Estimation of the Contouring Error Vector for the Cross-Coupled Control Design

Syh-Shiuh Yeh and Pau-Lo Hsu

***Abstract*—In biaxial motion systems, applying the cross-coupled control (CCC) significantly improves contouring accuracy for linear and circular contours. As geometrical and parametric curves become more popular in modern manufacturing, machining processes with multiaxis motion systems are required, however, the available biaxial CCC cannot be directly applied to arbitrary contours with multiaxis machining systems. In this paper, we propose a novel approach for arbitrary contours by estimating the contouring error vector to efficiently determine the variable gains for CCC. Experimental results for a biaxial motion system indicate that the proposed approach efficiently yields variable gains similar to those in traditional CCC. Furthermore, results on a three-axis CNC machining center show that the present approach significantly improves motion accuracy in multiaxis motion systems.**

***Index Terms*—Cross-coupled control, contouring error, cross-coupling gains, multiaxis motion systems.**

I. Introduction

IN MACHINING processes, motion precision depends on both tracking and contouring accuracy. Traditionally, tracking accuracy was improved by applying feedback and feedforward control loops to each axis individually. Poo *et al.* [1] analyzed relations between the feedback controller and the contouring error and concluded that the matched dc gain in feedback control design improves contouring precision. Feedforward control loops are also common in motion control design because they efficiently reduce the servo lag and, thus, decrease the contouring error [2]–[5]. In addition to feedback and feedforward control loops, the cross-coupled control (CCC) structure, which considers the mutual dynamic effects among all axes, was developed by Koren [6] to further reduce the contouring error. Various improved CCC designs have since been proposed [7]–[9].

Recently, the variable-gain CCC was proposed by Koren and Lo [10], [11] to provide more precise contouring results by estimating the magnitude and the direction of contouring errors for further compensation. For arbitrary contour applications, the variable-gain CCC estimates the contouring error by applying the circular contour approximation and compensates each axis along the direction of estimated contouring error vector. The variable-gain CCC works well for biaxial motion systems, but it is difficult to apply the available CCC design to multiaxis machines that are gaining popularity in modern industries. Also, CCC needs an efficient algorithm to determine the variable gains of arbitrary contours in real time.

To improve contouring accuracy for multiaxis motion systems, Lo [12] proposed an approach by transforming the coordinate to obtain the moving basis to form a feedback controller for a 3-axis motion system. Chiu and Tomizuka [13] proposed the task coordinated approach by considering all axes as the first-order loops to obtain the feedback and the feedforward control loops. However, Lo's approach is difficult to be applied to more than three axes and its tracking accuracy of the controller without a feedforward control loop can be further improved. On the other hand, performance of a simplified first-order design including an unreliable plant model by Chiu and Tomizuka is, thus, inherently limited as verified in [14].

In this paper, we propose a modified variable-gain CCC design based on the contouring error vector by applying the linear contour approximation. The design can be directly extended to multiaxis motion systems. Theoretically, the contouring error vector is defined as a vector from the actual position to the nearest point on the contour. However, its computation is very complicated. In our approach, a vector from the actual position to the nearest point on the line that passes through the reference position tangentially is adopted if the tracking error is minimized. Finally, experimental results on a 3-axis machining center show that the variable gains in multiaxis CCC are more efficiently obtained and contouring accuracy of the CNC is significantly improved by applying the proposed multiaxis CCC design.

II. The Variable-Gain CCC

The biaxial motion control system with the variable-gain CCC is shown in Fig. 1. $(K_{px},\ K_{py})$ are position loop controllers, (P_x, P_y) represent the controlled plants within the position loops, $(X_r,\ Y_r)$ and $(X_a,\ Y_a)$ denote reference position and the actual position, respectively. $(E_x,\ E_y)$ are the axial errors of each axis, $\hat{\varepsilon}$ denotes the estimated contouring error, $(C_x,\ C_y)$ are varying cross-coupling gains which depend on the tool path trajectory and K_c is the cross-coupled controller.

Since an arbitrary contour can be approximated by a circular contour, let ρ be the radius of curvature at the reference position R and θ be the traversal angle of a circular motion. The estimated contouring error $\hat{\varepsilon}$ is obtained as

$$\hat{\varepsilon} = \sqrt{(X_a - X_o)^2 + (Y_a - Y_o)^2} - \rho \tag{1}$$

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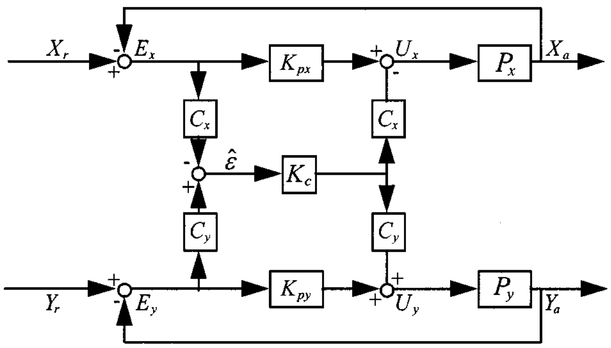

Fig. 1. Biaxial motion control system with variable-gain CCC [10], [11].

