Unconventional vortices in multicomponent Ginzburg-Landau theory

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Simulations of a three-component time-dependent Ginzburg-Landau (Abelian Higgs) model reveals that the dominant topological defects are vector vortices rather than conventional Abrikosov (Nielsen-Olesen) vortices or skyrmions. We describe in detail these vortices in the steady state and discuss their possible role in the dynamics. In particular, we conclude that the vector vortices have a superconducting core distinct from the superconducting bulk state. The profile of the vector order parameter and the magnetic field are calculated.

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I. INTRODUCTION

The formation of an equilibrium state in a system undergoing a symmetry-breaking phase transition due to a sudden change in the thermodynamic condition is a topic of wide theoretical[1] and experimental[2] interest. Recently much progress has been made in understanding the phase-transition dynamics in systems with global symmetry such as liquid crystals,[3] superfluid He^3,[4] and the atomic Bose-Einstein condensate.[5] The extremely important case of a symmetry-breaking phase transition with local gauge symmetry is also under intensive consideration. It includes the critical dynamics in conventional superconductors, as well as symmetry-breaking phenomena in the early Universe.[6,7] The transformation of this unstable state to a thermodynamic equilibrium state is accompanied by massive creation of topological defects. For example, Abrikosov vortices and antivortices emerge in a normal-to-superconductor phase transition.[8] The problem becomes much more involved in nonconventional superconductors. The order parameter describing Cooper pairs generally has several components.[9] Examples include the description of high T_c superconductors as a mixture of *d*-wave and *s*-wave components,[10] and *p*-wave superconductors a heavy fermion UPt_3[11] or newly discovered Sr_2RuO_4.[12] The symmetry of the order parameter is closely linked to the crystallographic symmetry group of the material. The common feature of theories describing these diverse systems remains the U(1) local gauge invariance. It is well known that, while in the one-component case the Abrikosov vortices (AV) are the only kind of topological defect, in the multicomponent case other types of defects can be formed during the phase transition.

In this paper we concentrate on a complex vector field model (describing, in particular, superconductors with *p*-wave pairing) that possesses an approximate global SO(3) symmetry. In this case the number of the components of the order parameter $n=3$ provided more sophisticated structure of the topological defects, in particular, coreless vortices typical for the He^3 rather than for superconductors. We consider this kind of topological defect in a U(1) gauge invariant multicomponent Ginzburg-Landau model: vortices with nonzero order parameter along the vortex axis that will be called "vector vortices" (VV) below. These defects are essentially different from both the usual Abrikosov vortices (which are simply one-component topological solitons with additional components vanishing), coreless magnetic skyrmions discovered in this system[13] and vortices in two-component order parameter models (see Ref. 9). In the latter case the theory predicts either usual Abrikosov vortices with zero of the order parameter in the vortex center or nonzero in the vortex center but zeros of the order parameter outside the vortex center. In contrast to AV, the core of VV is superconducting although the superconducting state inside the core is different from that outside the core. We calculated both the structure of the order parameter and magnetic field decaying from the vortex axis and predict that the vortex nucleation is dominated by vector vortices.

II. THE MODEL

We investigate the dynamics of a complex vector field $\psi(\mathbf{r})=(\psi^1,\psi^2,\psi^3)$ and of a vector potential $\mathbf{A}(\mathbf{r})$ in two dimensions, $\mathbf{r}=(x,y)$, with Hamiltonian

$$\mathcal{H}=\int d^2r\left[\frac{K}{2}|(\boldsymbol{\nabla}-i\mathbf{A})\psi^a|^2-\alpha|\psi^a|^2+\frac{\beta_1}{2}(\psi^a\psi^{*a})^2+\frac{\beta_2}{2}|\psi^a\psi^a|^2+\frac{(\boldsymbol{\nabla}\times\mathbf{A})^2}{8\pi}\right] \tag{1}$$

in the framework of the time-dependent Ginzburg-Landau (TDGL) equations, where $a=1,2,3$. It is convenient to define the dimensionless variables as follows:

$$r\rightarrow r/\xi,\quad \psi\rightarrow\psi/\psi_0,\quad \mathbf{A}\rightarrow\mathbf{A}/(\sqrt{2}\delta H_{cm}),$$
$$\xi=\sqrt{\frac{K}{2\alpha}},\quad \psi_0=\sqrt{\alpha/\beta_1}. \tag{2}$$

