

# Optimal Design and Placement of Omni-Cameras in Binocular Vision Systems for Accurate 3-D Data Measurement

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**Abstract**—A new problem of automatically designing an optimal stereo vision system using two omni-directional catadioptric cameras to yield the highest 3-D data accuracy is studied. Factors of the system configuration considered in the design include camera pose, field of vision, and mirror shape. To find the optimal vision system configuration, an analytic formula is derived to model the 3-D measurement error, which takes into consideration the variations of pixel-quantization precisions and angular resolutions in images by conducting error propagation analysis in the data computation process. The formula is then used in an optimization framework to find the optimal system configurations for different shapes of system setup environments. For regular cases with rectangular cuboid-shaped 3-D measurement and camera placement areas, two fast algorithms are proposed to solve the problem, one being bisection-based and relatively slower for deriving the optimal solution, and the other faster using analytic formulas for deriving a suboptimal solution that is proved to be close to the optimal one in precision. Experimental results of simulations and real application cases show the feasibility of the proposed method.

**Index Terms**—3-D data measurement accuracy, omni-cameras, optimal system configuration, stereo vision system.

## I. INTRODUCTION

**H**UMAN-computer interaction is an important research area of modern technologies, attracting more and more attention in recent years. To become a successful product, many commercial applications are seeking better interfaces for man-machine interaction. Two examples are the Nintendo Wii and the Microsoft Kinect in the home entertainment industry.

One way to offer friendly interfacing is to provide the 3-D data of the user's movement, making it possible to interact with the computer without wearing or holding any device, such as the case of using the Microsoft Kinect. In the meantime, the

provision of a wide field of view (FOV) by a vision system is also desired in order to offer faster and more comfortable interaction. One approach is to use two wide-FOV cameras (such as fisheye-lens or catadioptric omni-cameras) to compose a stereo vision system for capturing the visual information in the application environment. The main goal of this paper is to investigate methods for designing binocular vision systems that can be used to acquire accurate 3-D data. The type of system considered is composed of two catadioptric omni-cameras, as depicted in Fig. 1, with each omni-camera consisting of a hyperboloidal-shaped mirror and a perspective camera.

Related to this paper, Eisert and Girod [1] reconstructed 3-D objects from images acquired by uncalibrated cameras by simultaneously estimating the camera-motion parameters and refining the object shape. Wang and Wu [2] recovered the projective depth for perspective 3-D Euclidean reconstruction by perspective factorization [3]. Starck *et al.* [4] constructed a 3-D production studio using multiple cameras and gave suggestions for camera deployment. When constructing stereo vision systems, it is well known that the precision of the calculated 3-D data is strongly affected by the system configuration [4]–[19], including the placements, directions, and mirror shapes of the used cameras. Thus, the objective of this paper is to derive the optimal configuration of a stereo vision system to yield the most accurately measured data.

Several methods have been proposed to derive optimal vision system configurations. Mittal [5] proposed a formula to assess 3-D data errors in terms of the object distance and the cameras' baseline and focal lengths. Hanel *et al.* [6] derived the optimal vision system configuration that makes the visual hull of the detected object more accurate. Cowan and Kovesi [7] generated camera locations such that better image resolutions can be obtained and all the object surface points appear within the cameras' FOVs, unoccluded, and in focus. In these methods, the issues of the variations of pixel-quantization precisions and angular resolutions in images are not considered. In contrast, the method proposed in this paper takes these issues into consideration by analyzing the error propagations in the 3-D data measurement process.

Alternately, the optimal system configuration can be derived by considering the angular error produced by the cameras [8]–[11]. Bishop *et al.* [8] assessed the 3-D data measurement error using the Fisher information matrix. Zhao *et al.* [9], [10] used the frame theory to prove the existence of only two types

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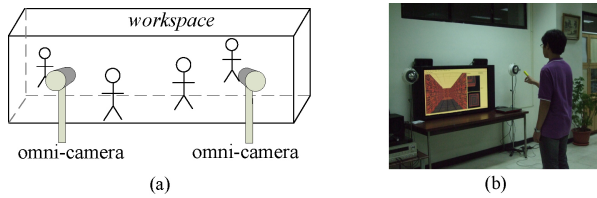


Fig. 1. Stereo vision system with two omni-cameras for this paper. (a) Illustration of the vision system. (b) User playing a game using the system.

of optimal placement of bearing-only sensors. Herath and Pathirana [11] analyzed the sensor-target geometry in terms of the Cramer–Rao inequality and the corresponding Fisher information matrix. However, in these studies, the angular information is assumed to include Gaussian noise. Also, the effects of the variations of pixel-quantization precisions and the angular resolutions yielded by the cameras are ignored. Less accurate modeling of 3-D data errors is so yielded. In contrast, the error model proposed in this paper takes such effects into consideration via analytic error propagation analysis.

One way to consider the effect of pixel-quantization precision variations is to assess the 3-D measurement error by the use of the covariance matrix [12]–[17]. For this, Wenhardt *et al.* [12] determined the locations of mobile cameras to yield the best 3-D model reconstruction by assessing the covariance of the resulting 3-D data in three ways, namely, using the determinant, eigenvalues, and trace of the covariance matrix, respectively. Hoppe *et al.* [15] used the eigenvalues of the covariance matrix to model the 3-D measurement error for precise camera localization and object modeling. Alsadik *et al.* [14] established a camera network for precise reconstruction of a cultural heritage object by the use of the trace of the covariance matrix. Olague and Mohr [16] proposed a multicellular genetic algorithm to decide camera locations, which yield minimal 3-D measurement errors, by the use of the maximum diagonal element of the covariance matrix. Zhang [13] determined the optimal 2-D spatial placement of multiple sensors participating in a robot perception task utilizing the determinant of the covariance matrix. Rivera-Rios *et al.* [17] analyzed 3-D measurement errors due to feature-point localization errors and found accordingly the optimal camera pose by the mean-square-error criterion using the covariance matrix of the 3-D measurement data. In these methods, the precisions of the 3-D measurements are all assessed by the use of the covariance matrix. Additionally, a local-affineness assumption is made when deriving the covariance matrix (as stated in [20]). Contrastively, an approach without making this assumption is proposed in this paper, which is achieved by detailed analysis of the error propagations in the 3-D data computation process. Furthermore, it is shown by experimental results in Section VII-B that the proposed approach yields more precise assessments by a factor of four than the covariance matrix-based methods.

Besides the use of the covariance matrix, some alternative methods have also been proposed to assess the 3-D measurement error. Ercan *et al.* [18] studied the problem of the optimal placement of multiple cameras and the selection of the best subset of the cameras for single target localization in a sensor

network. Their goal is to localize an object as accurately as possible in the ground plane, and the cameras are assumed to be oriented horizontally and far enough from the object. Blostein and Huang [19] analyzed the relationship between the geometry of a stereo camera setup and the accuracy in the obtained 3-D measurements. The probability that the position estimates obtained from triangulation are within a specified error tolerance was derived, with the error being modeled as known spatial image plane quantization. The error model is only derived for perspective cameras.

Briefly speaking, the method proposed in this paper differs from the existing ones in two aspects: 1) the 3-D measurement precision is assessed by error propagation analysis that considers the effect of pixel-quantization precision variations, and 2) the assessed 3-D measurement precision is related to the resolutions of the pixels in such a way that a feature point in a higher resolution area is with a lower error in the calculated 3-D measurements, and vice versa. As a result, the proposed method can be applied both to perspective cameras and to wide-FOV cameras. Contrarily, the above-mentioned methods make the assumption of uniform resolutions across the entire image, which is unreasonable when wide-FOV cameras are used.

Contributions of this paper include the following aspects.

- 1) Analytic formulas describing the error propagations in 3-D data computation using a binocular omni-camera vision system are derived for assessing the 3-D measurement precision.
- 2) Algorithms for determining the optimal system configuration, including the mirror shape, FOV, and the pose of the two omni-cameras, are derived for use in establishing a more precise stereo vision system.
- 3) Three optimization algorithms are designed for various environments to meet different applications: a) the first is applicable to general cases but slower; b) the second is applicable to regular cases but faster, yielding the optimal solution by an iterative scheme; and c) the third is applicable to regular cases and even faster, yielding a near-optimal solution using analytic formulas.

In the remainder of this paper, an optimization framework to find the optimal system configuration is proposed in Section II. Some formulas modeling the catadioptric omni-cameras are derived in Section III. A measure of 3-D data accuracy for use in the optimization process is derived in Section IV. The mentioned three optimization algorithms for finding the optimal system configuration are presented in Sections V and VI. Finally, experimental results and conclusions are given in Sections VII and VIII, respectively.

## II. OPTIMIZATION FRAMEWORK

An optimization framework is proposed in this paper, as illustrated in Fig. 2, to facilitate the determination of the optimal system configuration of a binocular omni-vision system, which includes the intrinsic parameters, locations, and orientations of the two omni-cameras of the system, for the purpose of acquiring the most accurate 3-D data. Some observable properties of this frame are as follows:

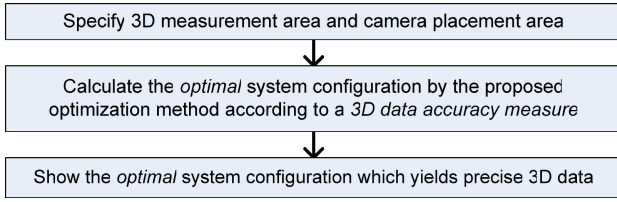


Fig. 2. Proposed optimization framework.

- 1) if the two omni-cameras are close to each other, the length of the line segment  $L$  connecting the two cameras, namely, the baseline, will become small, and the computed 3-D data accuracy will so be low;
- 2) if the baseline is very large, the space points in front of the cameras will become relatively close to  $L$ , and the resulting accuracy will so be low as well;
- 3) since the distortion of an image taken by an omni-camera is significant, the resolution also varies significantly in the taken image, so that, if a feature point is located in a higher resolution area, the computed 3-D data accuracy will become higher, and vice versa. According to these observed facts, it can be seen that optimal system configurations do exist; therefore, it is meaningful to propose an optimization framework, as conducted in this paper, for use in finding the optimal system configuration.

The proposed optimization framework, as depicted in Fig. 2, includes three main steps. First, an area where the 3-D data are measured is specified, called the 3-D measurement area; and an area where the cameras can be placed is also specified, called the camera placement area. Then, the optimal locations, optical axes, and intrinsic parameters of the two omni-cameras are derived according to one of the three proposed system optimization algorithms presented in Sections V-B, V-C, and VI-A, respectively. The three optimization algorithms are based on the use of an analytic formula indicating the degree of accuracy of the computed 3-D data, which is derived according to error propagation analysis as described in Section IV. The found optimal configuration is just the one with the highest degree of 3-D data accuracy, which is shown to the user to tell him/her how to place (and/or design) the cameras.

### III. OMNI-CAMERA STRUCTURE AND FORMULAS

#### A. Omni-Camera Structure

The catadioptric omni-camera is composed of a hyperboloid mirror and a perspective camera looking toward the mirror, as depicted in Fig. 3. A camera coordinate system (CCS)  $x$ - $y$ - $z$  and an image coordinate system  $u$ - $v$  are defined in such a way that the  $x$ - and  $y$ -axes are parallel to the  $u$ - and  $v$ -axes, respectively, and the two focal points of the mirror are at  $O(0, 0, 0)$  and  $O_c(0, 0, 2c)$ . In this way, the mirror shape can be expressed [21] by

$$(z - c)^2/a^2 - (x^2 + y^2)/b^2 = 1, \quad c = \sqrt{a^2 + b^2}, \quad z < c \quad (1)$$

with its eccentricity  $\varepsilon$  being defined by  $c/a > 1$ . A projection equation describing the relation between the complementary

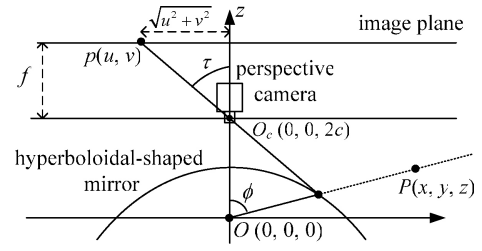


Fig. 3. Catadioptric omni-camera structure and its camera coordinate system.

elevation angle  $\phi$  of a space point  $P$  and image coordinates  $(u, v)$  of the projected pixel  $p$  can be expressed [22] as

$$\tan \phi = \frac{(\varepsilon^2 - 1) \sin \tau}{(\varepsilon^2 + 1) \cos \tau - 2\varepsilon} \quad (2)$$

where  $\cos \tau$  and  $\sin \tau$ , as seen from Fig. 3, can be expressed by

$$\cos \tau = \frac{f}{\sqrt{u^2 + v^2 + f^2}}; \quad \sin \tau = \frac{\sqrt{u^2 + v^2}}{\sqrt{u^2 + v^2 + f^2}}. \quad (3)$$

#### B. Determination of Intrinsic Parameters of Omni-Cameras

By assuming that the perspective camera in the omni-camera is well calibrated and distortion free, the only parameter of the perspective camera is its viewing angle  $2\tau_{\max}$ . Also, as specified in the projection (2), the eccentricity  $\varepsilon$  describes all the distortion effect produced by the mirror. Therefore, the intrinsic parameters of an omni-camera to be determined are the viewing angle  $2\tau_{\max}$  of the perspective camera and the eccentricity  $\varepsilon$  of the mirror. In the following, we first derive the formula for the eccentricity  $\varepsilon$  under the assumption that the viewing angle  $2\tau_{\max}$  of the perspective camera is fixed. Then, we provide a guideline to determine the angle  $2\tau_{\max}$ .

