TECHNICAL ARTICLE

Deformation of Multilayer Flexible Electronics Subjected to Torque

Y.C. Lee and T.S. Liu

Department of Mechanical Engineering, National Chiao Tung University, Hsinchu, Taiwan

Keywords
Flexible Electronics, Flexible Substrate, Multilayer Substrate, Twist, Deformation

Correspondence
T.S. Liu,
Department of Mechanical Engineering,
National Chiao Tung University,
Hsinchu 30010,
Taiwan
Email: tsliu@mail.nctu.edu.tw

Received: April 25, 2011; accepted: July 20, 2011

doi:10.1111/j.1747-1567.2011.00780.x

Abstract

Flexibility is one of remarkable characteristics of flexible electronics. Therefore, the analysis of flexible electronics deformation caused by external torques will help its design. In this work, analytical derivation is conducted for flexible electronics treated as multilayer structures. In experiments, an electric motor under computer control exerts a torque to generate torsion, whose twist angles are obtained by using both optical encoder measurement and theoretical calculation. Comparisons are made between experimental and theoretical results for bare PEN (polyethylene naphthalate) substrates of different sizes, multilayer PET (polyethylene terephthalate) substrates, and ITO (indium tin oxide)-coated PET substrates. Finally, a laser displacement sensor is used to measure the out-of-plane deformation for validating the proposed theoretical model.

Introduction

Flexible electronic devices are under development in modern technology. In flexible electronics, the flexibility depends on the substrate. Flexible substrates can be used to facilitate bendable electronic fabrics, smart tags, and conformal sensor arrays as they are lightweight, cost-effective, and deformable. Three kinds of substrates are considered to be flexible: thin glass, metal foil, and plastic. Plastic is the key material of choice, as it allows reasonable trade-offs in mechanical, optical, and chemical performance. It is an inexpensive and useful material for in line production via roll-to-roll processes. Polymers are promising materials for flexible electrics with many advantages. They are transparent, lightweight, flexible, and robust. Polymers are a good alternative to glass substrates that have been actively used for flat panel displays such as liquid crystal displays and plasma discharge panels.[1]

In practice, electronic patterns are printed on a thin film deposited on a polymer layer to fabricate a flexible electronic substrate. The residual stress on the film caused by the temperature variation during the processes affects its quality and lifetime. The tensile and bending test for film deposition on the polymer substrate is worth research. Yanaka et al.[2,3] investigated cracking effects caused by residual stresses for submicron thick glass films deposited on a polymer substrate by the tensile test. Tamulevicius et al.[4] measured changes of diffractive optical elements in photopolymer that is coated on polyethylene terephthalate (PET) substrates under tensile forces. Suo et al.[5] proposed an analytic equation for film-on-foil devices subjected to external force and thermally induced bending. Grego et al.[6] compared two measurement systems in bend tests of flexible substrates. Gleskova et al. and Park et al.[7,8] analyzed for two-layer structures of substrate and film mechanics of thin film devices on flexible substrates. Lewis[9] investigated material properties for flexible substrates with and without thin films under different curvatures in bending. However, not only bending but also twist may occur in daily usage of the flexible electronics. There is scarce literature on modeling and measurement of flexible layered structures subjected to a torque. Chen et al.[10,11] have developed flexible electronic products. They investigated flexible display module reliability with experiments and simulations, such as bending, twist, and ball drop.

Flexible electronics can be treated as multilayer thin plates having isotropic or orthotropic properties. Thin plates are common structural elements employed in engineering applications subjected to a wide variety of loading. Loads such as bending and torsion are fundamental concerns in engineering design, whose exact solution but limited to small deformation for the case of homogeneous, isotropic bars, and rectangular plates has been developed.[12,13] Whitney[14] measured the shear modulus of fiber-reinforced composites in plate-twist test and a torsion tube. Whitney and Kurtz[15] obtained an exact solution for rectangular laminated plates subjected to a torque. Chandra[16] analyzed large deflection behavior of an orthotropic square plate subjected to twist arising from four-corner forces. An expression relating load to deflection is derived by applying a variational method.

Twist of the flexible electronics may occur whenever users hold it. Hence, this study aims to investigate large deformation of flexible polymer substrates subjected to torques in the two opposite edges. Modeling of flexible layered devices subjected to torques is developed. In addition, polymer substrates like PET, polyethylene naphthalate (PEN) substrates, and indium tin oxide (ITO)-coated PET substrates are tested to compare the theoretical model by an electric motor and a laser displacement sensor to measure their twist angle and out-of-plane deformation, respectively.

