - 5y_2e_1 + \tilde{b}^2 \end{cases} \tag{4.2}$$

the simulation results of time history of errors and the time history of parametric errors, which are given in Figs. 10 and 11. The errors achieve the original points in 20 s via BGYC and in 50 s via the traditional one. On the other hand, the parametric errors achieve the original points within 0.4 s via BGYC and in 60 s via the traditional one. The efficiency of adaptive synchronization is truly increasing through the BGYC design.

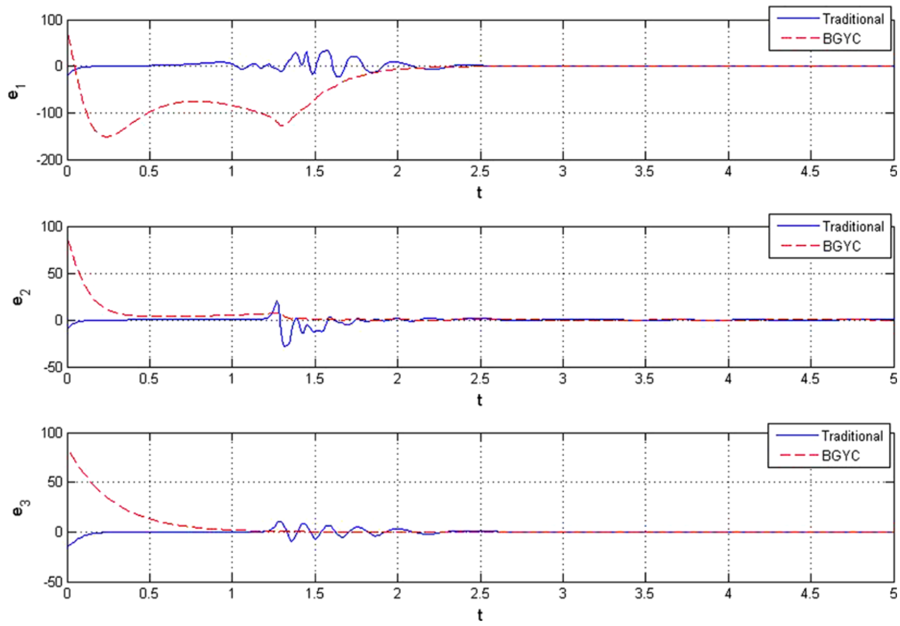


Fig. 8 Comparison of errors in Case I

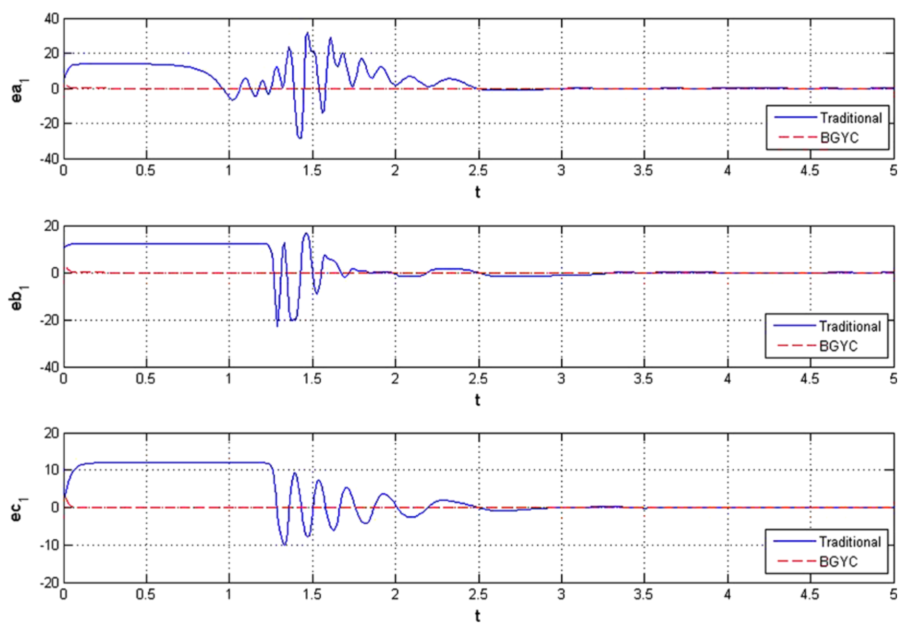


Fig. 9 Comparison of parametric errors in Case I

Through the comparison of figures in simulation results, our new approach—backstepping with the GYC partial region stability theory (BGYC) is demonstrated as an effective and powerful tool. It is not only increasing the converging speed to our goal enormously, but also having no complicated controller and update laws of parameters.

