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ORIGINAL ARTICLE

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Measuring process yield based on the capability index C_{pm}

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Abstract Process capability indices C_p, C_a, C_{pk} and C_{pm} have been proposed to the manufacturing industry as capability measures based on various criteria including variation, departure, yield, and loss. It has been noted in recent quality research and capability analysis literature that both the C_{pk} and C_{pm} indices provide the same lower bounds on process yield, that is, $Yield \geqslant 2\Phi(3C_{pk}) - 1 = 2\Phi(3C_{pm}) - 1$. In this paper, we investigate the behaviour of the actual process yield in terms of the number of nonconformities (in ppm) for processes with a fixed index value of $C_{pk} = C_{pm}$, but with different degrees of process centring, which can be expressed as a function of the capability index C_a. The results illustrate that it is advantageous to use the index C_{pm} over the index C_{pk} when measuring process capability, since C_{pm} provides better customer protection.

Keywords Nonconformities · Process capability index · Process yield

1 Introduction

Process capability indices, including the precision index C_p, the accuracy index C_a, and the yield-based index C_{pk} have been proposed in the manufacturing industry, as well as the service industry, providing numerical measures on whether a process is capable of reproducing items within specification limits preset in the factory. These indices have been defined as the following [1–4]:

$$C_p = \frac{USL - LSL}{6\sigma},$$
$$C_a = 1 - \frac{|\mu - m|}{d},$$
$$C_{pk} = \min\left\{\frac{USL-\mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right\},$$
$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}},$$

where USL is the upper specification limit, LSL is the lower specification limit, μ is the process mean, σ is the process standard deviation, $m = (USL + LSL)/2$ is the midpoint of the specification interval, and $d = (USL - LSL)/2$ represents the half-length of the specification tolerance.

The index C_p, which is a function of the process standard deviation σ and the specification limits, has been referred to as the precision index. It is defined to measure the consistency of the process quality characteristic relative to the manufacturing tolerance. The index C_a, a function of the process mean and the specification limits, has been referred to as the accuracy index, which is defined to measure the degree of process centring relative to the manufacturing tolerance. The index C_{pm}, often regarded as a loss-based index, may be rewritten as $C_{pm} = C_p/[1 + 3C_p(1 - C_a)]^{1/2}$, and is a function of the two basic indices C_p and C_a.

In recent quality research and capability analysis, it has been often noted that the C_{pm} index provides both a lower bound on the process yield, that is, $Yield = \Phi[(USL - \mu)/\sigma] - \Phi[(LSL - \mu)/\sigma] \geqslant 2\Phi(3C_{pm}) - 1$, and an upper bound on the fraction of the nonconformities, $P(\text{NC}) = 1 - \Phi[(USL - \mu)/\sigma] + \Phi[(LSL - \mu)/\sigma] \leqslant 2\Phi(-3C_{pm})$. In this paper, we investigate the behaviour of the actual process yield in terms of the number of nonconformities (in ppm) for processes with a fixed index value of $C_{pk} = C_{pm}$, but with different degrees of process centring, which can be expressed as a function of the capability index C_a. The results illustrate the advantage of using the index

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C_{pm} over the index C_{pk} when measuring process capability, since C_{pm} provides better customer protection.

2 Process yield measuring based on C_{pm}

In general, if the process characteristic, X, follows the normal distribution, $N(\mu, \sigma^2)$, then the fraction of the nonconformities (NC), may be expressed as:

$$\begin{aligned} P(\mathrm{NC}) &= 1 - Pr\,(LSL \leqslant X \leqslant USL) \\ &= Pr\,(X < LSL) + Pr\,(X > USL) \\ &= Pr\left(\frac{X-\mu}{\sigma} < \frac{LSL-\mu}{\sigma}\right) + Pr\left(\frac{X-\mu}{\sigma} > \frac{USL-\mu}{\sigma}\right) \\ &= \Phi\left(\frac{LSL-\mu}{\sigma}\right) + 1 - \Phi\left(\frac{USL-\mu}{\sigma}\right), \end{aligned}$$

where $\Phi(\cdot)$ is the cumulated distribution function of the standard normal distribution $N(0, 1)$. Since $USL = m + d$ and $LSL = m - d$, then

$$\begin{aligned} P(\mathrm{NC}) &= \Phi\left(\frac{m-d-\mu}{\sigma}\right) + 1 - \Phi\left(\frac{m+d-\mu}{\sigma}\right) \\ &= \Phi\left(\frac{m-d-\mu}{\sigma}\right) + \Phi\left(-\frac{m+d-\mu}{\sigma}\right) \\ &= \Phi\left(-\frac{d+\mu-m}{d}\cdot\frac{d}{\sigma}\right) + \Phi\left(-\frac{d-\mu+m}{d}\cdot\frac{d}{\sigma}\right). \end{aligned}$$