Theoretical Analysis

Flexible electronics is easy to deform and can be treated as a rectangular plate. One of its applications is flexible display and Fig. 1 shows a schematic diagram of the flexible display. Based on the von Karman plate theory,[13] this study investigates its deformation results undergoing twist arising from torques employing the large deformation theory.

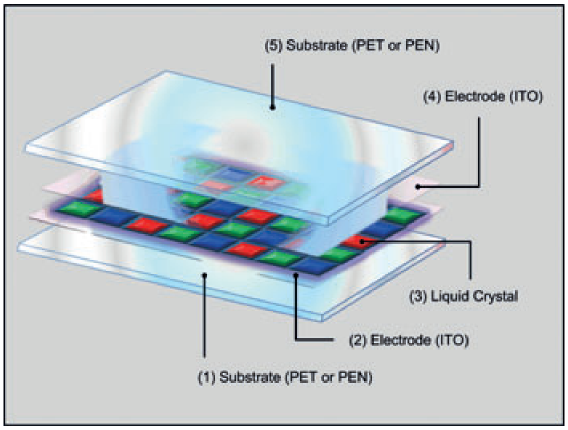

Figure 1 Schematic diagram of components in flexible display.

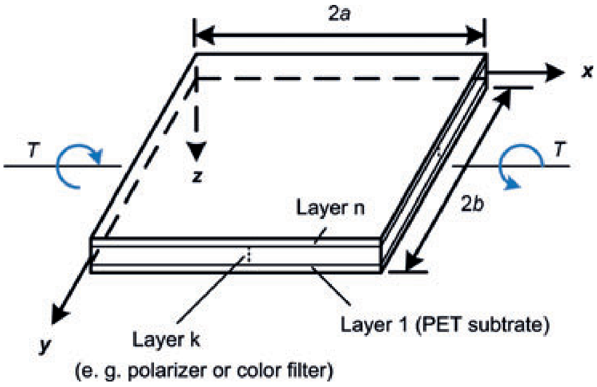

Figure 2 Multilayer flexible electronics subjected to torque.

Governing equation for multilayer substrate

We consider a n-layer flexible electronics of length $2a$, width $2b$, and total thickness h_{total} in a coordinate system as depicted in Fig. 2. Torques are applied in two opposite edges. Strain–displacement relations can be expressed by

$$\varepsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - z\frac{\partial^2 w}{\partial x^2} \tag{1}$$

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial \upsilon}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right) - z\frac{\partial^2 w}{\partial x \partial y} \tag{2}$$

$$\varepsilon_y = \frac{\partial \upsilon}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 - z\frac{\partial^2 w}{\partial y^2} \tag{3}$$

where ε_x and ε_y are normal strain components in x and y directions, respectively and ε_{xy} is normal strain component on the x–y plane. (u, υ, w) are midplane deformations in x, y, and z directions, respectively. These strains with nonlinear terms of $(\partial w/\partial x)^2$, $(\partial w/\partial x)(\partial w/\partial y)$, and $(\partial w/\partial y)^2$ are called von Karman strains[12] in the large deformation theory. Based on generalized Hooke's law, strains due to inplane forces N_x, N_y, and N_{xy} in the i-th layer are derived as

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{bmatrix}^{(i)} = \begin{bmatrix} \frac{1}{E_x} & \frac{-\nu_{xy}}{E_x} & 0 \\ \frac{-\nu_{xy}}{E_y} & \frac{1}{E_y} & 0 \\ 0 & 0 & \frac{1}{G_{xy}} \end{bmatrix} \begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix}^{(i)} \tag{4}$$

where E_x and E_y denote Young's modulus along x and y directions in the i-th layer, respectively. G_{xy} and ν_{xy} denote shear modulus and Poisson's ratio on the x–y plane, respectively. Direct forces are identically satisfied by an Airy stress function ϕ

$$N_x = h\frac{\partial^2 \phi}{\partial y^2}, N_y = h\frac{\partial^2 \phi}{\partial x^2}, N_{xy} = -h\frac{\partial^2 \phi}{\partial x \partial y} \tag{5}$$

