Optical Coating Designs Using the Family Competition Evolutionary Algorithm

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Abstract

A robust evolutionary approach, called the *Family Competition Evolutionary Algorithm* (FCEA), is described for the synthesis of optical thin-film designs. Based on family competition and adaptive rules, the proposed approach consists of global and local strategies by integrating decreasing mutations and self-adaptive mutations. The method is applied to three different optical coating designs with complex spectral quantities. Numerical results indicate that the proposed approach performs very robustly and is very competitive with other approaches.

Keywords

Evolutionary algorithms, family competition, infrared antireflection coating, multiple mutations, thin-film coatings, tristimulus filter.

1 Introduction

The optical thin-film coating is a vital technology in the field of modern optics. It can be broadly described as follows: any device or material deliberately used to change the spectral intensity distribution or the state of polarization of the electromagnetic radiation incident on it in order to satisfy performance specification and some constraints (Macleod, 1986; Dobrowolski, 1995). Optical thin-film coatings have numerous remarkable applications in many branches of science and technology, such as scientific instrument manufacturing, spectroscope, medicine, and astronomy (Dobrowolski, 1997).

Many different approaches have been proposed for designing optical coatings. They roughly include analytical, graphical, and numerical methods (Macleod, 1986; Dobrowolski, 1995). The numerical methods are particularly powerful because they can be applied to the design of coatings with complicated properties. In most numerical methods the design of optical coatings is formulated as an optimization problem based on the use of merit functions, which are often extremely difficult due to the large number of local minimum (Dobrowolski, 1989).

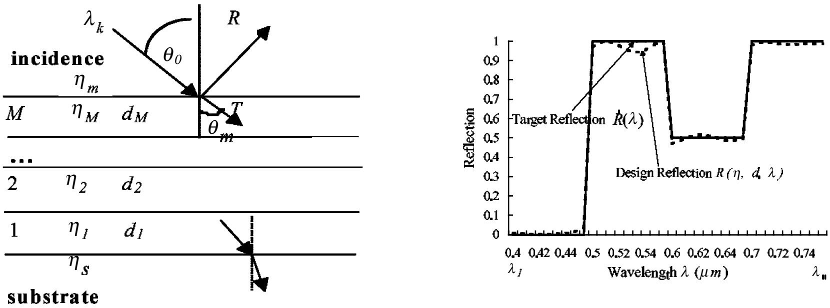
The two basic approaches to the design of numerical optical coatings are refinement methods (Aguilera et al., 1988; Dobrowolski and Kemp, 1990) and synthesis methods (Dobrowolski, 1986; Li and Dobrowolski, 1992; Tikhonravov, 1993; Bovard, 1988). Refinement methods normally require a starting design, which is gradually modified to a desired solution. The quality of the solution is highly dependent on the start point. Unfortunately, good starting designs are not readily available for many modern design problems. Choosing a good starting design is often a time-consuming and difficult task in a complex coating system. Contrary to refinement methods, synthesis methods automatically generate their own starting designs and, due to poor solution quality, are usually combined with numerical refinement methods. Therefore, developing a good synthesis method is an important research topic.

Recently, evolutionary algorithms (EAs) have been successfully applied to several problems encountered in optical filters and coatings that are inherently computationally complex (Eisenhammer et al., 1993; Martin et al., 1995; Greiner, 1996; Yang and Kao, 1998). EAs are an adaptable concept for problem solving and especially well suited for solving difficult optimization problems. They have been used to solve problems involving large search spaces, where traditional optimization methods are less efficient.

Genetic algorithms (GAs) (Goldberg, 1989), evolution strategies (ES) (Bäck et al., 1991), and evolutionary programming (EP) (Fogel, 1995) are independently developed but have related implementations. For GAs, the coding function of standard binary codes may introduce an additional multimodality, making the combined objective function more complex than the original function. To achieve better performance, gray-coded and real-coded GAs have been introduced (Eshelman and Schaffer, 1993; Mühlenbein and Schlierkamp-Voosen, 1993; Deb and Agrawal, 1995). In contrast, ESs and EP use mainly real-valued representation and focus on self-adaptive Gaussian mutations in the design of optical coatings (Greiner, 1996; Bäck and Schütz, 1995; Schutz and Sprave, 1996). This type of mutation has succeeded in continuous optimization and has been widely regarded as a good operator for local searches. Unfortunately, experiments show that self-adaptive Gaussian mutation leaves individuals trapped near local optima for rugged functions (Yang and Kao, 2000b).

Many modifications have been proposed to improve solution quality and to speed up convergence in EAs. In particular, one trend is to incorporate local search techniques into EAs. Martin et al. (1995) combined a gradient method into a real-coded genetic algorithm for synthesizing three complex inhomogeneous problems. This hybrid approach may make a better tradeoff between computational cost and the global optimality of the solution. However, for existing methods, local search techniques and genetic operators often work separately during the search process.