Therefore,

$$P(\mathrm{NC}) = \Phi\left[-\frac{(1+\delta)}{\gamma}\right] + \Phi\left[-\frac{(1-\delta)}{\gamma}\right],$$

where $\delta = (\mu - m)/d$ and $\gamma = \sigma/d$. Further, since $P(\mathrm{NC})$ is an even function of δ, then $P(\mathrm{NC})$ may be rewritten as:

$$P(\mathrm{NC}) = \Phi\left[-\frac{1+|\delta|}{\gamma}\right] + \Phi\left[-\frac{1-|\delta|}{\gamma}\right].$$

The yield as a function of C_{pm} is determined as follows. Since

$$C_{\mathrm{a}} = 1 - \frac{|\mu - m|}{d} = 1 - |\delta|,$$

then,

$$P(\mathrm{NC}) = \Phi\left[-\frac{2-C_a}{\gamma}\right] + \Phi\left[-\frac{C_a}{\gamma}\right].$$

Therefore,

$$C_{\mathrm{pm}} = \frac{d}{3\sqrt{\sigma^2 + (\mu - m)^2}} = \frac{1}{3\sqrt{\gamma^2 + \delta^2}},$$

and so

$$\begin{aligned} \gamma^2 &= \frac{1}{(3C_{\mathrm{pm}})^2} - \delta^2 = \left(\frac{1}{3C_{\mathrm{pm}}} + \delta\right)\left(\frac{1}{3C_{\mathrm{pm}}} - \delta\right), \\ \gamma &= \sqrt{\left(\frac{1}{3C_{\mathrm{pm}}} + \delta\right)\left(\frac{1}{3C_{\mathrm{pm}}} - \delta\right)} \\ &= \sqrt{\left(\frac{1}{3C_{\mathrm{pm}}} + |\delta|\right)\left(\frac{1}{3C_{\mathrm{pm}}} - |\delta|\right)} \end{aligned}$$

holds for $0 \leqslant |\delta| \leqslant 1/(3C_{\mathrm{pm}})$, i.e., for $1 - 1/(3C_{\mathrm{pm}}) \leqslant C_{\mathrm{a}} \leqslant 1$. Consequently, we have the following relationship between the

Table 1. The corresponding nonconformities (in ppm) for $C_{\mathrm{pm}} = 1.0, 1.25, 1.5, 1.75 and 2.0$ with various C_{a}

C_{a} C_{pm}	0.6667	0.6944	0.7222	0.7500	0.7778	0.8056	0.8333	0.8611	0.8889	0.9167	0.9444	0.9722	1.0000
1.0	0	0.09	44.34	334.87	872.99	1468.46	1972.77	2328.70	2542.34	2648.91	2689.62	2699.16	2699.80
C_{a} C_{pm}	0.7333	0.7556	0.7778	0.8000	0.8222	0.8444	0.8667	0.8889	0.9111	0.9333	0.9556	0.9778	1.0000
1.25	0	0	0.07	2.8	17.26	48.40	87.90	125.09	152.51	168.34	175.04	176.72	176.83
C_{a} C_{pm}	0.7778	0.7963	0.8184	0.8333	0.8519	0.8704	0.8889	0.9074	0.9259	.09444	0.9630	0.9815	1.0000
1.5	0	0	0	0.01	0.14	0.71	1.93	3.56	5.10	6.15	6.65	6.79	6.80
C_{a} C_{pm}	0.8095	0.8254	0.8413	0.8571	0.8730	0.8889	0.9048	0.9206	0.9365	0.9524	0.9683	0.9841	1.0000
1.75	0	0	0	0	0.0004	0.0046	0.0207	0.0530	0.0935	0.1276	0.1462	0.1517	0.1521
C_{a} C_{pm}	0.8333	0.8472	0.8611	0.8750	0.8889	0.9028	0.9167	0.9306	0.9444	0.9583	0.9722	0.9861	1.0000
2.0	0	0	0	0	0	0	0.0001	0.0004	0.0009	0.0015	0.0018	0.0020	0.0020

