Comments on "Availability of k-Coterie"

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Abstract— **Kakugawa *et al.* proposed the k-majority coterie for the distributed k-mutual exclusion problem (k-mutex). It was claimed that the k-majority coterie is a k-coterie, which is a general solution for k-mutex. In this comment, we show that the k-majority coterie is not necessary a k-coterie.**

Index Terms—**Coterie, mutual exclusion, distributed system.**

Kakugawa *et al.* proposed the k-majority coterie for the distributed k-mutual exclusion problem (k-mutex: at most k processes can enter the critical section at a time) [1]. The authors claimed that the k-majority coterie is a k-coterie, which is a general solution for k-mutex. Let $U = \{u_1, \cdots, u_n\}$ be the set of processes, where n is the number of processes. The definitions of k-coterie and k-majority coterie are shown in the following.

Definition 1 [1]: A nonempty set C of nonempty subsets Q of U is called a k-coterie if and only if all the following three condition holds.

A1) Nonintersection property: For any $h(< k)$ elements $Q_1, \cdots, Q_h \in C$ such that $Q_i \cap Q_j = \emptyset (i \neq j)$ for $1 \leq i, j \leq h$, there exists an element $Q \in C$ such that $Q \cap Q_i = \emptyset$ for $1 \leq i \leq h$.

A2) Intersection property: For any $k + 1$ elements $Q_1, \cdots, Q_{k+1} \in C$ there exists a pair Q_i and Q_j such that $Q_i \cap Q_j \neq \emptyset$.

A3) Minimality property: For any two distinct elements Q_i and Q_j in C, $Q_i \not\subseteq Q_j$.

Definition 2 [1]: Let $W = \lceil (n + 1)/(k + 1) \rceil$, where n is the number of processes. The set $Maj_k = \{Q_i (\subseteq U) : |Q_i| = W\}$ is called k-majority coterie.

The author claimed that the k-majority coterie is a k-coterie. However, we find that this is not true for all n and k. For example, consider $n = 8$, $k = 3$, and thus $W = 3$. Let Q_1, Q_2 be two elements in Maj_3 ($|Q_1| = |Q_2| = 3$) such that $Q_1 \cap Q_2 = \emptyset$. If there exists an element Q in C ($|Q| = 3$) such that $Q \cap Q_1 = \emptyset$ and $Q \cap Q_2 = \emptyset$, then

$$|Q \cup Q_1 \cup Q_2| = |Q| + |Q_1| + |Q_2| = 9 > n.$$

It is a contradiction.

That is Condition A1) does not hold and the 3-majority coterie is not a 3-coterie.

To satisfy the conditions in Definition 2, the following conditions must hold for the k-majority coterie:

B1) $kW \leq n$;

B2) $(k + 1)W > n$;

where W in an integer.

In other words, there must exist an integer in $(\frac{n}{k+1}, \frac{n}{k}]$, which means that $\lfloor \frac{n}{k+1} \rfloor < \lfloor \frac{n}{k} \rfloor$.

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References

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