5 Conclusions

In this paper, a new strategy—backstepping with the GYC partial region stability theory (BGYC) is proposed to achieve adaptive synchronization. There three main contributions in this new approach: (1) the new Lyapunov function can be designed as a simple linear

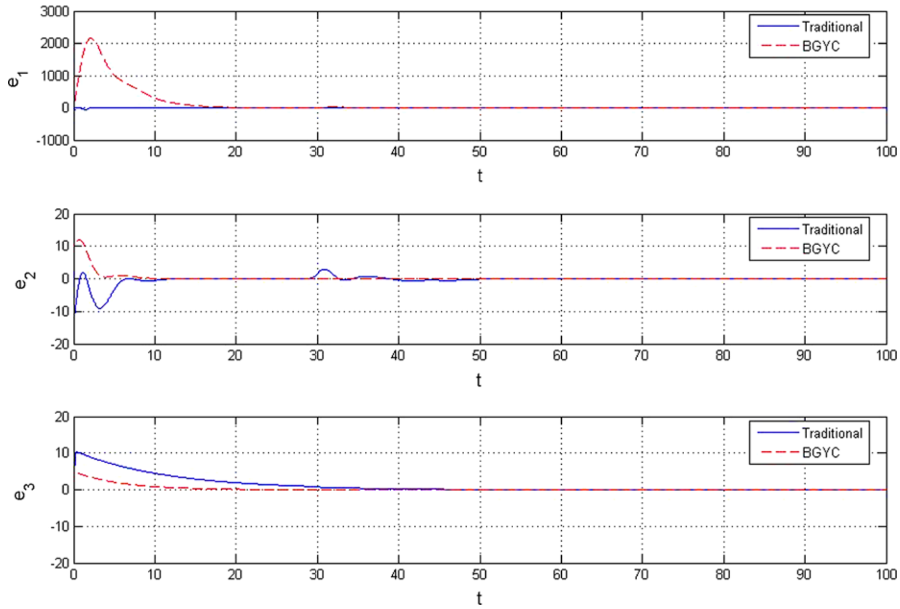


Fig. 10 Comparison of errors in Case II

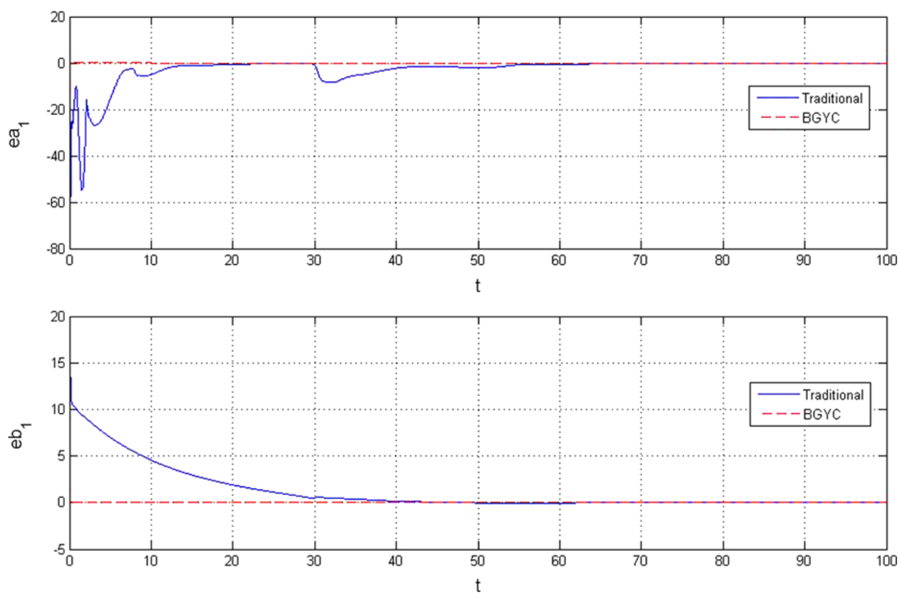


Fig. 11 Comparison of parametric errors in Case II

homogeneous function of states; (2) the update laws of parameters shall be simpler; (3) the performance of the adaptive synchronization is enormously raised up, especially in parameters adapting. This study gives another new strategy to achieve adaptive synchronization, and it can also be applied to various kinds of applications about parameters adapting or synchronization problems in advance.

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Appendix

For Case I: The error can be described as

$$\mathbf{e} = \begin{bmatrix} e_1(t) & e_2(t) & e_3(t) \end{bmatrix}. \tag{A.1}$$

From Eq. (A.1), we have the following error dynamics:

$$\dot{e}_1 = -x_2 e_3 - y_3 e_2 + a_1 e_1 + \tilde{a}_1 y_1 - u_1$$
$$\dot{e}_2 = x_1 e_3 + y_3 e_1 + b_1 e_2 + \tilde{b}_1 y_2 - u_2 \quad (A.2)$$
$$\dot{e}_3 = x_1 e_2/3 + y_2 e_1/3 + c_1 e_3 + \tilde{c}_1 y_3 - u_3$$

where $\tilde{a}_1 = a_1 - \hat{a}_1$, $\tilde{b}_1 = b_1 - \hat{b}_1$ and $\tilde{c}_1 = c_1 - \hat{c}_1$ are the error of parameters.

Step 1 For the first equation of Eq. (A.2), we choose the Lyapunov function as

$$V_1 = \frac{1}{2}\left(e_1^2 + \tilde{a}_1^2\right) \quad (A.3)$$