Here ξ is the coherence length, δ is the penetration depth of the magnetic field, H_{cm} is the thermodynamic critical magnetic field, and Φ_0 is the flux quantum. The corresponding dimensionless TDGL equations are

$$\Gamma\frac{\partial\psi^a}{\partial t}=\psi^a-(\psi^b\psi^{*b})\psi^a-\beta(\psi^b\psi^b)\psi^{*a}-(i\boldsymbol{\nabla}+\mathbf{A})^2\psi^a+\omega, \tag{3}$$

$$\frac{\partial\mathbf{A}}{\partial t}=-\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\mathbf{A}+\mathbf{J}_s, \tag{4}$$

where time is rescaled in the Ginzburg-Landau characteristic time units, $\beta=\beta_2/\beta_1$ and ω is the random force.

The superconducting current is defined by

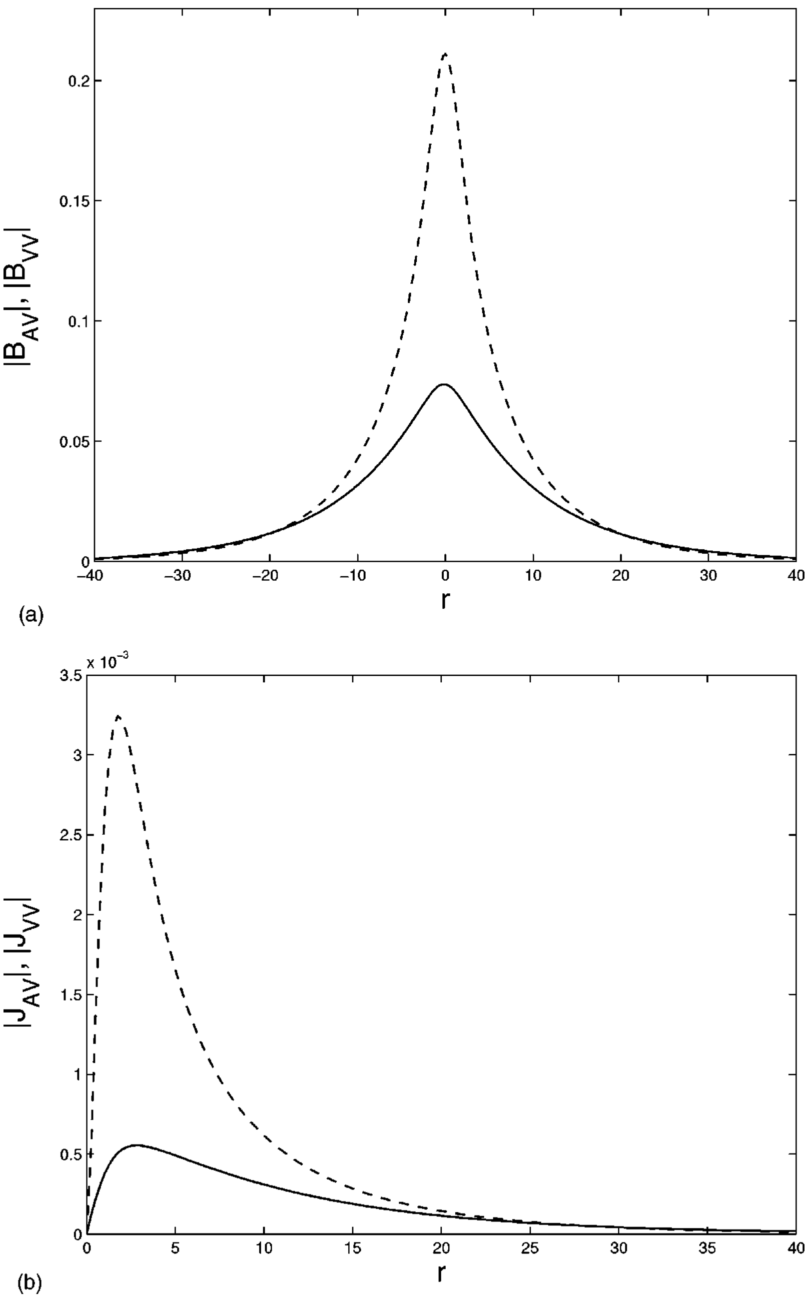

FIG. 1. (a) Magnetic-field dependence for single VV (solid line) and AV (dash line). (b) Azimuthal component of the superconducting current encircling the VV (solid line) and the AV (dash line).

$$\mathbf{J}_s = -\frac{i}{2\kappa^2}(\psi^{*a}\boldsymbol{\nabla}\psi^a - \psi^a\boldsymbol{\nabla}\psi^{*a}) - \frac{1}{\kappa^2}|\psi^a|^2\mathbf{A}, \qquad (5)$$

