Generation of photon-number-entangled soliton pairs through interactions

Ray-Kuang Lee,[1] Yinchieh Lai,[1,*] and Boris A. Malomed[2,†]
[1]*Department of Photonics and Institute of Electro-Optical Engineering, National Chiao-Tung University, Hsinchu 300, Taiwan*
[2]*Department of Interdisciplinary Studies, Faculty of Engineering, Tel Aviv University, Tel Aviv 69978, Israel*
(Received 24 May 2004; revised manuscript received 9 August 2004; published 21 January 2005)

Two simple schemes for generating macroscopic (many-photon) continuous-variable entangled states by means of continuous interactions (rather than collisions) between solitons in optical fibers are proposed. First, quantum fluctuations around two time-separated single-component temporal solitons are considered. Almost perfect correlation between the photon-number fluctuations can be achieved after passing a certain distance, with a suitable initial separation between the solitons. The photon-number correlation can also be achieved in a pair of vectorial solitons with two polarization components. In the latter case, the photon-number-entangled pulses can be easily separated by a polarization beam splitter. These results offer possibilities to produce entangled sources for quantum communication and computation.

DOI: 10.1103/PhysRevA.71.013816 PACS number(s): 42.50.Dv, 03.67.Mn, 05.45.Yv, 42.65.Tg

I. INTRODUCTION

Quantum-noise squeezing and correlations are two key quantum properties that can exhibit completely different characteristics when compared to the predictions of the classical theory. Almost all the proposed applications to quantum measurements and quantum information treatment utilize either one or both of these properties. In particular, solitons in optical fibers have been known to serve as a platform for demonstrating macroscopic quantum properties in optical fields, such as quadrature squeezing [1–5], amplitude squeezing [6,7], and both intrapulse and interpulse correlations [8,9].

It is well known that Kerr-induced interactions between pulses in *multisoliton* states are useful for quantum nondemolition (QND) measurements (those that do not disturb the quantum distribution of the variable being measured) [10–17]; see Refs. [18,19] for a review of previous work on quantum solitons. In these schemes, either soliton collisions or interactions through cross-phase modulation are utilized to produce quantum correlations between the two solitons and then QND measurements can be performed after the collisions or interactions.

Recently, experimental progress in demonstrating various quantum information processes by using two-mode squeezed states in optical solitons has been reported; see Refs. [20–22] and references therein. Motivated by the achievements in the experiment, the objective of the present work is to study the quantum properties of interacting two-soliton systems in optical fibers, both in single-polarization and bimodal settings. In these two-soliton systems, the two soliton states are either incompletely separated in the time domain with equal group velocity and same polarization or are formed to be symmetric and antisymmetric bound soliton pulses with different phase velocities and orthogonal polarizations. The quantum properties of these interacting (noncolliding) two-soliton states have not yet been studied in detail previously and may offer possibilities to produce entangled sources for quantum communications and computations.

In the field of quantum information processing and quantum computing, nonlocally entangled optical quantum states have been shown to be highly useful sources. The applications include quantum cryptography [23], teleportation [24], and algorithms [25,26]. Following Bohm's suggestion [27], the entangled pairs were mostly realized in terms of discrete quantum variables, such as spin, polarization, etc. However, the original *gedanken experiment* proposed by Einstein, Podolsky, and Rosen (EPR) utilized continuous variables (the coordinate and momentum of a particle) to argue that quantum mechanics is incomplete [28]. In 1992, Ou *et al.* used nondegenerate parametric amplification to demonstrate the EPR paradox with continuous variables [29]. Later, Vaidman proposed a generalized method for the teleportation of continuous-variable quantum states [30]. Braunstein and Kimble analyzed the entanglement fidelity of quantum teleportation with continuous variables [31]. Quantum teleportation of optical coherent states was experimentally realized by using the entanglement from squeezed states [32]. After that, quantum-information processing with continuous variables has attracted a lot of interest as an alternative to single-photon schemes.

In previous works, continuous-variable entangled beams have been generated by letting two squeezed fields (squeezed vacuum states [32] or amplitude-squeezed fields [33]) interfere through a beam splitter, which mathematically acts as the Hadamard transformation. By utilizing the continuous EPR-like correlations of optical beams, one can also realize quantum-key distributions [21] and entanglement swapping [22]. Thanks to these successful applications, squeezed states become essential for generating entangled continuous-variable quantum states and play an important role in the study of the quantum-information processing.