**Theorem 1:** If the viewing angle of the perspective camera is  $2\tau_{\max}$  and the viewing angle of the omni-camera is  $2\phi_{\max}$ , then the eccentricity  $\varepsilon$  of the hyperboloidal-shaped mirror is

$$\varepsilon = \frac{\sin \phi_{\max} + \sin \tau_{\max}}{\sin(\phi_{\max} - \tau_{\max})}. \quad (4)$$

**Proof:** According to the projection (2), we have

$$(\tan \phi_{\max} - \tan \tau_{\max}) \varepsilon^2 - 2 \tan \phi_{\max} \sec \tau_{\max} \varepsilon + (\tan \phi_{\max} + \tan \tau_{\max}) = 0. \quad (5)$$

Accordingly, two solutions of  $\varepsilon$  can be obtained to be

$$\varepsilon = \frac{\tan \phi_{\max} \sec \tau_{\max} \pm \tan \tau_{\max} \sec \phi_{\max}}{\tan \phi_{\max} - \tan \tau_{\max}} = \frac{\sin \phi_{\max} \pm \sin \tau_{\max}}{\sin(\phi_{\max} - \tau_{\max})}. \quad (6)$$

The solution  $\varepsilon_1$  with the minus sign is proved to be invalid as follows. First,  $\varepsilon_1$ , as an eccentricity, is larger than one, that is

$$\varepsilon_1 = (\sin \phi_{\max} - \sin \tau_{\max}) / \sin(\phi_{\max} - \tau_{\max}) > 1. \quad (7)$$

Next, since the viewing angle  $2\phi_{\max}$  of the omni-camera is larger than the viewing angle  $2\tau_{\max}$  of the perspective

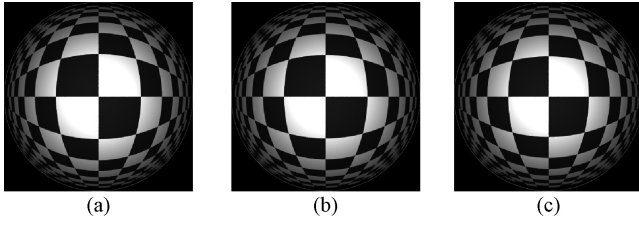


Fig. 4. Simulated omni-images using perspective cameras with different viewing angles. (a) 20°. (b) 40°. (c) 60°.

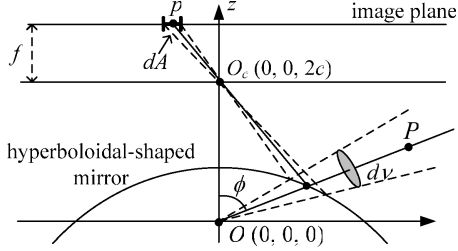


Fig. 5. Illustration for defining the resolution at a pixel.

camera, we have  $\pi > \phi_{\max} > \tau_{\max} > 0$ , leading to  $\cot(\phi_{\max}/2) < \cot(\tau_{\max}/2)$ , which, according to the cotangent half-angle formula [23], results in

$$\cot\left(\frac{\phi_{\max}}{2}\right) = \frac{\sin \phi_{\max}}{1 - \cos \phi_{\max}} < \frac{\sin \tau_{\max}}{1 - \cos \tau_{\max}} = \cot\left(\frac{\tau_{\max}}{2}\right) \quad (8)$$

or equivalently

$$\frac{\sin \phi_{\max} - \sin \tau_{\max}}{\sin(\phi_{\max} - \tau_{\max})} = \varepsilon_1 < 1 \quad (9)$$

which is a contradiction to (7). Therefore, the other solution described by (4) should be taken as the desired result. ■

The effect of choosing different viewing angle  $2\tau_{\max}$  of the perspective camera is shown by some images in Fig. 4 obtained from simulations with a checkerboard placed in front of the omni-camera, and the hyperboloidal mirror designed according to Theorem 1 in such a way that the entire checkerboard can be viewed. As seen from Fig. 4, the taken omni-images are not severely influenced by the magnitude of the viewing angle, implying that one may choose freely the viewing angle as long as the camera is distortion-free.

### C. Resolution Formula for Omni-Cameras

Baker and Nayer [24] proposed a formula to calculate the resolutions at different pixels in an omni-image as follows. Let  $dA$  be an infinitesimal area on the image plane near a pixel  $p$ , which, as illustrated in Fig. 5, is the projection of an area in the space described by an infinitesimal solid angle  $dv$  coming from a point  $P$ . The resolution of pixel  $p$  is formulated as

$$R(\varepsilon, f, \phi) = \frac{dA}{dv} = \frac{(\varepsilon^2 - 1)^2(\varepsilon^2 + 2\varepsilon \cos \phi + 1)}{[2\varepsilon + (\varepsilon^2 + 1) \cos \phi]^3} f^2 \quad (10)$$

where  $\varepsilon$  is the eccentricity of the mirror,  $f$  is the focal length of the camera, and  $\phi$  is the complementary elevation angle of  $P$ .

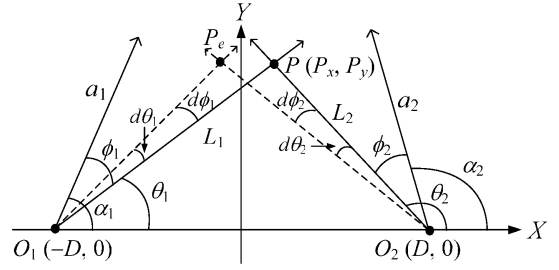


Fig. 6. Top-view illustration of triangulation process and error propagation.

## IV. FORMULA FOR DESCRIBING DEGREE OF ACCURACY

A criterion function measuring the degree of accuracy of the computed 3-D data is proposed in this section, for uses in the optimization algorithms proposed in Sections V and VI.

### A. 3-D Calculation Function and Its Differentials

A right-handed world coordinate system  $X$ - $Y$ - $Z$  is defined, as shown in Fig. 6, in such a way that the camera centers  $O_1$  and  $O_2$  are located at  $(-D, 0, 0)$  and  $(D, 0, 0)$ , respectively, and the  $XY$ -plane contains the space point  $P$ . An assumption that, the two optical axes  $a_1$  and  $a_2$  lie on the  $XY$ -plane, is made in the following derivations, and this assumption is analyzed more thoroughly in Section IV-C.

As depicted in Fig. 6, the two axes  $a_1$  and  $a_2$  are defined by the angles  $\alpha_1$  and  $\alpha_2$ , respectively. To calculate the 3-D data of  $P$ , two images are acquired first by the two omni-cameras, and a feature detection process is applied to extract the two pixels  $p_1$  and  $p_2$  corresponding to  $P$  in the two omni-images. Then, the complementary elevation angles  $\phi_1$  and  $\phi_2$  are derived by (2) using the coordinates of  $p_1$  and  $p_2$ , respectively. Finally, the angles  $\theta_1$  and  $\theta_2$  as depicted in Fig. 6 are computed by

$$\theta_1 = \alpha_1 - \phi_1, \text{ and } \theta_2 = \alpha_2 - \phi_2. \quad (11)$$

*Proposition 1:* As depicted in Fig. 6, the position  $(P_x, P_y)$  of space point  $P$  can be calculated in terms of  $\theta_1$  and  $\theta_2$  as

$$T(\theta_1, \theta_2) = (P_x, P_y) = \left( D \frac{\sin(\theta_2 + \theta_1)}{\sin(\theta_2 - \theta_1)}, 2D \frac{\sin \theta_2 \sin \theta_1}{\sin(\theta_2 - \theta_1)} \right) \quad (12)$$

and the differentials of  $P_x$  and  $P_y$  are

$$\begin{bmatrix} dP_x \\ dP_y \end{bmatrix} = \frac{2D}{\sin^2(\theta_2 - \theta_1)} \begin{bmatrix} \sin \theta_2 \cos \theta_2 d\theta_1 - \sin \theta_1 \cos \theta_1 d\theta_2 \\ \sin^2 \theta_2 d\theta_1 - \sin^2 \theta_1 d\theta_2 \end{bmatrix}. \quad (13)$$

*Proof:* As depicted in Fig. 6, the position of the feature point  $P$  can be calculated by two parametric equations

$$P = O_1 + s_1 \cdot (\cos \theta_1, \sin \theta_1); \quad P = O_2 + s_2 \cdot (\cos \theta_2, \sin \theta_2) \quad (14)$$

where  $P$ ,  $O_1$ , and  $O_2$  are regarded as 2-D coordinate vectors, and  $s_1$  and  $s_2$  are unknown parameters. Equation (14) is equivalent to

$$P_x = -D + s_1 \cos \theta_1 = D + s_2 \cos \theta_2; \quad P_y = s_1 \sin \theta_1 = s_2 \sin \theta_2 \quad (15)$$



which may be solved to get  $s_1$  and  $s_2$ , leading to the results

$$P_x = D \frac{\sin \theta_2 \cos \theta_1 + \cos \theta_2 \sin \theta_1}{\sin \theta_2 \cos \theta_1 - \cos \theta_2 \sin \theta_1} = D \frac{\sin(\theta_2 + \theta_1)}{\sin(\theta_2 - \theta_1)}$$

$$P_y = 2D \frac{\sin \theta_1 \sin \theta_2}{\sin \theta_2 \cos \theta_1 - \cos \theta_2 \sin \theta_1} = 2D \frac{\sin \theta_2 \sin \theta_1}{\sin(\theta_2 - \theta_1)}. \quad (16)$$

The differentials of  $P_x$  and  $P_y$  can be derived accordingly to be those described by (13). ■

### B. Considering Variations of Pixel-Quantization Precisions and Angular Resolutions

The infinitesimals  $d\theta_1$  and  $d\theta_2$  are derived by considering the varying pixel-quantization precisions and angular resolutions as follows. As can be seen from Fig. 6, if the feature detection process to extract  $p_1$  and  $p_2$  in the two images is inaccurate so that the angles  $\phi_1$  and  $\phi_2$  are with errors  $d\phi_1$  and  $d\phi_2$ , respectively, then an inaccurate triangulation result  $P_e$ , as depicted in Fig. 6, will be produced. Assume that the pixel quantization and feature detection process introduces an error within a small area  $dA$ , then, as depicted in Fig. 5, the measured angle  $\phi$  will be with an error  $d\phi$  related to the angular resolution function  $R$ . In the sense that  $d\phi$  is a 2-D solid angle and the angle  $d\phi$  is the corresponding 1-D angle, and under the assumption that the back-projected cone forming by  $d\phi$  is circular, the value of  $d\phi$  can be estimated by

$$d\phi = \pm \sqrt{d\phi} = \pm \sqrt{dA/R} \quad (17)$$

in which the term  $d\phi$  is expressed by the resolution formula (10). Also, by taking the differentiations of the equations in (11), one can get a relation between  $d\phi$  and  $d\theta$  as  $d\theta = -d\phi$ , which, after being combined with (17), leads to

$$d\theta = \pm \sqrt{dA/R}. \quad (18)$$

Accordingly, the errors  $d\theta_1$  and  $d\theta_2$  can be derived to be

$$d\theta_1 = \pm \sqrt{\frac{dA}{R(\varepsilon_1, f_1, \phi_1)}}, \text{ and } d\theta_2 = \pm \sqrt{\frac{dA}{R(\varepsilon_2, f_2, \phi_2)}}. \quad (19)$$

### C. Proposed Error Model and Its Limitations

To assess the 3-D measurement error of a feature point  $P$ , an error function  $E(P)$  is proposed in the following, and the degree of accuracy of the point  $P$  is defined as  $-E(P)$  in the sequel.