while the topological charge in this case has the form

$$P = \int_C \mathbf{A}\cdot dl = \frac{1}{2\pi}\sum_a \int_C |\psi^a|^2 \boldsymbol{\nabla}\Phi^a dl, \qquad (6)$$

where the phases are defined by $\psi^a = |\psi^a|\exp(i\Phi^a)$ and $\kappa = \delta/\xi$.

III. NUMERICAL SIMULATIONS OF PHASE TRANSITION

In order to study the topological defects in the p-wave superconductors the TDGL Eqs. (3) and (4) with $\beta = 1$ have been solved numerically. We start with the problem where superconductivity establishes itself in a normal domain in the bulk superconductor. The initial conditions, were chosen in the form

$$\boldsymbol{\psi}(\mathbf{r}, t=0) = \begin{cases} 0, & r < R-1 \\ \left(\frac{1}{\sqrt{2}}, \frac{i}{\sqrt{2}}, 0\right)\exp(iP\varphi)[r+1-R], & R-1 < r < R \\ \left(\frac{1}{\sqrt{2}}, \frac{i}{\sqrt{2}}, 0\right)\exp(iP\varphi), & r > R, \end{cases} \qquad (7)$$

$$\mathbf{A} = \{A_r; A_\varphi\}, \quad A_r(\mathbf{r}, t=0) = 0 \qquad (8)$$

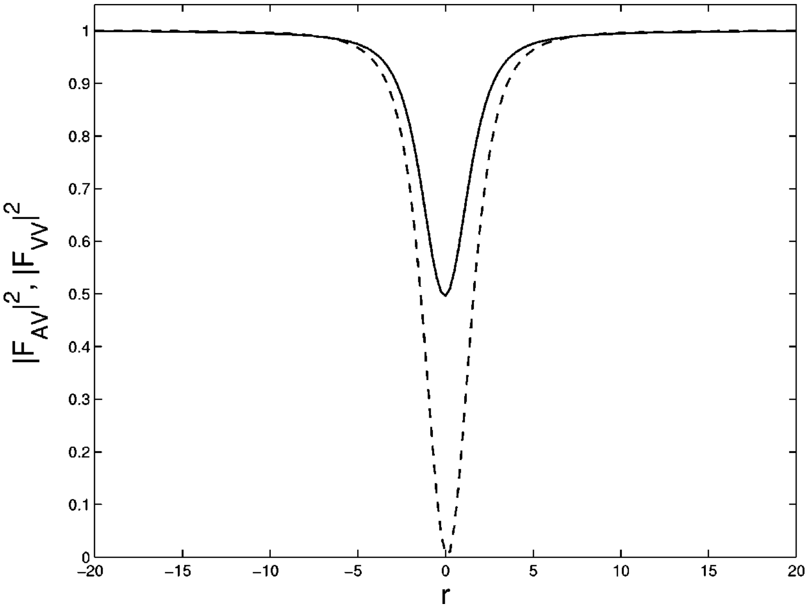

FIG. 2. Square of the absolute magnitude of the order parameter of the VV (solid line) and the AV (dash line).