It has been demonstrated that two independent squeezed pulse states can be simultaneously generated by using optical solitons in the Sagnac fiber loop configuration [20]. An EPR pulse source can be obtained by combining the two output pulse squeezed states by means of a 50:50 beam splitter. In

*Electronic address: yclai@mail.nctu.edu.tw
†Electronic address: malomed@eng.tau.ac.il

contrast to this known method for achieving the entanglement, in this work we propose simple schemes for generating continuous-variable entangled states through *continuous interaction* of two solitons, not collision, in single-mode and bimodal (two-component) systems, *without* using beam splitters. The quantum interaction of two time-separated solitons in the same polarization is described by the quantum nonlinear Schrödinger equation (NLSE), and in the bimodal system, including two polarizations, it is described by a system of coupled NLSE's. The photon-number correlation between the two solitons can be numerically calculated by using the *back-propagation method* [34]. In addition to the transient multimode correlations induced by cross-phase modulation [9], we also find nearly maximum photon-number entanglement in the soliton pair. By controlling the initial separation of the two solitons, one can achieve a positive quantum correlation with the correlation parameter taking values close to 1.

The paper is organized as follows. The pairs of quantum solitons in the single-mode and bimodal systems are considered, respectively, in Secs. II and III. Conclusions are formulated in Sec. IV.

II. SINGLE-MODE SYSTEM

Neglecting loss and higher-order effects, which are immaterial for the experimentally relevant range of the propagation distance z, temporal solitons in optical fibers are described by the NLSE in the normalized form

$$iU_z + \frac{1}{2}U_{tt} + |U|^2U = 0,$$

where t is the retarded time [35]. The input profile of the soliton pair is taken as

$$U(z,t) = \text{sech}(z,t+\rho) + \gamma\,\text{sech}(z,t-\rho)e^{i\theta}, \tag{1}$$

where γ, θ, and 2ρ are, respectively, the relative amplitude, phase, and separation of the solitons. If $\theta=0$ (the in-phase pair), the two solitons will form a breather, periodically colliding in the course of the propagation. Otherwise, they move apart due to repulsion between them. The interaction between the solitons is produced by the overlap of the "body" of each soliton with an exponentially decaying "tail" of the other one. A detailed account explaining the tail-mediated interaction in the NLSE with the instantaneous cubic (Kerr) nonlinearity can be found in review [36].

It should be mentioned that, rigorously speaking, two interacting solitons cannot be considered as independent modes (in particular, the creation and annihilation operators for the quanta belonging to the different solitons do not strictly satisfy the commutation relations for independent modes). Nevertheless, weakly interacting far-separated solitons may be treated, in the *lowest approximation* of the perturbation theory (the interaction itself being the perturbation), as effectively independent modes. While this fact has been firmly established for classical solitons [36,37], it pertains equally well to quantum fluctuations around the solitons. Indeed, one of the formalisms adopted in the classical perturbation theory, which, to the lowest approximation, is completely equivalent to other classical techniques [37], is based on the computation of fluctuation eigenmodes around classical solitons, and this is essentially tantamount to the way the quantum fluctuations are treated [38]. Thus, in the first approximation, one may rely on the notion of correlations between orthogonal number states of each soliton. The same pertains to the general case of two far-separated vectorial solitons in a two-component system; see Sec. III below.

As said above, one can investigate multimode quantum fluctuations around the solitons by solving the linearized quantum NLSE [38]. For the case of two-soliton collisions [9], König *et al.* used an exact classical solution for the in-phase soliton pair and found that the colliding solitons carry both intrapulse and interpulse photon-number correlations. However, the photon-number correlation between the colliding solitons is transient; i.e., the interpulse correlation vanishes after the collision. Unlike the case of the collision between two solitons moving with different velocities, the photon-number correlations caused by the *continuous interaction* between the solitons belonging to the pair initiated by the configuration in Eq. (1)—i.e., two solitons moving with the same velocity—persist with the propagation. Although the interaction between pump and probe solitons with a small relative velocity was dealt with in the QND schemes [12], this situation requires further consideration. General results for this case are given in the present paper, for both the single-mode and bimodal systems.