**Theorem 2:** With reference to Fig. 6, when the triangulation process yields an imprecise point  $P_e$  due to small errors  $d\theta_1$  and  $d\theta_2$ , the 3-D measurement error  $E(P)$ , which is the distance between the actual point  $P$  and the measured point  $P_e$ , is

$$E(P) = \max(E_1(P), E_2(P)) \quad (20)$$

where

$$E_1(P) = \frac{\sqrt{G_1^2(P) + 2G_1(P)G_2(P)\cos(\theta_2 - \theta_1) + G_2^2(P)}\sqrt{dA}}{\sin(\theta_2 - \theta_1)}$$

$$E_2(P) = \frac{\sqrt{G_1^2(P) - 2G_1(P)G_2(P)\cos(\theta_2 - \theta_1) + G_2^2(P)}\sqrt{dA}}{\sin(\theta_2 - \theta_1)}$$

$$G_1(P) = \frac{|\overline{O_1P}|}{\sqrt{R(\varepsilon_1, f_1, \phi_1)}}, \text{ and } G_2(P) = \frac{|\overline{O_2P}|}{\sqrt{R(\varepsilon_2, f_2, \phi_2)}}. \quad (21)$$

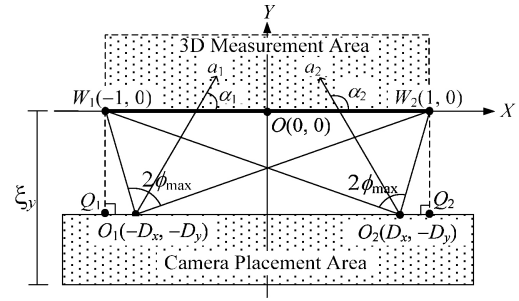


Fig. 7. Illustration of the regular cases.

**Proof:** The measurement error  $E$ , which by definition is the distance between  $P$  and  $P_e$ , may be computed from (13) to be

$$E(P) = \|dT\| = \sqrt{(dP_x)^2 + (dP_y)^2}$$

$$= \sqrt{\left(\frac{2D \sin \theta_2 d\theta_1}{\sin(\theta_2 - \theta_1)}\right)^2 - 2 \cos(\theta_2 - \theta_1) \frac{2D \sin \theta_2 d\theta_1}{\sin(\theta_2 - \theta_1)} \frac{2D \sin \theta_1 d\theta_2}{\sin(\theta_2 - \theta_1)} + \left(\frac{2D \sin \theta_1 d\theta_2}{\sin(\theta_2 - \theta_1)}\right)^2}$$

$$= \frac{\sqrt{\left(\frac{2D \sin \theta_2 d\theta_1}{\sin(\theta_2 - \theta_1)}\right)^2 - 2 \cos(\theta_2 - \theta_1) \frac{2D \sin \theta_2 d\theta_1}{\sin(\theta_2 - \theta_1)} \frac{2D \sin \theta_1 d\theta_2}{\sin(\theta_2 - \theta_1)} + \left(\frac{2D \sin \theta_1 d\theta_2}{\sin(\theta_2 - \theta_1)}\right)^2}}{\sin(\theta_2 - \theta_1)}. \quad (22)$$

From Fig. 6, it can be derived, from the law of sines, that

$$\frac{2D}{\sin(\theta_2 - \theta_1)} = \frac{|\overline{O_1P}|}{\sin \theta_2} = \frac{|\overline{O_2P}|}{\sin \theta_1}. \quad (23)$$

By combining (22) and (23), we get

$$E(P) = \frac{\sqrt{(|\overline{O_1P}| d\theta_1)^2 - 2 \cos(\theta_2 - \theta_1) (|\overline{O_1P}| d\theta_1)(|\overline{O_2P}| d\theta_2) + (|\overline{O_2P}| d\theta_2)^2}}{\sin(\theta_2 - \theta_1)} \quad (24)$$

which, when combined with (19), leads to (20) and (21). ■

## V. FAST CONFIGURATION OPTIMIZATION FOR REGULAR CASES

The optimization framework proposed in Section II can deal with general cases, in which the 3-D measurement area and the camera placement area can be of irregular shapes, and the two used perspective cameras can be different from each other. However, in regular indoor vision systems (called the regular cases), the two perspective cameras are of the same type, and the 3-D measurement area and the camera placement area can be specified by two rectangular cuboids as illustrated in Fig. 7. In the following, the formal definition of the optimization problem for such regular cases is derived in Section V-A. Then, the derivation of the proposed optimization algorithm for generating the corresponding optimal system configuration is proposed in Section V-B. Another suboptimal but analytic optimization method is derived in Section V-C, which is shown additionally to be a good approximation to the optimal solution.

### A. Problem Definition

As described in the optimization framework proposed in Section II, a system configuration includes all the necessary

parameters to design a vision system, and an optimization process needs to find the optimal system configuration that yields the best 3-D measurement accuracy. A criterion function  $E_w$  for the optimization is defined in this paper to be the maximum measurement error within the 3-D measurement area  $W$ , that is

$$E_w(W) = \max_{P \in W} (E(P)) \quad (25)$$

where  $E(P)$  is the measurement error of a feature point  $P$  as derived by (20) in Theorem 2. By choosing the maximum value, all the 3-D measurements errors are ensured to be lower than the value  $E_w$ . Next, as assumed, the two perspective cameras used in the omni-cameras are identical, so their focal lengths  $f_1$  and  $f_2$  are both equal to  $f$ , and their viewing angles  $2\tau_{\max 1}$  and  $2\tau_{\max 2}$  are both equal to  $2\tau_{\max}$ . The two omni-cameras are identical in structure and placed symmetrically, so the two optical axes  $a_1$  and  $a_2$  are coplanar so that the two optical axes can be defined by the two angles  $\alpha_1$  and  $\alpha_2$ , as shown in Fig. 7. Then, a system configuration can be defined to be the parameter set  $(D_x, D_y, \alpha_1, \alpha_2, \varepsilon_1, \varepsilon_2)$ , where 1) the omni-cameras are placed, as seen from the top, at  $O_1(-D_x, -D_y)$  and  $O_2(D_x, -D_y)$ , respectively; 2) the orientations of their optical axes are defined by  $\alpha_1$  and  $\alpha_2$ , respectively; and 3) the eccentricities of the mirrors are  $\varepsilon_1$  and  $\varepsilon_2$ , respectively. Hence, the optimization problem is just to find the optimal parameter set  $(D_x^*, D_y^*, \alpha_1^*, \alpha_2^*, \varepsilon_1^*, \varepsilon_2^*)$  derived in the following way

$$(D_x^*, D_y^*, \alpha_1^*, \alpha_2^*, \varepsilon_1^*, \varepsilon_2^*) = \arg \min_{(D_x, D_y, \alpha_1, \alpha_2, \varepsilon_1, \varepsilon_2)} (E_w(W)). \quad (26)$$

Since, it is desired that the captured image be fully filled up with the 3-D measurement area, the cameras should be oriented to face the 3-D measurement area. Accordingly, the optical axes  $a_1$  and  $a_2$  can be figured out to be just the bisectors of the angles spanned by the measurement area as depicted in Fig. 7, that is, the optical axis  $a_1$  of the left omni-camera is the bisector of the viewing angle formed by  $\overline{O_1 W_1}$  and  $\overline{O_1 W_2}$ , and the optical axis  $a_2$  is the bisector of the viewing angle formed by  $\overline{O_2 W_1}$  and  $\overline{O_2 W_2}$ . In view of these facts, the angle  $\phi_{\max}$  and the optical-axis angles  $\alpha_1$  and  $\alpha_2$  can be calculated in terms of  $D_x$  and  $D_y$  as follows. First, the fact  $\alpha_2 = \pi - \alpha_1$  can be derived from Fig. 7. Then, from the triangle formed by  $O_1$ ,  $W_2$ , and  $Q_2$ , we have  $D_y = (D_x + 1) \times \tan(\alpha_1 - \phi_{\max})$ . Similarly, from the triangle formed by  $O_1$ ,  $W_1$ , and  $Q_1$ , we have  $D_y = (D_x - 1) \times \tan(\alpha_1 + \phi_{\max})$ . Accordingly, the two unknowns  $\alpha_1$  and  $\phi_{\max}$  can be solved respectively to be

$$\begin{cases} \alpha_1 = 0.5 [\tan^{-1}(D_y/D_x - 1) + \tan^{-1}(D_y/D_x + 1)] \\ \phi_{\max} = 0.5 [\tan^{-1}(D_y/D_x - 1) - \tan^{-1}(D_y/D_x + 1)] \end{cases} \quad (27)$$

Then, based on Theorem 1, the eccentricities  $\varepsilon_1$  and  $\varepsilon_2$  are

$$\varepsilon_1 = \varepsilon_2 = \varepsilon = (\sin \phi_{\max} + \sin \tau_{\max}) / \sin(\phi_{\max} - \tau_{\max}) \quad (28)$$

where  $2\tau_{\max}$  is the viewing angle of the perspective cameras. To sum up, the optimization problem (26) is now reduced to

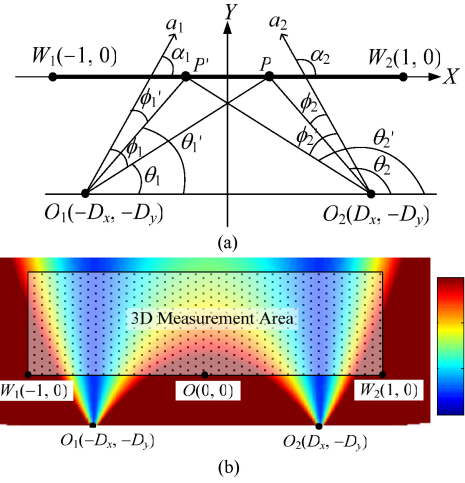


Fig. 8. Analysis of function  $E_w$ . (a) Illustrations of related terms. (b) Drawing of distribution of measurement errors  $E$  of a configuration.

include two parameters as follows:

$$(D_x^*, D_y^*) = \arg \min_{(D_x, D_y)} (E_w(W)) = \arg \min_{(D_x, D_y)} \left( \max_{P \in W} (E(P)) \right). \quad (29)$$

For further simplifications, two claims are given as follows.

*Claim 1:* The function  $E_w$  described by (25) can be rewritten as

$$E_w(W) = \max_{P \in W} (E(P)) = \max(E(O), E(W_2)) \quad (30)$$

if the two terms  $R(\varepsilon, f, \phi_1)$  and  $R(\varepsilon, f, \phi_2)$  are equal, where points  $O$  and  $W_2$  are located at  $(0, 0)$  and  $(1, 0)$ , respectively.

*Proof:* First, by referring to Fig. 8(a), the measurement error of a point  $P$  at coordinates  $(P_x, 0)$  can be derived using (20) and (21) with

$$\begin{aligned} \theta_1 &= \cos^{-1} \frac{P_x + D_x}{\sqrt{(P_x + D_x)^2 + D_y^2}}, \quad \theta_2 = \cos^{-1} \frac{P_x - D_x}{\sqrt{(P_x - D_x)^2 + D_y^2}}, \\ G_1(P) &= \frac{\sqrt{(P_x + D_x)^2 + D_y^2}}{\sqrt{R(\varepsilon, f, \phi_1)}}, \quad \text{and } G_2(P) = \frac{\sqrt{(P_x - D_x)^2 + D_y^2}}{\sqrt{R(\varepsilon, f, \phi_2)}}. \end{aligned} \quad (31)$$

Next, the function  $E(P)$  is proved to be an even function as follows. From Fig. 8(a), the angles  $\phi_1$  and  $\phi_2$  can be seen to be

$$\phi_1 = |\alpha_1 - \theta_1|, \text{ and } \phi_2 = |\alpha_2 - \theta_2|. \quad (32)$$

From the resolution formula (10), we can get the equality  $R(\varepsilon, f, \phi) = R(\varepsilon, f, -\phi)$  for any angle  $\phi$ . Accordingly, we have

$$R(\varepsilon, f, \phi_1) = R(\varepsilon, f, \alpha_1 - \theta_1); R(\varepsilon, f, \phi_2) = R(\varepsilon, f, \alpha_2 - \theta_2). \quad (33)$$

Let  $P'$  be the point located at  $(-P_x, 0)$ , and let the related angles  $\theta_1'$ ,  $\theta_2'$ ,  $\phi_1'$ , and  $\phi_2'$  be defined as those shown in Fig. 8(a). Since the triangles  $\triangle O_1 O_2 P'$  and  $\triangle O_1 O_2 P$  are similar, we get  $\theta_1' = \pi - \theta_2$  and  $\theta_2' = \pi - \theta_1$ . Combining these facts with (32),

we have

$$\begin{aligned}\phi'_1 &= |\alpha_1 - \theta'_1| = |(\pi - \alpha_2) - (\pi - \theta_2)| = |-\alpha_2 + \theta_2| = \phi_2; \\ \phi'_2 &= |\alpha_2 - \theta'_2| = |(\pi - \alpha_1) - (\pi - \theta_1)| = |-\alpha_1 + \theta_1| = \phi_1.\end{aligned}\quad (34)$$

Thus, the following equality can be derived:

$$G_1(P') = \frac{\sqrt{(-P_x + D_x)^2 + D_y^2}}{\sqrt{R(\varepsilon, f, \phi_1')}} = \frac{\sqrt{(D_x - P_x)^2 + D_y^2}}{\sqrt{R(\varepsilon, f, \phi_2)}} = G_2(P). \quad (35)$$

The function  $E_1(P)$  can be proved accordingly to be an even function by

$$\begin{aligned}E_1(P) &= \frac{\sqrt{G_1^2(P) + 2G_1(P)G_2(P)\cos(\theta_2 - \theta_1) + G_2^2(P)}\sqrt{dA}}{\sin(\theta_2 - \theta_1)} \\ &= \frac{\sqrt{G_2^2(P') + 2G_2(P')G_1(P')\cos(\theta_2' - \theta_1') + G_1^2(P')}\sqrt{dA}}{\sin(\theta_2' - \theta_1')} \\ &= E_1(P').\end{aligned}\quad (36)$$

Similarly, we can prove  $E_2(P) = E_2(P')$ , meaning that  $E_2(P)$  is also an even function.

