Energy Conversion and Management 52 (2011) 147–152

Contents lists available at ScienceDirect

Energy Conversion and Management

journal homepage: www.elsevier.com/locate/enconman

Forecasting energy consumption using a grey model improved by incorporating genetic programming

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ARTICLE INFO

Article history:
Received 28 October 2009
Accepted 22 June 2010

Keywords:
Energy consumption
Grey forecasting model
Genetic programming

ABSTRACT

Energy consumption is an important economic index, which reflects the industrial development of a city or a country. Forecasting energy consumption by conventional statistical methods usually requires the making of assumptions such as the normal distribution of energy consumption data or on a large sample size. However, the data collected on energy consumption are often very few or non-normal. Since a grey forecasting model, based on grey theory, can be constructed for at least four data points or ambiguity data, it can be adopted to forecast energy consumption. In some cases, however, a grey forecasting model may yield large forecasting errors. To minimize such errors, this study develops an improved grey forecasting model, which combines residual modification with genetic programming sign estimation. Finally, a real case of Chinese energy consumption is considered to demonstrate the effectiveness of the proposed forecasting model.

1. Introduction

Energy is an important source of economic development. Therefore, many countries are concerned with energy-related issues. In Asia, China is the country that products and consumers the most energy. The China statistical yearbook [1] states that the rate of production of energy increases 4.5% annually while the rate of consumption of energy increases 5.2% annually. Moreover, in 2006, about 69% of the total energy consumed was provided by coal; 20% was provided by oil, and 3% was provided by gas. With the rapid development of the economy and high-tech industries, energy consumption has become very high. Hence, forecasting energy consumption is important for every national government.

Some traditional forecasting models, such as regression models and artificial neural network (ANN)-based models, are frequently adopted in many fields. However, their predictive accuracy is low when the sample is small [2,3]. Chiang et al. [4] provided an overview of some conventional forecasting models and noted that the grey model (GM) can be employed for small data sets or data with limited information. The GM is based on grey theory [4] and requires at least four observations.

GM(1, 1) represents the first-order one-variable grey model. It has been utilized as a predictive model in many fields. However, the effectiveness of the residual series of GM(1, 1) depends on the number of data points with the same sign, which is generally low when the observations are few [5,6]. To enhance the effectiveness of residual sign of GM(1, 1), some studies have developed improved residual sign estimators for the GM(1, 1) model. For example, Hsu and Chen [5] proposed an improved grey forecasting model that combines residual modification and residual artificial neural network (ANN) sign estimation to forecast power demand. Hsu [6] modified the residual of GM(1, 1) model, and adopted Markov-chain sign estimation to forecast the value of the global integrated circuit industry.

To increase the accuracy of the GM(1, 1) model, this study proposes a novel approach that combines residual modification and residual genetic programming (GP) sign estimation to improve the precision of the residual sign estimator. GP is a strategy for evolving functions that effective perform designated tasks. Like genetic algorithms (GAs), GP can find the optimal solution using crossover, mutation and reproduction rules [2]. GP does not need to assume any particular relationship between dependent and independent variables [7] and it performs well on a small data set [2]. GP provides two advantages. First, it can obtain a mathematical equation through regression analysis. Second, GP can express a mathematical expression using the technique of a parse tree. Accordingly, GP can efficiently find the optimal residual sign estimation.

Motivated by the importance of forecasting the energy consumption, this study proposes an improved grey forecasting model that integrates GP and grey theory. This paper is organized as follows. Section 2 reviews the literature on the grey forecasting model and the concept of GP. Section 3 considers the improvement of

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doi:10.1016/j.enconman.2010.06.053

grey forecasting. One real-world example is utilized to demonstrate the proposed method. Section 4 compares the proposed method is compared with other forecasting models and Section 5 draws conclusions.

2. Literature review

The grey theory, proposed by Deng [8] has been applied in many fields including business [9–12], transportation [13], and electric power [5]. Grey theory concerns the grey generation, relational analysis, model construction, prediction, decision-making and system control [8]. The grey prediction model GM(1, 1) is commonly adopted with real-world data.