$$A_\varphi(\mathbf{r},t=0)=\begin{cases}(H_0/2)r, & r<R\\ -H_0\kappa(K_1(r/\kappa)/K_0(R/\kappa))+P/r, & r>R,\end{cases} \tag{9}$$

where

$$H_0=\frac{(P/R)K_0(R/\kappa)}{\kappa K_1(R/\kappa)+(R/2)K_0(R/\kappa)}. \tag{10}$$

Here K_0 and K_1 are modified Bessel functions.

Thus, initially, we have a normal spot ($|\psi|=0$) of radius R with P quanta of magnetic flux trapped inside with temperature $T_m>T_c$, while outside the temperature is still lower than the critical one ($T_0<T_c$). The phase of the order parameter increases linearly to provide the total topological charge P. The magnetic field is uniform across the spot and decays outside[14] (see plot of our initial configuration in Ref. 8). The magnetic field at the sample edges is assumed to be zero. The boundary conditions for the order parameter and the vector potential $\mathbf{A}$ are imposed in the form

$$\left(\frac{\partial\psi^a}{\partial\mathbf{n}}-i\mathbf{A}\psi^a\right)_{boundary}=0,\quad (\boldsymbol{\nabla}\times\mathbf{A})_{boundary}=0. \tag{11}$$

The equations were discretized using the link variables of the lattice gauge theory[15] in a domain $L\times L$ with $L=401$. Computations were performed using the semi-implicit Crank-Nicholson method. The total number of runs was 50. The system evolves into a steady state in which one can identify well separated topological defects. Our main objective in this paper is to describe these objects.

The results of the numerical simulations (with $P=1$) are presented in Figs. 1–4. In Figs. 1(a), 1(b), and 2 the single vortex spatial profile of magnetic field, supercurrent (J_φ, the azimuthal component), and the order parameter $F^2=|\psi_1|^2+|\psi_2|^2+|\psi_3|^2$, respectively, are presented and compared with those of AV. It shows that a single vortex solution has rotational $SO_r(2)$ symmetry. Surprisingly, the profile of the order parameter F^2 of each of the numerous vortices is very different from those of AV (see Fig. 2). In particular F does not vanish in the center of the vortex. On the other hand a gauge invariant, but not SO(3) invariant, quantity like $|\psi_1|^2$ has no $SO_r(2)$ rotation symmetry [see Fig. 3(a)]. However, one can find an SO(3) transformation R such that one of its components $(R\boldsymbol{\psi})_3$ becomes $SO_r(2)$ invariant [see Fig. 3(b)]. The remaining two components constitute a two-dimensional vector, while the sum $|\psi_1|^2+|\psi_2|^2$ possesses $SO_r(2)$ invariance [Fig. 3(b)]. Thus, in the one vortex state the $SO_r(2)$ symmetry is preserved while the global SO(3) symmetry is completely broken. The VV solution is parametrized below using a set of rotation invariant functions.