$(X_o,\ Y_o)$ denotes the center of curvature. Because the actual position $(X_a,\ Y_a)$ can be represented as

$$X_a = \rho \sin\theta + X_o - E_x \tag{2}$$
$$Y_a = -\rho\cos\theta + Y_o + E_y \tag{3}$$

by substituting (2) and (3) into (1), the estimated contouring error $\hat{\varepsilon}$ becomes

$$\hat{\varepsilon} = \sqrt{(\rho\sin\theta - E_x)^2 + (\rho\cos\theta + E_y)^2} - \rho. \tag{4}$$

If the radius of curvature ρ is large enough and the contouring error ε is much smaller than the axial errors $(E_x,\ E_y)$, the estimated contouring error $\hat{\varepsilon}$ can be approximated by the Taylor's expansion of (4), which is

$$\hat{\varepsilon} = \left(\cos\theta + \frac{E_y}{2\rho}\right)E_y - \left(\sin\theta - \frac{E_x}{2\rho}\right)E_x. \tag{5}$$

The cross-coupling gains $(C_x,\ C_y)$ are then

$$C_x = \sin\theta - \frac{E_x}{2\rho} \tag{6}$$
$$C_y = \cos\theta + \frac{E_y}{2\rho}. \tag{7}$$

For a linear contour, the radius of curvature ρ is infinite and the corresponding cross-coupling gains are

$$C_x = \sin\theta \tag{8}$$
$$C_y = \cos\theta \tag{9}$$

where θ denotes the incline angle of the linear contour.

For a circular contour, the cross-coupling gains are the same as (6) and (7), except θ denotes the circular contour traversal angle and ρ is replaced by the fixed radius r of the circular contour as

$$C_x = \sin\theta - \frac{E_x}{2r} \tag{10}$$
$$C_y = \cos\theta + \frac{E_y}{2r}. \tag{11}$$

In general, all curve contours can be approximated by circular contours with different radii.

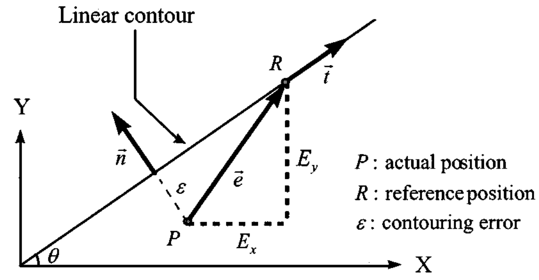

Fig. 2. Linear contour representation.

III. Biaxial Contouring Error Vector

A. Estimation of the Contouring Error Vector

In this section, we propose an estimation approach of the contouring error vector for obtaining the variable-gain vector $\vec{C} = \begin{bmatrix} -C_x \\ C_y \end{bmatrix}$ in the biaxial CCC. This approach will be extended to multiaxis motion systems as in Section IV. Consider the linear contour as shown in Fig. 2. θ is the incline angle of the linear contour. The normalized tangential vector of the linear contour is $\vec{t} = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$. $\vec{n} = \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$ is the normalized normal vector which is perpendicular to tangential vector $\vec{t} \cdot (E_x,\ E_y)$ are the axial errors. The tracking error vector is $\vec{e} = \begin{bmatrix} E_x \\ E_y \end{bmatrix}$. ε is the contouring error. P and R denote the actual and the reference position, respectively. The contouring error ε can be directly obtained as

$$\varepsilon = \cos\theta \cdot E_y - \sin\theta \cdot E_x = \begin{bmatrix} E_x \\ E_y \end{bmatrix}^T \cdot \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix} = \langle \vec{e},\ \vec{n}\rangle \tag{12}$$

where, $\langle\cdot,\cdot\rangle$ is the inner product operator.

Define the contouring error vector $\vec{\varepsilon}$ as a vector from the actual position P to the nearest point on the contour trajectory as

$$\vec{\varepsilon} = \varepsilon \cdot \vec{n} = \langle \vec{e},\ \vec{n}\rangle \cdot \vec{n}. \tag{13}$$

The contouring error vector $\vec{\varepsilon}$ is, thus, a vector with contouring error ε and direction $\vec{n}$. Furthermore, the contouring error ε is the inner product of tracking error vector $\vec{e}$ and the normalized normal vector $\vec{n}$, as shown in (12).