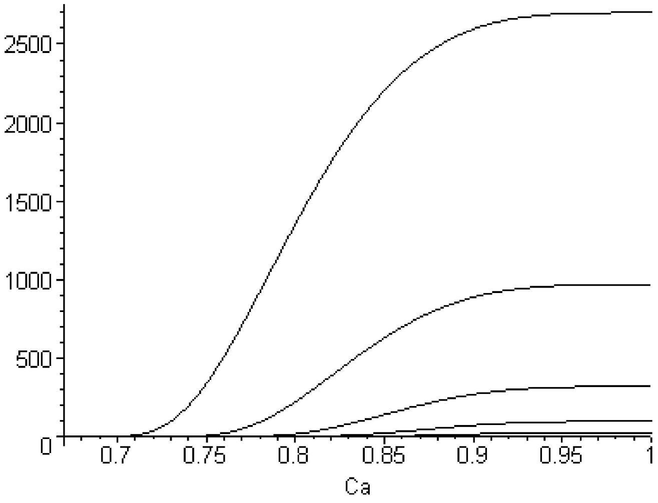

Fig. 1. Plot of the actual NC vs C_a for $C_\mathrm{pm} = 1.0, 1.1, 1.2, 1.3$ and 1.4 (*top* to *bottom*)

process yield and the index C_pm for $1 - 1/(3C_\mathrm{pm}) \leqslant C_a \leqslant 1$:

$$P(\mathrm{NC}) = \Phi\left[-\frac{2 - C_a}{\sqrt{\frac{1}{(3\,C_\mathrm{pm})^2} - (1 - C_a)^2}}\right] + \Phi\left[-\frac{C_a}{\sqrt{\frac{1}{(3\,C_\mathrm{pm})^2} - (1 - C_a)^2}}\right].$$

Table 1 displays the number of nonconformities (in ppm) for $C_\mathrm{pm} = 1.0, 1.25, 1.50, 1.75$ and 2.0 with various values of C_a satisfying $1 - 1/(3C_\mathrm{pm}) \leqslant C_\mathrm{a} \leqslant 1$. Figure 1 plots the actual number of nonconformities (in ppm) for $C_\mathrm{pm} = 1.0, 1.1, 1.2, 1.3$ and 1.4 (from top to bottom in the plot) with $1 - 1/(3C_\mathrm{pm}) \leqslant C_a \leqslant 1$. Note that for $C_\mathrm{pm} > 1.4$, the curves are almost indistinguishable.

Ruczinski [5] obtained a lower bound for the yield: *Yield* $\geqslant 2\Phi(3C_\mathrm{pm}) - 1$, or $P(\mathrm{NC}) \leqslant 2\Phi(-3C_\mathrm{pm})$ for $C_\mathrm{pm} > \sqrt{3}/3$. Table 2 displays the bound in ppm for $C_\mathrm{pm} = 0.99(0.01)2.00$. For example, for $C_\mathrm{pm} = 1.24$, the number of nonconformities is no greater than 200 ppm.

Table 2. Various values of $C_\mathrm{pm} = 0.99(0.01)2.00$ and the corresponding nonconformities (in ppm)

C_pm	ppm	C_pm	ppm	C_pm	ppm
0.99	2977.997	1.33	66.073	1.67	0.544
1.00	2699.796	1.34	58.198	1.68	0.466
1.01	2445.537	1.35	51.218	1.69	0.398
1.02	2213.370	1.36	45.036	1.70	0.340
1.03	2001.565	1.37	39.566	1.71	0.290
1.04	1808.510	1.38	34.731	1.72	0.247
1.05	1632.705	1.39	30.460	1.73	0.210
1.06	1472.751	1.40	26.691	1.74	0.179
1.07	1327.350	1.41	23.369	1.75	0.152
1.08	1195.297	1.42	20.443	1.76	0.129
1.09	1075.475	1.43	17.867	1.77	0.110
0.10	966.848	1.44	15.603	1.78	0.093
1.11	868.460	1.45	13.614	1.79	0.079
1.12	779.425	1.46	11.868	1.80	0.067
1.13	698.926	1.47	10.337	1.81	0.056
1.14	626.211	1.48	8.996	1.82	0.048
1.15	560.587	1.49	7.822	1.83	0.040
1.16	501.414	1.50	6.795	1.84	0.034
1.17	448.107	1.51	5.898	1.85	0.029
1.18	400.127	1.52	5.115	1.86	0.024
1.19	356.981	1.53	4.432	1.87	0.020
1.20	318.217	1.54	3.837	1.88	0.017
0.21	283.421	1.55	3.319	1.89	0.014
1.22	252.215	1.56	2.869	1.90	0.012
1.23	224.254	1.57	2.477	1.91	0.010
1.24	199.223	1.58	2.137	1.92	0.008
1.25	176.835	1.59	1.842	1.93	0.007
1.26	156.828	1.60	1.587	1.94	0.006
1.27	138.967	1.61	1.365	1.95	0.005
1.28	123.034	1.62	1.174	1.96	0.004
1.29	108.835	1.63	1.008	1.97	0.003
1.30	96.193	1.64	0.865	1.98	0.003
1.31	84.946	1.65	0.742	1.99	0.002
1.32	74.950	1.66	0.636	2.00	0.002

