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ORIGINAL ARTICLE

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# Bootstrap approach for estimating process quality yield with application to light emitting diodes

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**Abstract** Process capability indices have been widely used by quality professionals for measuring process performance. Although process yield is the most common criterion used in the manufacturing industry for measuring process performance, a more advanced measurement formula $Y_q$, called quality yield index, has been proposed as an alternative measure of process performance. Quality yield can be viewed as the classical process yield minus the truncated expected relative process loss, within the specifications, which focuses on customer satisfaction. By taking customer loss into consideration, the advantage of using the quality-yield measure as process performance is that the formula can be applied to processes with arbitrary distributions. Unfortunately, statistical properties of the estimated $Y_q$ are mathematically intractable. Therefore, capability testing cannot be performed. In this paper, a nonparametric but computer intensive method called bootstrap is used to obtain a lower confidence bound on quality yield for capability testing purposes. Simulation studies are conducted to examine the sampling distribution of the estimated $Y_q$. An application using the index $Y_q$ for the light emitting diode manufacturing process is presented for illustration purposes.

**Keywords** Bootstrap methods · Lower confidence bound · Process capability indices · Quality yield · Simulation

## 1 Introduction

Process capability indices are convenient and powerful tools for measuring process performance. In recent years, process capability indices have received substantial research attention in the quality assurance and statistical literature. Those indices quantify process performance by taking into consideration process location, process variation, and manufacturing specifications, which reflect process consistency, process accuracy, process yield, and process loss. The process indices $C_p$, $C_{pk}$, $C_{pm}$ and $C_{pmk}$ [1–3] have become popular as unitless measures, which combine natural process tolerance, manufacturing specifications, process centering, and the target value of the process. Those indices convey critical information regarding whether a process is capable of reproducing items satisfying the customer's requirement. In practice, a minimal capability requirement would be preset by the customers/engineers. If the prescribed minimum capability fails to be met, one would conclude that the process is incapable. Four basic well-known capability indices are:

$$C_p = \frac{USL - LSL}{6\sigma}, \tag{1}$$

$$C_{pk} = \min\left\{\frac{USL-\mu}{3\sigma},\ \frac{\mu-LSL}{3\sigma}\right\}, \tag{2}$$

$$C_{pm} = \frac{USL-LSL}{6\sqrt{\sigma^2+(\mu-T)^2}}, \tag{3}$$

$$C_{pmk} = \min\left\{\frac{USL-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}},\ \frac{\mu-LSL}{3\sqrt{\sigma^2+(\mu-T)^2}}\right\}. \tag{4}$$

Those indices are effective tools for process capability analysis and quality assurance. Two process characteristics including the process location in relation to its target value, and the process spread are used to establish the formula of those capability indices. A rough categorization of those indices is by consideration of the target value $T$. The first category includes $C_p$ and $C_{pk}$, which are independent of $T$. Process loss incurred by the departure from the target is, however, neglected. The second category includes $C_{pm}$ and $C_{pmk}$, which rectify the disadvantage by taking the target value into account. The limitation on using those indices defined above is that they require the assumption that the quality characteristic measurements must be coming from normal distributions. Process quality yield index $Y_q$ is proposed to remedy this disadvantage.

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Traditionally, process yield $Y$, is defined as the percentage of the processed product units passing the inspections, which has for a long time been the most common and standard criteria used in the manufacturing industries for judging process performance. According to the manufacturing specifications placed on various key product characteristics, units are inspected and sorted into two categories: accepted (conforming items) and rejected (defectives). For product units rejected during the inspection, additional costs would be incurred to the factory for scrapping or reworking. All passed product units are treated equally and accepted by the producer. No additional cost to the factory is required. The definition of $Y$index is

$$Y = \int_{LSL}^{USL} \mathrm{d}F(x), \tag{5}$$

where $USL$ and $LSL$ are the upper and the lower specification limits, respectively, and $F(x)$ is the cumulative distribution function of the measured characteristic $X$. The disadvantage of yield measure is that it does not distinguish the products that fall inside of the specification limits. Customers do notice unit-to-unit differences in these characteristics, especially if the variance is large and/or the mean is offset from the target. To rectify this, a more accurate, complete and customer-oriented measure of yield, which is referred to as quality yield $Y_q$, was proposed [4]. The index distinguishes the products within the specifications by increasing the penalty as the departure from the target increases. The quadratic loss function is incorporated with the yield measure. Johnson [5] developed the relative expected loss $L_e$ to provide comparisons between processes, defined as:

$$L_e = \int_{-\infty}^{\infty} \left[ \frac{(x-T)^2}{d^2} \right] \mathrm{d}F(x), \tag{6}$$

where $\sigma^2$ is the process variance, $\mu$ is the process mean, $T$ is the target value and $d = (USL - LSL)/2$ is the half specification width. The disadvantage of the $L_e$ index is the difficulty in setting a standard for the index since it increases from zero to infinity. The quality yield index $Y_q$ differs from the expected relative worth index defined by Johnson [5] by truncating the deviation outside the specifications. With this truncation, the quality yield index will be between zero and one and thus has better interpretation. To illustrate basic differences among yield $Y$, quality yield $Y_q$, and process capability indices $C_p$, $C_{pk}$, $C_{pm}$ and $C_{pmk}$, we calculated their index values for some cases, as presented in Table 1.

Quality yield can be treated as traditional yield minus truncated expected relative process loss within the specifications to offer an excellent opportunity to quantify how well a process can meet customer requirements. While yield is the proportion of conforming products, $Q$-yield can be interpreted as the proportion of "perfect" products. By relating to the yield measure, which is familiar to engineers, it is much easier for the engineers to understand and accept this capability measure. The advantage of the $Y_q$ index over the $L_e$ index is that the value of the former goes from zero to one. Similarly to the yield index, the $Y$ measure, the ideal value of $Y_q$ is one, which provides the user a clear concept about the standard. Similar to yield $Y$, the $Y_q$ index does not rely on the normality assumption. Current practices of measuring manufacturing capability by only evaluating the point estimates of capability indices have been severely criticized since it ignores sampling error. The sampling distribution and sampling errors of the estimated $Q$-yield have never been investigated due to their mathematical intractability. A decision maker, however, may be interested in the lower confidence bound on the quality yield rather than just the point estimate, which does not convey reliable information.