B. Determination of the Variable Gains

Comparing the cross-coupling gain vector $\vec{C} = \begin{bmatrix} -C_x \\ C_y \end{bmatrix}$ in (8)–(9) and the normalized normal vector $\vec{n} = \begin{bmatrix} n_x \\ n_y \end{bmatrix}$, note that the cross-coupling gain vector $\vec{C}$ contains the corresponding element of the normalized normal vector $\vec{n}$, represented as

$$\vec{n} = \begin{bmatrix} n_x \\ n_y \end{bmatrix} = \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix} = \begin{bmatrix} -C_x \\ C_y \end{bmatrix} = \vec{C}. \tag{14}$$

The contouring error vector $\vec{\varepsilon}$ of a linear command can be directly obtained by (13). For arbitrary contour applications, the geometric relations among the desired contour, the actual position P and the reference position R in a biaxial motion systems are as shown in Fig. 3. In the present approach, the estimated

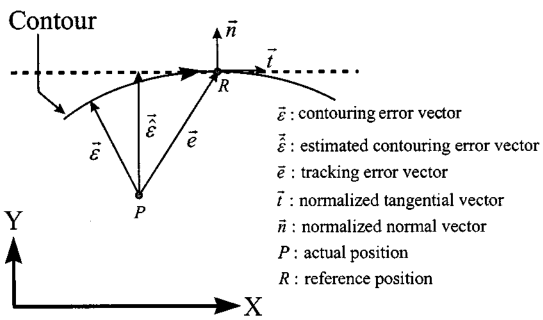

Fig. 3. Geometrical relations of biaxial motion systems [15].

contouring error vector $\vec{\hat{\varepsilon}}$ is defined as the vector from the actual position to the nearest point on the line that passes through the reference position tangentially with direction $\vec{t}$. Note that the estimated contouring error vector $\vec{\hat{\varepsilon}}$ approximates the contouring error vector $\vec{\varepsilon}$ well only with small tracking error [15].

Since the direction of the estimated contouring error vector $\vec{\hat{\varepsilon}}$ is parallel to the normalized normal vector $\vec{n}$ at the reference position R, the magnitude of $\vec{\hat{\varepsilon}}$ denoted as $\hat{\varepsilon}$ is, thus, defined as the inner product of the tracking error vector $\vec{e}$ and the normalized normal vector $\vec{n}$.

Let the tangential vector $\vec{t}$ and the normalized normal vector $\vec{n}$ be $\vec{t} = \begin{bmatrix} t_x \\ t_y \end{bmatrix}$ and $\vec{n} = \begin{bmatrix} n_x \\ n_y \end{bmatrix}$, respectively. The vector $\vec{n}$ can be directly derived as

$$\vec{n} = \begin{bmatrix} n_x \\ n_y \end{bmatrix} = \begin{bmatrix} \dfrac{-t_y}{\sqrt{t_x^2 + t_y^2}} \\ \dfrac{t_x}{\sqrt{t_x^2 + t_y^2}} \end{bmatrix} \tag{15}$$

and the estimated contouring error $\hat{\varepsilon}$ is

$$\hat{\varepsilon} = \langle \vec{e},\, \vec{n} \rangle . \tag{16}$$

The estimated contouring error vector $\vec{\hat{\varepsilon}}$ is then expressed as

$$\vec{\hat{\varepsilon}} = \hat{\varepsilon} \cdot \vec{n} = \langle \vec{e},\, \vec{n} \rangle \cdot \vec{n}. \tag{17}$$

By comparing (17) and (13), the cross-coupling gains $(-C_x,\, C_y)$ can be replaced by the elements of the normalized normal vector $\vec{n} = (n_x,\, n_y)$, as shown in (14).

IV. Multiaxis Contouring Error Vector

A. Estimation in Multiple-Dimensional Space

Consider the geometric relations among the arbitrary desired contour, the actual position P and the reference position R in three-dimensional space as shown in Fig. 4. Because it is difficult to obtain the contouring error vector $\vec{\varepsilon}$ exactly, we adopt the estimated contouring error vector $\vec{\hat{\varepsilon}}$ for obtaining the variable gains in the proposed multiaxis CCC. As shown in Fig. 4, the estimated contouring error vector $\vec{\hat{\varepsilon}}$ lies on the plane expanded by the tracking error vector $\vec{e}$ and the normalized tangential vector $\vec{t}$ and perpendicular to the normalized tangential vector $\vec{t}$. In fact, as the tracking error $\|\vec{e}\|$ is small enough, the contouring error vector $\vec{\varepsilon}$ can be closely approximated by the estimated contouring error vector $\vec{\hat{\varepsilon}}$. Define the normalized estimated contouring error vector $\vec{n}$