Finally, we prove in the following the property that if  $R(\varepsilon, f, \phi_1)$  and  $R(\varepsilon, f, \phi_2)$  are equal, the maximum of  $E(P)$  will occur at  $O(0, 0)$  or  $W_2(1, 0)$ . Firstly, let  $P_x^*$  be a value of  $P_x$  such that the case  $\theta_2 - \theta_1 = 90^\circ$  occurs. When  $0 \leq P_x \leq P_x^*$ , we get  $90^\circ \leq \theta_2 - \theta_1 \leq 180^\circ$  so that  $\cos(\theta_2 - \theta_1) \leq 0$ , implying that  $E_2(P) \leq E_1(P)$  according to (21), which leads to the following fact according to (20):

$$E(P) = E_2(P). \quad (37)$$

Furthermore, by applying the law of cosines and the assumption  $R(\varepsilon, f, \phi_1) = R(\varepsilon, f, \phi_2)$ ,  $E_2(P)$  can be reduced in the following way:

$$\begin{aligned}E_2(P) &= \frac{\sqrt{|O_1P|^2 - 2|O_1P||O_2P|\cos(\theta_2 - \theta_1) + |O_2P|^2}}{\sin(\theta_2 - \theta_1)} \\ &= \frac{|O_1O_2|}{\sin(\theta_2 - \theta_1)}.\end{aligned}\quad (38)$$

Accordingly, since the angle  $\theta_2 - \theta_1$  decreases from  $180^\circ$  to  $90^\circ$  when  $P_x$  increases from 0 to  $P_x^*$ , the maximum of  $E(P)$  occurs at  $P_x = 0$ . For the other case that  $P_x^* \leq P_x \leq 1$ , we can get  $0^\circ \leq \theta_2 - \theta_1 \leq 90^\circ$  so that  $\cos(\theta_2 - \theta_1) \geq 0$ , implying that  $E_1(P) \leq E_2(P)$  according to (21), which leads to the following fact according to (20):

$$E(P) = E_1(P). \quad (39)$$

Furthermore, according to (31),  $E_1(P)$  can be expressed as

$$\begin{aligned}E_1(P) &= \frac{\sqrt{|O_1O_2|^2 + 4\vec{PO}_1 \cdot \vec{PO}_2}}{\sin(\theta_2 - \theta_1)} \\ &= \frac{\sqrt{|O_1O_2|^2 + 4(P_x + D_x, D_y) \cdot (P_x - D_x, D_y)}}{\sin(\theta_2 - \theta_1)}.\end{aligned}\quad (40)$$

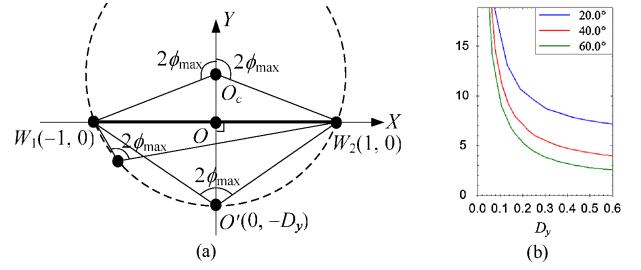


Fig. 9. Finding the optimal value of  $D_y$ . (a) Illustration to find the upper bound. (b) Plot of  $E_w$  for different values of  $2\tau_{\max}$ :  $20^\circ$ ,  $40^\circ$ , and  $60^\circ$ .

Accordingly, since the angle  $\theta_2 - \theta_1$  decreases from  $90^\circ$  to  $0^\circ$  when  $P_x$  increases, the maximum of  $E(P)$  occurs at  $P_x = 1$ . Combine the results of the two cases, we get the conclusion that the maximum occurs at  $O(0, 0)$  or  $W_2(1, 0)$ .

Finally, since both  $E_1(P)$  and  $E_2(P)$  are even functions, this conclusion may also be proved to hold for the left-side range  $-1 \leq P_x \leq 0$ . Therefore, the overall conclusion described by (30) can be drawn. ■

In the above proof, the assumption  $R(\varepsilon, f, \phi_1) = R(\varepsilon, f, \phi_2)$  is made, which is proved later by simulation results to be appropriate with very little affection on the 3-D measurement precision of the derived system configuration (Fig. 12).

**Claim 2:** A larger value of  $D_y$  always yields a smaller value of the criterion function  $E_w$ .

**Proof:** The inscribed angle theorem says that an angle  $\theta$  inscribed in a circle is a half of the central angle  $2\theta$  that subtends the same arc on the circle [25]. That is, if the viewing angle is  $2\phi_{\max}$ , the possible positions of the cameras can be figured out to be constrained on the dashed circle shown in Fig. 9(a), and the upper bound of  $D_y$  occurs at the bottom of the circle. Also, while recalling that the two cameras are omni-directional, we assume their viewing angle  $2\phi_{\max}$  to be larger than  $120^\circ$ . So, we have

$$\max(D_y) = \cot \phi_{\max} \cot(60^\circ) \approx 0.6. \quad (41)$$

With this upper bound, the function  $E_w$  is plotted in Fig. 9(b), which shows that a larger value of  $D_y$  yields a smaller value of  $E_w$ . ■

With Claim 1, Eq. (29) can be reformulated as

$$(D_x^*, D_y^*) = \arg \min_{(D_x, D_y)} (\max(E(O), E(W_2))). \quad (42)$$

Also, recall that the upper bound of  $D_y$  is limited by the camera deployment constraint, which we denoted as  $\xi_y$  in Fig. 7. With Claim 2, the optimal value  $D_y^*$  in (42) can be derived to be  $\min(\xi_y, 0.6)$ , leaving  $D_x^*$  to be the only parameter to be optimized according to the following constraint:

$$D_x^* = \arg \min_{D_x} (\max(E_{\text{mid}}(D_x, D_y^*), E_{\text{bound}}(D_x, D_y^*))) \quad (43)$$

where  $E_{\text{bound}}$  and  $E_{\text{mid}}$  are defined as

$$E_{\text{bound}}(D_x, D_y^*) = E(W_2); E_{\text{mid}}(D_x, D_y^*) = E(O). \quad (44)$$

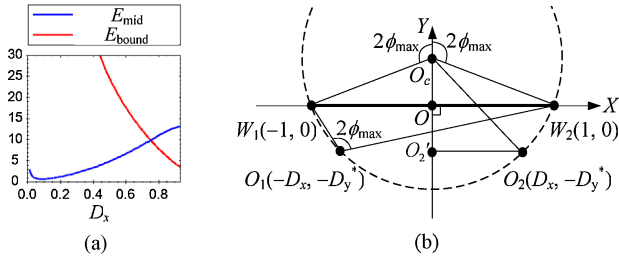


Fig. 10. Illustrations of finding the optimal solution  $D_x^*$  and its upper bound. (a) Plot of  $E_{\text{mid}}$  and  $E_{\text{bound}}$  versus  $D_x$  when  $D_y^* = 0.1$  and  $2\tau_{\text{max}} = 60.0^\circ$ . (b) Derivation of the upper bound of  $D_x$ .

### B. Optimization by Numerical Methods

An optimization algorithm to solve  $D_x^*$  in (43) by a bisection scheme is proposed in this section. By referring to the plots of  $E_{\text{mid}}$  and  $E_{\text{bound}}$  depicted in Fig. 10(a), the optimal solution  $D_x^*$  is found at the intersection of the two functions of  $E_{\text{mid}}$  and  $E_{\text{bound}}$ . So, if the two functions intersect each other, the intersection point may be defined to be the optimal solution  $D_x^*$ ; otherwise, the optimal solution  $D_x^*$  is defined to be the maximum value which will also be derived later in this section. In more details, at first we define a new function  $E_{\text{opt}}$  as

$$E_{\text{opt}}(D_x, D_y^*) = E_{\text{mid}}(D_x, D_y^*) - E_{\text{bound}}(D_x, D_y^*). \quad (45)$$

Then, the optimal solution  $D_x^*$  is just the root of  $E_{\text{opt}}$ , which can be derived by a bisection scheme. Before the scheme is conducted, the initial range of the root must be determined. The lower bound  $\text{lower}_{D_x}$  of  $D_x$  is obviously zero, and the upper bound  $\text{upper}_{D_x}$  is derived as follows. From Fig. 10(b), we have

$$|\overline{O_c O_2}| = |\overline{O_c W_2}| = \csc(\pi - 2\phi_{\text{max}}) = \csc(2\phi_{\text{max}}). \quad (46)$$

The coordinates of the circle center  $O_c$  are

$$O_c = (0, \cot(\pi - 2\phi_{\text{max}})) = (0, -\cot(2\phi_{\text{max}})). \quad (47)$$

According to the Pythagorean theorem, we have

$$\begin{aligned} D_x^2 &= |\overline{O_c O_2}|^2 - |\overline{O_c O_2^*}|^2 = \csc^2(2\phi_{\text{max}}) \\ &\quad - [D_y^* - \cot(2\phi_{\text{max}})]^2 \\ &= 1 - D_y^*[D_y^* - 2\cot(2\phi_{\text{max}})] \end{aligned} \quad (48)$$

and the first derivative of  $D_x^2$  with respect to  $2\phi_{\text{max}}$  is

$$\frac{\partial(D_x^2)}{\partial(2\phi_{\text{max}})} = 2D_y^*(-\csc^2(2\phi_{\text{max}})). \quad (49)$$

Since  $D_y^* > 0$ , the first derivative of  $D_x^2$  is smaller than zero, that is  $D_x$  decreases as  $2\phi_{\text{max}}$  increases, or equivalently, that the maximum of  $D_x$  occurs when  $\phi_{\text{max}}$  is minimized. So, the upper bound of  $D_x$  can be derived from (48) to be

$$\text{upper}_{D_x} = \sqrt{1 - D_y^*[D_y^* - 2\cot(2\phi_{\text{max}})]}. \quad (50)$$

A method to solve the optimization problem is proposed below.

### Algorithm 1 Finding the optimal configuration $(D_x^*, D_y^*)$ .

- Input:** the viewing angle  $2\tau_{\text{max}}$  and the focal length  $f$  of the cameras.
- Output:** the optimal system configuration  $(D_x^*, D_y^*)$ , meaning that the omni-cameras are placed at  $O_1(-D_x^*, D_y^*)$  and  $O_2(D_x^*, D_y^*)$ , and oriented as shown in Fig. 7.
- Steps.**
- Step 1. Calculate  $\xi_y$  according to the deployment size as stated in Section V-A, and set  $D_y^* = \min(\xi_y, 0.6)$ .
  - Step 2. Set variable  $\text{lower}_{D_x} = 0$  and calculate  $E_{\text{opt}}(\text{lower}_{D_x}, D_y^*)$  as follows and assign the result to the variable lower.
    - 2.1 Set  $D_x = \text{lower}_{D_x}$  and calculate  $\phi_{\text{max}}$  according to (27).
    - 2.2 Calculate the eccentricity  $\varepsilon$  according to (28).
    - 2.3 Calculate  $E_{\text{mid}}$  and  $E_{\text{bound}}$  by (44) with  $O_1 = (-D_x, D_y^*)$  and  $O_2 = (D_x, D_y^*)$ , and calculate  $E_{\text{opt}}$  by (45).
  - Step 3. Calculate the upper bound  $\text{upper}_{D_x}$  of  $D_x^*$  by (50).
  - Step 4. Calculate  $E_{\text{opt}}(\text{upper}_{D_x}, D_y^*)$  in a way similar to Steps 2.1 through 2.3, and assign the result to the variable upper.
  - Step 5. If lower and upper are with opposite signs, then find the root in a bisecting fashion as follows [26].
    - 5.1 Set variable  $\text{new}_{D_x} = (\text{lower}_{D_x} + \text{upper}_{D_x})/2$ .
    - 5.2 Calculate  $E_{\text{opt}}(\text{new}_{D_x}, D_y^*)$  in a way similar to Steps 2.1 through 2.3 and assign the result to the variable new.
    - 5.3 If  $(\text{new} < \text{lower})$ , then set  $\text{lower} = \text{new}$  and  $\text{lower}_{D_x} = \text{new}_{D_x}$ ; otherwise, set  $\text{upper} = \text{new}$  and  $\text{upper}_{D_x} = \text{new}_{D_x}$ .
    - 5.4 If  $(\text{upper}_{D_x} - \text{lower}_{D_x}) < \delta$ , where  $\delta$  is a predefined precision threshold, then take  $(D_x^*, D_y^*) = (\text{new}_{D_x}, \xi_y)$  as the output and exit; otherwise, go to Step 5.1.
  - Step 6. If  $\text{lower} > 0$  and  $\text{upper} > 0$  or if  $\text{lower} < 0$  and  $\text{upper} < 0$ , then choose  $D_x^*$  to be the upper bound  $\text{upper}_{D_x}$ , and take  $(D_x^*, D_y^*) = (\text{upper}_{D_x}, \xi_y)$  as the output.