Its time derivative is

$$\dot{V}_1 = e_1 \dot{e}_1 + \tilde{a}\dot{\tilde{a}}$$
$$= e_1(-x_2 e_3 - y_3 e_2 + a_1 e_1 + \tilde{a}_1 y_1 - u_1) + \tilde{a}_1 \dot{\tilde{a}}_1 \quad (A.4)$$

We assume e_2 as the virtual controller, choose the update laws of parameters and controller u_1 as

$$\begin{cases} e_2 = \alpha e_1 = 0 \quad (\alpha = 0) \\ \dot{\tilde{a}}_1 = -\dot{\hat{a}}_1 = -y_1 e_1 \\ u_1 = -x_2 e_3 + 2a_1 e_1 \end{cases} \quad (A.5)$$

Then we can obtain

$$\dot{V}_1 = -a_1 e_1^2 < 0 \quad (A.6)$$

This means that $e_1 = 0$ is asymptotically stable.

Step 2 For studying the (e_1, w_1) system:
According to $e_2 = \alpha e_1 = 0$, we have

$$w_1 = e_2 - \alpha e_1 = e_2 \quad (A.7)$$

then the $(e1, w2)$ system (A.8) can be described as follows:

$$\dot{e}_1 = -a_1 e_1 + \tilde{a}_1 y_1$$
$$\dot{w}_1 = x_1 e_3 + y_3 e_1 + b_1 w_1 + \tilde{b}_1 y_2 - u_2 \quad (A.8)$$

Choose the Lyapunov function as

$$V_2 = V_1 + \frac{1}{2}\left(w_1^2 + \tilde{b}_1^2\right) \quad (A.9)$$

Its time derivative is

$$\dot{V}_2 = \dot{V}_1 + w_1 \dot{w}_1 + \tilde{b}_1 \dot{\tilde{b}}_1$$
$$= \dot{V}_1 + w_1(x_1 e_3 + y_3 e_1 + b_1 w_1 + \tilde{b}_1 y_2 - u_2) + \tilde{b}_1 \dot{\tilde{b}}_1 \quad (A.10)$$

We assume e_3 as the virtual controller, and choose the update laws of parameters and controller u_2 as

$$\begin{cases} e_3 = \beta e_2 = 0 \\ \dot{\tilde{b}}_1 = -\dot{\hat{b}}_1 = -y_2 w_1 \\ u_2 = y_3 e_1 \end{cases} \quad (A.11)$$

Then we can obtain

$$\dot{V}_2 = -a_1 e_1^2 + b_1 w_1^2 < 0, \quad \text{where } b_1 = -10 \quad (A.12)$$

This means that $e_2 = 0$ is asymptotically stable.