Some studies have proposed the improved GM(1, 1) model, with increased accuracy. For example, Hsu and Wang [14] adopted the Bayesian method to estimate the parameters of the GM(1, 1) model. They used their model to forecast the value of the integrated circuit industry. Similarly, Wang and Hsu [15] combined the GM(1, 1) model and GAs to find the optimal parameters of the grey forecasting model. They utilized their model to forecast the value of the high-technology industry.

This study combines residual modification and residual GP sign estimation to elucidate an improved grey prediction model. The proposed model is developed for two main reasons. First, if the original data constitute a small time-series, then GP can increase the residual sign precision of the prediction. Second, the accuracy and reliability of GP exceed those of ANN in constructing a forecasting model or classification [2,3]. The energy consumption data in China are utilized as an empirical example to demonstrate the effectiveness of the proposed model.

3. Methodology

The motivations for using a grey model that is based on grey theory and GP instead of traditional forecasting methods is introduced. Then, an overview of the relevant principles of GM(1, 1) and the GP residual sign estimation method are presented.

3.1. GM(1, 1) model

The GM(1, 1) model is for forecasting time-series. It has been adopted in many fields, including the power industry [9,10], the integrated circuit industry [6,14], system control [16], and others. The GM(1, 1) model does not require as large a data set as the linear regression model or ANN. Generally, GM(1, 1) needs at least four observations [4]. GM(1, 1) model is constructed as follows.

The original positive time-series data sequence is assumed to be

$$y^{(0)} = \left(y^{(0)}(1), y^{(0)}(2), y^{(0)}(3), \ldots, y^{(0)}(n)\right), \quad n \geqslant 4 \tag{1}$$

where n represents the total number of periods.

To increase the precision of the GM(1, 1) model, the accumulated generating operator (AGO) is derived from the original time-series data. The AGO is derived as,

$$y^{(1)}(k) = \sum_{m=1}^{n} y^{(0)}(m), \quad k = 2, 3, \ldots, n \tag{2}$$

The accumulated time-series data sequence that is obtained using the AGO formation can be expressed as,

$$\begin{aligned} y^{(1)} &= \left(y^{(0)}(1), \sum_{m=1}^{2} y^{(0)}(m), \sum_{m=1}^{3} y^{(0)}(m), \ldots, \sum_{m=1}^{n} y^{(0)}(m)\right) \\ &= \left(y^{(1)}(1), y^{(1)}(2), y^{(1)}(3), \ldots, y^{(1)}(n)\right) \end{aligned} \tag{3}$$

The GM(1, 1) model can then be constructed using grey differential equation. The grey differential equation is,

$$y^{(0)}(k) + az^{(1)}(k) = u \tag{4}$$

where a is the development coefficient; u represents the grey input, and z represents the background value. $z^{(1)}$ can be obtained by applying the mean operator on the $y^{(1)}$ series of data. $z^{(1)}$ is defined as,

$$z^{(1)}(k) = \frac{1}{2} \times \left[y^{(1)}(k) + y^{(1)}(k-1)\right], \quad k = 2, 3, \ldots, n \tag{5}$$

Therefore, $y^{(1)}$ in Eq. (4) can be obtained using the ordinary least squares method:

$$y^{(1)}(k) = \left(y^{(0)}(1) - \frac{\hat{u}}{\hat{a}}\right) \times e^{-\hat{a}(k-1)} + \frac{\hat{u}}{\hat{a}} \tag{6}$$

where

$$\begin{bmatrix} \hat{a} \\ \hat{u} \end{bmatrix} = (A^T A)^{-1} A^T Y \tag{7}$$

and

$$A = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix} \tag{8}$$

$$Y = \left[y^{(0)}(2), y^{(0)}(3), \ldots, y^{(0)}(n)\right]^T \tag{9}$$

Finally, the inverse AGO method is employed to obtain the forecasting value. The forecasting equation is,

$$\begin{aligned} \hat{y}^{(0)}(n+p) &= \hat{y}^{(1)}(n+p) - \hat{y}^{(1)}(n+p-1) \\ &= \left(y^{(0)}(1) - \frac{\hat{u}}{\hat{a}}\right)(1 - e^{\hat{a}})e^{-\hat{a}(n+p-1)}, \quad p = 1, 2, 3, \ldots \end{aligned} \tag{10}$$

In Eq. (10), the time-series data, $\hat{y}^{(0)}(1), \hat{y}^{(0)}(2), \hat{y}^{(0)}(3), \ldots, \hat{y}^{(0)}(n)$ are called the GM(1, 1) fitted sequence, and $\hat{y}^{(0)}(n+1), \hat{y}^{(0)}(n+2), \hat{y}^{(0)}(n+3), \ldots, \hat{y}^{(0)}(n+p)$ are called the forecast values of GM(1, 1) in the period, p.