**Table 1.** Comparisons of yield, $Q$-yield and PCIs

| Case | $Y\%$ | $Y_q\%$ | $C_p$ | $C_{pk}$ | $C_{pm}$ | $C_{pmk}$ |
|---|---|---|---|---|---|---|
| $N(T, d)$ | 68.27 | 48.39 | 0.33 | 0.33 | 0.33 | 0.33 |
| $N(T, d/2)$ | 95.45 | 76.99 | 0.67 | 0.67 | 0.67 | 0.67 |
| $N(T, d/3)$ | 99.73 | 88.94 | 1.00 | 1.00 | 1.00 | 1.00 |
| $N(T, d/4)$ | 99.99 | 93.75 | 1.33 | 1.33 | 1.33 | 1.33 |
| $N(T \pm d/3, d/2)$ | 90.50 | 69.13 | 0.67 | 0.44 | 0.55 | 0.37 |
| $N(T \pm d/3, d/3)$ | 97.72 | 78.41 | 1.00 | 0.67 | 0.71 | 0.47 |
| $N(T \pm d/3, d/4)$ | 99.62 | 82.70 | 1.33 | 0.89 | 0.80 | 0.53 |
| $N(T \pm d/3, d/6)$ | 99.997 | 86.11 | 2.00 | 1.33 | 0.89 | 0.60 |

In this paper, we apply the bootstrap resampling technique to obtain the lower confidence bound on $\hat{Y}_q$ for practical purpose. Four types of bootstrap confidence intervals, including the standard bootstrap confidence interval (SB), the percentile bootstrap confidence interval (PB), the biased corrected percentile bootstrap confidence interval (BCPB), and the bootstrap-$t$ (BT) methods will be conducted. The practitioners can use the results to perform quality testing and determine the process can reproduce product items to meet the specified quality requirement. The lower confidence bound not only provides us information regarding actual process performance, which is tightly related to both the fractions of defective units and customer quality loss, but is also useful in making reliable decisions for capability testing and monitoring the performance of process departure for targets as well.

This paper is organized as follows. We first give a brief introduction on the quality yield index $Y_q$ and the sample estimator of $Y_q$. We then introduce the bootstrap estimation technique and the definitions of the four bootstrap confidence intervals in Sect. 3. Subsequently, in Sect. 4, some simulations on four distributions (normal, student's $t$, chi-square and lognormal) are conducted to examine the distribution behavior of the estimated $Y_q$. For illustrative purpose, a real-world application to the light emitting diode (LED) manufacturing process is presented in Sect. 5. An integrated computer program for calculating the bootstrap lower confidence bounds is given in the Appendix. Some concluding remarks are made in Sect. 6.

## 2 Estimation of yield and quality yield

The main idea of the quality yield index $Y_q$ is that it penalizes yield for the variation of the product characteristics from its target. It was suggested by Ng and Tsui [4] by connecting the proportion-conforming-based index $Y$ and loss-function-based index $L_e$. Unlike the yield index $Y$, the quality yield $Y_q$ focuses on the ability of the process to cluster around the target by taking the relative loss within the specifications into consideration. If the $USL$ and $LSL$ are the upper and lower specification limits, respectively, $T$ is the target value, $d$ is the half specification width, and $F(x)$ is the cumulative distribution function of the measured characteristic, then the index $Y_q$ is defined as

$$Y_q = \int_{LSL}^{USL} \left[1 - \frac{(x-T)^2}{d^2}\right] \mathrm{d}F(x). \tag{7}$$

In practical applications, sample data must be collected to estimate the index. A sample estimator based on a finite population of products was proposed by Ng and Tsui [4]. Suppose $X_1, X_2, \ldots, X_n$ denote the sample measurements of product characteristics. A natural estimator of $Y$ and $Y_q$ may be expressed as

$$\hat{Y} = \sum_{LSL \le X_i \le USL} \frac{1}{n}, \tag{8}$$

$$\hat{Y}_q = \sum_{LSL \le X_i \le USL} \left[\frac{1-(X_i-T)^2/d^2}{n}\right]. \tag{9}$$

In addition to point estimation, however, a decision maker may be interested in a lower limit on the quality yield from the process as well. The sampling distribution of $\hat{Y}_q$ is then required but, unfortunately, the derivation of the exact distribution of $\hat{Y}_q$ is mathematically intractable. Pearn et al. [6] constructed an approximate lower confidence bound of the estimator $\hat{Y}_q$ for very low fraction of defectives under the assumption of normality. However, the calculation of the approximation is rather messy and cumbersome to undertake. Further, the accuracy of the approximation has not been investigated.

Normal-based process capability indices such as $C_p$, $C_{pk}$, $C_{pm}$ and $C_{pmk}$ do not measure process fallout for non-normal process data accurately. In the literature, Somerville and Montgomery [7] presented an extensive study to illustrate how poorly the normally based capability indices perform as a predictor of process fallout when the process is non-normally distributed. If the normally based capability indices are still used to deal with non-normal process data, the values of the capability indices are incorrect and might misrepresent the actual product quality. Although new capability indices have been developed for non-normal distributions, those indices are harder to compute and interpret, and are sensitive to data peculiarities such as bimodality or truncation. Moreover, those indices do not explicitly account for the manufacturing cost or customer's loss. If a process is clearly non-normal, there is some question as to whether any process index is valid or should even be calculated. To illustrate the relationship between the squared loss function and some probability distributions, we plot four process distributions: normal distribution, lognormal distribution, student's $t$ distribution and chi-square distribution, respectively, with the loss function and under the true value of $Y_q = 0.6$ (see Figs. 1–4).

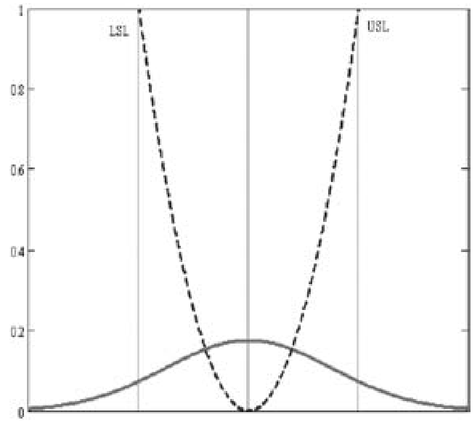

**Fig. 1.** Distribution plots of normal distribution with the loss function under true $Y_q = 0.6$

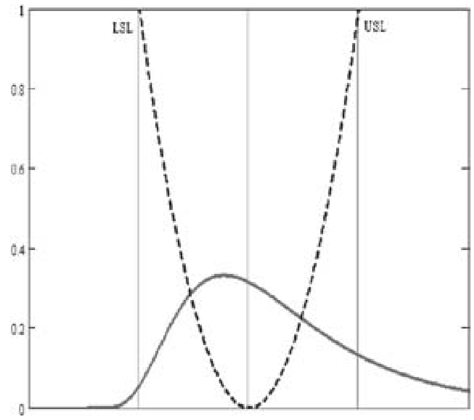

**Fig. 2.** Distribution plots of lognormal distribution with the loss function under true $Y_q = 0.6$

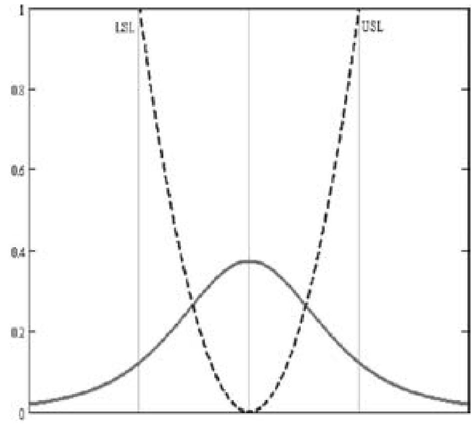

**Fig. 3.** Distribution plots of $t$ distribution with the loss function under true $Y_q = 0.6$

Note that most existing capability indices require the normality assumption and they are generally defined based on the specification limits rather than the customer's satisfactions. The advantage of using the $Q$-yield as process performance meas-