3 Comparisons between C_pm and C_pk

Using a similar technique as used for deriving the yield formula based on C_pm, we can obtain a yield measure formula based on C_pk . We first rewrite the definition of the C_pk index as:

$$C_\mathrm{pk} = \frac{d\ - |\mu - m|}{3\sigma} = \frac{1\ - |(\mu - m)/d|}{3(\sigma/d)} = \frac{1\ - |\delta|}{3\gamma} = \frac{C_a}{3\gamma}.$$

Then, for $\delta \geqslant 0$ and $C_\mathrm{pk} > 0$, we have $\delta = 1 - C_\mathrm{a}$, $\gamma = C_\mathrm{a}/(3C_\mathrm{pk})$, and

$$P(\mathrm{NC}) = \Phi\left[-\frac{3C_\mathrm{pk}(2 - C_a)}{C_a}\right] + \Phi\left[-\frac{3C_\mathrm{pk}C_a}{C_a}\right].$$

On the other hand, for $\delta < 0$ and $C_\mathrm{pk} > 0$, we have $\delta = C_\mathrm{a} - 1$, $\gamma = C_\mathrm{a}/(3C_\mathrm{pk})$, and

$$P(\mathrm{NC}) = \Phi\left[-\frac{3C_\mathrm{pk}C_a}{C_a}\right] + \Phi\left[-\frac{3C_\mathrm{pk}(2 - C_a)}{C_a}\right].$$

Consequently, for $C_\mathrm{pk} > 0$,

$$P(\mathrm{NC}) = \Phi\left[-3C_\mathrm{pk}\right] + \Phi\left[-\frac{3C_\mathrm{pk}(2 - C_a)}{C_a}\right].$$

We have $P(\mathrm{NC}) \leqslant 2\Phi(-3C)$ and $0 \leqslant C_\mathrm{a} \leqslant 1$, i.e., $LSL \leqslant \mu \leqslant USL$, for $C_\mathrm{pk} = C$. On the other hand, we have $P(\mathrm{NC}) \leqslant 2\Phi(-3C)$ and $1 - 1/(3C) \leqslant C_a \leqslant 1$, i.e., $m - d/(3C) \leqslant \mu \leqslant m + d/(3C)$, for $C_\mathrm{pm} = C$. For example, if $C_\mathrm{pk} = 1.00$ we only have the information of process yield through the upper bound $P(\mathrm{NC}) \leqslant 2699.796$ ppm. But, if $C_\mathrm{pm} = 1.00$ we have the information of process yield through the upper bound $P(\mathrm{NC}) \leqslant 2699.796$ ppm and the process centring measure $0.667 \leqslant C_a \leqslant 1$.

According to today's modern quality improvement theories based on Taguchi's quality philosophy, reduction of variation from the target value is the guiding principle. Therefore, attention should focus on meeting the target instead of meeting the tolerances. Following this principle, if μ is far away from the

Table 3. Bounds of P(NC) and C_a for $C_{pk} = C_{pm} = C$, respectively

C	C_{pk} Bound of P(NC)	Bound of C_a	C_{pm} Bound of P(NC)	Bound of C_a
1.00	2699.796 ppm	$0 \leqslant C_a \leqslant 1$	2699.796 ppm	$0.667 \leqslant C_a \leqslant 1$
1.25	176.835 ppm	$0 \leqslant C_a \leqslant 1$	176.835 ppm	$0.733 \leqslant C_a \leqslant 1$
1.50	6.795 ppm	$0 \leqslant C_a \leqslant 1$	6.795 ppm	$0.778 \leqslant C_a \leqslant 1$
2.00	0.002 ppm	$0 \leqslant C_a \leqslant 1$	0.002 ppm	$0.833 \leqslant C_a \leqslant 1$

