AIP Applied Physics Letters

Quasistationary states of a relativistic field-emission-limited diode employing a high-transparency mesh anode

Ming-Chieh Lin and Der-San Chuu

Citation: Applied Physics Letters **80**, 4262 (2002); doi: 10.1063/1.1482789
View online: http://dx.doi.org/10.1063/1.1482789
View Table of Contents: http://scitation.aip.org/content/aip/journal/apl/80/22?ver=pdfcov
Published by the AIP Publishing

APPLIED PHYSICS LETTERS VOLUME 80, NUMBER 22 3 JUNE 2002

Quasistationary states of a relativistic field-emission-limited diode employing a high-transparency mesh anode

Ming-Chieh Lin[a)] and Der-San Chuu
Department of Electrophysics, National Chiao Tung University, Hsinchu, Taiwan 30050, Republic of China

(Received 10 October 2001; accepted for publication 5 April 2002)

A relativistic field-emission-limited diode employing a high-transparency mesh anode is investigated via a self-consistent approach. The field emission process is described quantum mechanically by the Fowler–Nordheim equation. The cathode plasma and surface properties are considered within the framework of the effective work function approximation. Space-charge effects are described by Poisson's equation including relativistic effects. Ionization effects at the high-transparency mesh anode are ignored. The numerical calculations are carried out on a time scale much shorter than the emergence of the gap closure. The quasistationary state of the diode exhibits a cutoff voltage. The electric field on the cathode surface is found to be saturated in the high-voltage regime and determined by the effective work function only. [DOI: 10.1063/1.1482789]

In high-power vacuum electron devices such as virtual cathode oscillators,[1–3] SuperReltrons,[4,5] and electron injectors,[6] relativistic diodes are usually used to accelerate the electron beam. The basic features of nonrelativistic diodes can be illustrated with an idealized model, the Child–Langmuir diode in a parallel planar geometry.[7] This expression is based on the assumption that the cathode can supply enough current to be space-charge limited, in which case the electric field at the cathode surface is zero and the current is a maximum. Jory and Trivelpiece[8] considered the problem of current-limited emission in a one-dimensional planar diode including the relativistic corrections in the equation of motion of the electrons. They obtained an exact relativistic solution for the one-dimensional planar diode. Both current-limited and space-charge-limited solutions were found. While electron emission can result from any of several processes, including thermionic emission,[9] photoemission, secondary emission, field emission, and explosive emission,[10] the dominant mechanism of our concern is field emission. Although field emission can be microscopically enhanced by a field enhancement factor due to protrusions, contamination, oxide layers, dielectric inclusions, grain boundaries, or adsorbates, the current density is usually not enough to be space-charge-limited macroscopically. In this case, the current is field-emission limited. In recent years, due to the need for a high-quality electron beam, mesh anodes were widely employed in the high-power devices just mentioned.[1–6] It was found that high-transparency mesh anodes have the advantages of low interception, reduced anode plasma, and elimination of diode short.

In this letter, relativistic field-emission-limited diodes (RFELDs) employing high-transparency mesh anodes are investigated. The field-emission process is described quantum mechanically by the Fowler–Nordheim equation. The cathode plasma and surface properties are considered within the framework of the effective work function approximation. Space-charge effects are described by Poisson's equation including relativistic effects. Ionization effects at the high-transparency mesh anode are ignored.[11–20] Quasistationary states can be obtained via the following self-consistent approach.

Let us consider field emission of electrons in a relativistic planar diode. The phenomenon of field emission from a cold metal can be described as a quantum mechanical tunneling of conduction electrons through the potential barrier at the surface of the metal. The basic field-emission process is described by the Fowler–Nordheim equation,[11–18,21]

$$J=\frac{AE_s^2}{\phi t^2(y)}\exp\left(\frac{-B\nu(y)\phi^{3/2}}{E_s}\right), \tag{1}$$

where A and B are the Fowler–Nordheim constants, and ϕ is the effective work function assumed to be a constant dependent on the cathode material, surface roughness, and ionization effects. The normal electric field at the cathode surface, E_s, should be obtained from the Fowler–Nordheim equation to serve as a boundary condition for Poisson's equation. The functions $t(y)$ and $\nu(y)$ were introduced by Spindt *et al.*,[21] and approximated by

$$t^2(y)=1.1, \tag{2}$$

$$\nu(y)=0.95-y^2, \tag{3}$$

$$y=3.79\times10^{-5}E_s^{1/2}/\phi, \tag{4}$$

where y is the Schottky lowering of the effective work function barrier.

