Large Simulation of Hysteresis Systems Using a Piecewise Polynomial Function

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***Abstract*—Hysteresis is a memory effect frequently observed in physical research. The output of a hysteresis system is independent of input speed. This property, known as *rate independence*, significantly distinguishes hysteresis from short-term memory effects. This work conducts a numerical simulation to demonstrate that conventional models with short-term memories cannot properly simulate hysteresis trajectories. Subsequently, a novel model is developed to contribute to the field of system modeling. Experimental results confirm that the proposed model can model hysteresis behavior precisely.**

***Index Terms*—Hysteresis, rate independence, short-term memory.**

I. Introduction

TYPICAL hysteresis cycles are displayed in Fig. 1. As the input alternates between increasing and decreasing, the response curve diverges from its original path drawing a new curve. Importantly, only the previous extreme inputs and the current input value, regardless of input speed, determine the output. This property is known as rate independence [1], the defining characteristic of hysteresis behavior.

Hysteresis phenomena are frequently observed in physical research domains, including magnetism [2]–[4], plasticity [1], [5], electronics [6]–[8], thermodynamics [9]–[11], materials [12]–[14], and mechanics [15], [16]. However, the unique property of rate independence makes modeling hysteresis behaviors extremely difficult.

Existing hysteresis models can be categorized into nonlocal and local memory models [17]. Nonlocal memory models, such as Preisach [18], [19], Prandtl–Ishlinskiĭ (with stop and play types) [1], [18], as well as various partial differential equation (PDE) models [1], [20], globally refer to past extreme inputs while transducing the new input value to its corresponding output. Meanwhile, local memory models, such as the Madelung [17, page xvi] and Hysterys [21] models, consider the current input–output (I/O) values locally: a maximum of two curves pass through each working point in the I/O diagram. For an increasing input $u(t)$, the curve rises, causing the output $y(t)$ to respond to $u(t)$. If the input decreases, then a falling curve is traced. Both of these models have disadvantages. Nonlocal memory models require an amount of memory in order to record past extreme values and are computationally difficult, whereas local memory models cannot approximate actual systems as closely as do nonlocal memory models.

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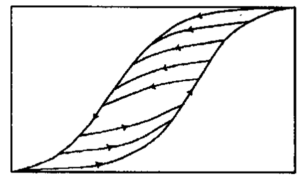

Fig. 1. Typical branches of hysteresis.

An attempt is made herein to examine whether conventional memory-related models, such as finite-impulse response (FIR) models [22], autoregressive external input (ARX) models [23], time delay neural networks (TDNN) [24], and recurrent neural networks (RNN) [24] can simulate hysteresis behavior. If they cannot, then this work intends to develop a new hysteresis model, which must include nonlocal memory but also must be computationally convenient, similar to a local memory model.

Conventional system models (FIR, ARX, TDNN, and RNN) use a sliding window or feedback links to arrange previous input data and, in so doing, form short-term memory. These models use short-term memories and built-in parameters to calculate output values. The parameters can be conveniently determined by the least squares estimation (LSE) and gradient descent methods [23]. However, architecture based on short-term memory likely prevents conventional models from simulating hysteresis behavior.

This work initially simulates a hysteresis trajectory using conventional models. Simulation results show that conventional models simulate hysteresis behavior inaccurately. Thereafter, a novel model is designed using a piecewise polynomial function of I/O sequences. Experimental results, obtained using the proposed model to simulate the same trajectory, reveal that the model presented herein can feasibly model hysteresis behavior.

II. Simulation of Hysteresis Behavior Using Short-Term Memory

The Preisach model is used to generate the target trajectory employed in the following tests. This model is the renowned hysteresis model of magnetics [17] and has attracted extensive attention [2], [25]–[27] since its publication in 1935. The Preisach model is described by superimposing a continuous set of relay operators. For a relay operator $\hat{\gamma}_{\alpha\beta}$, α, and β specify the operating limitations, each operator adopts the current input value u and causes bivalued hysteresis (returns -1 or 1) owing to the switching between ascending and descending paths.