In Fig. 1, we display the result of evaluation of the time-domain photon-number correlations for the out-of-phase ($\theta = \pi/2$) two-soliton pair. The correlation coefficients, which are defined through the normally ordered covariance,

$$C_{ij} \equiv \frac{\langle:\Delta\hat{n}_i\Delta\hat{n}_j:\rangle}{\sqrt{\Delta\hat{n}_i^2\Delta\hat{n}_j^2}}, \tag{2}$$

were calculated by means of the above-mentioned back-propagation method [34]. In Eq. (2), $\Delta\hat{n}_j$ is the photon-number fluctuation in the ith slot Δt_i in the time domain,

$$\Delta\hat{n}_i = \int_{\Delta t_i} dt[U(z,t)\Delta\hat{U}^{\dagger}(z,t) + U^*(z,t)\Delta\hat{U}(z,t)],$$

where $\Delta\hat{U}(z,t)$ is the perturbation of the quantum-field operator, $U(z,t)$ is the classical unperturbed solution, and the integral is taken over the given time slot. As could be intuitively expected, nonzero correlation coefficients are found solely in the diagonal region of the spectra (*intrapulse correlations*) if the interaction distance is short, as shown in Fig. 1(A). As the interaction distance increases, *interpulse correlations* between the two solitons emerge and grow, as shown in Figs. 1(B) and 1(C).

In addition to the time-domain photon-number correlation pattern, we have also calculated a photon-number *correlation parameter* between the two interaction solitons as