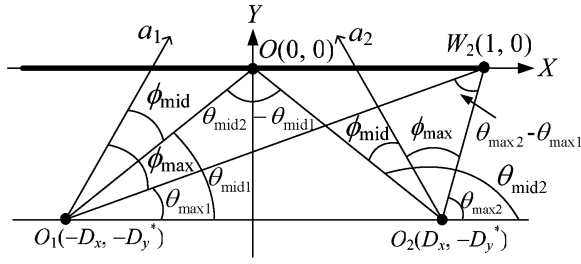
### C. Derivation of Analytic Formula for Suboptimal Solution

The method proposed in Algorithm 1 is further simplified to get an analytic formula for deriving a suboptimal solution. Let  $v_1$  and  $v_2$  be two vectors, and  $\theta$  be the included angle. We have

$$|v_1 \pm v_2|^2 = (v_1 \pm v_2) \cdot (v_1 \pm v_2) = |v_1|^2 \pm 2|v_1||v_2|\cos\theta + |v_2|^2. \quad (51)$$

By referring to Fig. 11, the formula of the measurement error  $E_{\text{mid}}$  can be derived, using (44), (20), (21), and (51), to be

$$\begin{aligned} E_{\text{mid}}(D_x, D_y^*) &= \frac{\max \left( \sqrt{\frac{|\overline{O_1 O}|^2 \pm 2|\overline{O_1 O}||\overline{O_2 O}| \cos(\theta_{\text{mid}2} - \theta_{\text{mid}1}) + |\overline{O_2 O}|^2}}{\cos(\theta_{\text{mid}2} - \theta_{\text{mid}1}) + \frac{|\overline{O_2 O}|^2}{|\overline{O_1 O}|^2}} \right) \sqrt{dA}}{\sin(\theta_{\text{mid}2} - \theta_{\text{mid}1}) \sqrt{R(\varepsilon, f, \phi_{\text{mid}})}} \\ &= \frac{\max \left( \left| \overline{O_1 O} + \overline{O_2 O} \right|, \left| \overline{O_1 O} - \overline{O_2 O} \right| \right) \sqrt{dA}}{\sin(\theta_{\text{mid}2} - \theta_{\text{mid}1}) \sqrt{R(\varepsilon, f, \phi_{\text{mid}})}} \\ &= \frac{\max(2D_y^*, 2D_x) \sqrt{dA}}{\sin(\theta_{\text{mid}2} - \theta_{\text{mid}1}) \sqrt{R(\varepsilon, f, \phi_{\text{mid}})}}. \end{aligned} \quad (52)$$

Fig. 11. Related parameters involved in  $E_{\text{mid}}$  and  $E_{\text{bound}}$ .

Referring to Fig. 11 and based on the double-angle formula of the sine function, we can get

$$\begin{aligned} \sin(\theta_{\text{mid2}} - \theta_{\text{mid1}}) &= 2 \sin\left(\frac{\theta_{\text{mid2}} - \theta_{\text{mid1}}}{2}\right) \cos\left(\frac{\theta_{\text{mid2}} - \theta_{\text{mid1}}}{2}\right) \\ &= \frac{2D_y^* D_x}{D_x^2 + (D_y^*)^2}. \end{aligned} \quad (53)$$

Thus, the function  $E_{\text{mid}}$  in (52) can be rewritten as

$$\begin{aligned} E_{\text{mid}}(D_x, D_y^*) &= \frac{[D_x^2 + (D_y^*)^2] \sqrt{dA}}{2D_y^* D_x \sqrt{R(\varepsilon, f, \phi_{\text{mid}})}} \\ &= \frac{[D_x^2 + (D_y^*)^2] \sqrt{dA}}{\max(2D_y^*, 2D_x) \sqrt{R(\varepsilon, f, \phi_{\text{mid}})}}. \end{aligned} \quad (54)$$

Similarly, the measurement error  $E_{\text{bound}}$  of the feature point  $W_2(1, 0)$  can be simplified, using (20), (21), (44), and (51), to be

$$\begin{aligned} E_{\text{bound}}(D_x, D_y^*) &= \frac{\max\left(\sqrt{\frac{|\vec{O_1 W_2}|^2 \pm 2|\vec{O_1 W_2}| |\vec{O_2 W_2}|}{\cos(\theta_{\text{max2}} - \theta_{\text{max1}}) + |\vec{O_2 W_2}|^2}}\right) \sqrt{dA}}{\sin(\theta_{\text{max2}} - \theta_{\text{max1}}) \sqrt{R(\varepsilon, f, \phi_{\text{max}})}} \\ &= \frac{\max\left(2\sqrt{1 + (D_y^*)^2}, 2D_x\right) \sqrt{dA}}{\sin(\theta_{\text{max2}} - \theta_{\text{max1}}) \sqrt{R(\varepsilon, f, \phi_{\text{max}})}}. \end{aligned} \quad (55)$$

From the geometry shown in Fig. 11, we have

$$\begin{aligned} \sin(\theta_{\text{max2}} - \theta_{\text{max1}}) &= \sqrt{1 - \left(\frac{\vec{O_1 W_2} \cdot \vec{O_2 W_2}}{|\vec{O_1 W_2}| |\vec{O_2 W_2}|}\right)^2} \\ &= \frac{2D_x D_y^*}{\sqrt{(D_x^2 + (D_y^*)^2 + 1)^2 - 4D_x^2}}. \end{aligned} \quad (56)$$

Thus, the function  $E_{\text{bound}}$  in (55) can be rewritten to be

$$\begin{aligned} E_{\text{bound}}(D_x, D_y^*) &= \frac{\sqrt{(D_x^2 + (D_y^*)^2 + 1)^2 - 4D_x^2} \max\left(\sqrt{1 + (D_y^*)^2}, D_x\right) \sqrt{dA}}{D_x D_y^* \sqrt{R(\varepsilon, f, \phi_{\text{max}})}}. \end{aligned} \quad (57)$$

By combining (54) and (57), the function  $E_{\text{opt}}$  can be reformulated as

$$\begin{aligned} E_{\text{opt}}(D_x, D_y^*) &= \frac{(D_x^2 + (D_y^*)^2) \sqrt{dA}}{\min(D_y^*, D_x) \sqrt{R(\varepsilon, f, \phi_{\text{mid}})}} \\ &\quad - \frac{\max\left(\sqrt{1 + (D_y^*)^2}, D_x\right) \sqrt{(D_x^2 + (D_y^*)^2 + 1)^2 - 4D_x^2} \sqrt{dA}}{D_x D_y^* \sqrt{R(\varepsilon, f, \phi_{\text{max}})}}. \end{aligned} \quad (58)$$

To calculate the optimal system configuration, the value  $D_y^*$  is firstly derived in the same way as stated in Algorithm 1. Then, the optimal solution  $D_x^*$  is just the root of the function  $E_{\text{opt}}$  described by (58). However, the root does not have an analytic formula since the involved terms  $\phi_{\text{mid}}$ ,  $\phi_{\text{max}}$ , and  $\varepsilon$  are all with complicated formulas with respect to  $D_x$ . In order to derive an analytic solution for the root, we first propose a skillful approximation method to produce a new function  $E'_{\text{opt}}$  to approximate the original one  $E_{\text{opt}}$ , and then derive an analytic formula to compute a suboptimal solution  $D_x'$ . This suboptimal solution  $D_x'$  is a good approximation to the optimal one  $D_x^*$  as will be shown later in this section.

To simplify the function  $E_{\text{opt}}$  described by (58), we assume further

$$R(\varepsilon, f, \phi_{\text{mid}}) \approx R(\varepsilon, f, \phi_{\text{max}}) \quad (59)$$

which is a special case of the assumption  $R(\varepsilon, f, \phi_1) = R(\varepsilon, f, \phi_2)$  made before in the proof of Claim 1. This new assumption can be proved as well later by simulation results to be proper with very little affection on the 3-D measurement precision of the derived system configuration (Fig. 12). Consequently, (58) may now be simplified to be

$$\begin{aligned} E'_{\text{opt}}(D_x, D_y^*) &= \left(\frac{\sqrt{dA}}{\sqrt{R(\varepsilon, f, \phi_{\text{max}})}}\right) \times \left(\frac{D_x^2 + (D_y^*)^2}{\min(D_y^*, D_x)}\right) \\ &\quad - \frac{\max\left(\sqrt{1 + (D_y^*)^2}, D_x\right) \sqrt{[D_x^2 + (D_y^*)^2 + 1]^2 - 4D_x^2}}{D_x D_y^*}. \end{aligned} \quad (60)$$

Thus, the root  $D_x'$  of the equation  $E'_{\text{opt}}(D_x, D_y^*) = 0$  satisfies

$$\frac{(D_x')^2 + (D_y^*)^2}{\min(D_y^*, D_x')} = \max\left(\sqrt{1 + (D_y^*)^2}, D_x'\right) \frac{\sqrt{[(D_x')^2 + (D_y^*)^2 + 1]^2 - 4(D_x')^2}}{D_x' D_y^*}. \quad (61)$$

**Theorem 3:** The solution of  $D_x'$  in (61) is

$$D_x' = \sqrt{\frac{-4(D_y^*)^4 - (C - 2)(D_y^*)^2 - C^2 + C + 5}{3C}} \quad (62)$$

where

$$\begin{aligned} C &= \sqrt[3]{0.5Q - 8(D_y^*)^6 - 48(D_y^*)^4 - 46.5(D_y^*)^2 - 5.5}; \\ Q &= \sqrt{27(1 + (D_y^*)^2)(128(D_y^*)^8 + 352(D_y^*)^6 + 288(D_y^*)^4 + 75(D_y^*)^2 + 23)}. \end{aligned} \quad (63)$$



*Proof:* To deal with the involved min/max function in (61), four cases are discussed separately, which are: 1)  $D_x' < D_y^*$  and  $D_x' \leq (1 + (D_y^*)^2)^{0.5}$ ; 2)  $D_x' < D_y^*$  and  $D_x' > (1 + (D_y^*)^2)^{0.5}$ ; 3)  $D_x' \geq D_y^*$  and  $D_x' \leq (1 + (D_y^*)^2)^{0.5}$ ; and 4)  $D_x' \geq D_y^*$  and  $D_x' > (1 + (D_y^*)^2)^{0.5}$ . It is proved that only Case (3) is valid.

For Case (1), we have the assumptions  $D_x' < D_y^*$  and  $D_x' \leq (1 + (D_y^*)^2)^{0.5}$ , so that (61) can be reduced to be

$$\frac{(D_x')^2 + (D_y^*)^2}{D_x'} = \sqrt{1 + (D_y^*)^2} \frac{\sqrt{[(D_x')^2 + (D_y^*)^2 + 1]^2 - 4(D_x')^2}}{D_x' D_y^*} \quad (64)$$

or equivalently, to be

$$\begin{aligned} D_y^* [(D_x')^2 + (D_y^*)^2] \\ = \sqrt{1 + (D_y^*)^2} \sqrt{[(D_x')^2 + (D_y^*)^2 + 1]^2 - 4(D_x')^2}. \end{aligned} \quad (65)$$

Defining  $A = (D_x')^2$  and  $B = (D_y^*)^2$ , (65) can be expressed as

$$A^2 - 2A + (3B^2 + 3B + 1) = 0. \quad (66)$$

Since, the discriminant of (66) is  $4 - 4(3B^2 + 3B + 1) < 0$ , the solution of  $A$  does not exist, or equivalently,  $D_x'$  does not exist. As a result, the assumptions made for Case (1) are invalid.

The assumptions made for Case (2) are  $D_x' < D_y^*$  and  $D_x' > (1 + (D_y^*)^2)^{0.5}$ . Since these two inequalities are contradictory to each other, Case (2) is also out of consideration.

For Case (4), the two assumptions are  $D_x' \geq D_y^*$  and  $D_x' > (1 + (D_y^*)^2)^{0.5}$ . Thus, (61) can be rewritten to be

$$(D_x')^2 + (D_y^*)^2 = \sqrt{[(D_x')^2 + (D_y^*)^2 + 1]^2 - 4(D_x')^2}. \quad (67)$$

Defining  $A = (D_x')^2$ ,  $B = (D_y^*)^2$  and taking the squares of both sides of the above equation, we have  $(A + B)^2 = (A + B + 1)^2 - 4A$ , or equivalently,  $2A = 2B + 1$ . However, from the second assumption  $D_x' > (1 + (D_y^*)^2)^{0.5}$ , we get  $A > B + 1$ , which is a contradiction to the equation  $2A = 2B + 1$  derived previously. Therefore, the assumptions made for Case (4) are also invalid.