When combining operators, such as $\hat{\gamma}_{\alpha_1\beta_1}$ and $\hat{\gamma}_{\alpha_2\beta_2}$, an entire system can be expressed by the following:

$$y(t) = \iint_{\alpha\geq\beta} \mu(\alpha, \beta) \cdot \hat{\gamma}_{\alpha\beta}[u(t)]\, d\alpha\, d\beta \tag{1}$$

where y is the output signal that corresponds to the input signal, u, and $\mu(\alpha, \beta)$ is the weight function collaborating with each relay operator $\hat{\gamma}_{\alpha\beta}$.

The target trajectory included 6000 I/O pairs. Beginning with constant zero, the input values of the target trajectory were sequentially set up by repeating the following two steps: a) an intermediary pursued value p was randomly selected in the interval $[-1, 1]$ and b) n equidistant successive values were produced from the current input to the pursued value p, where n is another random variable uniformly selected from one to ten. This process was continued until 6000 values were generated in the input sequence. Restated, the considered input signal was continuous and piecewise linear, each piece of which had a random slope and a random duration.

Thereafter, the input sequence was input to the given Preisach model with weight function $\mu(\alpha, \beta) = 2.4 - (\alpha + \beta)^2$ to produce the output signal. For these 6000-length I/O pairs of data, the first 100 data points sufficiently assigned the initial values to the tested models. The next 3900 and 2000 I/O pairs were used for training and testing.

The simulations involved four conventional models:

1) FIR model with fifth-order moving average;
2) ARX model with third-order autoregression and fifth-order moving average;
3) TDNN with only one input node, only one output node, two hidden layers (both with three nodes), and where the memory depths of the hidden layers are five and three;
4) fully connected RNN with seven hidden nodes.

The structures of these models were built after comprehensive testing, and the training processes of the network models were adequately convergent. Fig. 2 plots the simulated outcomes of the listed models, where the dashed line represents input sequence u; the gray line represents desired output sequence y; and the solid line represents the actual output z.

The maximum input at the time index 4968, followed by a period of slowly decreasing inputs, is observed. Notably, the extreme input value has a prolonged influence on the hysteresis system, since this input alters the path in the I/O relation. For the same input as in the tested models, however, a steeply falling output sequence is generated in the interval $[4968, 4998]$. This outcome remarkably indicates that conventional short-term memory models fail to satisfy the property of rate independence.

III. Simulation of Hysteresis Behavior Using Newly Proposed Model

This letter develops a new model of hysteresis phenomena. Fig. 3 presents the framework. Combining three major blocks—the gradient investigator (GI), extreme-value template (ET), and output function (OF)—the proposed model approximates hysteresis systems conveniently, by determining an active polynomial function once an extreme input value is reached.

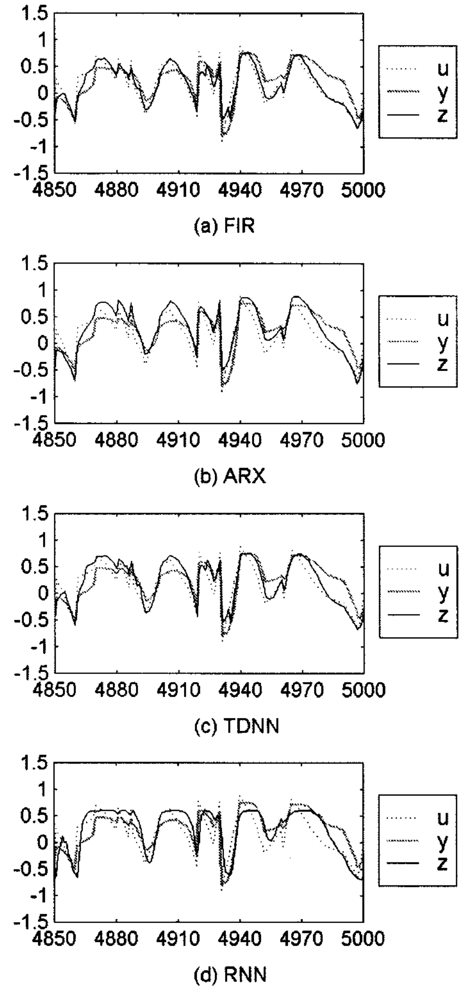

Fig. 2. Simulating hysteresis behavior by conventional models. The Preisach model with weight function $\mu(\alpha, \beta) = 2.4 - (\alpha + \beta)^2$ generates the target trajectory. Plots (a)–(d) are the simulation results of the FIR model, ARX model, TDNN, and RNN, respectively. Sequences u, y and z are the input, desired output, and actual output signals.