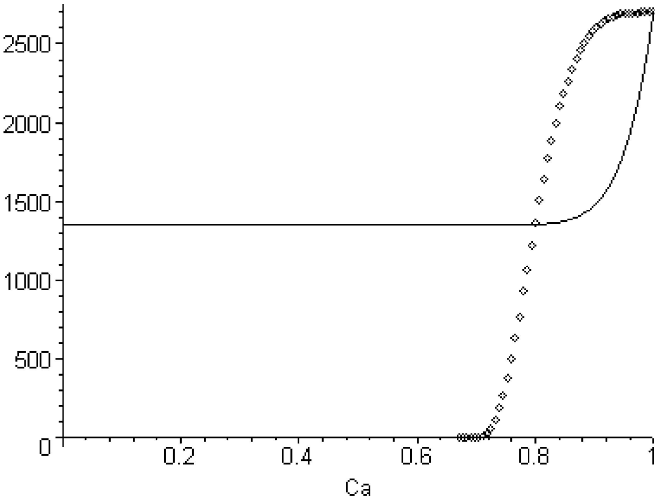

Fig. 2. The actual nonconformities curves for $C_{pk} = 1.0$ (*line*) and $C_{pm} = 1.0$ (*point*), with various allowed C_a

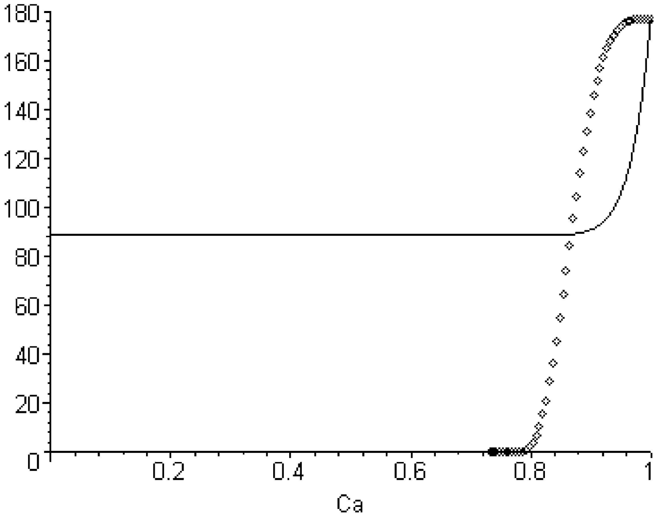

Fig. 3. The actual nonconformities curves for $C_{pk} = 1.25$ (*line*) and $C_{pm} = 1.25$ (*point*), with various allowed C_a

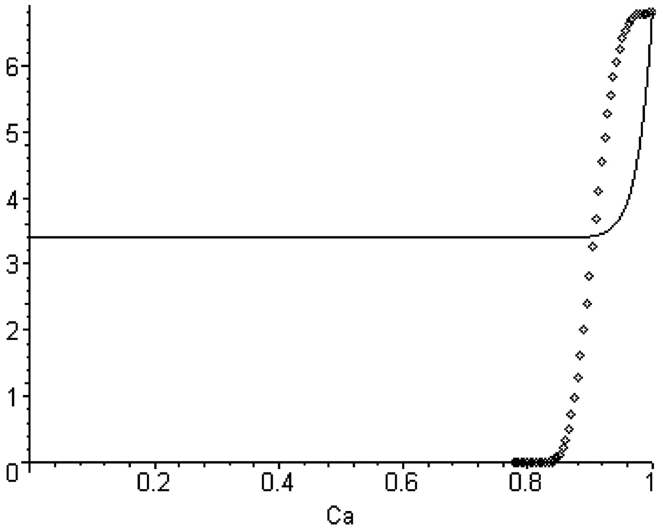

Fig. 4. The actual nonconformities curves for $C_{pk} = 1.5$ (*line*) and $C_{pm} = 1.5$ (*point*), with various allowed C_a

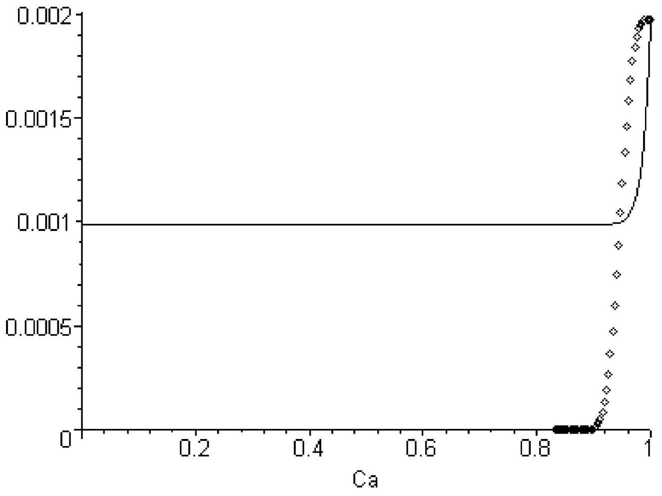

Fig. 5. The actual nonconformities curves for $C_{pk} = 2.0$ (*line*) and $C_{pm} = 2.0$ (*point*), with various allowed C_a