As a result, Case (3) is the only valid one, for which the two assumptions are  $D_x' \geq D_y^*$  and  $D_x' \leq (1 + (D_y^*)^2)^{0.5}$ . Accordingly, (61) can be rewritten to be

$$\frac{(D_x')^2 + (D_y^*)^2}{D_y^*} = \sqrt{1 + (D_y^*)^2} \frac{\sqrt{[(D_x')^2 + (D_y^*)^2 + 1]^2 - 4(D_x')^2}}{D_x' D_y^*} \quad (68)$$

or equivalently, to be

$$\alpha A^3 + bA^2 + cA + d = 0 \quad (69)$$

where  $A = (D_x')^2$ ,  $B = (D_y^*)^2$ ,  $a = 1$ ,  $b = B - 1$ ,  $c = 2 - B^2$ , and  $d = -(B + 1)^3$ . To find the solution of  $A$  for the cubic function (69), we first calculate its discriminant  $\Delta$ , according to [27], as

$$\begin{aligned} \Delta &= 18abcd - 4b^3d + b^2c^2 - 4ac^3 - 27a^2d^2 \\ &= -(B + 1)(128B^4 + 352B^3 + 288B^2 + 75B + 23) \end{aligned} \quad (70)$$

**Algorithm 2** Finding a suboptimal configuration  $(D_x', D_y^*)$  by analytic formulas.

**Input:** the viewing angle  $2\tau_{\max}$  and the focal length  $f$  of the cameras.

**Output:** the suboptimal configuration  $(D_x', D_y^*)$ , meaning that the omni-cameras are placed at  $O_1(-D_x', D_y^*)$  and  $O_2(D_x', D_y^*)$ , and oriented as shown in Fig. 7.

**Steps.**

- Step 1. Calculate  $\xi_y$  according to the deployment size as stated in Section V-A, and set  $D_y^* = \min(\xi_y, 0.6)$ .
- Step 2. Calculate the upper bound  $\text{upper}_{D_x}$  of  $D_x^*$  by (50).
- Step 3. Calculate  $D_x'$  by (62) derived in Theorem 3.
- Step 4. Set  $D_x' = \min(D_x', \text{upper}_{D_x})$ .
- Step 5. Output the optimal system configuration  $(D_x', D_y^*)$ .

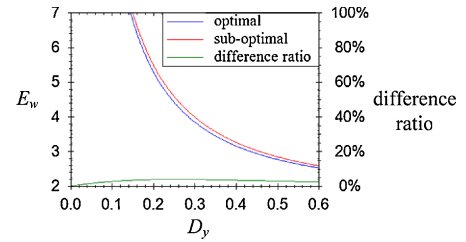


Fig. 12. Comparison of optimal configuration  $(D_x', D_y^*)$  and suboptimal configuration  $(D_x', D_y^*)$  with viewing angle  $2\tau_{\max} = 60^\circ$ .

which is smaller than zero because  $B = (D_y^*)^2 > 0$ . Thus, we get to know that the cubic polynomial equation has only one real root that can be described by (62) according to [27]. ■

The above-described process of generating the suboptimal configuration  $(D_x', D_y^*)$  is summarized as an algorithm below.

The suboptimal configuration  $(D_x', D_y^*)$  is shown to be a good approximation to the optimal one  $(D_x^*, D_y^*)$  as follows. Recalling that the goal of the optimization is to minimize the measurement error  $E_w$  defined by (43), we use the function  $E_w$  as a criterion to analyze the precision of the approximate one. In Fig. 12, we plot the curves of the measurement error values of the optimal and suboptimal configurations for all the possible values of  $D_y$ , from which we see that the measurement errors are very close to each other, meaning that the suboptimal configuration also yields precise 3-D measurement results as the optimal configuration does. Also, we use a difference ratio defined by

$$(E_w' - E_w^*)/E_w^* \quad (71)$$

to determine the goodness of the performance of the suboptimal configuration, where  $E_w^*$  and  $E_w'$  denote the measurement errors using the optimal and suboptimal configurations, respectively. As shown in the figure, the difference ratio is smaller than 0.4%, showing that the suboptimal solution is indeed a good approximation.

## VI. OPTIMIZATION FOR GENERAL CASES

### A. Idea of System Optimization for General Cases

To design a system configuration for the general case, the 3-D measurement area and the camera placement area

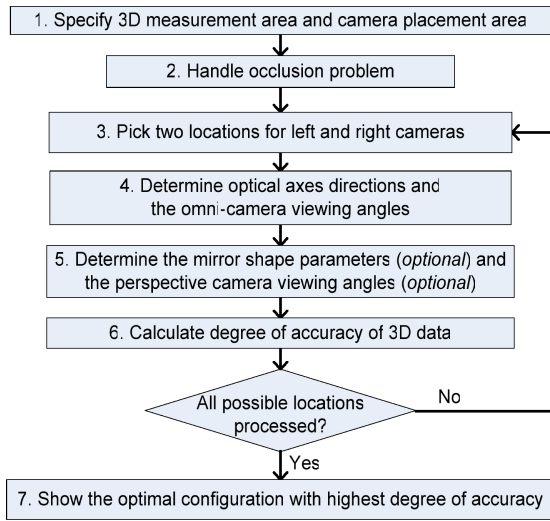


Fig. 13. Proposed optimization process to deal with general cases.

are specified first. To make the descriptions of the possibly irregular shapes of the two areas easy, each area is described by multiple sampled points, called the 3-D measurement locations and camera placement locations, respectively. For example, a cuboid can be described by 10,000 evenly-distributed points. The occlusion problem can be handled by just eliminating the camera locations where the 3-D measurement locations will be partially occluded if the camera was placed there [29].

After the identification of both the 3-D measurement locations and the camera placement locations, the optimal system configuration can be found out as follows (see Fig. 13 for a flowchart). At first, two locations are chosen from the camera placement locations to be the positions of the left and right cameras, respectively. Then, the cameras are oriented to face the 3-D measurement area as described next in Section VI-B to determine the extrinsic parameters of the omni-cameras. Also, their intrinsic parameters, including the mirror-shape parameter and the viewing angle of the perspective camera, are determined by the method proposed in Section III.

To decide which configuration yields the most precise 3-D measurements, a measure indicating the degree of accuracy of the computed 3-D data defined as  $\min(-E(M_i))$  is calculated, where  $-E(M_i)$  is the degree of accuracy of a 3-D measurement location  $M_i$  as derived in Theorem 2. It is noted that, by choosing the minimum value, all the 3-D measurement locations are ensured to be at least with this degree of accuracy. Then, to find the optimal system configuration, the previously-described steps are executed repeatedly for all possible camera locations as shown by the loop in Fig. 13, and the configuration with the highest degree of accuracy is picked out finally as the desired result.

#### B. Determination of Camera's Optical Axes and View Angles

After the positions of the two omni-cameras are decided, it is necessary to determine the optical axis and the viewing angle of each omni-camera. To solve this problem, the camera should face the 3-D measurement locations, and this in turn determines the orientations of the camera. Specifically, let  $M_i$

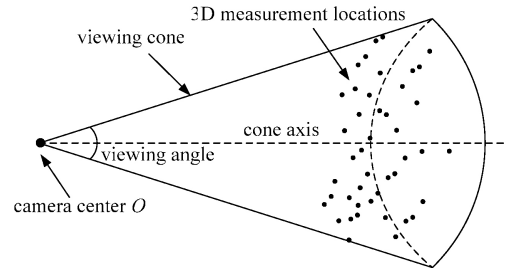


Fig. 14. Illustration for finding the optical axis and the viewing angle.

denote the points in the 3-D measurement locations, and  $O$  be the chosen location of each omni-camera (i.e., the camera center). Then, the problem may be solved, as depicted in Fig. 14, by the three steps: 1) find the smallest viewing cone containing all the 3-D measurement locations  $M_i$  and with  $O$  as its apex; 2) set the optical axis as the viewing cone axis; and 3) take the viewing angle  $2\phi_{\max}$  of the omni-camera to be the aperture of the cone.

#### C. Speed-Up Techniques

To speed up the optimization method described in Fig. 13, three techniques are proposed as follows.

- 1) *Longest baseline first*: When picking up the locations of the cameras in Step 3, the ones with large baselines (i.e., the distance between the cameras) are picked up first.
- 2) *Farthest 3-D point first*: When calculating the 3-D data accuracy in Step 6, the 3-D points that are farther from the two cameras are calculated first.
- 3) *Early stopping*: If the computed 3-D data accuracy is larger than that of the so-far best configuration, then the current configuration cannot be a better one, and so the algorithm continues to perform Step 3 for the next configuration.

These techniques were implemented and tested by experiments, and the results show that they reduced the running time from 56.6 s down to 22.3, indicating a speedup of about 2.5 times.

#### D. Choosing Optimization Methods for Different Applications

Three optimization methods have been proposed in Sections V-B, V-C, and VI, for the regular-optimal, regular-suboptimal, and general cases, respectively. These methods have their own advantages and disadvantages as described in the following.

- 1) If the 3-D measurement area and the camera placement area are both approximately of rectangular shapes, the first and second methods can be used; otherwise, the third one.
- 2) If the optimization process can be done in an off-line fashion, the third method is suitable; otherwise, the first two methods should be used.
- 3) If it requires a fast computation time, or the system has a low computation capability, the first two methods are suitable; and among the two, the second is the faster one but a little bit less accurate.

Some possible applications are stated next to demonstrate the uses of the three optimization methods. When designing a

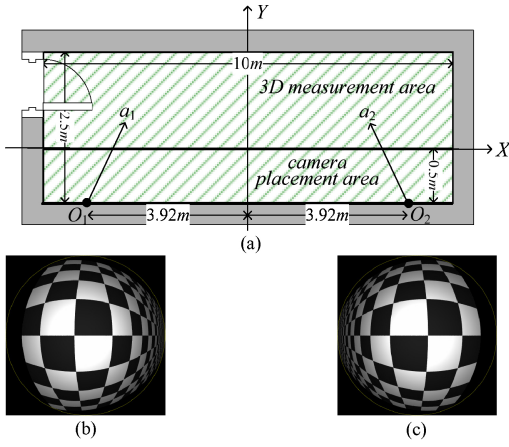


Fig. 15. Optimal system configuration for the general case derived by the optimization process proposed in Section VI. (a) Illustration of the optimal system configuration. (b) and (c) Simulated images taken by the two cameras, with 3-D measurement area drawn as a checkerboard cube.

vision system for home entertainment, exhibitions, wide-area video surveillance, etc., since the camera positions can be derived in advance and the environment may be irregular-shaped and possibly with occlusions, the third optimization method should be used. However, if the cameras can be oriented automatically by computer, the first optimization method may be used in an online fashion to achieve better 3-D measurement accuracies. On the other hand, if the cameras are mounted, for example, on unmanned vehicles to collect wide-area 3-D information in realtime, the second optimization method should be used because fast computations according to analytic formulas can be conducted, and the saved computation power can be used for navigation, learning, event recognition, etc.

## VII. EXPERIMENTAL RESULTS

In this section, the experimental results of a case study of finding the optimal system configuration in a simulated indoor environment are described first in Section VII-A. The proposed method is then compared with four existing methods by experimental results for a real laboratory environment in Section VII-B.

### A. Case of Constructing Indoor Stereo Vision System

A room with size  $10\text{m} \times 2.5\text{m} \times 3\text{m}$ , as depicted in Fig. 15(a), was considered, and the 3-D data of a user's body moving within the 3-D measurement area were required to be calculated accurately. Also, the omni-cameras need be designed and placed in the camera placement area. In the following, the optimal system configuration for this simulation case study is derived first using the proposed method. Then, comparisons of the 3-D measurement accuracies yielded by the optimal system configuration and some nonoptimal ones are presented.

1) *Finding Optimal System Configuration:* To find the optimal system configuration, a coordinate system is defined first, as depicted in Fig. 15(a), with the floor being taken to be the plane  $Z=0$ , and the viewing angles  $2\tau_{\max}$  of the

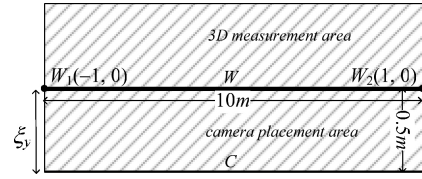


Fig. 16. Corresponding 2-D problem of the case study.

perspective cameras chosen to be  $60^\circ$ . Next, about 100 evenly distributed points were generated in the 3-D measurement area, and about 1000 similarly distributed points were generated in the camera placement area. Then, the optimization process proposed in Section VI-A (depicted in Fig. 13) was applied to such a general case of environment. And the generated configuration with the minimum 3-D measurement error was chosen finally to be the desired optimal system configuration  $S_1^*$ , which, as illustrated in Fig. 15(a), includes: 1) the locations  $O_1$  and  $O_2$  of the two cameras at  $(\mp 3.92\text{ m}, -0.5\text{ m}, 2.5\text{ m})$ , respectively; 2) the optical axes  $a_1$  and  $a_2$  oriented in accordance with the vectors  $(\pm 0.093, 0.996, 0.0)$ , respectively; and 3) the eccentricities  $\varepsilon_1$  and  $\varepsilon_2$  of the mirrors both being 1.8967. The images taken by the two cameras were simulated by a ray-tracing program POV-Ray, with two examples shown in Fig. 15(b) and (c).