3.2. GP-based GM(1, 1) model

The difference between the target values $y^{(0)}$ and the predicted values $\hat{y}^{(0)}$ is called the residual series. To increase the forecasting accuracy of the GM(1, 1) model, the residual GM(1, 1) model must be developed [8]. The modified forecasting values is obtained by combining original GM(1, 1) and residual GM(1, 1). However, the effectiveness of the residual series depends on the number of data points with the same sign [5,6]. If the number of data points with the same sign does not exceed four, the residual GM(1, 1) model cannot be constructed [5].

To solve the problems, Hsu and Chen [5] proposed the grey forecasting model by combining residual modification with ANN residual sign estimation. However, in ANN, a large data set is required [2,3] and the hidden layer is difficult to justify. Hence, this study develops an improved GM(1, 1) by combining residual modification with GP sign estimation to enhance the power of forecasting residual sign. Fig. 1 depicts the construction of the proposed prediction model. The following subsection describes in detail the construction of the proposed model.

3.2.1. Residual GM(1, 1) model

Let the original absolute values of the residual sequence be denoted $r^{(0)}$, which is given by,

$$r^{(0)} = \left(r^{(0)}(2), r^{(0)}(3), r^{(0)}(4), \ldots, r^{(0)}(n)\right) \tag{11}$$

where

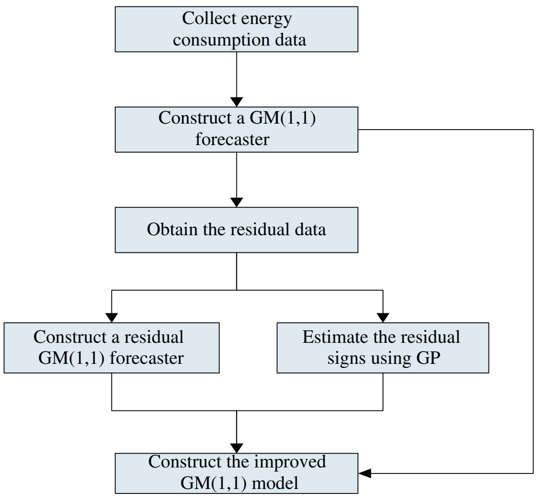

Fig. 1. The process of constructing the proposed model.

$$r^{(0)}(k) = |\varepsilon^{(0)}(k)| = |y^{(0)}(k) - \hat{y}^{(0)}(k)|, \quad k = 2, 3, \ldots, n \tag{12}$$

The GM(1, 1) model of $r^{(0)}$ can be constructed using Eqs. (1)–(12). The forecast residual series model is given by,

$$\hat{r}^{(0)}(n+p) = \left(r^{(0)}(2) - \frac{\hat{u}_r}{\hat{a}_r}\right)(1 - e^{\hat{a}_r})e^{-\hat{a}_r(n+p-1)}, \quad n = 1, 2, 3, \ldots, \quad p = 1, 2, 3, \ldots \tag{13}$$

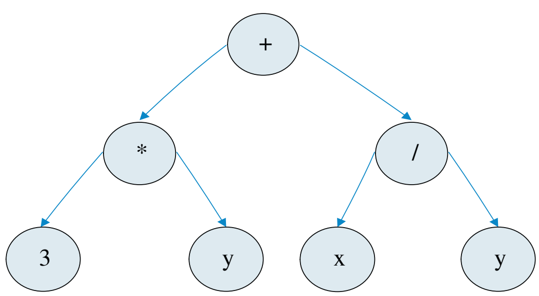

Fig. 2. Example of GP parse tree representation.