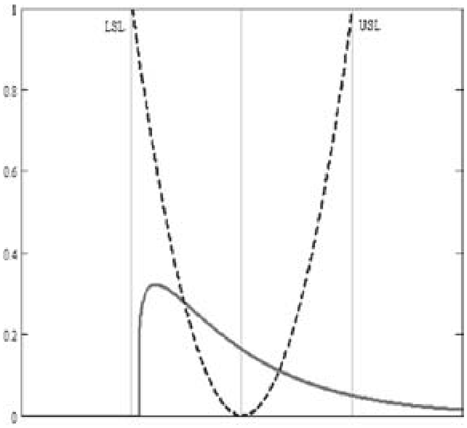

**Fig. 4.** Distribution plots of chi-square distribution with the loss function under true $Y_q = 0.6$

ure is that it does not rely on the normal distribution assumption. High values of $Q$-yield are desirable, which can be viewed as improving product quality from the customer's viewpoint. Furthermore, $Q$-yield is more flexible because it compares the quality of different characteristics of a product on a single percentage scale, and indicates how close a product comes to meeting 100% customer satisfaction.

## 3 The bootstrap methodology

Traditionally, statistical research work has relied on the central limit theorem and normal approximations to obtain standard errors and confidence intervals. These techniques are valid only when the statistic, or some known transformation of the statistic, is asymptotically normally distributed. Unfortunately, many real world processes are not normally distributed and this departure from normality could potentially affect these estimates. A major motivation for the traditional reliance on normal-theory methods has been computational tractability. Access to powerful computation enables the use of statistics in new and varied ways. Idealized models and assumptions can now be replaced with more realistic modeling or by virtually model-free analyses. Much statistical work and data analysis is undertaken today by computers in ways that are too complicated for practical analytical treatment. The new effects of these computational advances are probably best reflected in the recent enormous success of bootstrap methodology, which shows that many problems, previously difficult to solve, can be conquered. For either normal or non-normal distributions, the bootstrap method could be applied to return valid inferential results required.

The essence of bootstrapping is the idea that in the absence of any other knowledge about a population, the distribution of values found in a random sample of size $n$ from the population is the best guide to the distribution in the population. By resampling observations from the observed data, the process of sampling observations from the population is mimicked. Instead of using a sample statistic to estimate a population parameter, as is done within the framework of conventional parametric statistical tests, the bootstrap uses multiple samples derived from the original data to provide what in some instances may be a more accurate measure of the population parameter. Therefore, to approximate what would happen if the population was resampled, it is sensible to resample the sample. In other words, the infinite population that consist of the $n$ observed sample values, each with probability $1/n$, is used to model the unknown real population. The sampling is with replacement, which is the only difference in practice between bootstrapping and randomization in many applications.

The bootstrap, a data-based simulation technique for statistical inference which introduced by Efron [8, 9] is a nonparametric, computationally intensive but effective estimation method. The most common application of the bootstrap involves estimating a population standard error and/or confidence interval. In particular, one can use the sampling distribution of a statistic, while assuming that the sample is only representative of the population from which it is drawn, and that the observations are independent and identically distributed. The main merit of the nonparametric bootstrap is that it does not rely on any distributional assumptions about the underlying population. The more ambiguous the information is to the researcher regarding the underlying population distribution, the more likely it is that the bootstrap may prove useful. Rather than using distribution frequency tables to compute approximate $p$ probability values, the bootstrap method generates a unique sampling distribution based on the actual sample rather than the analytic methods. The formulation detail follows.

In this method, $B$ new samples, each of the same size as the observed data, are drawn with replacement from the available sample. The statistic of interest is then calculated for each new set of resampled data, in our case say $\hat{Y}^*_{q1}, \hat{Y}^*_{q2}, \ldots, \hat{Y}^*_{qB}$, yielding a bootstrap distribution for the statistic, say $\hat{Y}_q$. Four types of bootstrap confidence intervals, including the standard bootstrap confidence interval (SB), the percentile bootstrap confidence interval (PB), the biased corrected percentile bootstrap confidence interval (BCPB), and the bootstrap-$t$ (BT) method introduced by Efron [10] and Efron and Tibshiraniwill [11] will be conducted in this paper. Assume the observations $x_1, x_2, \ldots, x_n$ to be a random sample of size $n$ taken from a process. A bootstrap sample, denoted by $x^*_1, x^*_2, \ldots, x^*_n$, is a sample of size $n$ drawn with replacement from the original sample. There are possibly a total of $n^n$ such resamples. Each such sample is called a "bootstrap sample." In our case, these resamples would then be used to calculate $n^n$ values of $\hat{Y}^*_q$. Each of these would be an estimate of $Y_q$ and the entire collection would constitute the (complete) bootstrap distribution for $\hat{Y}_q$. Bootstrap sampling is equivalent to sampling (with replacement) from the empirical probability distribution function. Thus, the bootstrap distribution of $Y_q$ is estimator of the distribution of $Y_q$.

Due to the overwhelming computation time, it is not of practical interest to choose $n^n$ such samples. Usually, in practice, only a random sample of $n^n$ possible resamples is drawn, the statistic is calculated for each of these, and the resulting empirical distribution is referred to as the bootstrap distribution of the statistic. Empirical work [11] indicated that only rough minimum of 1000 bootstrap resamples are required for the procedure to be useful to calculate valid confidence limits for population parameters. Throughout our discussion, it is assumed that $B = 10\,000$ bootstrap resamples (each of the same size as the available data) are taken and $B = 10\,000$ bootstrap estimate of $Y_q$ are calculated and ordered from smallest to largest. The generic notations $\hat{Y}_q$ and $\hat{Y}_q^*(i)$ will be used to denote the estimator of a $Q$-yield index and the associated ordered bootstrap estimate. Construction of a two-sided $(1-2\alpha)100\%$ confidence limit will be described. We note that a lower $(1-\alpha)100\%$ confidence limit can be obtained by using only the lower limit. If the calculated bootstrap lower confidence limit is found to be smaller than the predetermined index value, we would judge that the process is incapable. Quality improvement activities will be initiated. Otherwise, the process is considered to be capable. Four kinds of confidence intervals can be derived.