From Eqs. 1–5, the governing equation in the i-th layer is expressed by

$$\frac{E_x^{(i)}}{E_y^{(i)}}\frac{\partial^4\phi^{(i)}}{\partial x^4}+\left(\frac{E_x^{(i)}}{G_{xy}^{(i)}}-2\nu_{xy}^{(i)}\right)\frac{\partial^4\phi^{(i)}}{\partial x^2\partial y^2}+\frac{\partial^4\phi^{(i)}}{\partial y^4}=\left(\frac{\partial^2 w}{\partial x\partial y}\right)-\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} \tag{6}$$

Energy formulation

The total energy of a laminated plate with N layers under loading is given by

$$U_{\text{total}}=\sum_{i=1}^{N}\int_{V_i}(U+V+W)\mathrm{d}V_i \tag{7}$$

where U denotes the strain energy, and V and W are potential energies due to inplane loads and external forces, respectively. The strain energy U of the i-th layer is written as[13]

$$U=\frac{1}{2}\iiint(\sigma_x\varepsilon_{xx}+\sigma_y\varepsilon_{yy}+\sigma_{xy}\varepsilon_{xy})\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z \tag{8}$$

Substituting Eq. 4 into Eq. 8 and integrating with respect to the z coordinate lead to

$$U=\frac{1}{2}\cdot\frac{E_xKh^3}{12(K-\nu_{xy}^2)}\int_0^b\int_0^a \times\left(w_{,xx}^2+2\frac{\nu_{xy}}{K}w_{,xx}w_{,yy}+\frac{1}{K}w_{,yy}^2+4\delta w_{,xy}^2\right)\mathrm{d}x\,\mathrm{d}y \tag{9}$$

where $\delta=(K-\nu_{xy}^2)/\eta K$, $\eta=E_x/G_{xy}$, modulus ratio $K=E_x/E_y$ denotes the ratio of Young's modulus for the i-th layer in the x direction to that in the y direction, and h is the thickness of the i-th layer. The potential energy V arising from inplane forces at the i-th layer is formulated as[13]

$$V=\frac{1}{2}\int_0^b\int_0^a(N_x\varepsilon_x'+N_y\varepsilon_y'+N_{xy}\varepsilon_{xy}')\mathrm{d}x\,\mathrm{d}y \tag{10}$$

where N_x, N_y, and N_{xy} denote inplane force resultants per unit length applied to the midplane, while ε_x', ε_y', and ε_{xy}' denote inplane strains due to inplanes force resultants. According to Hooke's law, the potential energy caused by the inplane force can be written as

$$V=\frac{1}{2}\int_0^b\int_0^a\frac{1}{E_xh}[N_x^2-2\nu_{xy}N_xN_y+KN_y^2+\eta N_{xy}^2]\mathrm{d}x\,\mathrm{d}y \tag{11}$$

Flexible electronics subjected to torques

This study deals with deformation of a rectangular multilayer plate arising from torques of magnitude T at both $x=0$ and $x=a$ due to geometric symmetry. Boundary conditions for this laminated substrate of four edges are

$$M_x=M_y=V_x=V_y=0 \tag{12}$$

where V_x and V_y denote an effective transverse force per unit length, M_x and M_y are both resultant bending moments. M_{xy} is a resultant twist moment that is not equal to zero.

A deflection function satisfying the geometric boundary is assumed as

$$w(x,y)=A(x-a)(y-b) \tag{13}$$

In the i-th layer, satisfying zero normal stress and zero average shear stress on plate edges, an Airy stress function is written as

$$\phi=B\sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right) \tag{14}$$

Substituting Eqs. 13 and 14 into Eq. 6 and expanding A^2 into double Fourier series for ranges $x\in[0,a]$ and $y\in[0,b]$ lead to

$$B=\left(\frac{16}{\pi^2N}\right)A^2 \tag{15}$$

where

$$N=\left(\frac{1}{E_x}\right)\left[K\left(\frac{\pi}{a}\right)^4+(\eta-2\nu_{xy})\left(\frac{\pi}{a}\right)^2\left(\frac{\pi}{b}\right)^2+\left(\frac{\pi}{b}\right)^4\right] \tag{16}$$

At the center of edges subjected to twist, applying Taylor's series expansion, the twist angle Φ is expressed by

$$\Phi=2\tan^{-1}\left(\frac{\partial w}{\partial y}\right)=\left[\left(\frac{\partial w}{\partial y}\right)-\left(\frac{\partial w}{\partial y}\right)^3\Big/3\right]+\text{H.O.T.} \tag{17}$$