3.2.2. Model for estimating GP residual sign

Koza [17] developed GP which is an algorithm for forecasting and clustering data, which can be implemented using computer programs. GP has been used in the identification of model structures and symbolic regression [18]. The basic concepts are related to those of GAs-crossover, mutation and reproduction. Unlike GAs, GP utilizes the generic parse tree representation instead of the logic number of the genetic state (0 and 1). Accordingly, GP has become more popular than conventional linear forecasting methods because it can be employed to search complex non-linear spaces. GP is also extensively employed in practical applications, such as predicting coastal algal blooms [19], building credit scoring models [2,3] and modeling the rainfall-runoff process [20].

The functions or statements of GP include operators ({+, −, ×, ÷, log, exp}), trigonometric functions ({sin, cos, tan}), and conditional statements (If, then). As a simple example, $3y + x/y$ can be expressed using the GP parse tree in Fig. 2.

Furthermore, GP operation system can construct an optimal predicting function using the generic parse tree and symbolic regression. Fig. 3 depicts the crossover operator in GP. In the selection of input variables, GP automatically finds the variables that contribute most to the model [19] and, unlike ANN, it can be used with any sample size [2,3].

Unlike the residual sign estimator developed by Hsu and Chen [5], this study utilizes GP instead of ANN sign estimation, to estimate the sign of the residual. When the GP model is used, not only can construct a forecasting model for a small data set but also a forecasting equation can be obtained [7,19]. To reduce the forecasting error of GP, the following objective function can be used.

$$\text{Minimize}: \sum_{i=1}^{n} |(\widehat{F}_i - A_i)| \tag{14}$$

where $\widehat{F}_i$ represents the forecasted value and A_i represents the actual value. In this study, a two-stage GP model is employed to forecast the signs of the residual series. First, a dummy variable $c(t)$ is adopted to reveal the sign of the residual in the tth year. If the sign of the tth year residual is positive, then the value of $c(t)$ is one; otherwise it is zero. Next, the GP model parameters are determined by predicting the value of $c(t+1)$ from $c(t-1)$ and $c(t)$. Table 1 presents the GP parameters.

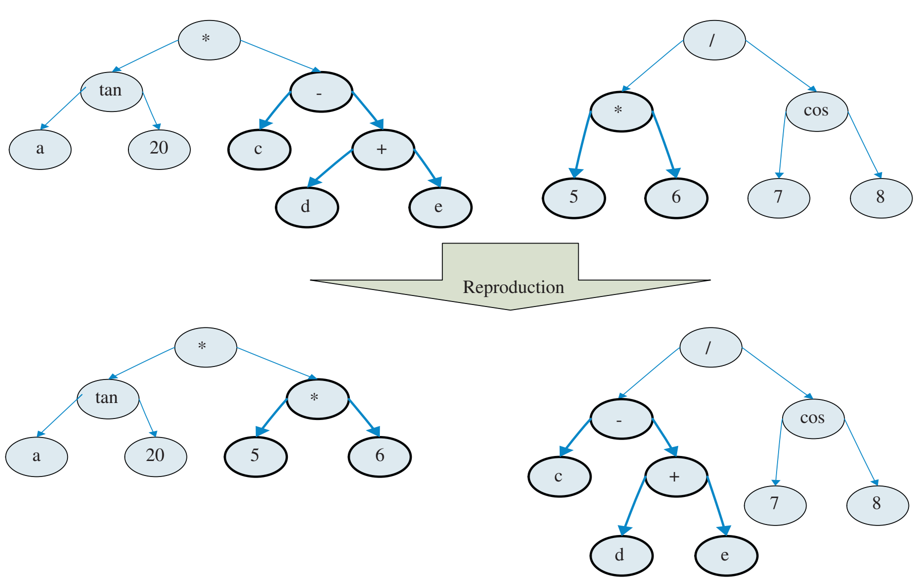

Fig. 3. Crossover operator in GP representation.