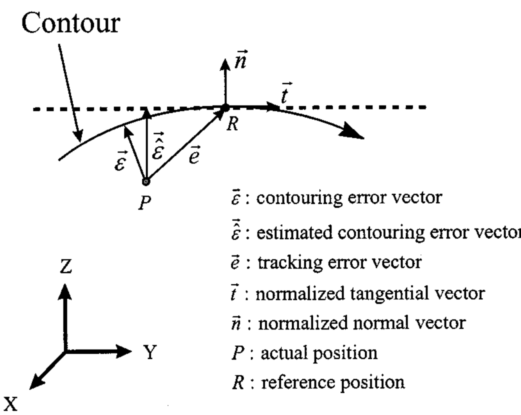

Fig. 4. Geometrical relations of 3-axis motion control systems.

$$\vec{n} = \alpha_1 \vec{t} + \alpha_2 \vec{e} \tag{18}$$

where

$$\langle \vec{n}, \vec{t} \rangle = 0 \tag{19}$$

$$\|\vec{n}\| = 1 \text{ or } \langle \vec{n}, \vec{n} \rangle = 1 \tag{20}$$

$$\|\vec{t}\| = 1 \tag{21}$$

and $\langle \cdot, \cdot \rangle$ is an inner product operator and $\|\cdot\|$ is a 2-norm operator. The relation between α_1 and α_2 can be derived from (19)–(21) as

$$\alpha_1 = -\alpha_2 \cdot \langle \vec{e},\, \vec{t} \rangle . \tag{22}$$

By substituting (22) into (20), α_1 and α_2 are obtained as

$$\alpha_1 = \mp \frac{\langle \vec{e},\, \vec{t} \rangle}{\sqrt{\|\vec{e}\|^2 - \langle \vec{e},\, \vec{t} \rangle^2}}$$

$$\alpha_2 = \pm \frac{1}{\sqrt{\|\vec{e}\|^2 - \langle \vec{e},\, \vec{t} \rangle^2}}$$

in which the signs of α_1 and α_2 determine the direction of the normalized estimated contouring error vector $\vec{n}$. Because the angle between the normalized estimated contouring error vector $\vec{n}$ and the tracking error vector $\vec{e}$ is in general within $[-90^\circ, +90^\circ]$, the following condition holds:

$$\langle \vec{n},\, \vec{e} \rangle \geq 0. \tag{23}$$

From (23), α_1 and α_2 can be further determined as

$$\alpha_1 = -\frac{\langle \vec{e},\, \vec{t} \rangle}{\sqrt{\|\vec{e}\|^2 - \langle \vec{e},\, \vec{t} \rangle^2}} \tag{24}$$

$$\alpha_2 = \frac{1}{\sqrt{\|\vec{e}\|^2 - \langle \vec{e},\, \vec{t} \rangle^2}}. \tag{25}$$

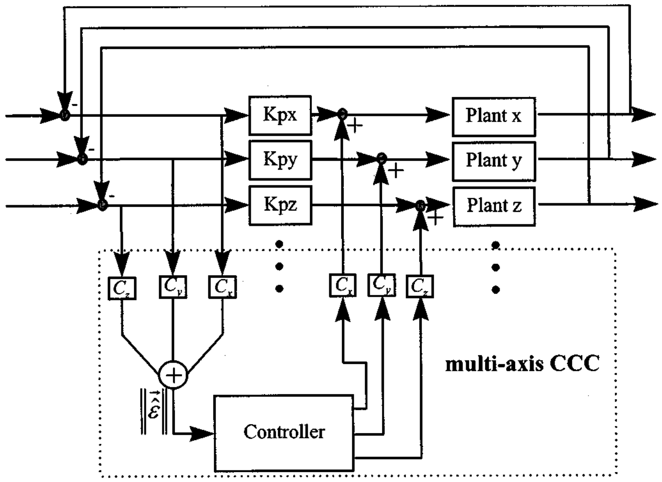

Fig. 5. Multiaxis motion control systems with multiaxis CCC.

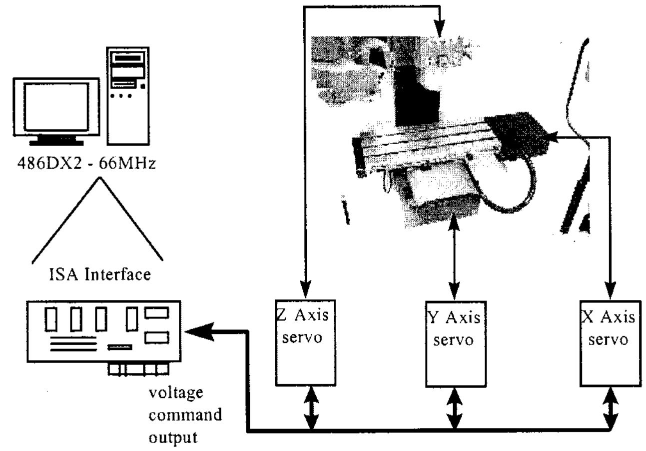

Fig. 6. Experimental setup.