IV. SINGLE VORTEX SOLUTION

We look for a solution of Eqs. (3) and (4) in the following $SO_r(2)$ symmetric ansatz in the gauge in which $\Phi^a=[0,\pi/2,\chi(r,\varphi)]$:

$$\psi^1=[c(r)\cos\varphi+d(r)\sin\varphi],$$

$$\psi^2=i[c(r)\sin\varphi-d(r)\cos\varphi],\quad \psi^3=f(r)\exp[i\chi(r,\varphi)], \tag{12}$$

$$A_x=a(r)\cos\varphi+b(r)\sin\varphi,\quad A_y=a(r)\sin\varphi-b(r)\cos\varphi. \tag{13}$$

Here φ is the azimuthal angle, $c(r),d(r),f(r),a(r),b(r)$ are even real functions of the radial coordinate, $\chi(r,\varphi)=\varphi$ at $r\geqslant\xi$.

Substituting the ansatz Eqs. (12) and (13) into Eq. (5), we obtain the components of the superconducting current

$$\kappa^2J_\varphi=-\frac{f^2}{r}-F^2br \tag{14}$$

and

$$\kappa^2J_r=F^2ar, \tag{15}$$

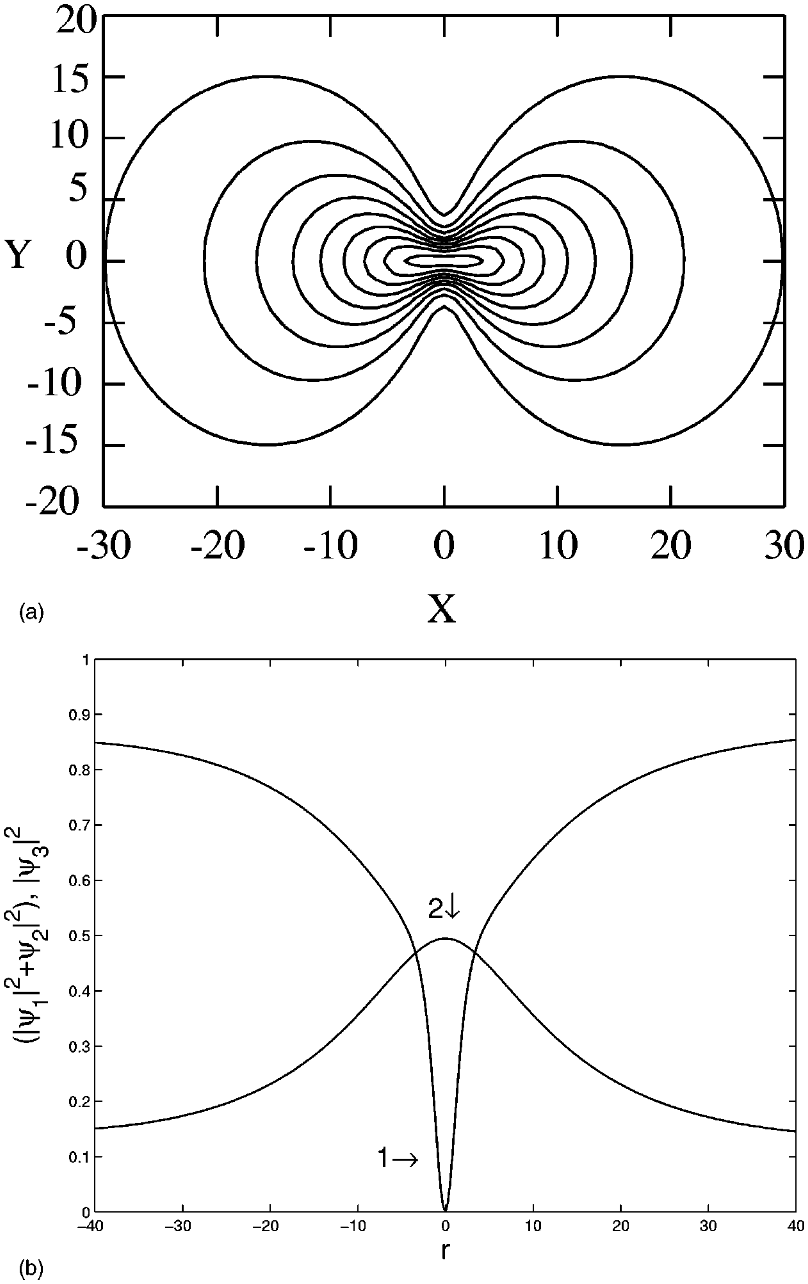

FIG. 3. (a) Contour lines of spatial distribution of $|\psi_1|^2$ show the absence of $SO_r(2)$ rotational symmetry. (b) Radial dependence of the $(|\psi_1|^2+|\psi_2|^2)$ (curve 1) and the $|\psi_3|^2$ (curve 2) functions after the R transformation.

where $F^2=(f^2+c^2+d^2)$. The necessary condition $J_r=0$ implies $A_r=0$ and hence $a(r)=0$.

It should be noted that the azimuthal angle in the phase exponent of the ψ_3 component of the ansatz coincides with that obtained numerically over the distances $1<r\ll\kappa$ where our analytical results have been obtained. Functions $c(\rho)$, $f(\rho)$, $d(\rho)$, and $b(\rho)$ at small $\rho=r/\kappa$ $(1\gg\rho\gg\kappa^{-1})$ behave as