Step 3 For studying the (e_1, w_1, w_2) system:
According to $e_3 = \beta e_2 = 0$, we have

$$w_2 = e_3 - \beta e_2 = e_3 \quad (A.13)$$

then $(e1, w2)$ system (A.14) can be described as follow:

$$\dot{e}_1 = -a_1 e_1 + \tilde{a}_1 y_1$$
$$\dot{w}_1 = b_1 w_1 + \tilde{b}_1 y_2 \quad (A.14)$$
$$\dot{w}_2 = x_1 e_2/3 + y_2 e_1/3 + c_1 w_2 + \tilde{c}_1 y_3 - u_3$$

Choose the Lyapunov function as

$$V_3 = V_1 + V_2 + \frac{1}{2}\left(w_2^2 + \tilde{c}_1^2\right) \quad (A.15)$$

Its time derivative is

$$\dot{V}_3 = \dot{V}_1 + \dot{V}_2 + w_2 \dot{w}_2 + \tilde{c}_1 \dot{\tilde{c}}_1$$
$$= \dot{V}_1 + \dot{V}_2 + w_2(x_1 e_2/3 + y_2 e_1/3 + c_1 w_2 + \tilde{c}_1 y_3 - u_3) + \tilde{c}_1 \dot{\tilde{c}}_1 \quad (A.16)$$

We choose the update laws of parameters and controller u_3 as

$$\begin{cases} \dot{\tilde{c}}_1 = -\dot{\hat{c}}_1 = -y_3 w_2 \\ u_3 = x_1 e_2/3 + y_2 e_1/3 \end{cases} \quad (A.17)$$

Then we can obtain

$$\dot{V}_3 = -a_1 e_1^2 + b_1 w_1^2 + c_1 w_2^2 < 0, \quad \text{where } b_1 = -10 \text{ and } c_1 = -3.8 \quad (A.18)$$

This means that $e_3 = 0$ is asymptotically stable.

For Case II: The error can be described as

$$\mathbf{e} = \begin{bmatrix} e_1(t) & e_2(t) & e_3(t) \end{bmatrix}. \quad \text{(A.19)}$$

From Eq. (A.19), we have the following error dynamics:

$$\begin{aligned}
\dot{e}_1 &= -ae_1 - \tilde{a}y_1 + e_2 + 10x_2e_3 + 10y_3e_2 - u_1 \\
\dot{e}_2 &= -e_1 - 0.4e_2 + 5x_1e_3 + 5y_3e_1 - u_2 \\
\dot{e}_3 &= be_3 + \tilde{b}y_3 - 5x_1e_2 - 5y_2e_1 - u_3
\end{aligned} \quad \text{(A.20)}$$

where $\tilde{a} = a - \hat{a}$ and $\tilde{b} = b - \hat{b}$ are the error of parameters.

Step 1 For the first equation of Eq. (A.20), we choose the Lyapunov function as

$$V_1 = \frac{1}{2}\left(e_1^2 + \tilde{a}^2\right) \quad \text{(A.21)}$$

Its time derivative is

$$\begin{aligned}
\dot{V}_1 &= e_1\dot{e}_1 + \tilde{a}\dot{\tilde{a}} \\
&= e_1(-ae_1 - \tilde{a}y_1 + e_2 + 10x_2e_3 + 10y_3e_2 - u_1) \\
&\quad + \tilde{a}\dot{\tilde{a}}
\end{aligned} \quad \text{(A.22)}$$

We assume e_2 as the virtual controller, and choose the update laws of parameters and controller u_1 as

$$\begin{cases} e_2 = \alpha e_1 = 0 \quad (\alpha = 0) \\ \dot{\tilde{a}} = -\dot{\hat{a}} = y_1e_1 + \tilde{a}e_1 \\ u_1 = 10x_2e_3 + \tilde{a}^2 \end{cases} \quad \text{(A.23)}$$

Then we can obtain

$$\dot{V}_1 = -ae_1^2 < 0 \quad \text{(A.24)}$$

This means that $e_1 = 0$ is asymptotically stable.