To analyze the flow of electrons in a relativistic planar diode, we solve Poisson's equation coupled with the law of energy conservation:

$$\nabla^2\Phi=-\frac{\rho}{\epsilon_0}, \tag{5}$$

$$e\Phi=(\gamma-1)mc^2, \tag{6}$$

where ρ is the charge density and γ is the relativistic factor. The current density of electrons in the z direction is

[a)]Author to whom correspondence should be addressed; also at: Stanford Linear Accelerator Center; electronic mail: mclin.ep87g@nctu.edu.tw

$$J = -\rho v. \tag{7}$$

Combining Eqs. (5)–(7) yields

$$\frac{d^2\gamma}{dz^2} = \frac{eJ}{\epsilon_0 mc^3 \sqrt{1-\gamma^{-2}}}. \tag{8}$$

This equation can be integrated to give

$$\frac{d\gamma}{dz} = \left[\frac{2eJ}{\epsilon_0 mc^3} (\gamma^2 - 1)^{1/2} + \frac{e^2}{m^2c^4} E_s^2 \right]^{1/2}. \tag{9}$$

The constant of integration E_s is the electric field at the cathode surface where $z=0$ and $\Phi=0$. Integrating from cathode surface, $z=0$, to anode surface, $z=d$, Eq. (4) becomes

$$J = \frac{\epsilon_0 mc^3}{2ed^2} \int_1^{\gamma_0} \left[(\gamma^2-1)^{1/2} + \frac{\epsilon_0 e}{2mcJ} E_s^2 \right]^{-1/2} d\gamma, \tag{10}$$

where $\gamma_0 (= 1 + e\Phi_0 / mc^2)$ is the relativistic factor of electrons at the mesh anode due to the applied diode voltage (Φ_0). Actually, Eq. (10) should be modified for gap closure due to the expanding cathode plasma by replacing d as $(d - v_c t)$, where v_c represents the closure velocity. However, we consider the system on a time scale much shorter than the emergence of the gap closure, i.e., $v_c t \ll d$, so Eq. (10) is adequate to describe the quasistationary behavior of the diode. This integral can be integrated from standard table when $E_s = 0$, and with transformation by the change of variable to a more suitable form to integrate when $E_s \neq 0$. For a given E_s, the current density can be obtained by solving Eq. (10).

With an initial guess of the current density J, an initial approximation of the surface electric field E_s can be determined from the Fowler–Nordheim equation. This E_s then serves as a boundary condition for the Poisson's equation to solve for a better approximation of J. Thus, Eqs. (1) and (10) are solved iteratively until we arrive at a self-consistent solution of both the Fowler–Nordheim equation and the Poisson's equation, as shown in Fig. 1.

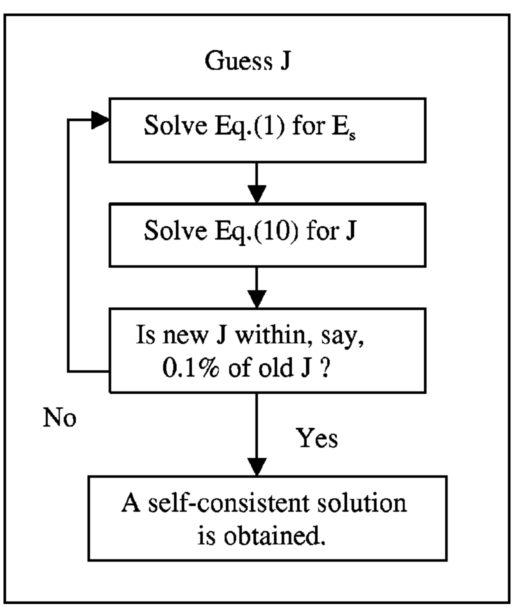

FIG. 1. Iterative solution of the Fowler–Nordheim equation and Poisson's equation.