The sign of the tth year residual, $i(t)$, can be expressed as

$$i(t) = \begin{cases} 1 & \text{if } c(t) = 1 \\ -1, & \text{if } c(t) = 1 \end{cases}, \quad t = 1, 2, 3, \ldots \tag{15}$$

Therefore, the improved forecasting method, called the GP-based grey forecasting model or GPGM(1, 1), can be obtained using Eqs. (1)–(15) as follows.

$$\hat{y}'^{(0)}(t) = \hat{y}^{(0)}_{GM} + i(t)\hat{r}^{(0)}_{GM} = \left(y^{(0)}(1) - \frac{\hat{u}}{\hat{a}}\right)(1 - e^{\hat{a}})e^{-\hat{a}(t-1)} + i(t) \times \left(r^{(0)}(2) - \frac{\hat{u}_r}{\hat{a}_r}\right)(1 - e^{\hat{a}_r})e^{-\hat{a}_r(t-1)} \tag{16}$$

where $t = 1, 2, 3 \ldots$

Table 1
The parameter settings of GP.

Parameter	Value
Input variable	$c(t-1)$, $c(t)$
Dependent variable	$c(t+1)$
Population size	50
Maximum number of generation	50
Objective function	Minimize: $\sum_{t=1}^{n} \lvert(\hat{c}(t) - c(t))\rvert$
Function set	+, −, ×, ÷, sin, cos, exp, log
Crossover rate	0.3
Mutation rate	0.3

Table 2
MAPE criteria for model examination. *Source*: Lewis [21].

MAPE (%)	Forecasting ability
<10	High forecasting
10–20	Good forecasting
20–50	Reasonable forecasting
>50	Weak forecasting

4. Empirical study

To demonstrate the effectiveness of the proposed grey forecasting model, the real case of energy consumption in China is considered as an example. The historical annual energy consumption of China from 1990 to 2003 is employed as the model-fitting and the data for 2004–2007 are utilized as ex post testing. Of 18 observations, 14 are utilized as training data while others are testing data.

Four forecasting models, GM(1, 1), GPGM(1, 1), grey forecasting method of Hsu and Chen [5] , and simple linear regression, are as follows.

1. GM(1, 1) forecasting equation:

$$y^{(0)}_{GM}(k) = 105193.9e^{(0.0328422743(k-1))}, \quad k = 2, 3, \ldots \tag{17}$$

2. GPGM(1, 1) forecasting equation:

$$y^{(0)}_{GPGM}(k) = 105193.9e^{(0.0328422743(k-1))} + i(k)3211.102e^{(0.09593455(k-1))} \tag{18}$$

where

$$i(k) = \begin{cases} 1 & \text{if } c(k) = 1 \\ -1, & \text{if } c(k) = 0 \end{cases}, \quad k = 1, 2, 3, \ldots$$

3. Improved grey forecasting equation of Hsu and Chen [5]:

$$\hat{y}^{(0)}(k) = 105193.9e^{(0.0328422743(k-1))} + s(k)3211.102e^{(0.09593455(k-1))}, \quad k = 1, 2, \ldots \tag{19}$$

where $s(k)$ represents the estimated sign value obtained using ANN.

4. Simple linear regression:

$$y^{(0)}_{LR}(k) = 97269.758 + 4486.813k, \quad k = 1, 2, 3, \ldots \tag{20}$$

To compare the predictive accuracy of the above four forecasting models, two evaluation indices are employed. The first index is

Table 3
Forecasted values and errors among models (unit: 10^4 tons of SCE).