We ignore higher order terms and the potential energy W resulting from applied twist moment T is hence written as

$$W=\frac{1}{4}T(\Phi)_{\substack{x=0\\y=0}}=-\frac{1}{2}T\left(aA-\frac{1}{3}a^3A^3\right) \tag{18}$$

Based on the previous equations and some derivations, it gives

$$U_{\text{total}}=\sum_{i=1}^{N}\left[\frac{abh_iN^{(i)}}{8}B^2+\frac{G_{xy}^{(i)}}{6}h_i^3abA^2\right]-\frac{1}{2}T\left(aA-\frac{1}{3}a^3A^3\right) \tag{19}$$

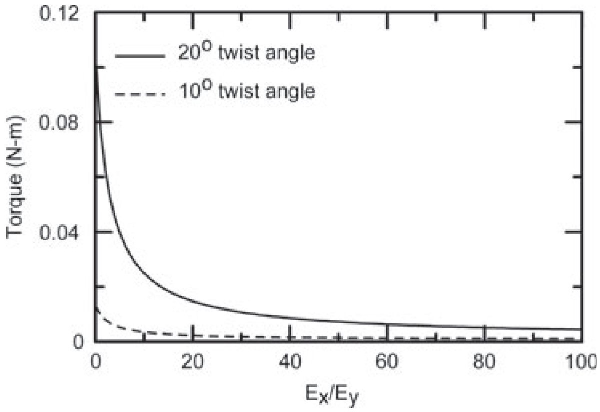

Figure 3 Torque versus modulus ratio curves for substrates of aspect ratios $a/b = 1$ at 10° and 20° twist angles, respectively.

where (i) denotes the i-th layer. The first term in Eq 19 represents strain energy due to axial force. After substituting Eq. 15 into 19 and taking variation to Eq. 19 yields

$$\sum_{i=1}^{N}\left[\frac{128abh^{(i)}}{\pi^4N^{(i)}}A^3+\frac{G_{xy}^{(i)}}{3}(h^{(i)})^3abA\right]-\frac{1}{2}T(a-a^3A^2)=0 \tag{20}$$

After solving Eq. 20, we can obtain deflection w and the twist angle Φ from Eqs. 13 and 17, respectively.

Simulation Results

Structures of polymer substrates have been developed for the flexible electronics due to their flexibility. Consider a substrate of length 10 cm, width 10 cm, and 200 μm thick subjected to torques with different modulus ratio as depicted in Fig. 2. Figure 3 shows torque versus $K(=E_x/E_y)$ curves for substrates of aspect ratio 1 (i.e., aspect ratio = substrate length/substrate width) at 10° and 20° twist angles, respectively. It shows that the required torque for unit K decreases. The difference between two curves is large when K is small. Required torques at both twist angles become smaller with increasing K. Therefore, it is easier to be twisted at larger K. Figure 4 shows torque versus K curves for substrates of aspect ratios 1 and 1.2 at 20° twist angle, respectively. The torque required for the aspect ratio 1 substrate is larger than the aspect ratio 1.2 one when K is smaller. On the other hand, when K is larger, the torque required for aspect ratio 1.2 substrate is close to that for aspect ratio 1 substrate. Substrates with aspect ratio 1 than 1.2 are easily affected by K magnitude.

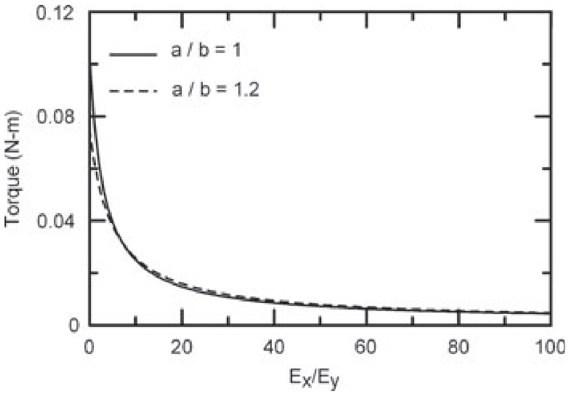

Figure 4 Torque versus modulus ratio curves at 20° twist angle for substrates of aspect ratios $a/b = 1$ and $a/b = 1.2$, respectively.