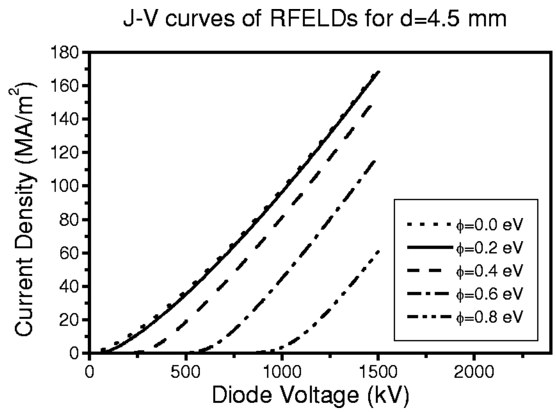

FIG. 2. $J-V$ curves of the RFELDs for gap length $d=4.5$ mm and effective work function $\phi=(0,0.2,0.4,0.6,0.8)$ eV. The curve corresponds to $\phi=0$ eV represents space-charge limited.

We have found the quasistationary states of relativistic field-emission-limited diodes via the self-consistent approach presented above. Figure 2 shows our calculated current density–voltage $(J-V)$ curves of the RFELDs for gap length $d=4.5$ mm and effective work function $\phi=(0,0.2,0.4,0.6,0.8)$ eV. The $J-V$ curve corresponding to $\phi=0$ eV, i.e., explosive emission, approaches the correct limit to be space-charge limited, and the other curves are field-emission limited. From Fig. 2, one can see that the field-emission-limited current corresponding to $\phi=0.2$ eV is nearly space-charge limited for the case. However, there exist cutoff voltages in the $J-V$ curves of the field-emission-limited diodes. There is almost no emission current for the four cases when diode voltages are below the cutoff voltages. That is different from space-charge-limited diodes.

Figure 3 shows the E_s-V curves of the RFELDs for $d=4.5$ mm and $\phi=(0.2,0.4,0.6,0.8,\text{Infinity})$ eV. The E_s-V curve corresponding to $\phi=\text{Infinity}$ approaches the correct limit to be the nonemission case. The surface electric field E_s is trivially proportional to the diode voltage. For the other four cases, the surface electric fields of RFELDs increase more slowly due to space-charge effects when the diode voltages are higher than the cutoff voltages. In even higher voltage regime, saturation of surface electric fields is quickly achieved. The surface electric fields are independent of the applied diode voltage in the saturated regions. Figure 4

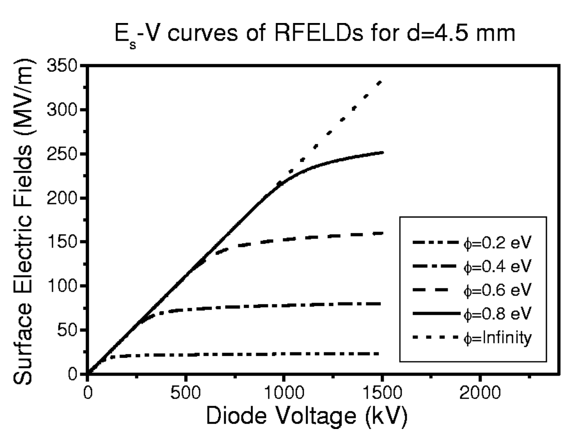

FIG. 3. E_s-V curves of the RFELDs for $d=4.5$ mm and $\phi=(0.2,0.4,0.6,0.8,\text{Infinity})$ eV. The curve corresponds to $\phi=\text{Infinity}$ represents nonemission case.

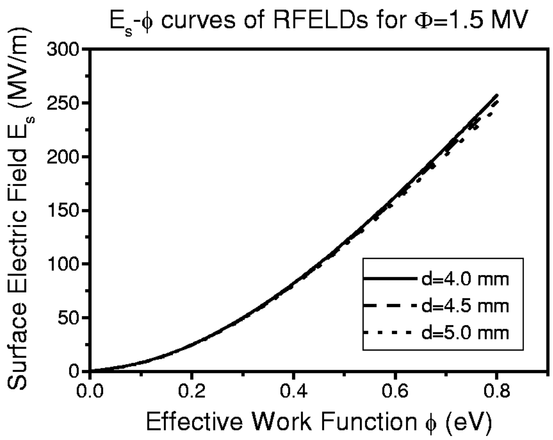

FIG. 4. E_s-ϕ curves of the RFELDs for $\Phi = 1.5$ MV and $d = (4.0,4.5,5.0)$ mm.

shows the $E_s - \phi$ curves of the RFELDs for $\Phi = 1.5$ MV and $d = (4.0,4.5,5.0)$ mm. The surface electric fields are determined by the effective work function, independent of the gap length. So, one can treat the surface electric field as a function of the effective work function only.