### 3.1 Standard bootstrap (SB)

From the $B$ bootstrap estimates $\hat{Y}_q^*(i)$, the sample average and the sample standard deviation can be obtained as

$$\bar{\hat{Y}}_q^* = \frac{1}{B}\sum_{i=1}^{B}\hat{Y}_q^*(i), \tag{10}$$

$$S_{Y_q}^* = \sqrt{\frac{1}{B-1}\sum_{i=1}^{B}\left[\hat{Y}_q^*(i)-\bar{\hat{Y}}_q^*\right]^2}. \tag{11}$$

where $\hat{Y}_q^*(i)$ is the $i$th bootstrap estimate. Actually the quantity $S_{Y_q}^*$ is an estimator of the standard deviation of $\hat{Y}_q$ if the distribution of $\hat{Y}_q$ is approximately normal. Thus, the $(1-2\alpha)100\%$ SB confidence interval for $Y_q$ can be constructed as

$$\left[\hat{Y}_q - z_\alpha S_{Y_q}^*,\, \hat{Y}_q + z_\alpha S_{Y_q}^*\right], \tag{12}$$

where $\hat{Y}_q$ is the estimated $Y_q$ for the original sample, and $z_\alpha$ is the upper $\alpha$ quantile of the standard normal distribution.

### 3.2 The percentile bootstrap (PB)

From the ordered collection of $\hat{Y}_q^*(i)$, the $\alpha$ percentage and $1-\alpha$ percentage points are used to obtain the $(1-2\alpha)100\%$ PB confidence interval for $Y_q$,

$$\left[\hat{Y}_q^*(\alpha B),\, \hat{Y}_q^*((1-\alpha)B)\right]. \tag{13}$$

### 3.3 Biased-corrected percentile bootstrap (BCPB)

While the percentile confidence interval is intuitively appealing it is possible that due to sampling errors, the bootstrap distribution may be biased. In other words, it is possible that bootstrap distributions obtained only using a sample of the complete bootstrap distribution may be shifted higher or lower than would be expected. A three steps procedure is suggested to correct for the possible bias [9]. First, using the ordered distribution of $\hat{Y}_q^*$, calculate the probability $p_0 = P[\hat{Y}_q^* \le \hat{y}_q]$. Second, we compute the inverse of the cumulative distribution function of a standard normal based upon $p_0$ as $z_0 = \Phi^{-1}(p_0)$, $p_L = \Phi(2z_0 - z_\alpha)p_U = \Phi(2z_0 + z_\alpha)$, where $\Phi(\cdot)$ is the standard normal cumulative distribution function. Finally, executing these steps to obtain the BCPB confidence interval,

$$\left[\hat{Y}_q^*(p_L B),\, \hat{Y}_q^*(p_U B)\right]. \tag{14}$$

### 3.4 Bootstrap-$t$ (BT)

By using bootstrapping to approximate the distribution of a statistic of the form $T = (\hat{Y}_q - Y_q)/S_{Y_q}$, where $\hat{Y}_q$ is an estimate of $Y_q$, with estimated standard error $S_{Y_q}$. The bootstrap approximation in this case is obtained by taking bootstrap samples from the original data values, calculating the corresponding estimates $\hat{Y}_q^*$ and their estimated standard error, and hence finding the bootstrapped $T$-values $T = (\hat{Y}_q^* - \hat{Y}_q)/S_{Y_q}^*$. The hope is then that the generated distribution will mimic the distribution of $T$. The $(1-2\alpha)100\%$ BT confidence interval for $Y_q$ may constitute as

$$\left[\hat{Y}_q - t_\alpha^* S_{Y_q}^*,\, \hat{Y}_q - t_{1-\alpha}^* S_{Y_q}^*\right], \tag{15}$$

where $t_\alpha^*$ and $t_{1-\alpha}^*$ are the upper $\alpha$ and $1-\alpha$ quantile of the bootstrap $t$-distribution respectively, i.e., by finding the values that satisfy the two equations $P[(\hat{Y}_q^* - \hat{Y}_q)/S_{Y_q}^* > t_\alpha^*] = \alpha$ and $P[(\hat{Y}_q^* - \hat{Y}_q)/S_{Y_q}^* > t_{1-\alpha}^*] = 1-\alpha$, for the generated bootstrap estimates.

In the literature, Franklin and Wasserman [12] investigated the lower confidence bounds for the capability indices, $C_p$, $C_{pk}$ and $C_{pm}$ using the first three bootstrap methods. Some simulations were conducted and a comparison was made among the three bootstrap methods based on the parametric estimates. The simulation results indicate that for normal processes the bootstrap confidence limits perform equally well as results obtained by Chou, Owen and Borego [13], Bissell [14], and Boyles [15]. And for non-normal processes the bootstrap estimates performed significantly better than other methods.

## 4 Distribution plot of the $Q$-yield estimator

In this section, some Monte Carlo simulations are conducted to study the behavior of the sampling distribution of the estimated $Y_q$, for several cases where the underlying process distributions are normal, skewed, or heavy tailed. We consider two levels of $Y_q$, say, $Y_q = 0.9$, $Y_q = 0.6$, with underlying process distributions set to

1. Normal distribution with probability density function

$$f(x) = (\sqrt{2\pi}\sigma)^{-1} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right], \tag{16}$$

with mean $\mu$ and variance $\sigma^2$, for $-\infty < x < \infty$.

2. Lognormal distribution with probability density function of

$$f(x) = (x\sqrt{2\pi}\sigma)^{-1} \exp\left[-\frac{(\ln x-\mu)^2}{2\sigma^2}\right], \tag{17}$$

with mean $\mu = e^{\alpha+\beta^2/2}$ and variance $\sigma^2 = e^{2\alpha+\beta^2}(e^{\beta^2}-1)$, for $x > 0$.

3. Student's $t$ distribution with degree of freedom $k$, where the probability density function $t_k$ is,

$$f(x) = \left[\frac{\Gamma\left((k+1)/2\right)}{\Gamma(k/2)}\right]\frac{1}{\sqrt{k\pi}}\left(1+\frac{x^2}{k}\right)^{-(k+1)/2}, \tag{18}$$

with mean $\mu = 0$, for $k > 1$ and variance $\sigma^2 = k/(k-2)$, for $k > 2$, $-\infty < x < \infty$.