$$c(\rho)\approx(c_0\rho+c_1\rho^3),\quad f(\rho)\approx f_0-f_1\rho^2,\quad b(\rho)\approx b_0\rho,$$

$$d(\rho)\approx(d_0\rho+d_1\rho^3). \tag{16}$$

Substituting these expansions into Eqs. (3) and (4) one obtains the lowest-order relation,

$$-2f_1+2f_0b_0-f_0+2f_0^3=0, \tag{17}$$

which is in excellent agreement with the result of the numerical simulation reading that for $\beta=1$ $f_1\approx0.01$, $f_0^2\approx0.49$, $b_0\approx0.07$ (see Fig. 2) [for arbitrary $\beta>0$, $f_0^2\approx(1+\beta)^{-1}$]. The ansatz solution (12) is degenerate with respect to the azimuthal angle and hence

$$c=g\cos\phi \quad\text{and}\quad d=-g\sin\phi \tag{18}$$

is also a solution for arbitrary angle ϕ. The angle ϕ indicates direction of the residual SO(2) global symmetry breaking. To conclude, the vortex solution is invariant under rotational $SO_r(2)$ but completely breaks SO(3). This is in contrast with

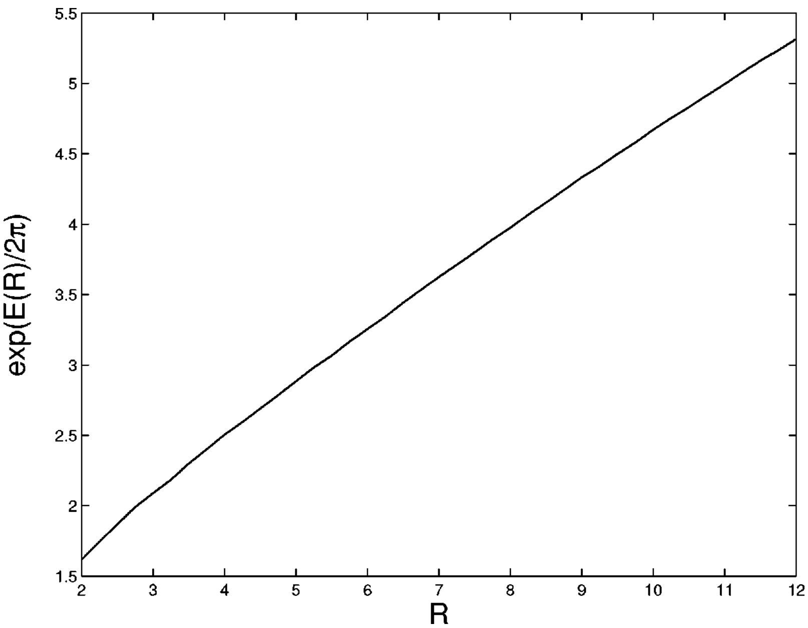

FIG. 4. The energy $E(R)$ of the VV has a logarithmic dependence on R.

the AV solution for which the SO(3) symmetry is broken down to its SO(2) subgroup only. To use a simple analogy with mechanics, the AV is like a symmetric top while the VV is similar to an asymmetric rigid body.