Alternatively, the above problem of vision system design can be seen as a 2-D one, in which only the  $XY$ -plane is considered, and the two cameras are installed at the middle height of 2.5 m (recalling that the room is with a height of 5 m). As depicted in Fig. 16, let  $W$  be the boundary line  $\overline{W_1W_2}$  of the 3-D measurement area that is nearer to the camera placement area, and let  $C$  be the boundary line of the camera placement area that is farther to the workspace. The 2-D coordinate system is defined in such a way that the coordinates of  $W_1$  and  $W_2$  are  $(-1, 0)$  and  $(1, 0)$ , respectively. In this sense, a unit in the coordinate system represents 5 m in real space, so that the distance  $\xi_y$  between the line segments  $W$  and  $C$ , as depicted in Fig. 16, can be derived to be 0.1. Then, with the use of  $\xi_y$ , the two proposed optimization schemes described by Algorithms 1 and 2 in Section V were performed to such a regular case of environment to derive the optimal two camera locations.

The optimal camera locations derived by Algorithm 1 are  $(\mp 0.753, -0.1)$  in the 2-D coordinate system, which then were mapped back to the world coordinate system. Subsequently, the methods proposed in Sections VI-B and III-B were applied to find the optical axes directions and eccentricities, yielding the optimal system configuration  $S_2^*$ , which includes: 1) the locations  $O_1$  and  $O_2$  of the two cameras at  $(\mp 3.766\text{ m}, -0.5\text{ m}, 2.5\text{ m})$ , respectively; 2) the optical axes  $a_1$  and  $a_2$  oriented in accordance with the vectors  $(\pm 0.163, 0.987, 0.0)$ , respectively; and 3) the eccentricities  $\varepsilon_1$  and  $\varepsilon_2$  of the mirrors both being 2.0067. In a similar way, the suboptimal system configuration  $S_3^*$  were derived by applying Algorithm 2 and the methods proposed in Sections VI-B and III-B, which includes: 1) the locations  $O_1$  and  $O_2$  of the two cameras at  $(\mp 3.822\text{ m}, -0.5\text{ m}, 2.5\text{ m})$ ; 2) the optical axes  $a_1$  and  $a_2$  oriented in accordance with the vectors  $(\pm 0.172, 0.985, 0.0)$ , respectively; and 3) the

TABLE I  
3-D MEASUREMENT ERRORS

| System config. | 3D measurement errors |               |           | Estimated 3D measurement error (by proposed method) |                  |                   |
|----------------|-----------------------|---------------|-----------|---|------------------|-------------------|
|                | min [cm]              | max [cm]      | std. [cm] | min <sup>a</sup>                                    | max <sup>a</sup> | std. <sup>a</sup> |
| $S_1^*$        | 0.395                 | <b>12.287</b> | 1.965     | 4.127   | <b>12.499</b>    | 1.965             |
| $S_2^*$        | 0.393                 | <b>12.673</b> | 2.000     | 3.860   | <b>11.799</b>    | 2.000             |
| $S_3^*$        | 0.385                 | <b>12.379</b> | 1.985     | 4.004   | <b>11.579</b>    | 1.985             |
| $S_1$          | 0.342                 | <b>22.773</b> | 3.235     | 1.655   | <b>17.159</b>    | 3.235             |
| $S_2$          | 0.362                 | <b>15.196</b> | 2.223     | 2.704   | <b>12.950</b>    | 2.223             |
| $S_3$          | 0.314                 | <b>36.685</b> | 5.326     | 1.078   | <b>27.236</b>    | 5.326             |

<sup>a</sup>The values are scaled such that the standard deviations are the same as the ones derived with Gaussian noise added.

eccentricities  $\varepsilon_1$  and  $\varepsilon_2$  both being 2.0194. It can be seen that the data of the two configurations  $S_2^*$  and  $S_3^*$  are quite close as expected.

2) *Comparison of 3-D Measurement Accuracies:* To compare the 3-D measurement accuracies yielded by the three different optimal system configurations  $S_1^*$ ,  $S_2^*$ , and  $S_3^*$  derived in the previous section, three additional system configurations  $S_1$ ,  $S_2$ , and  $S_3$  were chosen arbitrarily, in which the two cameras are located at  $(\pm 2.5 \text{ m}, -0.5 \text{ m}, 2.5 \text{ m})$  in configuration  $S_1$ , at  $(\pm 1.67 \text{ m}, -0.5 \text{ m}, 2.5 \text{ m})$  in  $S_2$ , and at  $(\pm 3.33 \text{ m}, -0.5 \text{ m}, 2.5 \text{ m})$  in  $S_3$ . Their optical axes' directions were calculated as proposed in Section VI-B, and the eccentricities of the two mirrors were calculated by the scheme described in Section III-B.

Similar to the experiments described in 1) of this section, about 10,000 points were uniformly generated in the 3-D measurement area. Each of the 10,000 point was firstly back-projected onto the two omni-images with coordinates  $(u_1, v_1)$  and  $(u_2, v_2)$ . Then, Gaussian noise with zero means and standard deviations 1.0 (pixel) were applied to the four coordinate values, and the estimated location of the 3-D point was calculated accordingly by mid-point triangulation [28]. The distance between the ground-truth 3-D point and the estimated 3-D point was then computed as the 3-D measurement error. The minimum, maximum, and standard deviation of the 3-D measurement errors resulting from the 10,000 points are listed in Table I. Also, the function  $E$  proposed in Section IV was used to estimate the maximum 3-D measurement errors, whose minimum, maximum, and standard deviation values are also listed in Table I for comparison. Note that, since the values calculated by the use of  $E$  are unit less, they were scaled in such a way that the standard deviations are the same as the one derived with Gaussian noise added.

Recall that a good system configuration is one with a small maximum 3-D measurement error (listed in the third column with bold fonts in Table I). Accordingly, we can see that the three optimal system configurations  $S_1^*$ ,  $S_2^*$ , and  $S_3^*$  are better. Also, recall that the function  $E$  is proposed to estimate these maximum values for use as a criterion to find the optimal system configuration. The effectiveness of  $E$  in this aspect can be seen from the similarity of the maximum values max listed in the third columns of Table I to those listed in the sixth column.

## B. Comparisons with Existing Methods

1) *Introduction to Existing Methods:* The proposed method to assess the 3-D measurement error is based on analytic error propagation analysis, as described in Section IV. Another approach found popular in the literature is to use the covariance matrix to assess the 3-D measurement error [12]–[17], as surveyed in Section I. Four different methods of this approach were implemented by programs in this paper. They are briefly introduced first here, and then compared with the proposed method by experimental results in this section.

When using the covariance matrix to assess the 3-D measurement error yielded by a binocular vision system, let  $P$  be a feature point in the space,  $(u_1, v_1)$  be the coordinates of the pixel  $P'$  corresponding to  $P$  in the left-camera image, and  $(u_2, v_2)$  be those of  $P'$  in the right-camera image. By mid-point triangulation [28], if the 3-D location of  $P$  is calculated by a function  $f(u_1, v_1, u_2, v_2)$ , then, according to [30], the covariance matrix  $\Sigma_P$  of the measured 3-D location data of  $P$  can be assessed by

$$\Sigma_P = (\partial f / \partial p) \Sigma_p (\partial f / \partial p)^T \quad (72)$$

where  $p$  is the vector  $(u_1, v_1, u_2, v_2)$ ,  $\Sigma_p$  is the covariance matrix of  $p$ , and  $T$  denotes the operation of matrix transpose. The covariance matrix  $\Sigma_p$  can be estimated in complicated ways [12]–[16] or by constant values [13], [17]. For simplicity, the matrix was estimated by an identity matrix in our implementations, and the first-order derivatives of  $f$  were obtained by a finite difference approach [31] with the difference taken to be  $10^{-10}$ . After the covariance matrix  $\Sigma_P$  is derived, the four implemented methods use the following data of  $\Sigma_P$  to assess the 3-D measurement error: 1) the determinant [12] and [13]; 2) the trace [12] and [14]; 3) the maximum eigenvalue [12] and [15]; and 4) the maximum diagonal element [16]. These four different methods are named DET, TR, MAXEIG, and MAXDIAG, respectively, subsequently.

2) *Comparisons of Proposed Error Model with Others:* A simulation environment for comparisons of the proposed error model with others is constructed as follows. The 3-D measurement area is defined to be rectangular shaped with two corners located at  $(-1, 0, -1)$  and  $(1, 0, 1)$ , including about 100,000 equidistant points for use as the 3-D measurement locations. The two omni-cameras were placed at  $(\pm 0.7, -0.1, 0)$ , and the viewing angles of the used perspective cameras are  $60^\circ$  and the resolution of acquired images is  $600 \times 600$ . In each simulation, two omni-images were taken firstly, and the projections of each 3-D measurement location  $L_i$  were extracted as two pixels  $l_i$  and  $r_i$  in the left and right omni-images, respectively. To simulate the imprecision introduced by the feature detection process, noise values within the range from  $-1.0$  to  $1.0$  were introduced into the coordinates of the extracted pixels  $l_i$  and  $r_i$ . The mid-point triangulation process [28] was then conducted to compute the 3-D position  $L'_i$  of each landmark point located at  $L_i$  using the coordinates of image pixels  $l_i$  and  $r_i$ . Since the coordinate values were interfered with noise, the calculated 3-D position  $L'_i$  is slightly different from the ground truth  $L_i$ . With the recall that the measurement error is defined as the distance between the actual point and

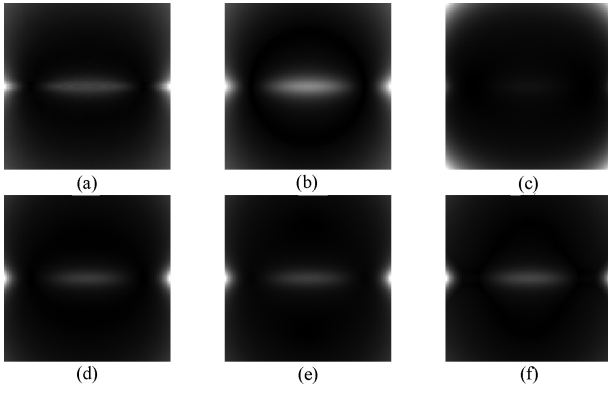


Fig. 17. Images for 3-D measurement errors (with darker colors indicating smaller errors) calculated by (a) simulations, (b) proposed method, (c) method *DET* [12] and [13], (d) method *TR* [12] and [14], (e) method *MAXEIG* [12] and [15], and (f) method *MAXDIAG* [16].

TABLE II  
PSNR VALUES AND RUNNING TIMES IN THE SIMULATION

| Method                        | PSNR    | Running Time        |
|-------------------------------|---------|---------------------|
| Proposed error model          | 23.67dB | 8.56 milliseconds   |
| Method <i>DET</i> [12][13]    | 12.07dB | 273.22 milliseconds |
| Method <i>TR</i> [12][14]     | 17.60dB | 273.97 milliseconds |
| Method <i>MAXEIG</i> [12][15] | 15.32dB | 344.03 milliseconds |
| Method <i>MAXDIAG</i> [16]    | 15.24dB | 277.24 milliseconds |

the measured one (Section IV), the measurement error yielded by the simulation was computed accordingly to be  $\|L_i - L'_i\|$ .

The above simulation was conducted several times, and an average measurement error was calculated for each landmark point as plotted in Fig. 17(a). These average measurement errors are considered as ground-truth values, and were compared with the measurement errors calculated by the proposed error model and those proposed by the four-implemented existing methods shown in Fig. 17(b)–(f), respectively. The peak signal-to-noise ratio (PSNR) values and the running times of the five methods are listed in Table II, from which one can see that the proposed error model yields the highest PSNR, and the *TR* method is the best of the four existing methods in this aspect but worse than the proposed model by a factor of  $10^{(2.367 - 1.760)} = 10^{0.607} \approx 4.04$ , and the running time of the proposed method is smaller than that of the *TR* method by a factor of  $273.97 / 8.56 \approx 32.01$ .

3) *Comparisons of the Optimization Algorithms:* In this section, we describe the derivation of the optimal configuration of a vision system for a real laboratory environment. In the environment, a user was allowed to move freely in a specified 3-D measurement area, and the two omni-cameras were placed within a specified camera placement area. The optimal positions and orientations of the two omni-cameras were computed for this environment by the proposed method and the four existing methods mentioned previously.