Accordingly, this model is referred to as the *polynomial output using local extreme-value template* (POLET) model. Notably, this model includes nonlocal memory in the ET block and is as computationally easy as local memory models of hysteresis.

The GI block generates a bivalued gradient signal d, using a function of u_k, u_{k-1}, and d_{k-1}, as described by (2)

$$d_k = \begin{cases} 1, & \text{if } u_k > u_{k-1} \\ -1, & \text{if } u_k < u_{k-1} \\ d_{k-1}, & \text{otherwise.} \end{cases} \tag{2}$$

The ET block contains a vector structure X termed the template. Template entries begin with a constant of one and the gradient signal d_k. The rest of the template is made into a shift register that records extreme input values and corresponding output values. This letter considers the standard notation of the C/C++ programming language. Let the first element of vector X be $X[0]$. Assuming that the template has a memory depth of order m, we have the following:

$$X_k = \begin{cases} [1\, d_k\, u_{k-1}\, y_{k-1}\, X_{k-1}[2]\, X_{k-1}[3] \\ \quad \cdots X_{k-1}[2m-1]]^T, & \text{if } d_k d_{k-1} \leq 0 \\ X_{k-1}, & \text{otherwise.} \end{cases} \tag{3}$$

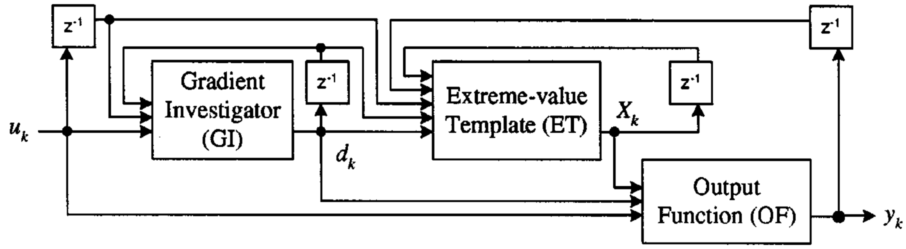

Fig. 3. POLET model. This model involves three major blocks: GI, ET, and OF.

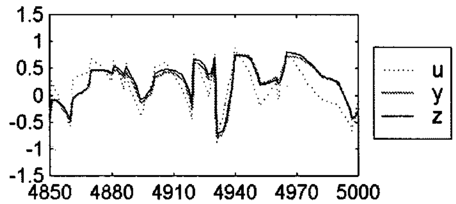

Fig. 4. Simulating hysteresis behavior by the proposed model.

TABLE I
MSE IN SIMULATING HYSTERESIS BEHAVIOR. THE TARGET TRAJECTORY IS GENERATED BY THE PREISACH MODEL WITH WEIGHT FUNCTION $\mu(\alpha, \beta) = 2.4 - (\alpha + \beta)^2$

	FIR	ARX	TDNN	RNN	POLET
Training Error	0.038	0.042	0.036	0.061	4.96×10^{-3}
Testing Error	0.033	0.041	0.032	0.061	4.33×10^{-3}

Restated, X_k is designed as a $(2m + 2)$-length vector $[1\, d_k\, \hat{u}_0\, \hat{y}_0 \cdots \hat{u}_n\, \hat{y}_n \cdots \hat{u}_{m-1}\, \hat{y}_{m-1}]^T$, with the entry pair $(\hat{u}_n, \hat{y}_n)$ recording the I/O values that correspond to the $(n+1)$th nearest extreme input. Consequently, under the condition $d_k d_{k-1} \le 0$, $X_k[p] = X_{k-1}[p-2]$, for $4 \le p \le 2m+1$.