4. Chi-square distribution with degree of freedom $k$, where the probability density function of $\chi_k^2$ is

$$f(x) = \left[\frac{1}{\Gamma(k/2)}\right]\left(\frac{1}{2}\right)^{k/2} \chi^{k/2-1}e^{-x/2}, \tag{19}$$

with mean $\mu = k$ and variance $\sigma^2 = 2k$, $k = 1, 2, \ldots$.

For each distribution, we randomly generate $N = 20\,000$ samples of sizes $n = 25$, 50, 100, 300, 500, then calculate the estimated capability index $Y_q$. Figures 5–12 plot the distribution of $\hat{Y}_q$ for the two levels of $Y_q$, $Y_q = 0.9$, and $Y_q = 0.6$, with four process distributions, normal distribution, lognormal distribution, student's $t$ distribution and chi-square distribution, respectively. For moderate and large sample size $n$, the distributions of the estimated $Q$-yield index all appear to be normal. Therefore, for processes where large sample data may be collected (product items may be inspected by automatic inspection machines) and normal approximations may be used for capability testing. Otherwise, the proposed bootstrap methodology seems to be more

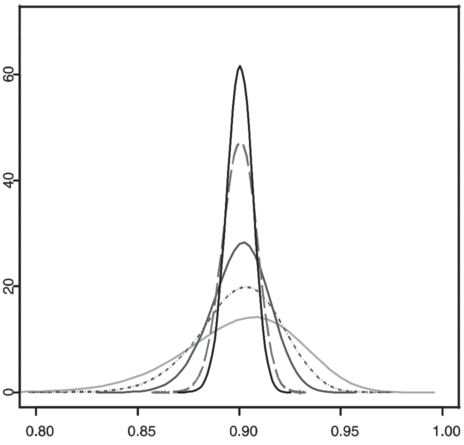

**Fig. 5.** Distribution plots of $\hat{Y}_q$ for normal distribution with $n = 25$, 50, 100, 300, 500 (bottom to top) under true $Y_q = 0.9$

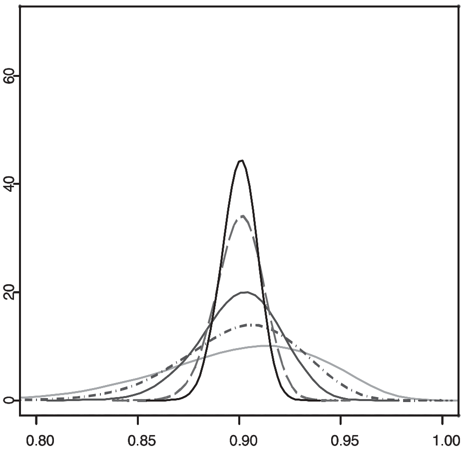

**Fig. 7.** Distribution plots of $\hat{Y}_q$ for lognormal distribution with $n = 25$, 50, 100, 300, 500 (bottom to top) under true $Y_q = 0.9$

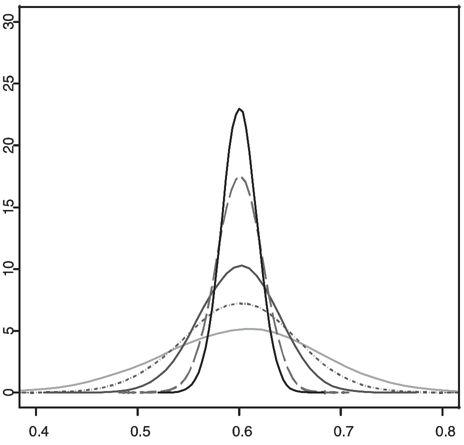

**Fig. 6.** Distribution plots of $\hat{Y}_q$ for normal distribution with $n = 25$, 50, 100, 300, 500 (bottom to top) under true $Y_q = 0.6$

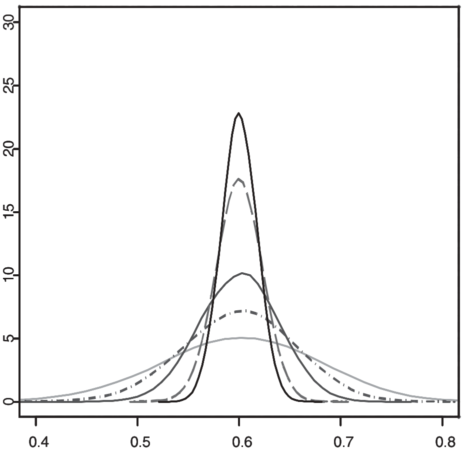

**Fig. 8.** Distribution plots of $\hat{Y}_q$ for lognormal distribution with $n = 25$, 50, 100, 300, 500 (bottom to top) under true $Y_q = 0.6$

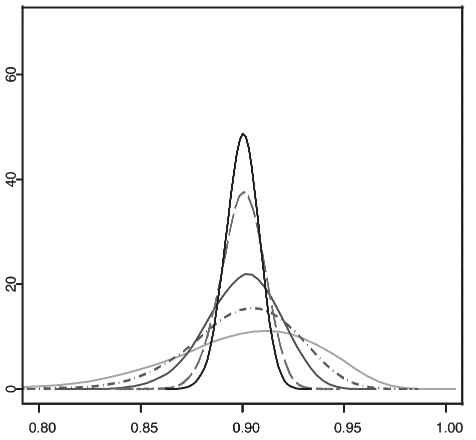

**Fig. 9.** Distribution plots of $\hat{Y}_q$ for $t$ distribution and $n = 25$, 50, 100, 300, 500 (bottom to top) under true $Y_q = 0.9$

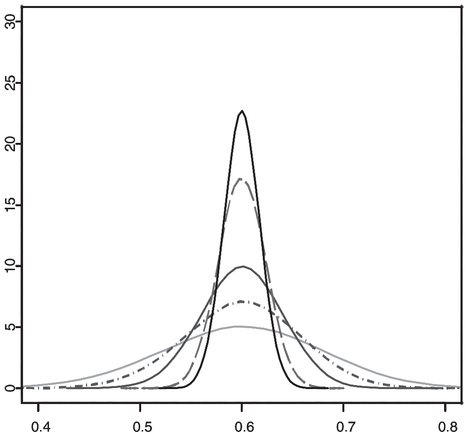

**Fig. 10.** Distribution plots of $\hat{Y}_q$ for $t$ distribution and $n = 25$, 50, 100, 300, 500 (bottom to top) under true $Y_q = 0.6$

reliable to make statistical inference on the estimated $Y_q$, when one has no idea what the underlying distribution really is. The bootstrap method especially is superior to other methods when the process distribution significantly deviates from normality and the size of sample data is small.