B. Determination of Multiaxis Variable Gains

As shown in (18) and Fig. 4, the magnitude of the estimated contouring error vector $\left\|\vec{\hat{\varepsilon}}\right\|$ is the inner product of the normalized estimated contouring error vector $\vec{n}$ and the tracking error vector $\vec{e}$

$$\left\|\vec{\hat{\varepsilon}}\right\| = \langle \vec{n},\, \vec{e} \rangle . \tag{26}$$

The estimated contouring error vector $\vec{\hat{\varepsilon}}$ is, thus, obtained as

$$\vec{\hat{\varepsilon}} = \left\|\vec{\hat{\varepsilon}}\right\| \cdot \vec{n} = \langle \vec{n},\, \vec{e} \rangle \cdot \vec{n}. \tag{27}$$

Following our analysis of the biaxial variable-gain CCC in Section II, the magnitude of the estimated contouring error vector $\left\|\vec{\hat{\varepsilon}}\right\|$ is modulated by a controller and then is used to compensate for each axis along the direction of the estimated contouring error vector. The compensation direction for each axis is suitably determined by the cross-coupling gain vector in the present variable-gain CCC. Therefore, the cross-coupling gains can be obtained directly from the elements of the normalized estimated contouring error vector. Let the normalized estimated contouring error vector be $\vec{n} = [n_x \quad n_y \quad n_z \quad \ldots]^T$. The cross coupling gains $(C_x, \quad C_y, \quad C_z \quad \ldots)$ are, thus, directly determined as

$$C_i = n_i, \qquad i = x,\, y,\, z, \ldots .$$

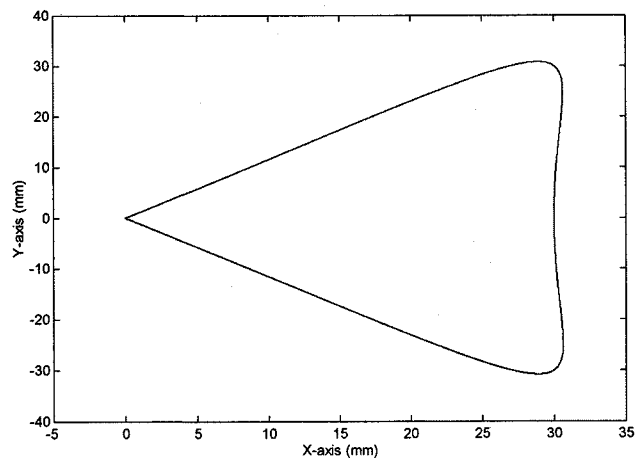

Fig. 7. Contour trajectory used in the biaxial experiments.

The proposed multiaxis CCC motion control system is shown in Fig. 5 and (Kpx, Kpy, Kpz) denote the position loop controllers for each axis of the motion system. From (21), we have

$$\begin{aligned} \|\vec{e}\|^2 - \langle \vec{e},\, \vec{t} \rangle^2 &= \|\vec{e}\|^2 - \left\|\vec{t}\right\|^2 \cdot \|\vec{e}\|^2 \cdot \cos^2\theta \\ &= \|\vec{e}\|^2 \left(1 - \cos^2\theta\right) \\ &= \|\vec{e}\|^2 \cdot \sin^2\theta \geq 0 \end{aligned}$$

where θ is the angle between the tracking error vector $\vec{e}$ and the normalized tangential vector $\vec{t}$. The singularity condition occurs when the tracking error vector $\vec{e}$ is parallel to the normalized tangential vector $\vec{t}$. In practice, one should check that $(\|\vec{e}\|^2 - \langle \vec{e},\, \vec{t} \rangle^2) \geq \delta > 0$ holds to avoid the singularity.

$$P_1\left(z^{-1}\right) = \frac{-0.0056z^{-1} + 0.0421z^{-2} + 0.1213z^{-3} + 0.0922z^{-4}}{1 - 1.1087z^{-1} - 0.2199z^{-2} + 0.1578z^{-3} + 0.0452z^{-4} + 0.1484z^{-5} - 0.0228z^{-6}}$$

$$P_2\left(z^{-1}\right) = \frac{-0.0015z^{-1} + 0.0445z^{-2} + 0.1251z^{-3} + 0.0586z^{-4}}{1 - 1.2360z^{-1} - 0.1549z^{-2} + 0.2173z^{-3} + 0.0723z^{-4} + 0.2628z^{-5} - 0.1616z^{-6}}$$

$$P_3\left(z^{-1}\right) = \frac{-0.0150z^{-1} + 0.0522z^{-2} + 0.1404z^{-3} - 0.0207z^{-4}}{1 - 1.6425z^{-1} + 0.3038z^{-2} + 0.4394z^{-3} - 0.1691z^{-4} + 0.1897z^{-5} - 0.1213z^{-6}}$$