Experiment

Required torque with single-layer substrate

Figure 5(a–d) shows a twist apparatus, where two jigs are used to hold the flexible polymer substrate subjected to a torque at 0°, 5°, 10°, and 15° twist angles, respectively. The motor is SmartMotor-made servo motor with type No. SM2315D. In computer control, the torque mode of the electric motor is used. The twist angle is measured by an optical encoder. At the onset of motor axis rotation, friction force of the electric motor is overcome by inputting a large torque. Polymer substrates of 200 μm thick PEN with two different sizes are used (1) 10 cm in length and 10 cm in width; (2) 12 cm in length and 10 cm in width. Material properties of PEN are given in Table 1. Based on measurement results, torque versus twist angle curves are depicted in Figs. 6 and 7. Figures 6 and 7 show relationships between the torque and twist angle for PEN substrates of 10 cm long and 10 cm wide and of 12 cm in long and 10 cm wide, respectively. They show that the theoretical model can validate experimental results for single-layer substrates. Multilayer substrates will be investigated in the next subsection. In addition, they show that, based on curve slopes, when substrate deflection becomes large, torques or forces needed for unit deformation also increase. Accordingly, it shows that the rectangular substrate is easier to deform than the square one.

Required torque with multilayer substrate

Two polymer substrates are used in the torsion measurement; namely 125 μm thick PET and 125 μm thick PET with 100 nm ITO on it. Material properties of PET and ITO are given in Table 1. Figure 8 shows the schematic diagram for three kinds of multilayer samples. Figure 9 shows simulation results between

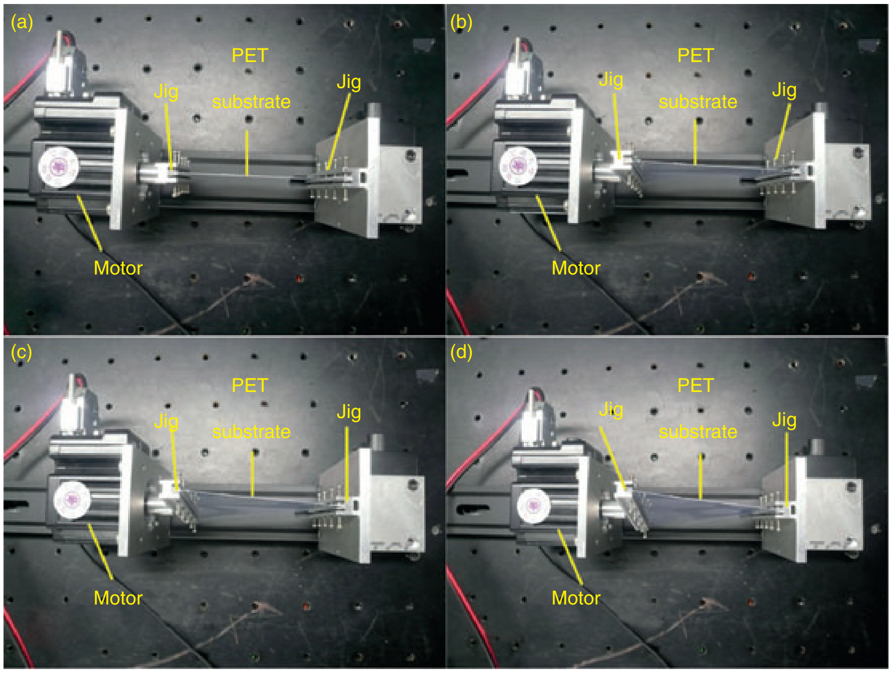

Figure 5 Photos in experiments where a flexible substrate is subjected to increasing motor torque with twist angles of (a) 0°, (b) 5°, (c) 10°, and (d) 15°.

Table 1 Material properties of PET, PEN, and ITO

Material	Young's modulus (GPa)	Poisson's ratio
PET	4	0.3
PEN	5	0.3
ITO	118	0.2

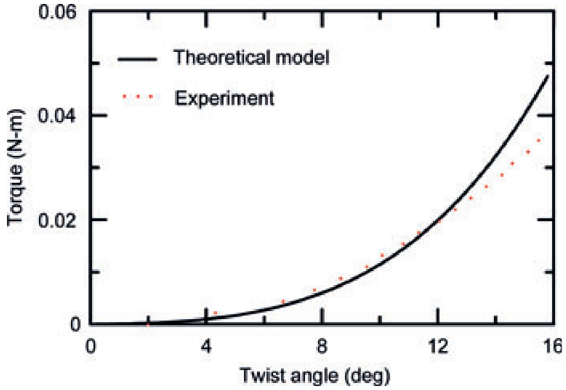

Figure 6 Torque versus twist angle curves for PEN substrate of 10 cm in length and 10 cm in width.