## 5 An application for LEDs

We present a case study on the light emitting diode (LED) manufacturing process to illustrate the usage of the bootstrap lower confidence bound on $Y_q$. The case we investigated was taken from a manufacturing factory located in the Science-Based Industrial Park, Taiwan, making LEDs. The application of LEDs is expanding rapidly since high intensity LEDs with a wide range of colors have been recently developed and become available, which enabled application of LEDs in a wide variety of areas including color displays, traffic signals, roadway signs (barricade lights), airport signaling and lighting. Two typical LED applications including font display and white LED lamps are shown in Fig. 13 and Fig. 14. As various LED applications are developed, accurate specifications of LED characteristics become increasingly important. However, serious discrepancy in measurement is gathered from different LED manufacturers and users. LEDs are unique light sources that are very different from lamps in

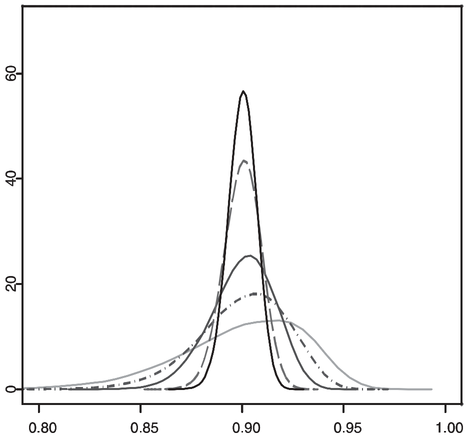

**Fig. 11.** Distribution plots of $\hat{Y}_q$ for chi-square distribution and $n = 25$, 50, 100, 300, 500 (bottom to top) under true $Y_q = 0.9$

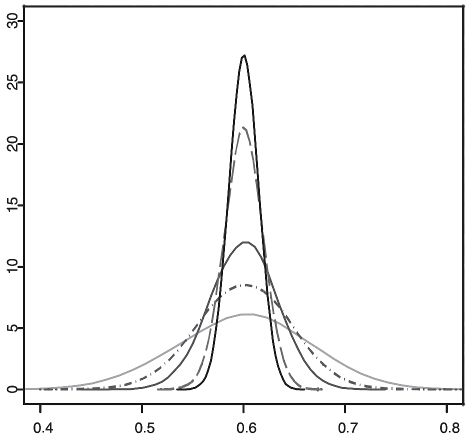

**Fig. 12.** Distribution plots of $\hat{Y}_q$ for chi-square distribution and $n = 25$, 50, 100, 300, 500 (bottom to top) under true $Y_q = 0.6$

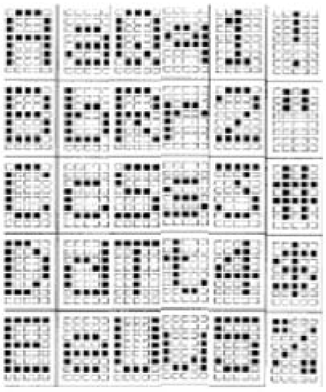

**Fig. 13.** LED application on font display

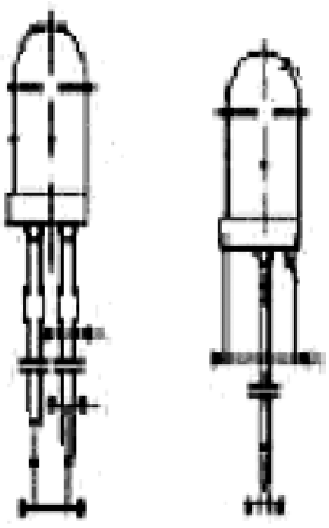

**Fig. 14.** LED application on white lamps

terms of physical size, flux level, spectrum, and spatial intensity distribution. A transfer of photometric scales from traditional luminous intensity standard lamps to LEDs is not a trivial task, and large uncertainties are involved. The temperature-dependent characteristics and a large variety of optical designs of LEDs make it even more difficult to reproduce measurements.

In order to solve this problem, the factory was requested to provide calibrated standard LEDs for luminous intensity and luminous flux, which should dramatically improve the accuracy of measurement at industry level. Thus, the factory develops the measurement technology and standards for LED luminous intensity and luminous flux measurements, and to establish calibration services for LEDs, thereby improving the accuracy and uniformity of LED measurements among optoelectronics and other industry. A photometric technique has been developed to determine the effective reference plane of a photometer with an uncertainty of 0.2 mm, using a photometric bench and a stable integrating sphere source instead of a tungsten filament lamp. With this method, any photometer head with unknown reference plane position can be calibrated for LED measurements at any distances longer than 10 cm within an uncertainty of less than 1%. The alignment of LEDs is still a major uncertainty component for luminous intensity. As described above, LEDs generally do not follow the inverse-square law, so setting the distances accurately is critical to achieve reproducible results. One method of setting the alignment is permanently mounting an LED in a mount that has a reference surface. The distance from the tip of the LED to the reference surface can be measured accurately. The angular alignment will not change because the reference surface will align the LED with the apparatus.

Typically, LEDs are not mounted in a permanent fixture, they are just bare LEDs. The widely accepted method of aligning the bare LEDs is along their mechanical axis, mainly because it can be done quickly. The factory tried two different methods of aligning bare LEDs, one using a mount that physically holds the LED by the sides and another using an optical aligning procedure. A mount that physically holds the sides has the advantages of the permanent mount once the LED is in the fixture. The fixture can be reproducibly placed in and out of a holder such that the distances are well known. The LED is easily centered along the detector axis and switching from the test LED to a standard LED can be done very quickly. However, we found reproducibly mounting the bare LED in the fixture was difficult. The fixture relied on placing pressure on the sides of the LED, which caused the sides of the LEDs to become scratched and damaged. In addition, a new fixture had to be fabricated for each different style or size of LED.

A better method is aligning the bare LEDs optically. Using a fixed telescope, a point in space is defined along the detector axis. The detector is on a translational stage with an optical encoder. The reference plane of the detector is moved to the point in space and then translated 100 mm or 316 mm away depending on the condition. The bare LED is mounted by its contacts on a stage that has five degrees of freedom. The stage can rotate, translate in the X, Y, and Z directions and tip and tilt about the point in space defined by the fixed telescope. By examining the LED from the side, the tip of the LED is translated to the point in space, set parallel to the detector axis and adjusted vertically. An LED application on LCD backlighting package dimension are depicted in Figs. 15 and 16.