The energy of the VV is about three times smaller then that of the AV. The energy $E(R)$

$$E(R)=\int_0^{2\pi}d\varphi\int_0^R F\{\boldsymbol{\psi}(\mathbf{r}),\mathbf{A}\}r\,dr \tag{19}$$

of the VV is presented in Fig. 4. Here $F\{\boldsymbol{\psi}(\mathbf{r}),\mathbf{A}\}$ is the Hamiltonian density.

V. EFFECTS OF VV ON DYNAMICS AND CONCLUSION

In this work we described the vector vortex solution in the Ginzburg-Landau (GL) theory with O(3) symmetry. Dependence of the order parameter and the magnetic field for a single VV, presented in Figs. 1(a), 1(b), and 2, shows that the magnetic field in the center of the VV core is small and cannot destroy superconductivity, but decreases the magnitude of the order parameter only.

Now we discuss the available results on the influence of the VV on dynamics. It should be noted that during all the runs in our numerical simulation the dynamics of the symmetry-breaking phase transition in the multicomponent GL theory was dominated by the superconducting vector vortices. In particular we simulated the normal-superconducting phase transition and calculated the averaged number of nucleated topological defects $n(t)$ both in multicomponent model and for usual scalar (one-component) GL theory. Here

$$n(t)=\sum_{\omega_i} W(\omega_j)\int_{t_0(\omega_j)}^{t} n(j,t')dt', \tag{20}$$

where $t_0(\omega_j)$ is the initial time of the random process depending on random force in jth realization ($j_{\max}=20$ in our runs), $n(j,t')$ is the local time number of vortices in jth realization, and $W(\omega_j)$ is the distribution function of the random force ω_j (white noise). The results presented in Fig. 5 demonstrate that vector vortices are essentially dominated the usual Abrikosov vortices. The reason may be comparatively small energy and large entropy of the VV due to degeneracy of the SO(2) global symmetry. On a basis of this observation we conjecture that in any multicomponent gauge theory with a symmetry-breaking phase transition the dominant topological defects are maximally asymmetric. This is in contrast with most analytical investigations of topological defects in which high symmetry of defects is generally postulated (due to simplicity of their identification).

The nontrivial structure of the vortex core might have various static and dynamical implications on the physics of the p-wave superconductors. For example, the existence of a superconducting component in the core must significantly decrease the pinning force and thereby the transport properties.

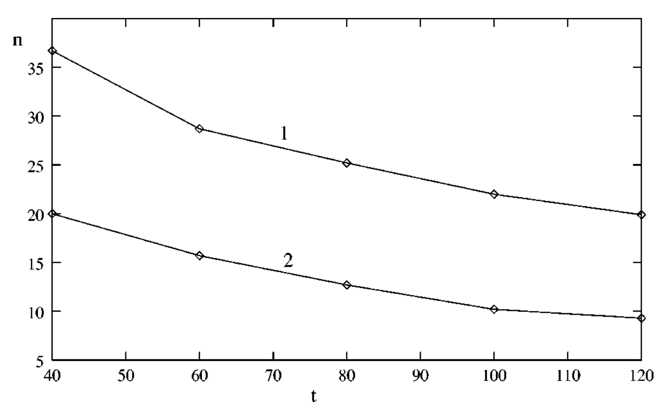

FIG. 5. The mean number of vortices and antivortices vs time for multicomponent (1) and usual scalar Ginzburg-Landau (2) models correspondingly. The initial topological charge $P=0$.

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