In this paper, we propose the *Family Competition Evolutionary Algorithm* (FCEA) to synthesize optical thin-film systems with various numbers of layers. The FCEA combines decreasing-based Gaussian mutation, self-adaptive Gaussian mutation, and self-adaptive Cauchy mutation. The FCEA will incorporate family competition (Yang et al., 1997) and adaptive rules to construct the relationship between mutations, whose performance heavily depends on the same factor called *step size*. The self-adaptive mutations adapt the step sizes with a stochastic mechanism. On the other hand, decreasing-based mutations decrease the step sizes with a fixed rate γ, where $\gamma < 1$. In order to balance exploration and exploitation, these mutation operators are designed to cooperate. The family competition is inspired from the $(1 + \lambda)$-ES (Bäck, 1996) and is similar



(a) The construction parameters (see text) of an optical coating system

(b) Target specification and the performance of a real coating system

Figure 1: (a) The construction parameters of a coating system and (b) profiles of the target specification and a real coating system.

to a local search procedure. The FCEA was successfully applied to solve flexible ligand docking (Yang and Kao, 2000a) and to train neural networks (Yang et al., 2000).

We illustrate features of our FCEA using three different thin-film design problems. The first is an infrared antireflection coating problem. Over 60 published solutions to this problem exist (Dobrowolski et al., 1996). Furthermore, we studied a filter with 0.0, 0.5, and 1.0 transmission regions within the $0.4 - 0.75\mu$m wavelength range. This problem was used to compare the FCEA to several synthesis methods (Dobrowolski, 1986; Li and Dobrowolski, 1992) on the performance and computation speed. The final problem is the synthesis of a tristimulus filter matching the CIE$_{\bar{x}_\lambda}$ used in the tristimulus colorimeters (Sullivan and Dobrowolski, 1996). Experimental results demonstrated that the FCEA is an encouraging synthesis approach for optical thin-film coatings.

This paper is organized as follows. Section 2 describes the problem of optical thin-film coatings. Section 3 introduces the evolutionary nature of the FCEA. In Section 4, three syntheses of thin-film optical coatings are presented to illustrate the performance of the FCEA. We also compare the FCEA with various approaches on these problems. Section 5 investigates the main characteristics of the FCEA. We conclude in Section 6.

2 Problem Definition

The goal in designing optical multilayer coatings is to find the construction parameters of systems that satisfy the desired optical specification. The construction parameters include the number of layers M, the thickness d, and refractive indices η of the medium, substrate, and layers. In general, a coating method finds the thickness $(d_1, \cdots, d_M)$ and indices $(\eta_l, \cdots, \eta_M)$ of layers of a M-layer system shown in Figure 1(a).

To design a multilayer coating system, it is necessary to specify the desired optical requirement, which is often defined as the target transmittance $\hat{T}$ or target reflectance $\hat{R}$ at a number of wavelengths in the interesting spectral region. $\hat{T}$ equals $1 - \hat{R}$ if the materials of a multilayer coating system are all nonabsorbing. Designing a thin-film system involves finding the number of layers M, the refractive indices η_j, and the thickness d_j of the jth layer, in order to match closely with the specified performance, where $1 \leq j \leq M$. A coating system is called a *normal-incidence coating* if the incident angle θ_0 is zero (see Figure 1(a)). Otherwise it is called an *oblique-incidence coating*. The dashed line in Figure 1(b) indicates a designed reflection. In this paper, we consider normal-incidence coating and nonabsorbing materials.

Let the spectral reflectance of the M-layer system shown in Figure 1(a) be denoted as $R(\eta, d, \lambda)$, where λ is the wavelength region of interest. The desired spectral reflectance profiles are fitted by minimizing a suitable merit function (Macleod, 1986) that is composed of an appropriate function of $R(\eta, d, \lambda)$ defined within the wavelength range of region: $[\lambda_l, \lambda_u]$, where λ_l and λ_u are the lower and upper bound wavelengths, respectively. A merit function is

$$f(\eta, d, \lambda) = \int_{\lambda_l}^{\lambda_u} \{[R(\eta, d, \lambda) - \hat{R}(\lambda)]^2\}^{1/2} d\lambda, \tag{1}$$

where $\hat{R}(\lambda)$ is the target reflectance. Figure 1(b) shows an example of a target reflection ($\hat{R}(\lambda)$) with a respective design reflection ($R(\eta, d, \lambda)$). In practical applications, this integral can be approximated by a summation over a discrete number W, and Equation (1) is represented as