We have established a capability for calibrating the luminous intensity of LEDs using the detector-based method. We have built a tentative measurement set up for LED measurements in the photometric bench and made the calibration service available for submitted LEDs. The measurement of LED luminous intensity currently has an overall uncertainty of 1.5% for LEDs with a special fixture, and 3% for normal bare LEDs with no alignment aids. A dedicated small photometric bench for LED measurements is to be built. Long-term stability and temperature dependence of these LEDs will be studied and standard LEDs for luminous intensity are to be developed. LEDs are unique light sources and are very different from traditional lamps in terms of physical size, flux level, spectrum and spatial distribution. The transfer of photometric scales from luminous intensity standard lamps to LEDs has not been trivial and

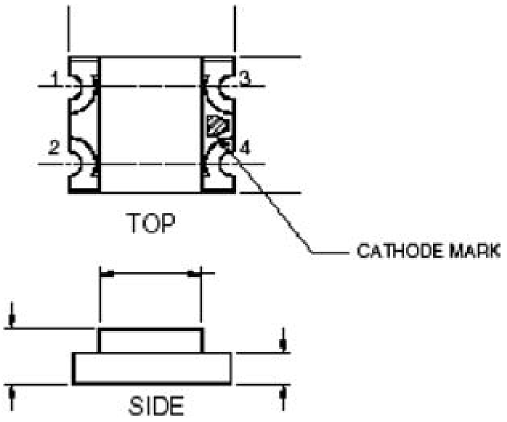

**Fig. 15.** The package dimensions drawing (top and side) of an LCD backlighting application

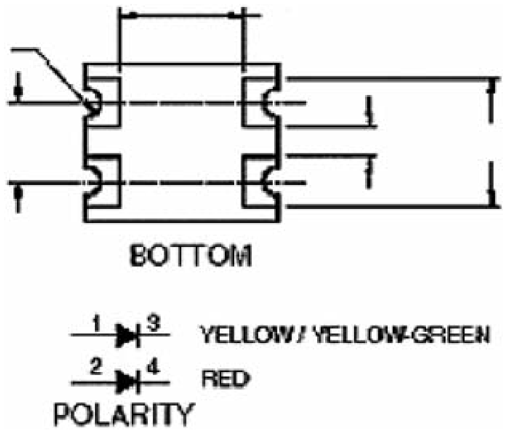

**Fig. 16.** The package dimensions drawing (bottom and polarity) of an LCD backlighting application

large discrepancies among companies have been measured. The factory has established two measurement conditions for single element LEDs with diameters less than 10 mm. These two measurement techniques compare LED luminous intensities without strictly using point source conditions. The factory has started research programs to establish appropriate measurement methods and calibration standards for all photometric quantities of LEDs. In particular, the measurement of luminous intensity of LED sources will be focused in our study. We investigated a particular model of the LED product with the upper and the lower specification limits of luminous intensity are set to $USL = 90$ mcd, $LSL = 40$ mcd, and the target value is set to $T = 65$ mcd. If the characteristic data does not fall within the tolerance $(LSL, USL)$, the LED is said to be defective.

For the purpose of making use of the methodology more convenient and accelerate the computation, an integrated S-PLUS computer program is developed (see Appendix) to calculate the bootstrap lower confidence bounds. The practitioners only need to input the manufacturing specification limits, $USL$, $LSL$, target value $T$, and the collected sample data of size $n$. Then the estimated values $\hat{Y}$, $\hat{Y}_q$ and the four bootstrap lower confidence bounds (SB, PB, BCPB, BT) of $\hat{Y}_q$ may be obtained. Thus, whether or not the process is capable may be determined.

A total of 100 observations were collected from a stable process in the factory and are displayed in Table 2. Figure 17 displays the histogram, and Fig. 18 displays the normal probability plot of these sample data. From Figs. 17 and 18, it is evident to conclude the data collected from the factory are not normal distributed. The data analysis results justify that the process is significantly away from the normal distribution. Proceeding with the calculations by running the integrated S-PLUS program with 95% confidence, we obtain the values of the sample estimators $\hat{Y}_q = 0.7477$ and the corresponding bootstrap lower confidence bound (LCB) as Table 3.

**Table 2.** A total of 100 observations

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 62 | 58 | 52 | 55 | 58 | 48 | 76 | 69 | 86 | 55 |
| 55 | 44 | 49 | 57 | 55 | 45 | 51 | 57 | 89 | 45 |
| 66 | 67 | 58 | 49 | 68 | 69 | 69 | 59 | 71 | 45 |
| 68 | 65 | 57 | 75 | 56 | 68 | 47 | 55 | 56 | 68 |
| 62 | 68 | 61 | 68 | 88 | 41 | 70 | 68 | 57 | 45 |
| 59 | 63 | 85 | 56 | 45 | 66 | 67 | 64 | 53 | 41 |
| 78 | 78 | 56 | 43 | 64 | 55 | 46 | 59 | 51 | 79 |
| 67 | 88 | 68 | 48 | 69 | 55 | 88 | 48 | 67 | 88 |
| 85 | 57 | 57 | 57 | 43 | 65 | 49 | 59 | 86 | 68 |
| 57 | 46 | 57 | 64 | 60 | 55 | 75 | 72 | 49 | 67 |