target T such that the corresponding C_a is small, then the process should not be considered capable even if σ is so small that the P(NC) is also small. Table 3 displays the bounds of P(NC) and C_a for $C_{pk} = C_{pm} = C$, respectively. Figures 2–5 plot the actual number of nonconformities (in ppm) for $C_{pm} = C_{pk} = 1.0, 1.25, 1.5$ and 2.0, with the restrictions $1 - 1/(3C_{pm}) \leqslant C_a \leqslant 1$ for C_{pm}, and $0 \leqslant C_a \leqslant 1$ for C_{pk}. The results illustrate the advantage of using the index C_{pm} over the index C_{pk} when measuring process capability, as C_{pm} provides better customer protection in terms of product quality loss.

4 Estimating and testing C_{pm}

The index C_{pm} can be rewritten as:

$$C_{pm} = \frac{\mathrm{d}}{3\sqrt{\sigma^2 + (\mu - T)^2}},$$

where $d = (USL - LSL)/2$ is half the length of the specification interval. In general, both the process mean μ and the process variance σ^2 are unknown. The estimated index $\hat{C}_{pm}$ is obtained by replacing μ and σ^2 by their estimators. Chan et al. [2] and Boyles [6] proposed two different estimators for C_{pm}, respectively defined as:

$$\hat{C}_{pm(CCS)} = \frac{d}{3\sqrt{S^2 + (\bar{X} - T)^2}},$$

$$\hat{C}_{pm(B)} = \frac{d}{3\sqrt{S_n^2 + (\bar{X} - T)^2}},$$

where $\bar{X} = \sum_{i=1}^{n} X_i/n$, $S^2 = \sum_{i=1}^{n} (X_i - \bar{X})^2/(n-1)$ and $S_n^2 = \sum_{i=1}^{n} (X_i - \bar{X})^2/n$. In fact, the two estimators, $\hat{C}_{pm(CCS)}$ and $\hat{C}_{pm(B)}$, are asymptotically equivalent. We note that $\bar{X}$ and S_n^2 are the MLEs of μ and σ^2, respectively. Hence, the estimated index $\hat{C}_{pm(B)}$ is the MLE of C_{pm}. Further, the term $S_n^2 + (\bar{X} - T)^2$ in the denominator of $\hat{C}_{pm(B)}$ is the uniformly minimum variance unbiased estimator (UMVUE) of the term $\sigma^2 + (\mu - T)^2$ in the denominator of C_{pm}, where $S_n^2 + (\bar{X} - T)^2 = \sum_{i=1}^{n} (X_i - T)^2/n$ and $\sigma^2 + (\mu - T)^2 = E[(X - T)^2]$. Therefore, it is reasonable for reliability purposes, that we let the estimator $\hat{C}_{pm(B)}$ evaluate the performances of normally distributed processes in this paper and define the estimated index $\hat{C}_{\mathrm{pm}} = \hat{C}_{pm(B)}$.

Using a method similar to that presented by Vännman [7], we obtain an exact form of the cumulative distribution function of $\hat{C}_{\mathrm{pm}}$. Under the assumption of normality, the cumulative distribution function of $\hat{C}_{\mathrm{pm}}$ can be expressed in terms of a mixture of the chi-square distribution and the normal distribution (see Lin and Pearn [8]):

$$F_{\hat{C}_{\mathrm{pm}}}(x) = 1 - \int_{0}^{b\sqrt{n}/(3x)} G\left(\frac{b^2 n}{9x^2} - t^2\right)\left[\phi(t + \xi\sqrt{n}) + \phi(t - \xi\sqrt{n})\right] dt, \quad (1)$$

for $x > 0$, where $b = d/\sigma$, $\xi = (\mu - T)/\sigma$, $G(\cdot)$ is the cumulative distribution function of the chi-square distribution χ^2_{n-1}, and $\phi(\cdot)$ is the probability density function of the standard normal distribution $N(0, 1)$.

