Reply to comment by A. F. Moench on "Aquifer parameter identification using the extended Kalman filter"

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[1] *Moench*'s [2004] comment on our paper [*Leng and Yeh*, 2003] arises from the issue of using the extended Kalman filter (EKF) to estimate parameters for confined and unconfined aquifers. In the confined aquifer case we chose a synthetic set of pumping test data given by *Todd* [1980, p. 127] for the study. The EKF gave better estimates of the parameters with only about 2.5 hours of drawdown data when compared with parameters estimated by graphical methods such as Cooper-Jacob and Chow using all data points, i.e., 4 hours of drawdown data as indicated in Tables 1 and 4 of *Leng and Yeh* [2003]. It is assumed that the drawdown happens instantaneously in response to the release of water from storage due to pumping. Thus there is no doubt that we can get stable and good estimates of the confined aquifer parameters by the EKF with only 2.5 hours of drawdown data. *Moench* [2004] states

> With the selected theoretical data set, equally accurate parameter estimates can be obtained by any existing approach using only 5–10 min of drawdown data (see *Todd* [1980, Figure 4.10] and apply, for example, the Cooper-Jacob method). EKF methodology as implemented by *Leng and Yeh* [2003] does not improve upon this.

[2] These comments are true because the selected theoretical data set falls on the curve of the Theis equation. However, existing approaches such as graphical methods or other computer methods may give inaccurate estimates if the selected data points are taken from field observation and contain measurement errors. In contrast, we had shown that EKF can obtain accurate aquifer parameters with only part of the drawdown data, even if the drawdown data contain a bump, white noise, or temporally correlated noise [*Leng and Yeh*, 2003]. This is one of the major merits of EKF over other existing methods in the area of parameter identification.

[3] Two sets of drawdown data, given by *Batu* [1998], were obtained from an unconfined aquifer for a pumping test conducted at Saint Pardon de Conques, Gironde, France. The first data set was taken from the observation well located at a distance of 10 m (r = 10 m) from the pumped well, whereas the second data set was taken from the observation well located at r = 30 m. For the first data set the aquifer parameters estimated by EKF give smaller prediction errors than those of the graphical approaches such as the *Neuman* [1975] type curve method and Neuman's semilogarithmic method [*Batu*, 1998], as shown in Tables 6 and 8 of *Leng and Yeh* [2003]. For the second data set the EKF-estimated parameters are $K_r = 1.95 \times 10^{-3}$ m s^{-1}, $K_z = 2.44 \times 10^{-5}$ m s^{-1}, $S = 5.55 \times 10^{-3}$, and $S_y = 10.2 \times 10^{-2}$. Figure 1 shows a comparison of the pumping test data obtained from Saint Pardon de Conques to the predicted drawdown curves (at r = 10 m and r = 30 m) obtained from the aquifer parameters estimated using EKF for the second data set. The predicted drawdown curves obtained from the aquifer parameters given by *Moench* [2004] and estimated using graphical composite plots [*Moench*, 1994] and nonlinear least squares (NLS) [*Heidari and Moench*, 1997] are also drawn in Figure 1. Figure 1 clearly demonstrates that the predicted drawdown curves using parameters estimated by EKF (at r = 30 m) and NLS fit these two data sets reasonably well and that the predicted drawdown curves using parameters estimated by the graphical composite plots deviate slightly from the observed drawdowns.

[4] With the advantage of eliminating the need for long-term aquifer tests the EKF is applicable for cases where the aquifer drawdown produced by the pumping test does not reach impervious boundaries or surface water bodies. If the objective of a pumping test is to detect or locate possible nearby impervious boundaries, then a long-term test may be needed, and the method of images can be used to solve the problem [*Batu*, 1998].

[5] The purpose of our paper [*Leng and Yeh*, 2003] was to demonstrate the use of EKF in identifying parameters from confined and unconfined aquifers. Parameter analysis of a second data set does not enhance understanding and appreciation for the use of EKF. In addition, the drawdowns at times 127.63 min, 484.90–608.35 min, and 1276.28 min are not accurate because they are smaller than the observed drawdowns from the previous time steps [*Batu*, 1998, p. 536]. Therefore the parameter analysis for the second data set was not included in our paper.

[6] There are three reasons why we are not in favor of analyzing all observation well data simultaneously (otherwise known as composite analysis) as advocated by *Moench* [2004]. First, the principle of least squares assumes that the measurement errors are independent [*McCuen*, 1985]. For sampling points very close together or at the same location with varying depths, the assumption of independent errors may not be warranted. Second, in the case of Saint Pardon de Conques [*Batu*, 1998] the estimated parameters from the second data set represent average aquifer properties over a circular area with r = 30 m. On the other hand, the estimated

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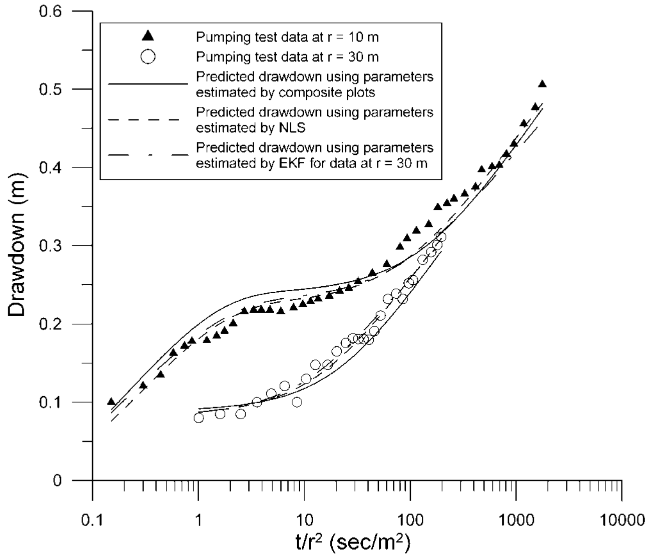

Figure 1. Comparison of the pumping test data obtained from Saint Pardon de Conques, France [*Batu*, 1998], and the predicted drawdown curves from aquifer parameters estimated using EKF, composite plots [*Moench*, 1994], and nonlinear least squares (NLS) [*Heidari and Moench*, 1997].

parameters from the first data set represent average aquifer properties over an area which has been covered in the second data set. For a multiple-observation well system the aquifer response in those observation wells located far away from the pumping well should also reflect the influence of the geological properties near the pumping well. Thus composite analysis attributes much more weight to the geology near the pumping well than that far away from the pumping well. Third, the mathematical model representing the response of an aquifer to well pumping generally assumes that the aquifer material is homogeneous. If the aquifer is fairly homogeneous, the estimated parameters from the single-well data analysis and the composite analysis should be about the same. In contrast, if the aquifer formation is very heterogeneous, one should perform a slug test at each well and analyze each data set separately using the Kansas Geological Survey model [*Hyder et al.*, 1994] for an unconfined aquifer system. Therefore we think that the validity of the use of all observation well data simultaneously to estimate averaging aquifer parameters is doubtful from statistical and mathematical viewpoints.

[7] In conclusion, Moench's criticisms on the inaccurate results of estimated parameters by EKF and other single-well data analyses by *Neuman* [1975] and *Batu* [1998] are mainly based on his assertion that one should use composite analysis. We have shown in Figure 1 that EKF-estimated parameters using single-well data yield better fits than those of *Moench*'s [1994] graphical composite plots. Obviously, Moench's comment on our paper [*Leng and Yeh*, 2003] implying that the EKF yields inaccurate estimates of aquifer parameters is inadequate.

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