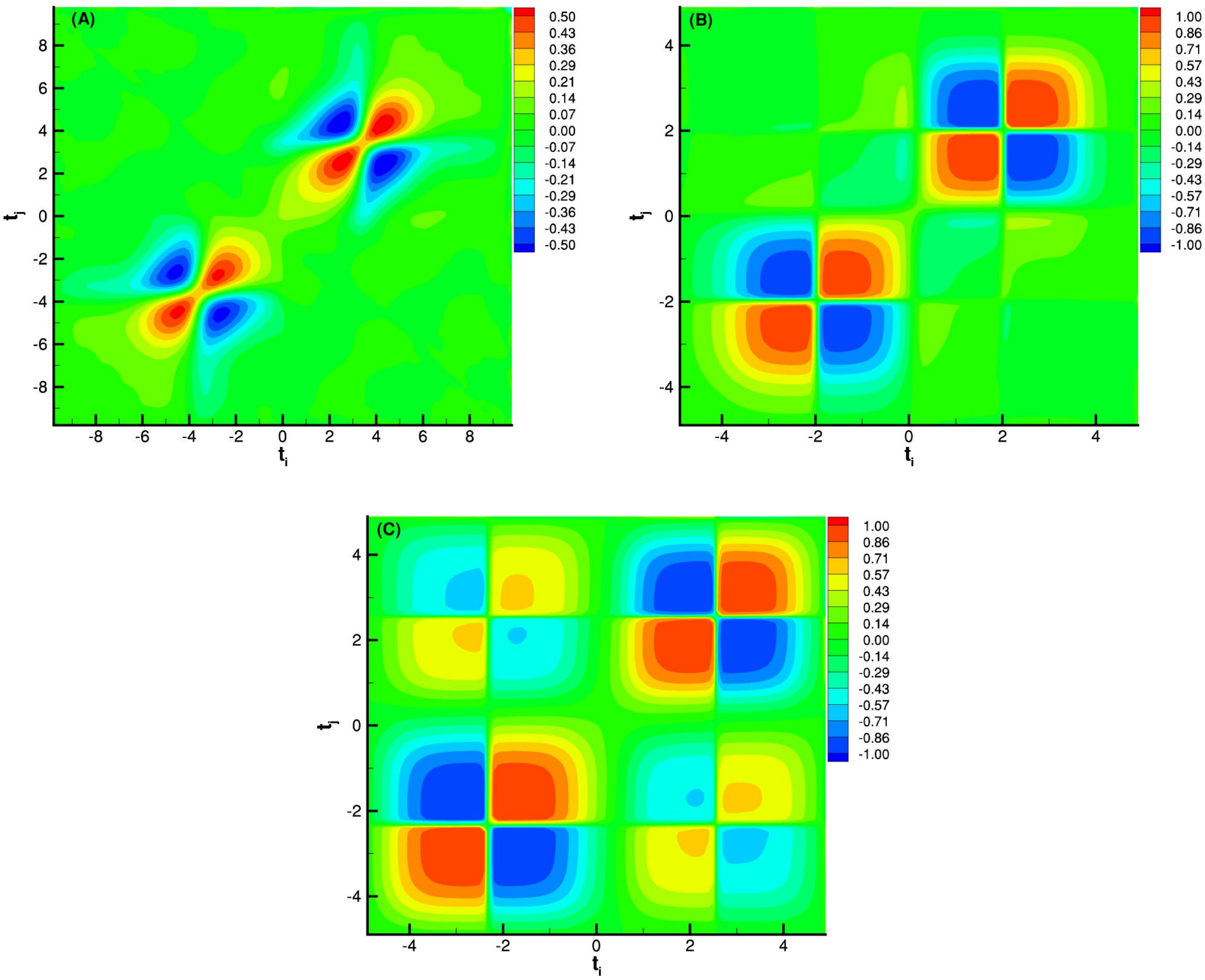

FIG. 1. (Color online) The pattern of time-domain photon-number correlations C_{ij} of two interacting out-of-phase solitons, with $\theta = \pi/2$, $\rho = 3.5$, and $\gamma = 1.0$ in Eq. (1). The propagation distance is $z = 6$ (a), 30 (b), and 50 (c), in normalized units. The width of the time slots is $\Delta t = 0.1$. Note the difference in the bar-code scales in the panels (A) and (B), (C).

$$C_{12} = \frac{\langle : \Delta \hat{N}_1 \Delta \hat{N}_2 : \rangle}{\sqrt{\Delta \hat{N}_1^2 \Delta \hat{N}_2^2}}.$$

Here $\Delta \hat{N}_{1,2}$ are the perturbations of the photon-number operators of the two solitons, which are numbered (first and second) according to their position in the time domain.

In Fig. 2(A), we show the coefficient C_{12} for the soliton pair (1) with the initial relative phase $\theta = \pi/2$, equal amplitudes ($\gamma = 1.0$), and different values of the separation ρ. At the initial stage of the interaction, the photon-number fluctuations are uncorrelated between the solitons, $C_{12} \approx 0$. After passing a certain distance, the photon-number correlations between the two solitons gradually build up, and the pair may become a nearly maximum-positive-correlated one. In accordance with the fact that the interaction between the solitons, which gives rise to the correlations, is mediated by their exponentially decaying tails, the propagation distance needed to achieve the maximum positive photon-number correlation decreases with the initial separation between the solitons.

On the other hand, one can fix the initial separation but vary the initial relative phase between the solitons. For this case, the results are shown in Fig. 2(B). Similar to the case of soliton-soliton collisions [9], the photon-number correlation coefficient oscillates, as a function of the propagation distance, with the period equal to that of the two-soliton breather, if the solitons are, initially, in phase. Note that in the case which may be regarded as intermediate between the in-phase and out-of-phase ones, $\theta = \pi/4$, the correlation coefficient first becomes negative and then positive.

Unless $\theta = 0$ (when the two solitons form a quasibound state in the form of a breather), the two solitons belonging to the initial configuration (1) will separate as a result of the propagation. Therefore, the interaction between them eventually vanishes, and thus the photon-number correlation coefficient may saturate before it has a chance to reach the value corresponding to the total positive correlation, which is clearly seen in the inset to Fig. 2(A).