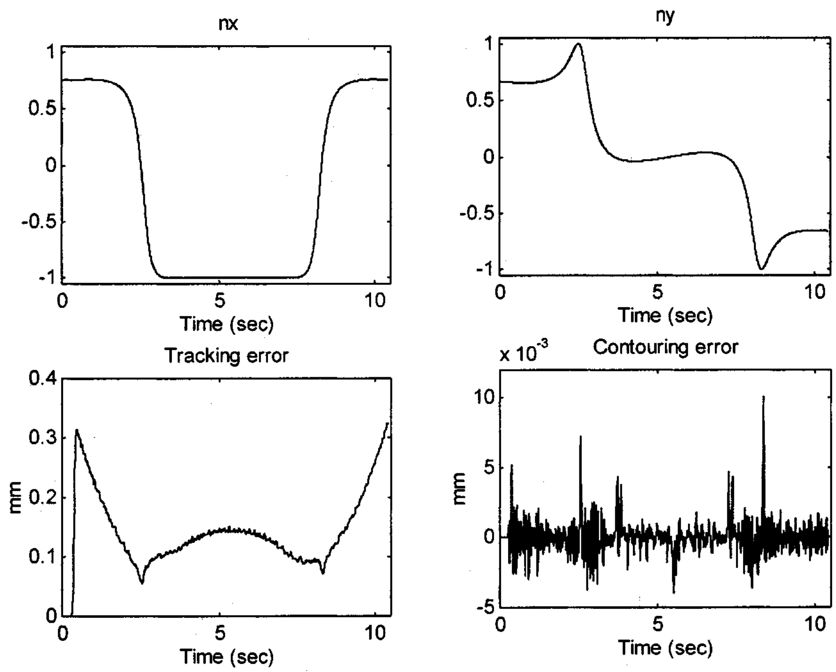

Fig. 8. Experimental results for the proposed variable-gain CCC.

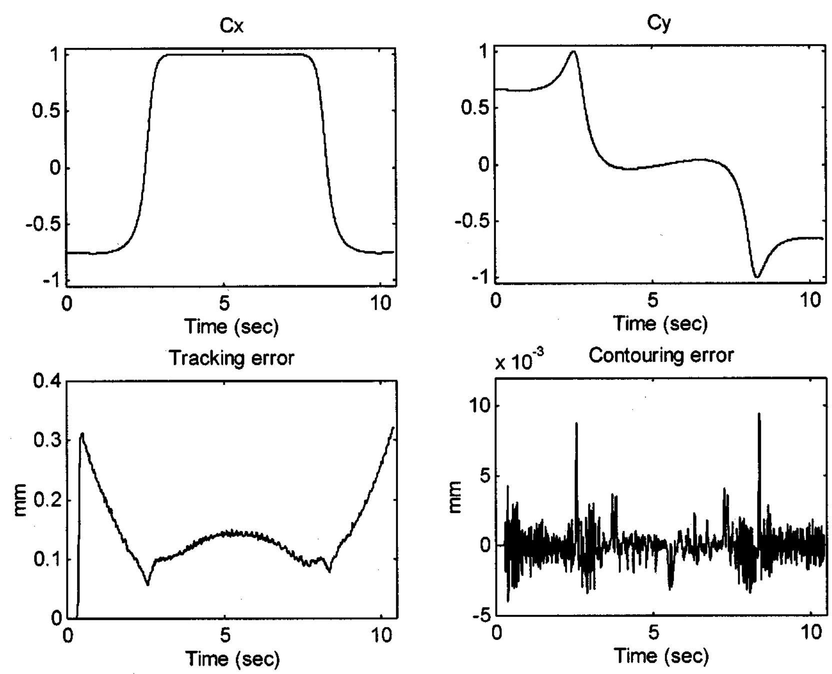

Fig. 9. Experimental results for the original variable-gain CCC.

V. Experiments

The experimental setup of a three-axis DYNA 1007 CNC machining center is shown in Fig. 6. A PC-486 generated the main control commands and recorded the signals including: the input commands for different contours, the implementation of a variable-gain CCC controller and the control inputs to the velocity loop of the AC servo motors. The PC-486 interface utilized an AD/DA card to send and receive the control inputs and position output, respectively, at a sampling period of 1 ms.