In summary, we have investigated relativistic field-emission-limited diodes. The field-emission process is described quantum mechanically by the Fowler–Nordheim equation. The cathode plasma and surface properties are considered within the framework of the effective work function approximation. The space-charge effects are described by Poisson's equation including relativistic effects. Ionization effects at the high-transparency mesh anode are ignored. The quasistationary states are obtained via the self-consistent approach. One of the differences between RFELDs and relativistic space-charge-limited diodes is that the former case exhibits a cutoff voltage. The other is that the surface electric fields of RFELDs are not zero. The surface electric field is found to be saturated in the high-voltage regime and is simply determined by the effective work function.

The authors would like to acknowledge the support and encouragement of George Caryotakis, Glenn Scheitrum, Daryl W. Sprehn, and Alex Burke, and the computer assistance of Cecilio Vazquez and Rafael J. Gomez, Stanford Linear Accelerator Center (SLAC), Stanford University, Stanford, CA. This work was partially supported by the National Science Council, Taiwan, under Grant No. NSC 90-2112-M-009-018.

[1] C. S. Hwang, M. S. Yang, F. C. Lin, M. W. Wu, and W. S. Hou, *Proceedings of the Seventh Pulse Power Conference* (IEEE, New York, 1989), p. 951.
[2] M. W. Wu, T. C. Guung, C. Y. Chen, and C. S. Hwang, *Proceedings of the Eighth Pulse Power Conference* (IEEE, New York, 1991), p. 815.
[3] E.-H. Choi, M. C. Choi, Y. Jung, M. W. Chong, J. J. Ko, Y. Seo, G. Cho, H. S. Uhm, and H. Suk, IEEE Trans. Plasma Sci. **28**, 2128 (2000).
[4] R. B. Miller, W. F. McCullough, K. T. Lancaster, and C. A. Muehlenweg, IEEE Trans. Plasma Sci. **20**, 332 (1992).
[5] R. B. Miller, IEEE Trans. Plasma Sci. **26**, 340 (1998).
[6] G. Mamaev, Proceedings of the 1997 Part. Accel. Conference 1998, Vol. 1, 1263.
[7] C. D. Child, Phys. Rev. **32**, 492 (1911); I. Langmuir, *ibid.* **2**, 450 (1913).
[8] H. R. Jory and A. W. Trivelpiece, J. Appl. Phys. **40**, 3924 (1969).
[9] S. Dushman, Phys. Rev. **21**, 623 (1923).
[10] R. B. Miller, *An Introduction to the Physics of Intense Charged Particle Beams* (Plenum, New York, 1982).
[11] R. H. Fowler and L. W. Nordheim, Proc. R. Soc. London, Ser. A **119**, 173 (1929).
[12] L. W. Nordheim, Proc. R. Soc. London, Ser. A **121**, 626 (1928).
[13] L. W. Nordheim, Z. Phys. **30**, 177 (1929).
[14] W. Schottky, Z. Phys. **14**, 63 (1923).
[15] N. H. Frank and L. A. Young, Phys. Rev. **38**, 80 (1931).
[16] E. Guth and C. J. Mullin, Phys. Rev. **61**, 339 (1942).
[17] D. V. Gogate and D. S. Kothari, Phys. Rev. **61**, 349 (1942).
[18] S. Gasiorowicz, *Quantum Physics*, 2nd ed. (Wiley, New York, 1996).
[19] S. A. Goldstein and R. Lee, Phys. Rev. Lett. **35**, 1079 (1975).
[20] E. H. Choi, H. M. Shin, and D. I. Choi, J. Appl. Phys. **61**, 2160 (1986).
[21] C. A. Spindt, I. Brodie, L. Humphrey, and E. R. Westerberg, J. Appl. Phys. **47**, 5248 (1976).