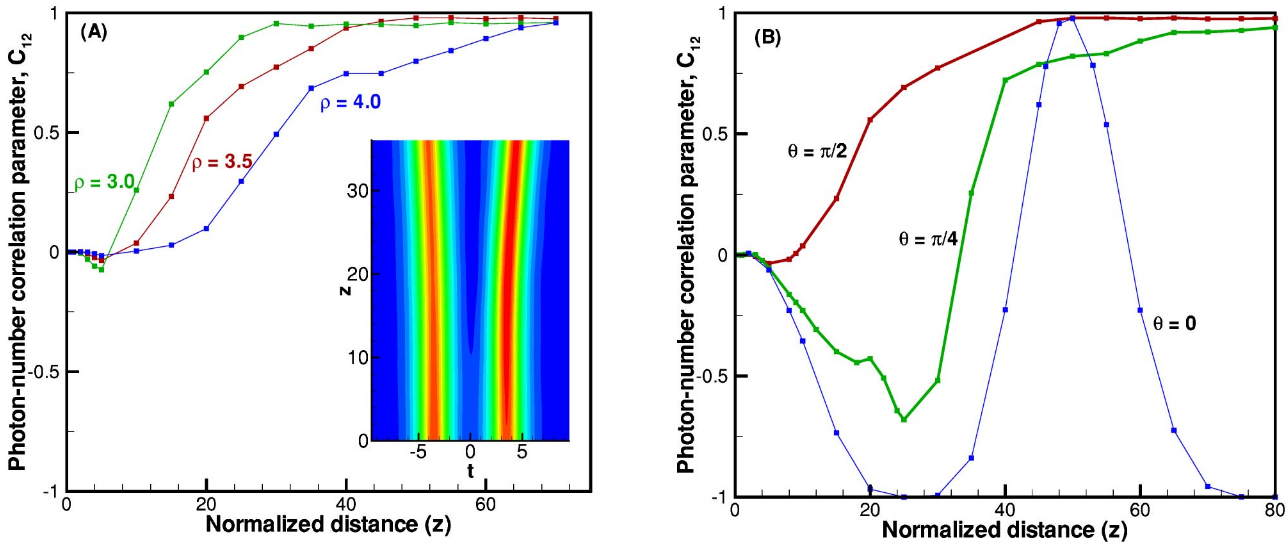

FIG. 2. (Color online) The photon-number correlation parameter C_{12} for the soliton pair with different values of the separation ($\rho = 3.0$, 3.5, 4.0, while $\theta = \pi/2$ and $\gamma = 1.0$) in (A) and different values of the relative phase ($\theta = 0$, $\pi/4$, $\pi/2$, while $\rho = 3.5$ and $\gamma = 1.0$) in (B). The inset in (A) shows the evolution of the interaction solitons by means of contour plots.

III. BIMODAL SYSTEM

The time-division entangled soliton pair in the single-mode system, considered above, can be separated by an optical switch. Since the actual time separation between the two solitons is, typically, on the order of a few picoseconds, a lossless ultrafast optical switch will be required for the actual implementation of the scheme. The experimental difficulties can be greatly reduced if another scheme is used, which utilizes vectorial solitons in two polarizations. The model is based on the well-known system of coupled NLSE's [35],

$$i\frac{\partial U}{\partial z} + \frac{1}{2}\frac{\partial^2 U}{\partial t^2} + A|U|^2 U + B|V|^2 U = 0, \tag{3}$$

$$i\frac{\partial V}{\partial z} + \frac{1}{2}\frac{\partial^2 V}{\partial t^2} + A|V|^2 V + B|U|^2 V = 0. \tag{4}$$

Here U and V are the fields in orthogonal circular polarizations, A and B being the self-phase- and cross-phase-modulation coefficients, respectively, with the relation $A:B = 1:2$ in the ordinary optical fibers [35]. We take the following initial configuration for the soliton pair [cf. Eq. (1)]:

$$U = \mathrm{sech}(t + t_1) + \mathrm{sech}(t - t_1), \tag{5}$$

$$V = \mathrm{sech}(t + t_1) - \mathrm{sech}(t - t_1). \tag{6}$$

It is relevant to mention that interactions between far-separated classical solitons belonging to the different polarizations are also mediated by their tails. However, the character of the interaction is quite different from that in the single-component model: the interaction is incoherent, and it decays faster with the increase of the separation between the solitons [39].