the torque and twist angle for three kinds of samples with 10 cm in length and 10 cm in width. It shows that torques required for these three samples are nearly the same due to small thickness layers of ITO films. Figures 10 and 11 show relationships between the torque and twist angle subjected to torque for samples A and B, respectively. It shows that

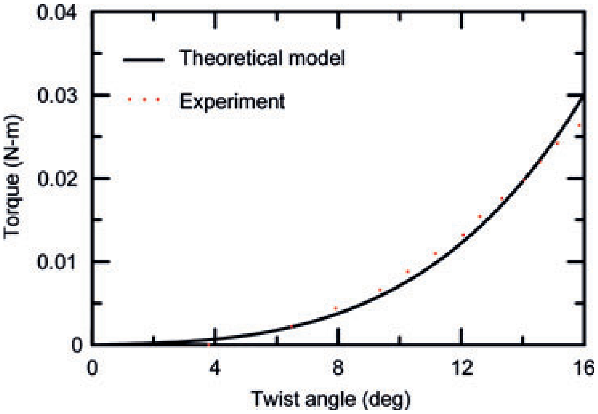

Figure 7 Torque versus twist angle curves for PEN substrate of 12 cm in length and 10 cm width.

the theoretical model can analyze multilayer flexible electronics subject to torques. Experimental results validate the theoretical model in the twist angle 10°. There was a discrepancy between theoretical models and experimental results when the twist angle is larger than 10°. This is due to sliding between contact substrates for samples A and B. Therefore, it shows that a larger torque will cause fracture of devices.

Deformation measurement by laser displacement sensor

A laser displacement sensor with type No. LK-G35 of KEYENCE was used to measure out-of-plane

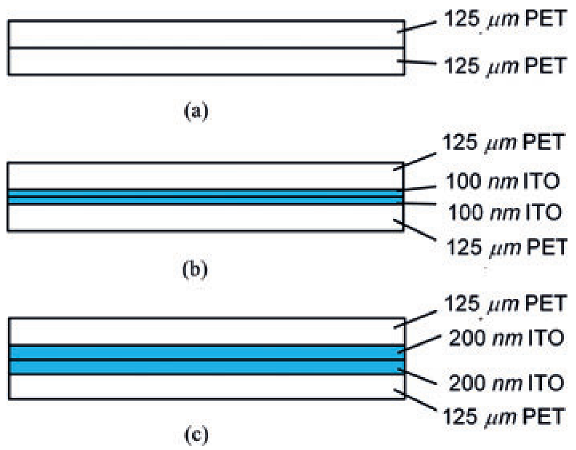

Figure 8 Schematic diagram for three kinds of multilayer samples.

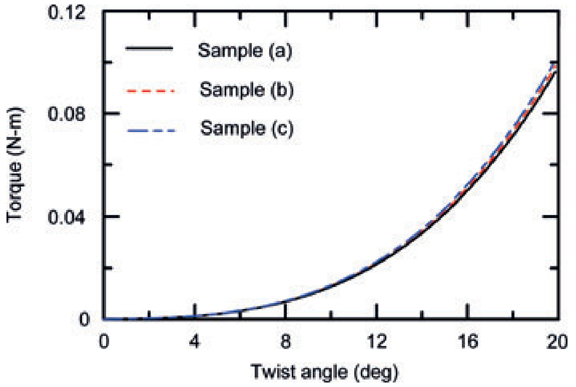

Figure 9 Torque versus twist angle curves for three kinds of samples.

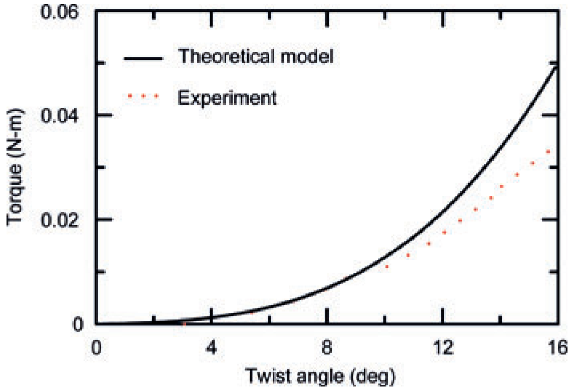

Figure 10 Torque versus twist angle curves for sample A of 10 cm in length and 10 cm in width.