Step 2 For studying the (e_1, w_1) system:
According to $e_2 = \alpha e_1 = 0$, we have

$$w_1 = e_2 - \alpha e_1 = e_2 \quad \text{(A.25)}$$

then the $(e1, w2)$ system (A.26) can be described as follows:

$$\begin{aligned}
\dot{e}_1 &= -ae_1 - \tilde{a}y_1 - \tilde{a}^2 \\
\dot{w}_1 &= -e_1 - 0.4e_2 + 5x_1e_3 + 5y_3e_1 - u_2
\end{aligned} \quad \text{(A.26)}$$

Choose the Lyapunov function as

$$V_2 = V_1 + \frac{1}{2}w_1^2 \quad \text{(A.27)}$$

Its time derivative is:

$$\begin{aligned}
\dot{V}_2 &= \dot{V}_1 + w_1\dot{w}_1 \\
&= \dot{V}_1 + w_1(-e_1 - 0.4w_1 + 5x_1e_3 + 5y_3e_1 - u_2)
\end{aligned} \quad \text{(A.28)}$$

We assume e_3 as the virtual controller, and choose the update laws of parameters and controller u_2 as

$$\begin{cases} e_3 = \beta e_2 = 0 \quad (\beta = 0) \\ u_2 = -e_1 + 5y_3e_1 \end{cases} \quad \text{(A.29)}$$

Then we can obtain

$$\dot{V}_2 = -ae_1^2 - 0.4w_1^2 < 0 \quad \text{(A.30)}$$

This means that $e_2 = 0$ is asymptotically stable.

Step 3 For studying the (e_1, w_1, w_2) system:
According to $e_3 = \beta e_2 = 0$, we have

$$w_2 = e_3 - \beta e_2 = e_3 \quad \text{(A.31)}$$

then the $(e1, w2)$ system (A.32) can be described as follows:

$$\begin{aligned}
\dot{e}_1 &= -ae_1 - \tilde{a}y_1 - \tilde{a}^2 \\
\dot{w}_1 &= -0.4w_1 \\
\dot{w}_2 &= bw_2 + \tilde{b}y_3 - 5x_1e_2 - 5y_2e_1 - u_3
\end{aligned} \quad \text{(A.32)}$$

Choose the Lyapunov function as

$$V_3 = V_1 + V_2 + \frac{1}{2}\left(w_2^2 + \tilde{b}^2\right) \quad \text{(A.33)}$$

Its time derivative is

$$\begin{aligned}
\dot{V}_3 &= \dot{V}_1 + \dot{V}_2 + w_2\dot{w}_2 + \tilde{b}\dot{\tilde{b}} \\
&= \dot{V}_1 + \dot{V}_2 + w_2(bw_2 + \tilde{b}y_3 - 5x_1e_2 \\
&\quad - 5y_2e_1 - u_3) + \tilde{b}\dot{\tilde{b}}
\end{aligned} \quad \text{(A.34)}$$

We choose the update laws of parameters and controller u_3 as

$$\begin{cases} \dot{\tilde{b}} = -\dot{\hat{c}}_1 = -y_3w_2 + \tilde{b}w_2 \\ u_3 = (b+1)w_2 - 5x_1e_2 - 5y_2e_1 + \tilde{b}^2 \end{cases} \quad \text{(A.35)}$$

Then we can obtain

$$\begin{aligned}
&\dot{V}_3 = -a_1e_1^2 - 0.4w_1^2 - w_2^2 < 0, \\
&\quad \text{where } a = 0.4 \text{ and } b = 0.175
\end{aligned} \quad \text{(A.36)}$$

This means that $e_3 = 0$ is asymptotically stable.

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