Using the methods for the analysis of the classical vectorial solitons developed in Refs. [39–42], we calculated the respective quantum fluctuations and the photon-number correlations numerically. It should be noted the total intensity of the vectorial solitons, defined in terms of the circular polarizations, remains unchanged during the propagation, but the intensities of the linearly polarized (x and y) components, $E_x = (U+V)/\sqrt{2}$ and $E_y = -i(U-V)/\sqrt{2}$, evolve periodically, as shown in the inset of Fig. 3. In this figure, we display the evolution of the photon-number correlation between the x and y components of the vectorial solitons, which are originally uncorrelated, and then become negatively correlated. It should be stressed that if one uses the polarization fields proper, U and V, as the projection basis, there are no strong photon-number correlations between these two fields. It is necessary to employ an appropriate basis—for example, the polarizations E_x and E_y in the present case—to identify

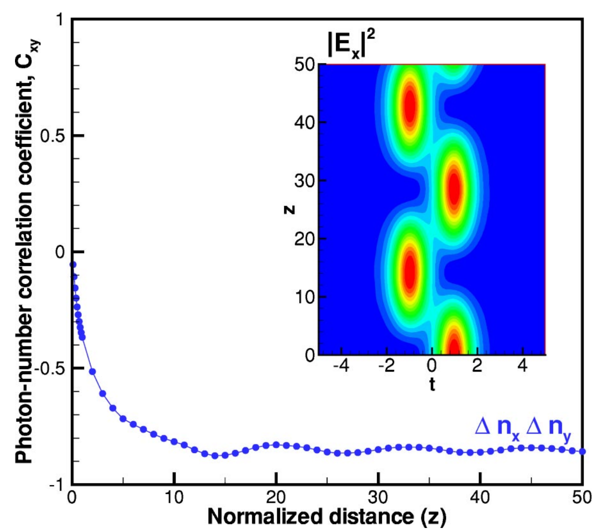

FIG. 3. (Color online) The photon-number correlation coefficient of interacting vectorial solitons. The inset displays the evolution of the x component of the classical field.

highly photon-number-correlated pairs. Recently, Lantz *et al.* [43] have shown that vectorial solitons in the *spatial domain* can also develop an almost perfect negative correlation between quantum fluctuations around an incoherently coupled soliton pair.

IV. CONCLUSION

In this work, we have studied the quantum photon-number correlations induced by the interactions between two solitons, mediated by their tails, in the time-division and polarization-division pairs. In the former case, using the pair with suitable initial separation and relative phase, one can generate positive or negative photon-number-correlated soliton pairs. An ultrafast optical switch will be needed to separate the two entangled solitons into different channels. On the other hand, by using the vectorial solitons with two polarization components, pairs with negative photon-number correlations between the solitons can be generated. For this case, a simple polarization beam splitter will be sufficient to separate the two entangled solitons.

Such photon-number-correlated soliton pairs feature unique entanglement properties, which may offer new possibilities for applications to quantum communications and computation. The applications will be considered in detail elsewhere.