The OF block accepts the current template X_k and determines an active polynomial. This polynomial acts on the upcoming inputs to generate the output values. A parametric matrix M is prescribed. The product of M and X_k yields the coefficients of the active polynomial. Supposing that the polynomials are of the rth degree, the output function can be represented as

$$h(X, u) = [(u - X[2])(u - X[2])^2 \cdots (u - X[2])^r] \cdot M \cdot X + X[3] \quad (4)$$

where the current template X is a $(2m + 2)$-length vector; u is the forthcoming input value; and M is a $(r) \times (2m+2)$ matrix. This function retains the first continuity of the output sequence, since $(X[2], X[3])$ is the I/O pair that corresponds to the nearest extreme input.

The parametric matrices M can be estimated through an online (recursive) LSE algorithm as follows:

Step 1. Initialize N by a scaled $r(2m+2) \times r(2m+2)$ identity matrix, θ to be a zero vector of dimension $r(2m+2)$.

Step 2. At each time step, determine the current template vector X from (3). Obtain a vector ϕ with $\phi_i = (u - X[2])^{p+1} \cdot X[q]$, where u is the current input value, $p = \lceil (i+1)/(2m+2) \rceil - 1$, and $q = i - p(2m+2)$, for $i \in \{0, 1, \ldots, r(2m+2) - 1\}$. Then, reset N and θ to $N - (N \cdot \phi \cdot \phi^T \cdot N / (1 + \phi^T \cdot N \cdot \phi))$ and $\theta + N \cdot \phi \cdot (y - X[3] - \phi^T \cdot \theta)$, where y is the current output.

Step 3. After estimates have been made in all the time steps, determine the parametric matrix M by $M_{i,j} = \theta_{(2m+2)i+j}$, for $0 \le i \le r-1$ and $0 \le j \le 2m+1$.

The same trajectory is simulated using the POLET model with second-order memory and a fourth-degree polynomial.

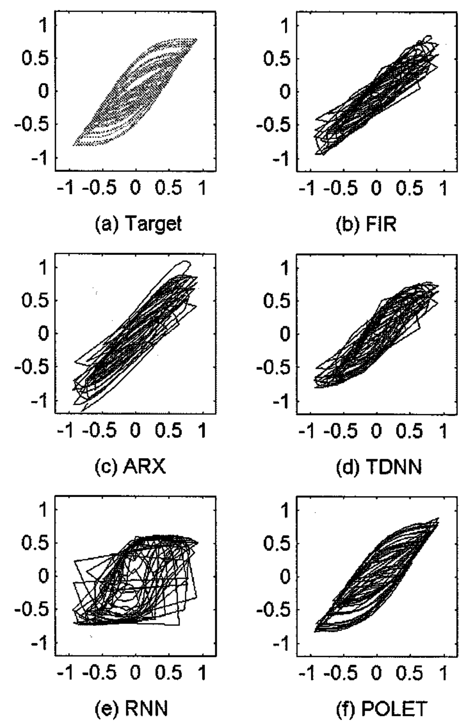

Fig. 5. I/O diagrams of simulating the Preisach model. Plot (a) shows the target trajetory. Plots (b) to (f) present trajectories obtained from the FIR model, ARX model, TDNN, RNN, and POLET model.

Fig. 4 presents the outcomes of the proposed model, demonstrating that this model can exactly simulate the target hysteresis trajectory.

Table I specifies the mean square errors (MSE) in all these simulations. The MSE is defined as

$$\text{MSE} = \frac{1}{b-a+1} \sum_{i=a}^{b} (y_i - z_i)^2 \quad (5)$$

where y and z are the desired and actual output sequences, with $(a, b) = (100, 3999)$ for training, and $(a, b) = (4000, 5999)$ for testing. Additionally, Fig. 5 presents I/O diagrams, where Fig. 5(a) shows the desired I/O diagram of the Preisach model, and Fig. 5(b)–(f) presents the tested I/O diagrams of the FIR, ARX, TDNN, RNN, and POLET models in order. Experimental results, shown in Table I and in Figs. 2, 4, and 5, reveal that the proposed model can follow given hysteresis trajectories with a slight error, while conventional models cannot.

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