To test whether a given process is capable, we consider the following statistical testing hypotheses:

$$H_0 : C_{\mathrm{pm}} \leqslant C \quad \text{(process is not capable)},$$
$$H_1 : C_{\mathrm{pm}} > C \quad \text{(process is capable)}.$$

Based on a given $\alpha(c_0) = \alpha$, the chance of incorrectly concluding an incapable process ($C_{\mathrm{pm}} \leqslant C$) as capable ($C_{\mathrm{pm}} > C$), the decision rule is to reject $H_0(C_{\mathrm{pm}} \leqslant C)$ if $\hat{C}_{\mathrm{pm}} > c_0$ and fails to reject H_0 otherwise. For processes with target value settings in the middle of the specification limits ($T = m = (USL + LSL)/2$), which are fairly common situations, the index may be rewritten as: $C_{\mathrm{pm}} = b/[3(1 + \xi^2)^{1/2}]$. Given that $C_{\mathrm{pm}} = C$, $b = d/\sigma$ can be expressed as $b = 3C(1 + \xi^2)^{1/2}$. Given a value of C (the capability requirement), $Pr(\hat{C}_{\mathrm{pm}} \geqslant c^* | C_{\mathrm{pm}} = C)$, the p-value corresponding to c^*, a specific value of $\hat{C}_{\mathrm{pm}}$ calculated from the sample data, is:

$$p - \text{value} = \int_{0}^{b\sqrt{n}/(3c^*)} G\left(\frac{b^2 n}{9(c^*)^2} - t^2\right)\left[\phi(t + \xi\sqrt{n}) + \phi(t - \xi\sqrt{n})\right] dt \quad (2)$$

Given values of α and C, the critical value c_0 can be obtained by solving the equation $Pr(\hat{C}_{\mathrm{pm}} \geqslant c_0 | C_{\mathrm{pm}} = C) = \alpha$. Hence, given values of capability requirement C, parameter ξ, sample size n, and risk α, the critical value c_0 can be obtained by solving the following equation:

$$\int_{0}^{b\sqrt{n}/(3c_0)} G\left(\frac{b^2 n}{9c_0^2} - t^2\right)\left[\phi(t + \xi\sqrt{n}) + \phi(t - \xi\sqrt{n})\right] dt = \alpha \quad (3)$$

Lin and Pearn [8] then implemented the testing hypothesis theory using Eqs. 2 and 3, and provided efficient Maple programs to calculate the p-values as well as the critical values.

The Maple program reads the sample data and the preset capability requirement C, and outputs the corresponding p-value and/or a critical value c_0. Also, the decision is made to reject the null hypothesis $H_0 : C_{\mathrm{pm}} \leqslant C$, or to not reject the null hypothesis. Based on the test, Lin and Pearn [8] developed a simple step-by-step procedure, which can be used for in-plant applications. The practitioners can use the proposed procedure to determine whether their process meets the preset capability requirement, and make reliable decisions.

5 Application example

The example presented below concerns the capability of a process that produces electrically erasable programmable read-only memory (EEPROM), which is user-modifiable read-only memory that can be erased and reprogrammed (written to) repeatedly through the application of higher electrical voltage. The product investigated here is a 128-bit EEPROM organised as 16 × 8 with a 2-wire serial interface. This EEPROM supports a bi-directional 2-wire bus and data transmission protocol. The output leakage current is an essential product characteristic, which has a significant impact on product quality. For the output leakage current of a particular model of EEPROM, the upper specification limit, USL, is set to 8 mA, the lower specification limit, LSL, is set to −8 mA, and the target, T, is set to 0 mA. Sample data are collected from 100 EEPROM chips, which are displayed in Table 4.

Figure 6 displays the histogram of the 100 observations. Figure 7 displays the normal probability plot of the sample data. The sample data appears to be normal. The Shapiro-Wilk test for normality is also performed, obtaining $W = 0.9917$. Thus, the sample data can be regarded as taken from a normal process. The sample mean $\bar{X} = 0.54$ and sample standard deviation $S_n = 1.64$ are calculated first. Hence, we can calculate the value of the estimator $\hat{C}_{\mathrm{pm}} = d/(3(S_n^2 + (\bar{X} - T)^2)^{1/2}) = 8/(3((1.64)^2 +$