Specifically, as shown in Fig. 18(a), the floor of the environment is the *XY*-plane, the 3-D measurement area is the cuboid with two diagonal points being (5.0, 2.0, 2.0) and (-5.0, 0.0, 0.0), and the camera placement area is the rectangle on the plane *Z*=0 with two diagonal points being (5.0,

TABLE III  
OPTIMIZATION RESULTS OF THE METHODS

| Method                        | Two Cameras' Positions (unit: meter) | Maximum 3D Measurement Error (unit: cm) | Run Time (unit: sec) |
|-------------------------------|--------------------------------------|---|----------------------|
| Proposed method               | ( $\pm 3.78, -0.5, 1.0$ )            | 26.290                                  | 11.413               |
| Method <i>DET</i> [12][13]    | ( $\pm 4.02, -0.5, 1.0$ )            | 29.185                                  | 216.927              |
| Method <i>TR</i> [12][14]     | ( $\pm 3.95, -0.5, 1.0$ )            | 28.007                                  | 217.241              |
| Method <i>MAXEIG</i> [12][15] | ( $\pm 3.94, -0.5, 1.0$ )            | 27.843                                  | 254.599              |
| Method <i>MAXDIAG</i> [16]    | ( $\pm 3.88, -0.5, 1.0$ )            | 26.877                                  | 220.423              |

0.0, 1.0) and (-5.0, -0.5, 1.0). The goal of the optimization algorithm is to find the optimal positions  $O_1$  and  $O_2$  of the two omni-cameras, and their optical axes directions  $a_1$  and  $a_2$ , such that the 3-D measurements of an object located in the 3-D measurement area are as accurate as possible. The two cameras are hyperboloidal catadioptric ones with eccentricity  $\varepsilon$  being 1.6571, the viewing angle of the perspective camera is  $2\tau_{\max} = 38^\circ$ , and the omni-image size is  $600 \times 600$ .

The optimization process was conducted firstly for a general case as stated in Section VI-A. At first, some points, for use as 3-D measurement locations, were sampled within the 3-D measurement area with a fixed interval 10 cm, and some points, for use as the camera placement locations, were sampled within the camera placement area with a fixed interval 1 cm. Then, for each possible positions  $O_1$  and  $O_2$  of the cameras, the directions of the optical axes were derived as stated in VI-B, and the 3-D measurement error were assessed by the error model proposed in Section IV. Finally, the poses of the two omni-cameras that yield the minimum 3-D measurement error were taken to be the parameters of the best system configuration. The result of this process says that two cameras should be placed at ( $\pm 3.78, -0.5, 1.0$ ), respectively, and the optical axes be oriented in accordance with the vectors ( $\mp 0.14, 0.99, 0.0$ ), respectively.

To test the 3-D measurement accuracy using such a system configuration, a checker board was placed on the planes with  $Y=0.0, 1.0$ , and  $2.0$  in the 3-D measurement area, as shown in Fig. 19(a) and (b). The image pixels corresponding to all the cross points were manually picked out from the captured omni-images, and the obtained coordinates of these pixels were disturbed with additive noise within the range  $[-5, 5]$  to simulate errors introduced by the feature detection process. Then, by mid-point triangulation [28], the 3-D data of the cross points were derived, called the measured data. Finally, the 3-D measurement errors were taken to be the distances between the measured data and the ground-truth data, the latter being measured manually in advance. The 3-D measurement errors of the cross points on the calibration board at plane  $Y=0.0$  are drawn in Fig. 19(c), whose shape, as can be found, is consistent with that of Fig. 8(b) or Fig. 17(b), though depicted in different ways. Also, these results of the proposed method are listed in Table III for comparison with those obtained similarly of the aforementioned four existing methods. As can be seen from the table, the proposed method yields the minimum measurement errors, and runs faster than the others for about 20 times.



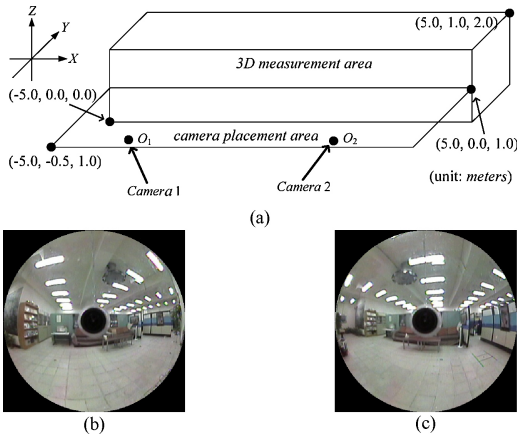


Fig. 18. Environment where experiments were conducted. (a) Illustration. (b) and (c) Two omni-images captured in the environment.

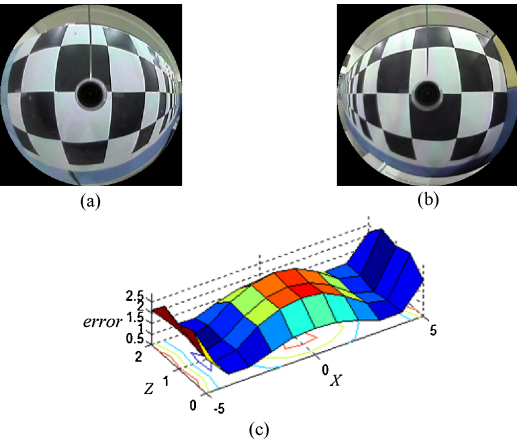


Fig. 19. Testing the 3-D measurement accuracy of the derived best system configuration. (a) and (b) Checkerboard is placed at  $Y=0$  to test the accuracy. (c) 3-D measurement errors of the points on the board.

### VIII. CONCLUSION

The issue of designing the optimal configuration of a stereo vision system with two catadioptric omni-cameras to compute 3-D data with minimum errors was investigated in this paper. The solution includes the poses and the mirror-shape parameters of the omni-cameras. An analytic formula was derived to model the 3-D measurement error, which takes into consideration the error propagation in the data computation process. Two fast and elegant optimization algorithms were designed accordingly for regular environments with rectangular cuboid-shaped 3-D measurement and camera placement areas. One of them, based on a bisection scheme, was optimal but relatively slower, which may be used for off-line applications. The other, using analytic formulas to calculate approximate solutions, was faster for realtime applications with the computed precision being suboptimal but close to that of the former. An algorithm for dealing with general environments with irregular-shaped 3-D measurement and camera placement areas was also developed for general uses. Experimental results showed the feasibility of the proposed method.

In real applications, a manufacturer may produce omni-cameras according to the derived optimal mirror shape. Then,

a consumer may bring them back and deploy them in the optimal or nearly-optimal pose using the proposed algorithms. As a result, a stereo vision system that yield precise 3-D measurement results can be set up. Future studies may be directed to generalizing the proposed optimization method to a stereo vision system with more than two omni-cameras.

### REFERENCES

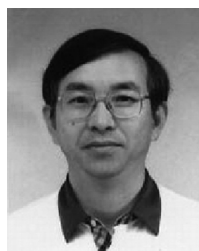
- [1] P. Eisert and B. Girod, "Automatic reconstruction on stationary 3-D objects from multiple uncalibrated camera views," *IEEE Trans. Circuits Syst. Video Technol.*, vol. 10, no. 2, pp. 261–277, Feb. 2000.
- [2] G. Wang and Q. M. J. Wu, "Perspective 3-D Euclidean reconstruction with varying camera parameters," *IEEE Trans. Circuits Syst. Video Technol.*, vol. 19, no. 12, pp. 1793–1803, Dec. 2009.
- [3] P. F. Sturm and B. Triggs, "A factorization based algorithm for multi-image projective structure and motion," in *Proc. 4th Eur. Conf. Comput. Vision*, vol. 2, 1996, pp. 709–720.
- [4] J. Starck, A. Maki, S. Nobuhara, A. Hilton, and T. Matsuyama, "The multiple-camera 3-D production studio," *IEEE Trans. Circuits Syst. Video Technol.*, vol. 19, no. 6, pp. 856–869, Jun. 2009.
- [5] A. Mittal, "Generalized multi-sensor planning," in *Proc. Eur. Conf. Comput. Vision*, 2006, pp. 522–535.
- [6] M. Hanel, S. Kuhn, D. Henrich, J. Pannek, and L. Grune, "Optimal camera placement to measure distances conservatively regarding static and dynamic obstacles," *Int. J. Sensor Netw.*, vol. 12, no. 1, pp. 25–36, 2012.
- [7] C. K. Cowan and P. D. Kovesi, "Automatic sensor placement from vision task requirements," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 10, no. 3, pp. 407–416, May 1998.
- [8] A. N. Bishop, B. Fidan, B. D. O. Anderson, K. Dogancay, and P. N. Pathirana, "Optimality analysis of sensor-target localization geometries," *J. Automatica*, vol. 46, no. 3, pp. 479–492, 2010.
- [9] S. Zhao, B. M. Chen, and T. H. Lee, "Optimal placement of bearing-only sensors for target localization," in *Proc. Amer. Control Conf.*, 2012, pp. 5108–5113.
- [10] S. Zhao, B. M. Chen, and T. H. Lee, "Optimal sensor placement for target localization and tracking in 2-D and 3-D," *Int. J. Control*, to be published.
- [11] S. C. K. Herath and P. N. Pathirana, "Optimal sensor separation for AoA based localization via linear sensor array," in *Proc. Int. Conf. Intell. Sensors Sensor Netw. Inform.*, 2010, pp. 187–192.
- [12] S. Wenhardt, B. Deutsch, E. Angelopoulou, and H. Niemann, "Active visual object reconstruction using D-, E-, and T-optimal next best views," in *Proc. IEEE Conf. Comput. Vision Pattern Recognit.*, 2007, pp. 1–7.
- [13] H. Zhang, "Two-dimensional optimal sensor placement," *IEEE Trans. Syst., Man, Cybern.*, vol. 25, no. 5, pp. 781–792, May 1995.
- [14] B. Alsadik, M. Gerke, and G. Vosselman, "Automated camera network design for 3-D modeling of cultural heritage objects," in *Proc. ISPRS Ann. Photogrammetry, Remote Sens. Spatial Inf. Sci.*, vols. 1–3, Melbourne, Australia, Sep. 2012, pp. 7–12.
- [15] C. Hoppe, A. Wendel, S. Zollmann, and S. Kluckner, "Photogrammetric camera network design for micro aerial vehicles," in *Proc. SPIE*, vol. 8791, pp. 879107-1–879107-11.
- [16] G. Olague and R. Mohr, "Optimal camera placement for accurate reconstruction," *Pattern Recognit.*, vol. 35, no. 4, pp. 927–944, 2002.
- [17] A. H. Rivera, F. L. Shih, and M. Marefat, "Stereo camera pose determination with error reduction and tolerance satisfaction for dimensional measurements," in *Proc. IEEE Int. Conf. Robot. Autom.*, Apr. 2005, pp. 423–428.
- [18] A. O. Ercan, D. B. Yang, A. E. Gamal, and L. J. Guibas, "Optimal placement and selection of camera network nodes for target localization," in *Proc. IEEE Int. Conf. Distrib. Comput. Sensor Syst.*, 2006, pp. 389–404.
- [19] S. D. Blostein and T. S. Huang, "Error analysis in stereo determination of 3-D point positions," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 9, no. 6, pp. 752–765, Nov. 1987.
- [20] R. Hartley and A. Zisserman, *Multiple View Geometry in Computer Vision*. Cambridge, U.K.: Cambridge Univ. Press, 2000.
- [21] C. Geyer and K. Daniilidis, "Catadioptric projective geometry," *Int. J. Comput. Vision*, vol. 45, no. 3, pp. 223–243, 2001.
- [22] C. J. Wu and W. H. Tsai, "An omni-vision based localization method for automatic helicopter landing assistance on standard helipads," in *Proc. Int. Conf. Comput. Autom. Eng.*, vol. 3, Feb. 2010, pp. 327–332.

- [23] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions With Formulas, Graphs, and Mathematical Tables*. Washington, DC, USA: U.S. Department of Commerce, National Bureau of Standards, 1964, p. 72.
- [24] S. Baker and S. Nayar, "A theory of single-viewpoint catadioptric image formation," *Int. J. Comput. Vision*, vol. 35, no. 2, pp. 175–196, 1999.
- [25] D. Pedoe, *Circles: A Mathematical View (Spectrum)*, 2nd ed. Washington, DC, USA: Math. Assoc. America, 1997.
- [26] T. H. Cormen, C. E. Leiserson, and R. L. Rivest, *Introduction to Algorithms*. New York, NY, USA: McGraw-Hill, 1990.
- [27] R. S. Irving, *Integers, Polynomials, and Rings*. New York, NY, USA: Springer, 2004.
- [28] R. Hartley and P. Sturm, "Triangulation," in *Proc. ARPA Image Understanding Workshop*, 1994, pp. 957–966.
- [29] M. Berg, M. Kreveld, M. Overmars, and O. Schwarzkopf, *Computational Geometry: Algorithms and Applications*. New York, NY, USA: Springer, 1997.
- [30] O. Faugeras, *Three-Dimensional Computer Vision: A Geometric Viewpoint*. Cambridge, MA, USA: MIT Press, 1996.
- [31] J. E. Dennis and R. B. Schnabel, *Numerical Methods for Unconstrained Optimization and Nonlinear Equations*. Englewood Cliffs, NJ, USA: Prentice-Hall, 1983.



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