[1] S. J. Carter, P. D. Drummond, M. D. Reid, and R. M. Shelby, Phys. Rev. Lett. **58**, 1841 (1987).
[2] P. D. Drummond and S. J. Carter, J. Opt. Soc. Am. B **4**, 1565 (1987).
[3] Y. Lai and H. A. Haus, Phys. Rev. A **40**, 844 (1989); **40**, 854 (1989).
[4] M. Rosenbluh and R. Shelby, Phys. Rev. Lett. **66**, 153 (1991).
[5] K. Bergman and H. A. Haus, Opt. Lett. **16**, 663 (1991).
[6] S. R. Friberg, S. Machida, M. J. Werner, A. Levanon, and T. Mukai, Phys. Rev. Lett. **77**, 3775 (1996).
[7] R.-K. Lee and Y. Lai, Phys. Rev. A **69**, 021801(R) (2004).
[8] S. Spölter, N. Korolkova, F. König, A. Sizmann, and G. Leuchs, Phys. Rev. Lett. **81**, 786 (1998).
[9] F. König, M. A. Zielonka, and A. Sizmann, Phys. Rev. A **66**, 013812 (2002).
[10] H. A. Haus, K. Watanabe, and Y. Yamamoto, J. Opt. Soc. Am. B **6**, 1138 (1989).
[11] S. R. Friberg, S. Machida, and Y. Yamamoto, Phys. Rev. Lett. **69**, 3165 (1992).
[12] P. D. Drummond, J. Breslin, and R. M. Shelby, Phys. Rev. Lett. **73**, 2837 (1994).
[13] J.-M. Courty, S. Spälter, F. König, A. Sizmann, and G. Leuchs, Phys. Rev. A **58**, 1501 (1998).
[14] A. B. Matsko, V. V. Kozlov, and M. O. Scully, Phys. Rev. Lett. **82**, 3244 (1999).
[15] S. R. Friberg, T. Mukai, and S. Machida, Phys. Rev. Lett. **84**, 59 (2000).
[16] V. V. Kozlov and D. A. Ivanov, Phys. Rev. A **65**, 023812 (2002).
[17] F. König, B. Buchler, T. Rechtenwald, G. Leuchs, and A. Sizmann, Phys. Rev. A **66**, 043810 (2002).
[18] P. D. Drummond, R. M. Shelby, S. R. Friberg, and Y. Yamamoto, Nature (London) **365**, 307 (1993).
[19] I. Abram, Phys. World **2**, 21 (1999).
[20] Ch. Silberhorn, P. K. Lam, O. Weib, F. König, N. Korolkova, and G. Leuchs, Phys. Rev. Lett. **86**, 4267 (2001).
[21] Ch. Silberhorn, N. Korolkova, and G. Leuchs, Phys. Rev. Lett. **88**, 167902 (2002).
[22] O. Glöckl, S. Lorenz, C. Marquardt, J. Heersink, M. Brownnutt, C. Silberhorn, Q. Pan, P. van Loock, N. Korolkova, and G. Leuchs, Phys. Rev. A **68**, 012319 (2003).
[23] R. Raz, in *Proceedings of the 31st ACM Symposium on Theory of Computing*, edited by J. S. Vitter, L. Larmore, and T. Leighton (ACM Press, New York, 1999), p. 358.
[24] C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. K. Wooters, Phys. Rev. Lett. **70**, 1895 (1993).
[25] P. W. Shor, in *Proceedings of the 35th Annual Symposium on the Foundations of Computer Science*, edited by S. Goldwasser (IEEE Computer Society, Los Alamitos, CA, 1994), p. 124.
[26] L. Grover, in *Proceedings of the 28th Annual ACM Symposium on the Theory of Computation*, edited by G. L. Miller (ACM Press, New York, 1996), p. 212.
[27] D. Bohm, *Quantum Theory* (Prentice-Hall, Englewood Cliffs, NJ, 1951).
[28] A. Einstein, P. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).
[29] Z. Y. Ou, S. F. Pereira, H. J. Kimble, and K. C. Peng, Phys. Rev. Lett. **68**, 3663 (1992).
[30] L. Vaidman, Phys. Rev. A **49**, 1473 (1994).
[31] S. L. Braunstein and H. J. Kimble, Phys. Rev. Lett. **80**, 869 (1998).
[32] A. Furusawa, J. L. Sørensen, S. L. Braunstein, C. A. Fuchs, H. J. Kimble, and E. S. Polzik, Science **282**, 706 (1998).
[33] T. C. Ralph and P. K. Lam, Phys. Rev. Lett. **81**, 5668 (1998).
[34] Y. Lai and S.-S. Yu, Phys. Rev. A **51**, 817 (1995).
[35] G. P. Agrawal, *Nonlinear Fiber Optics* (Academic, San Diego, 1995); Yu. S. Kivshar and G. P. Agrawal, *Optical Solitons: From Fibers to Photonic Crystals* (Academic, San Diego, 2003).
[36] B. A. Malomed, in *Progress in Optics*, edited by E. Wolf (Elsevier, Amsterdam, 2002), Vol. 43, p. 71.
[37] Yu. S. Kivshar and B. A. Malomed, Rev. Mod. Phys. **61**, 763 (1989).
[38] H. A. Haus and Y. Lai, J. Opt. Soc. Am. B **7**, 386 (1990).
[39] B. A. Malomed, A. I. Maimistov, and A. Desyatnikov, Phys. Lett. A **254**, 179 (1999).
[40] M. Haelterman, A. P. Sheppard, and A. W. Snyder, Opt. Lett. **18**, 1406 (1993).
[41] D. J. Kaup, B. A. Malomed, and R. S. Tasgal, Phys. Rev. E **48**, 3049 (1993).
[42] B. A. Malomed and R. S. Tasgal, Phys. Rev. E **58**, 2564 (1998).
[43] E. Lantz, T. Sylvestre, H. Maillotte, N. Treps, and C. Fabre, J. Opt. B: Quantum Semiclassical Opt. **6**, S295 (2004).