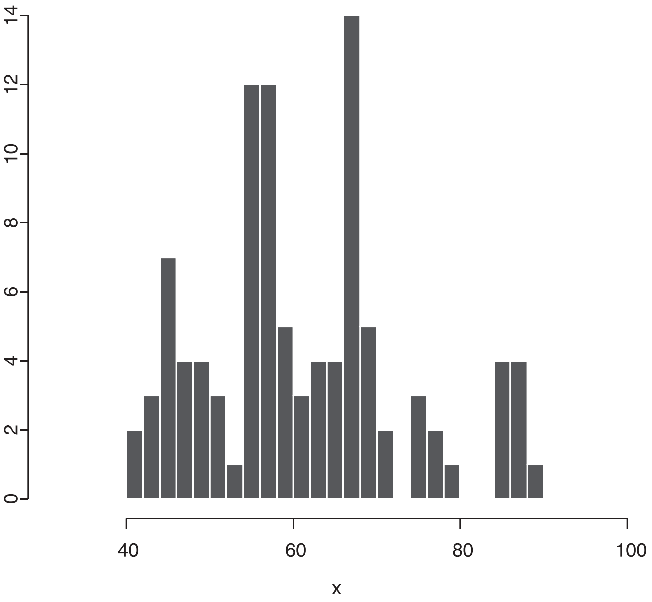

**Fig. 17.** Histogram plot of the sample data of size $n = 100$

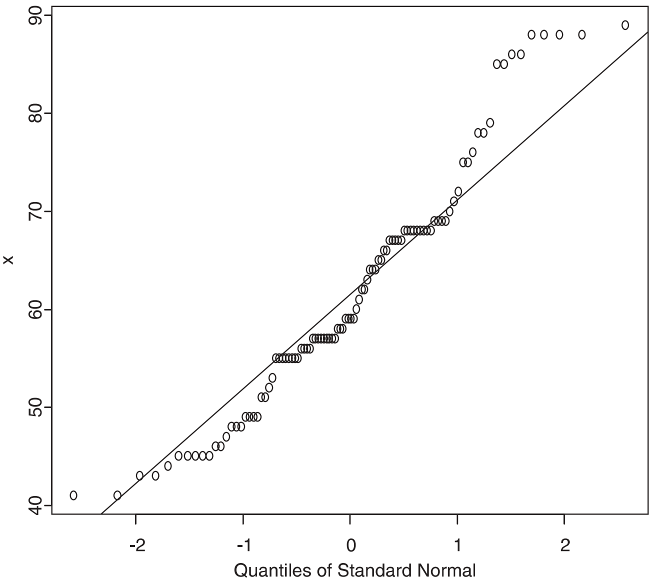

**Fig. 18.** Normal probability plot of the sample data of size $n = 100$

We note that the estimated index values for all the four extensions are greater than 0.7. In fact, all 100 observations fall within the specification interval $(LSL, USL)$ resulting that sample estimators of yield $\hat{Y} = 1$. From the producer's point of view, the

**Table 3.** Summary of the four bootstrap lower confidence bounds

| Type | SB | PB | BCPB | BT |
|---|---|---|---|---|
| LCB | 0.7010 | 0.7005 | 0.7027 | 0.7015 |

proportion of conforming products is 100%. However, to quantify how well a process can meet customer requirements, the lower confidence bound of $\hat{Y}_q$ is approximately 0.7 and can be interpreted as the proportion of "perfect" products being approximately 70%. From the corresponding lower confidence bounds on $Y_q$ based on four bootstrap methods, 0.7010, 0.7005, 0.7027, and 0.7015, an example of capability testing is that if the $Q$-yield requirement preprint on the contract $Y_q$ is set to 0.7, we may only conclude that the process is marginally capable, with 95% of confidence.

## 6 Conclusions

Quality yield is a flexible index because it compares the quality of different characteristics of a product on a single percentage scale, and indicates how close a product comes to meeting 100% customer satisfaction. Furthermore, comparing with the existing capability indices, these capability indices rely on the underlying assumption of normal distribution. Although new capability indices have been developed for non-normal distributions, those indices are harder to compute and interpret, and are sensitive to data peculiarities such as bimodality or truncation. Second, these indices do not explicitly account for the manufacturing cost or customer's loss. Capability indices are generally defined with respect to the specification limits rather than the customer's functional limits. If a process is clearly non-normal, there is some question as to whether any process index is valid or should even be calculated. In this paper, the nonparametric is computationally intensive but an effective estimation bootstrap method is applied to the $Q$-yield measure $\hat{Y}_q$ to obtain the lower confidence bounds. The lower confidence bound provides information regarding actual process performance for both the fractions of defectives units and customer quality loss. The proposed approach makes it feasible for the engineers to perform approximate process quality testing using the calculated $Y_q$.

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## Appendix

S-PLUS program for four bootstrap lower confidence bounds

```
#--------------------------------------------
# Input manufacturing specification limits
# USL, LSL and the target value T
#--------------------------------------------
USL_90
LSL_40
Target_65
#--------------------------------------------
# Input the original sample data of size
# n = 100 collected from the factory
#--------------------------------------------
data0_c(
62, 58, 52, 55, 58, 48, 76, 69, 86, 55,
55, 44, 49, 57, 55, 45, 51, 57, 89, 45,
66, 67, 58, 49, 68, 69, 69, 59, 71, 45,
68, 65, 57, 75, 56, 68, 47, 55, 56, 68,
62, 68, 61, 68, 88, 41, 70, 68, 57, 45,
59, 63, 85, 56, 45, 66, 67, 64, 53, 41,
78, 78, 56, 43, 64, 55, 46, 59, 51, 79,
67, 88, 68, 48, 69, 55, 88, 48, 67, 88,
85, 57, 57, 57, 43, 65, 49, 59, 86, 68,
57, 46, 57, 64, 60, 55, 75, 72, 49, 67)
#--------------------------------------------
# Function to calculate the estimated Y
# and Yq based on the given data
#--------------------------------------------
delta_(USL-LSL)/2
Q.yield_function(data){
 N_length(data)
 indata_data[data<USL&data>LSL]
 m_length(indata)
 Y_m/N
```

```
 Yq_Y-(sum(((indata-Target)/delta)^2)/N)
return(Y,Yq)
 }
#------------------------------------------
# Calculate the estimate of Y and Y_q based on
# the original sample data
#------------------------------------------
Y.Estimate_Q.yield(data0)$Y
Yq.Estimate_Q.yield(data0)$Yq
Y.Estimate
Yq.Estimate
#------------------------------------------
# Generate B = 10000 bootstrap resamples from
# the original sample data
#------------------------------------------
B_10000
Y.B_rep(0,B)
Yq.B_rep(0,B)
for (i in 1:B){
 dataS_sample(data0,length(data0),replace=T)
 Y.B[i]_Q.qield(dataS){\$}Y
 Yq.B[i]_Q.qield(dataS){\$}Yq
}
#------------------------------------------
# Calculate the four bootstrap lower confidence
# bounds based on resampled data
#------------------------------------------
Yq.SB.bootstrap.95LCB_Yq.Estimate
  -qnorm(0.95)*var(Yq.B)^0.5
Yq.PB.bootstrap.95LCB_quantile
  (Yq.B, probs = 0.05)
p0_mean(Yq.B<=q.Estimate)
z0_qnorm(1-p0)
pL_pnorm(2*z0-qnorm(0.95))
Yq.BCPB.bootstrap.95LCB_quantile
  (Yq.B, probs = floor(pL*B)/B)
qtB095_quantile((Yq.B-Yq.Estimate)
  /(var(Yq.B)^0.5), probs = 0.95)
Yq.BT.bootstrap.95LCB_Yq.Estimate
  -qtB095*var(Yq.B)^0.5
Yq.SB.bootstrap.95LCB
Yq.PB.bootstrap.95LCB
Yq.BCPB.bootstrap.95LCB
Yq.BT.bootstrap.95LCB
```

The output of the S-PLUS program is:

*The estimated Y and $Y_q$ based on the original sample data*:

```
> Y.Estimate = 1
> Yq.Estimate = 0.747744
```

*Four bootstrap lower confidence bounds of $Y_q$ based on resampled data*:

```
> Yq.SB.bootstrap.95LCB = 0.7010094
> Yq.PB.bootstrap.95LCB = 0.700512
> Yq.BCPB.bootstrap.95LCB = 0.70272
> Yq.BT.bootstrap.95LCB = 0.7015304
```