deformation of flexible substrates subjected to torque. The measurement principle of the laser displacement sensor uses triangulation as shown in Fig. 12. The position of the reflected light on a CCD moves as the target moves. The target deformation is measured by detecting this change. Figures 13 and 14 show deformations along a diagonal from the center for a 200-μm thick PEN substrate and sample B of 10 cm long and 10 cm wide, respectively. Five points along the diagonal of both substrates are measured. Both theoretical and experimental results are consistent. Hence, the theoretical model of Eq. 20 can be used to analyze the out-of-plane deformation of multilayer flexible substrates subjected to torque.

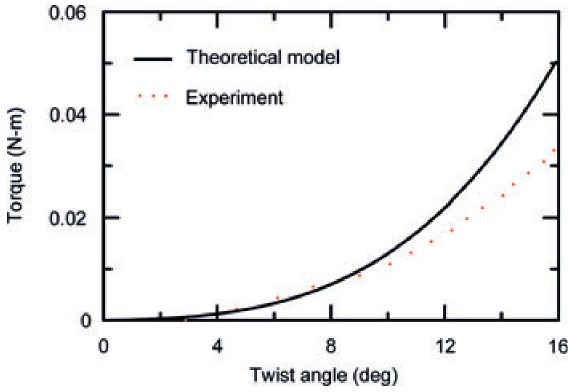

Figure 11 Torque versus twist angle curves for sample B of 10 cm in length and 10 cm width.

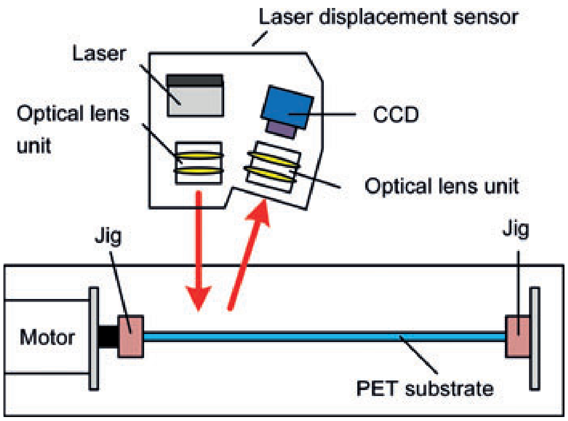

Figure 12 Substrate deformation measurement by using laser displacement sensor.

Conclusion

In this work, an analytical model has been presented in this study for multilayer flexible electronics subjected to twist arising from torques. Simulation results show that modulus ratios play an important role in flexible electronics, and can be used in design by choosing larger modulus ratios to reduce the required

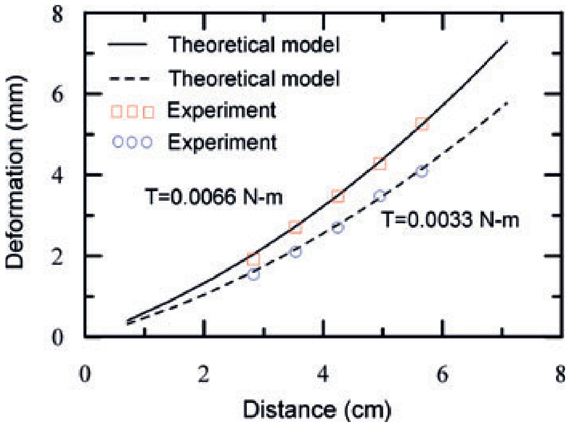

Figure 13 Deformation versus distance along a diagonal from the PEN substrate center.

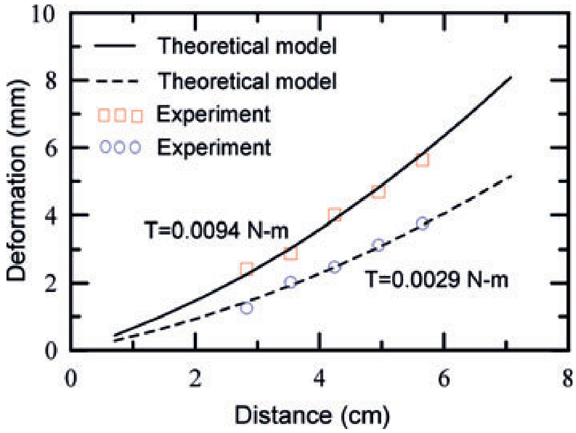

Figure 14 Deformation versus distance along a diagonal from sample B center.

torque per unit twist angle. In experiments, an electric motor under computer control exerts a torque to generate flexible substrate twist. The twist angle and out-of-plane deformation are measured by an optical encoder and a laser displacement sensor, respectively. In addition, comparisons have been made between theoretical models and experimental results for bare PEN substrates, multilayer PET substrates and ITO-coated PET substrates. A square substrate is not as easy to be twisted as rectangular one with the same width. Finally, experimental results validate the proposed theoretical model.