Table 4. The sample data of 100 observations

0.14	−0.45	0.10	3.13	−1.60	−3.32	1.65	0.71	3.95	1.28
−1.34	−1.97	0.04	−1.85	−0.69	2.80	1.09	1.86	2.79	1.54
0.97	−2.21	2.64	1.42	−1.71	−1.35	−0.83	1.91	2.58	0.92
2.54	−0.89	0.47	1.97	0.05	−0.39	0.23	−0.37	0.77	−0.96
2.42	−0.58	0.32	3.52	0.55	1.75	0.80	−0.80	0.60	2.48
3.23	−2.62	−0.18	0.47	0.64	1.31	1.45	2.29	1.29	0.13
−2.72	−0.26	3.60	−0.20	0.11	1.93	0.81	−1.23	0.56	−0.68
0.24	−0.01	1.92	1.63	3.94	1.51	−0.78	−1.77	−1.00	−0.65
2.26	0.80	4.21	0.02	−2.05	−1.49	3.46	1.68	−2.10	−0.05
−1.05	0.04	−0.40	1.75	−0.52	−1.10	1.34	1.57	1.86	−0.09

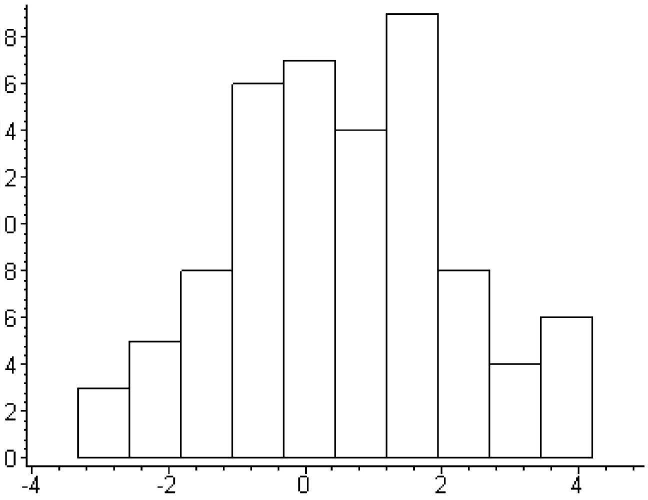

Fig. 6. Histogram of the sample data

$(0.54-0)^2)^{1/2}) = 1.54$. Using Maple computer software to calculate Eq. 2, we obtain the corresponding $p-\text{value} = 0.026$ for the preset capability requirement $C = 1.33$ and sample size $n = 100$. We conclude that the EEPROM manufacturing process meets the capability requirement "$C_{\text{pm}} > 1.33$" if the α-risk is set to be larger than 0.026. In this case, we believe that the process is capable, the number of the nonconformities is less than 67 ppm (from Table 2), and $C_{\text{a}} \geqslant 1 - 1/(3C) = 0.75$.

6 Conclusions

The process capability indices C_{p}, C_{a}, C_{pk} and C_{pm}, have been proposed to the manufacturing industry as capability measures based on various criteria including variation, departure, yield, and loss. Both the C_{pk} and C_{pm} indices provided the same lower bounds on process yield, that is, *Yield* $\geqslant 2\Phi(3C_{\text{pk}}) - 1 = 2\Phi(3C_{\text{pm}}) - 1$. In this paper, we investigated the behaviour of the actual process yield, in terms of the number of nonconformities (in ppm), for processes with fixed index value of $C_{\text{pk}} = C_{\text{pm}}$, but with different degrees of process centring, which can be expressed as a function of the capability index C_{a}. We also compared the two indices C_{pk} and C_{pm} in terms of process yield and process loss. The results illustrated the advantage of using the index C_{pm} over the index C_{pk} when measuring process capability, as C_{pm} provides better customer protection. We investigated a real example taken from a factory to illustrate how to measure the process yield based on the index C_{pm}.

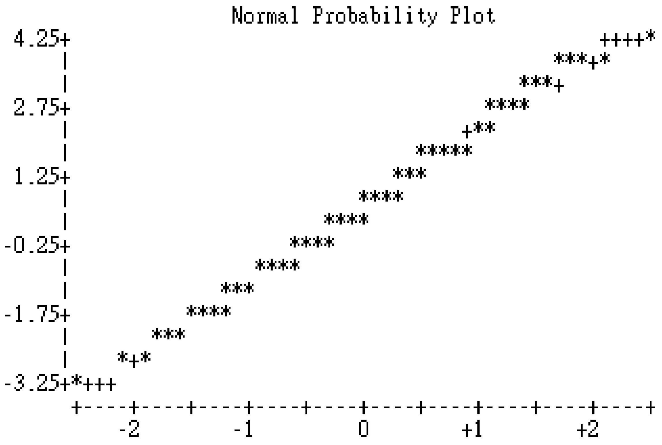

Fig. 7. The normal probability plot

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