To identify the controlled plant for each axis, the axial control input was given a pseudorandom binary sequence (PRBS) and the velocity control loops of the three axes was obtained using the ARX model as shown in the equations at the bottom of the previous page. To achieve both stable motion and matched gains for the uncoupled system [1], the feedback loop proportional gains (K_{p1}, K_{p2}, K_{p3}) were chosen as

$$K_{p1} = 0.07, \quad K_{p2} = 0.0694, \quad K_{p3} = 0.0665.$$

To achieve significantly reduced tracking errors in applying the proposed approach, the optimal zero phase error tracking control (ZPETC) was also applied to each axis [5].

TABLE I
EXPERIMENTAL RESULTS FOR THE BIAXIAL MOTION CONTROL SYSTEM

Performance / CCC	Contouring error (*mm*)				Tracking error (*mm*)			
	Max	*Mean*	*IAE*	*RMS*	*Max*	*Mean*	*IAE*	*RMS*
Proposed variable-gain CCC	0.0101	1.4 × 10^{-6}	1.4874	0.0011	0.3243	0.1402	291.52	0.1536
Original variable-gain CCC [10-11]	0.0094	-3.82 × 10^{-6}	1.6221	0.0012	0.3212	0.1404	292.04	0.1536

A. Verification in Biaxial Motion

The well-tuned PID cross-coupled controller K_c for the motion control system is chosen to be [14]

$$K_c\left(z^{-1}\right) = \frac{1.1935 - 2.1525z^{-1} + 0.98z^{-2}}{1 - z^{-1}}.$$

The contour trajectory of the motion control system is shown in Fig. 7. The average speed is 853 mm/min, the maximum speed is 2500 mm/min and the minimum speed is 250 mm/min. Experimental results obtained by applying the proposed variable-gain CCC approach and the original variable-gain CCC are also shown in Figs. 8 and 9, respectively. Fig. 8 shows the cross-coupling gains ($n_x \quad n_y$), the tracking error and the measured contouring error of the proposed approach. Fig. 9 shows the cross-coupling gains ($C_x \quad C_y$), the tracking error, and the contouring error as in [10], [11]. Results are summarized in Table I.

As shown in Figs. 8, 9, and Table I, the experimental results for our proposed variable-gain CCC are similar to those of the original variable-gain CCC by Koren and Lo [10], [11] except for the opposite sign of cross-coupling gain n_x and C_x, as indicated in (14). Compared to the operators for obtaining the variable gains for arbitrary contours, Table II indicates that implementing the proposed cross-coupling gains $(n_x,\ n_y)$ requires fewer operators than implementing $(C_x,\ C_y)$ in the original variable-gain CCC with circular approximation.

B. Application to 3-Axis Motion

The robust cross-coupled controller K_c is designed by applying the quantitative feedback theory (QFT) algorithm [16], [17] as

$$K_c\left(z^{-1}\right) = \frac{0.05 - 0.09z^{-1} + 0.040\,375z^{-2}}{1 - 1.03z^{-1} + 0.0302z^{-2} - 0.0002z^{-3}}.$$

The position commands for each axis of the 3-axis motion control system are shown in Fig. 10. The commands perform an inclined circular contour with an 18.75 mm radius at a speed of 600 mm/min. The cross-coupling gains shown in Fig. 11 are the direction components of the estimated contouring error vector. The experimental results for the 3-axis CCC system are compared to those for the control system without CCC in Fig. 12 and in Table III. Because of the friction effect, results of the cross-coupling gains $(C_x,\ C_y,\ C_z)$ are discontinuous as the slip-stick phenomenon occurs as speed of any axis approaches

TABLE II
NUMBER OF OPERATORS USED IN IMPLEMENTATION OF THE CROSS-COUPLING GAINS

CCC / Operator	present variable-gain CCC	variable-gain CCC [10-11]
$+,\ -$	2	4
×	4	7
÷	2	3
$\sqrt{\ }$	1	1
sin, cos, abs	-	3
1st derivative	1	1
2nd derivative	-	1