Year	Original value	GM(1, 1)		Hsu and Chen [5]		GPGM(1, 1)		Linear regression	
		Model value	Error[a]	Model value	Error[a]	Model value	Error[a]	Model value	Error[a]
1990	98,703	98703.00	0.00	98703.00	0.00	98,703	0.00	101756.57	3.09
1991	103,783	108706.11	4.74	103783.00	0.00	103,783	0.00	106243.38	2.37
1992	109,170	112335.53	2.90	116225.80	6.46	108445.2	−0.66	110730.19	1.43
1993	115,993	116086.14	0.08	111804.10	−3.61	111804.1	−3.61	115217.01	−0.67
1994	122,737	119961.97	−2.26	115248.80	−6.10	124675.1	1.58	119703.82	−2.47
1995	131,176	123967.21	−5.50	129154.80	−1.54	129154.8	−1.54	124190.63	−5.33
1996	138,948	128106.16	−7.80	133816.10	−3.69	133816.1	−3.69	128677.45	−7.39
1997	137,798	132383.31	−3.93	138668.20	0.63	138668.2	0.63	133164.26	−3.36
1998	132,214	136803.27	3.47	143721.00	8.70	129885.5	−1.76	137651.07	4.11
1999	133,831	141370.79	5.63	133756.50	−0.06	133756.5	−0.06	142137.89	6.21
2000	138,553	146090.81	5.44	137709.80	−0.61	137709.8	−0.61	146624.70	5.83
2001	143,199	150968.42	5.43	141743.60	−1.02	141743.6	−1.02	151111.51	5.53
2002	151,797	156008.89	2.77	145855.20	−3.91	145855.2	−3.91	155598.32	2.50
2003	174,990	161217.64	−7.87	150041.60	−14.26	172393.5	−1.48	160085.14	−8.52
MAPE (%) (1990–2003)			4.13		3.61		2.59		4.20
2004	203,227	166600.20	−18.02	178901.50	−11.97	178901.5	−11.97	164571.95	−19.02
2005	224,682	172162.60	−23.37	185702.40	−17.35	185702.4	−17.35	169058.76	−24.76
2006	264,270	177910.70	−32.68	192813.80	−27.04	192813.8	−27.04	173545.58	−34.33
2007	265,583	183850.70	−30.77	200254.30	−24.60	200254.3	−24.60	178032.39	−32.97
MAPE (%) (2004–2007)			26.21		20.23		20.23		27.76

[a] $ER = \frac{\hat{y}^{(0)}(k) - y^{(0)}(k)}{y^{(0)}(k)} \times 100\%$.

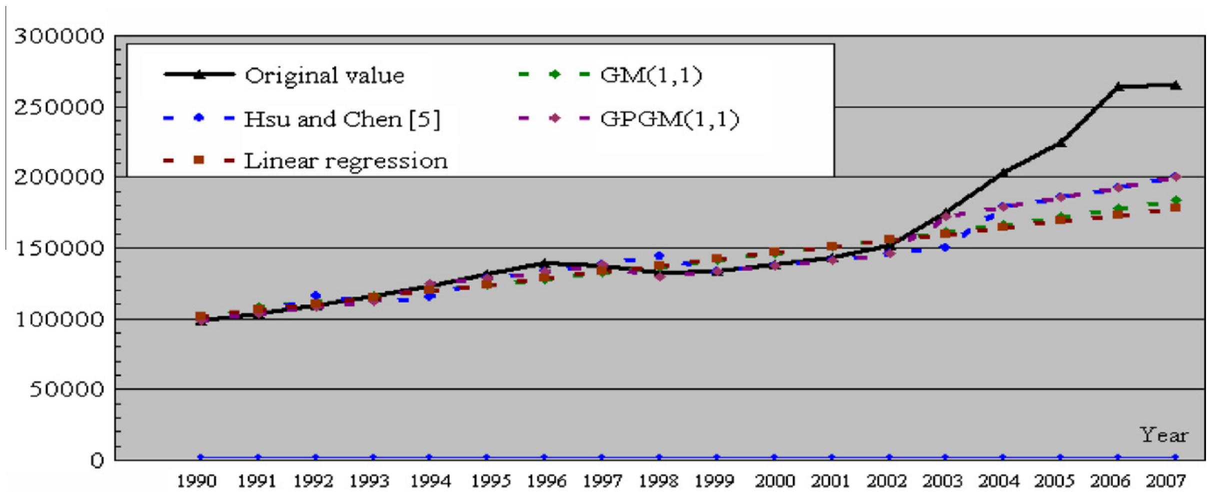

Fig. 4. The distribution of forecast values and target values from 1990 to 2007.