Acknowledgment

This work was supported by the National Science Council in Taiwan under Grant Number NSC98-2221-E009-008.

References

1. Choi, M.C., Kim, Y., and Ha, C.S., ''Polymers for Flexible Displays: From Material Selection to Device Applications,'' *Progress in Polymer Science* **33**(6): 581–630 (2008).
2. Yanaka, M., Tsukahara, Y., Nakasa, N., and Takeda, N., ''Cracking Phenomena of Brittle Films in Nanostructure Composites Analysed by a Modified Shear Lag Model with Residual Strain,'' *Journal of Material Science* **33**(8):2111–2119 (1998).
3. Yanaka, M., Kato, Y., Tsukahara, Y., and Takeda, N, ''Effects of Temperature on the Multiple Cracking Progress of Sub-micron Thick Glass Films Deposited on a Polymer Substrate,'' *Thin Solid Films* **355–366**(1):337–342 (1999).
4. Tamulevicius, S., Augulis, L., Janusas, G., Guobiene, A., Puodziukynas, L., and Vanagas, G., ''Mechanical and Surface Topography Changes During Mechanical Testing of Diffraction Optical Elements in Polymer,'' *Experimental Techniques* **34**(1):55–62 (2010).
5. Suo, Z., Ma, E.Y., Gleskova, H., and Wagner, S., ''Mechanics of Rollable and Foldable Film on Foil Electronics,'' *Applied Physics Letters* **74**(8):1177–1179 (1999).
6. Grego, S., Lewis, J., Vick, E., and Temple, D., ''Development and Evaluation of Bending- Testing Techniques for Flexible-Display Applications,'' *The Journal of the Society for Information Display* **13**(7): 575–581 (2005).
7. Gleskova, H., Cheng, J.C., Wagner, S., Sturm, J.C., and Suo, Z., ''Mechanics of Thin-Film Transistors and Solar Cells on Flexible Substrates,'' *Solar Energy* **80**(6):687–693 (2005).
8. Park, S.K., Han, J.I., Moon, D.G., and Kim, W.K., ''Mechanical Stability of Externally Deformed Indium–Tin–Oxide Films on Polymer Substrates,'' *Japanese Journal of Applied Physics* **42**(2A):623–629 (2003).
9. Lewis, J., ''Material Challenge for Flexible Organic Devices,'' *Materials Today* **9** (4):38–45 (2006).
10. Chen, Q., Xu, L., Salo, A., Neto, G., and Freitas, G., ''Reliability Study of Flexible Display Module by Experiments,'' *International Conference on Electronic Packaging Technology & High Density Packaging*, Singapore, pp. 1–6 (2008).
11. Chen, Q., Xu, L., Salo, A., Neto, G., and Freitas, G., ''Reliability of Flexible Display by Simulation and Strain Gauge Test,'' *The 10th Electronic Packaging Technology Conference*, Shanghai, China, pp. 322–327 (2008).
12. Reismann, H., and Pawlik, P.S., *Elasticity: Theory and Application*, John Wiley, New York (1980).
13. Ugural, A.C., *Stresses in Plates and Shells*, McGraw-Hill, New York (1981).

14. Whitney, J.M., ''Experimental Determination of Shear Modulus of Laminated Fiber- Reinforced Composites,'' *Experimental Mechanics* **7**(10):447–448 (1967).
15. Whitney, J.M., and Kurtz, R.D., ''Analysis of Orthotropic Laminated Plates Subjected to Torsional Loading,'' *Composites Engineering* **3**(1):83–97 (1993).
16. Chandra, R., ''On twisting of Orthotropic Plates in a Large Deflection Regime,'' *AIAA Journal* **14**(8): 1130–1131 (1976).
17. Reddy, J.N., *Mechanics of Laminated Composite Plates and Shells: Theory and Analysis*, CRC Press, New York (2004).