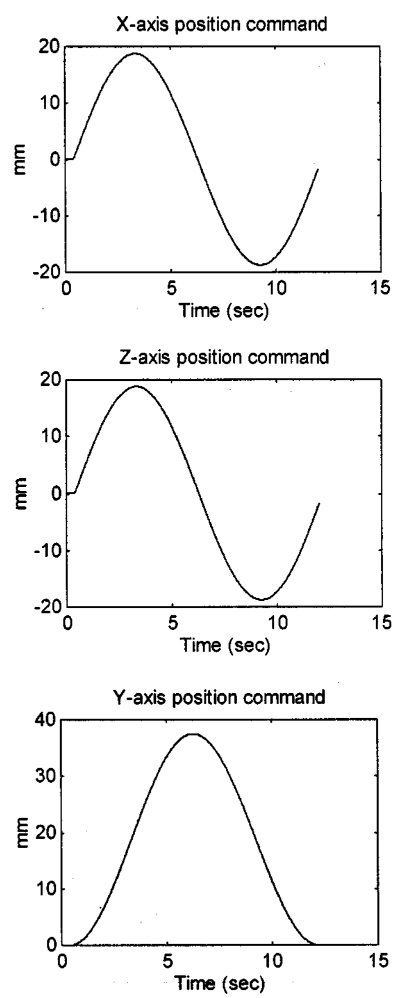

Fig. 10. The circular contour commands of the 3-axis CNC.

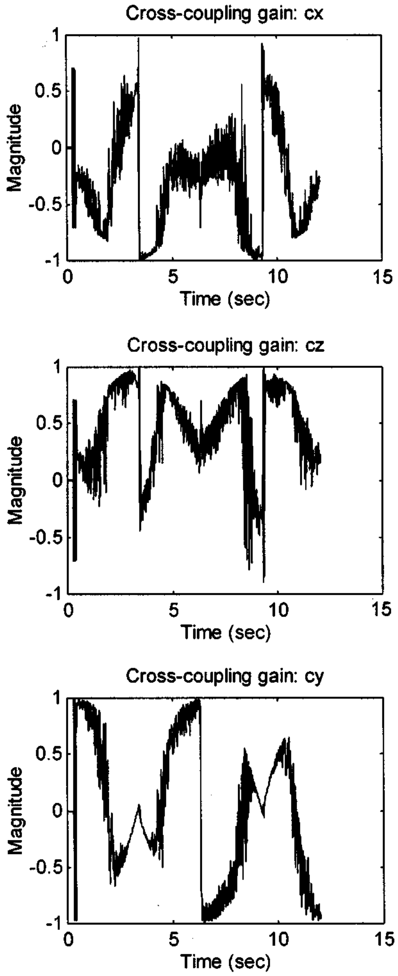

Fig. 11. Cross coupling gains of the 3-axis CNC.

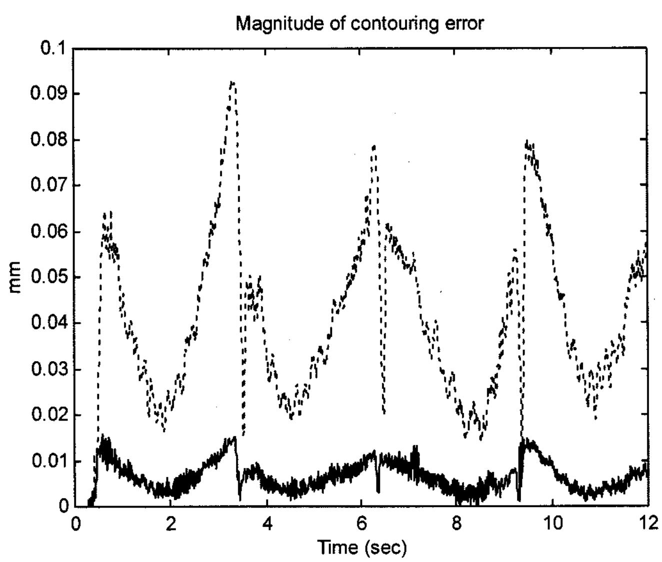

Fig. 12. Experimental results of the 3-axis motion control system.

zero. Fig. 12 and Table III indicate that the proposed 3-axis CCC effectively improves contouring accuracy. Note that the proposed algorithm can be directly applied to multiaxis motion systems.

TABLE III
EXPERIMENTAL RESULTS FOR THE 3-AXIS MOTION CONTROL SYSTEM

Performance index / Control system	Magnitude of contouring error (*mm*)			
	Max	*Mean*	*IAE*	*RMS*
without CCC	0.090	0.040	78.632	0.043
with CCC	0.016	0.006	12.669	0.007

VI. CONCLUSIONS

Although the CCC is known to effectively improve contouring accuracy in motion systems, it is difficult to apply the original biaxial variable gains to multiaxis CCC. Also, the original CCC is inefficient to obtain the variable gains for arbitrary contours. In this paper, we developed a multiaxis variable-gain CCC design which is based on the contouring error vector approach. Experimental results for a biaxial motion system show that the proposed CCC approach and the original variable-gain CCC obtain similar variable gains. However, the present approach is more efficient. The proposed multiaxis CCC system was also applied to a 3-axis CNC machining center. Experimental results indicate that with the proposed variable gains, the multiaxis CCC control significantly improves contouring accuracy.

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