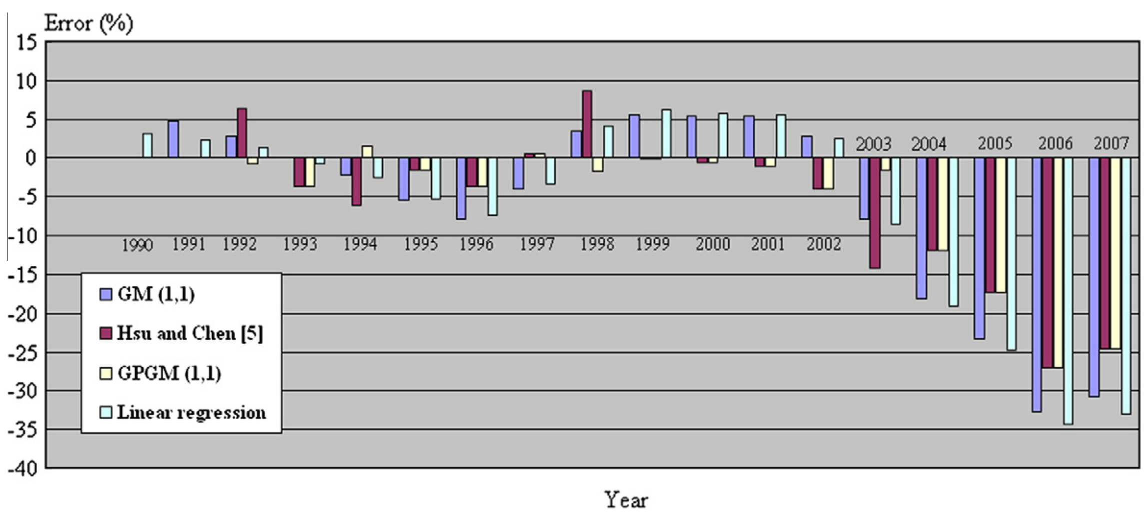

Fig. 5. Trends of percentage of predicting error for forecasting models from 1990 to 2007.

the percentage error (PE), which is used to compare the forecast and real values to the time-series data. The PE is defined as,

$$\mathrm{PE} = \frac{\hat{y}^{(0)}(k) - y^{(0)}(k)}{y^{(0)}(k)} \times 100\% \qquad (21)$$

which is the modeled forecasting error rate; $\hat{y}^{(0)}(k)$ is the forecast value and $y^{(0)}(k)$ the target value. The second index is the mean absolute percentage error (MAPE), which measures the forecasting accuracy of model using a statistical method. MAPE is defined as,

$$\mathrm{MAPE} = \frac{\sum_{k=1}^{n} |(\hat{y}^{(0)}(k) - y^{(0)}(k))/y^{(0)}(k)|}{N} \times 100\% \qquad (22)$$

Lewis [21] presented that the MAPE criteria for evaluating a forecasting model, which are listed in Table 2.

Table 3 and Fig. 4 present the forecast results obtained using the GM(1, 1) model, the GPGM(1, 1) model, the forecasting method of Hsu and Chen [5] and the linear regression model. Fig. 5 displays the error rate of the forecasting model. In Table 3, the MAPE of the GM(1, 1) model, the model of Hsu and Chen [5], the GPGM(1, 1) model and the linear regression model applied to the training data (1990–2003) are 4.13%, 3.61%, 2.59%, and 4.20%, respectively. Similarly, for the testing data, the MAPE are 26.21%, 20.23%, 20.23%, and 27.76% in years 2004–2007, respectively. The above results indicate that the improved forecasting approach, GPGM(1, 1), has a higher precision of forecasting than other forecasting methods. The GPGM(1, 1) model exhibits high forecasting ability in training data and nearly good forecasting ability in testing data according to the MAPE criteria. Thus, the GPGM(1, 1) model can enhance the model forecasting precision effectively.

5. Conclusion

Forecasting the energy consumption is generally difficult, since it is affected by the rapid development of the economy, technology and government decisions, among other factors. Therefore, the development of an accurate prediction model is very important. Although the GM(1, 1) model forecasts well using a small time-series of data, the proposed improved grey forecasting model, GPGM(1, 1) predicts with greater accuracy and reliability than GM(1, 1), the model of Hsu and Chen [5], and the linear regression model overall. GPGM(1, 1) has a lower forecasting error than the GM(1, 1) model, the model of Hsu and Chen [5] and the linear regression model when the data set is small. Accordingly, the proposed model has substantially higher predictive precision substantially than other models.

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