$$f(\eta, d, \lambda_k) = \{\frac{1}{W}\Sigma_{k=1}^{W} \frac{[R(\eta, d, \lambda_k) - \hat{R}(\lambda_k)]^2}{\delta R_k}\}^{1/2}, \tag{2}$$

where δR_k is the tolerance at wavelength λ_k, and in general, δR_k is set to 0.01. The most used method of calculating $R(\eta, d, \lambda_k)$ is a matrix method, which is especially useful when the number of optimizing parameters is large (Dobrowolski, 1995). According to the matrix method, the reflectance $R(\eta, d, \lambda_k)$ at wavelength λ_k is given by

$$R(\eta, d, \lambda_k) = \left| \frac{\mathcal{V}_m E_k - H_k}{\mathcal{V}_m E_k + H_k} \right|^2 \tag{3}$$

where $\mathcal{V}_m$ is the effective refractive index of the incident medium. E_k and H_k, the electric and magnetic vector, respectively, are defined as

$$\begin{bmatrix} E_k \\ H_k \end{bmatrix} = \left(\prod_{j=1}^{M} \begin{bmatrix} cos\phi_j & i\mathcal{V}_j^{-1} sin\phi_j \\ i\mathcal{V}_j sin\phi_j & cos\phi_j \end{bmatrix} \right) \begin{bmatrix} 1 \\ \mathcal{V}_s \end{bmatrix} \tag{4}$$

where $\phi_j = \frac{2\pi}{\lambda_k}(n_j d_j cos\theta_j)$. $\mathcal{V}_j$, d_j, and θ_j are the effective refractive index, thickness, and angle of incidence of the jth layer, respectively. $\mathcal{V}_s$ is the effective refractive index of the substrate medium. In Equations (3) and (4), $\mathcal{V}$ ($\mathcal{V}_m$, $\mathcal{V}_j$, or $\mathcal{V}_s$) is given by

$$\mathcal{V} = \begin{cases} \frac{\eta}{cos\theta} & p - \text{polarization} \\ \eta \, cos\theta & s - \text{polarization} \end{cases} \tag{5}$$

depending on whether the incident radiation is polarized parallel (p) or perpendicular (s) to the plane of incidence. The $\mathcal{V}$ is $\mathcal{V}_m$ in Equation (3). In Equation (4), $\mathcal{V}$ is $\mathcal{V}_j$ or $\mathcal{V}_s$ when η is η_j or η_s, respectively. At normal light incidence $\mathcal{V}$ is equal to η because the value of θ is zero.

In designing an optical thin-film system, we should consider several practical limitations. First, according to the maximum principle (Tikhonravov, 1993), it is not advantageous to use more than two materials of the lowest η_l and highest η_h refractive indices at normal light incidence. That is, the best result will be achieved with the pair

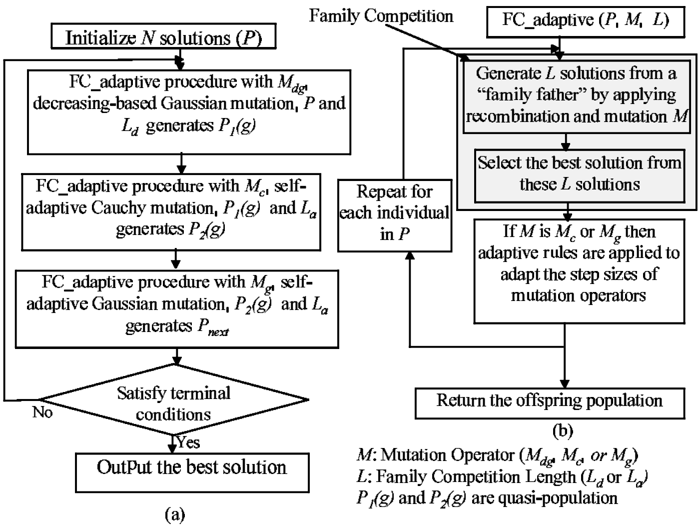


Figure 2: Overview of our algorithm: (a) FCEA (b) FC_adaptive procedure.

of materials where both have either the lowest and highest refractive indices. Second, the number of layers may be limited because the cost of coatings increases with the number of layers. Finally, the thickness cannot be negative, and very thin layers are difficult to control for some deposition processes.

According to the above discussions, we eliminate a layer if its thickness is less than 0.001 μm. We also use only one pair of materials with η_l and η_h for the design of optical coatings.

Generally, the implementation (merit function minimization procedure) of an evolutionary algorithm to design a thin-film coating with only one pair of materials with η_l and η_h can be described as

1. Initialize N coating systems as the initial population. The number of layers and the thickness of each coating system are randomly generated. Evaluate the objective value (Equation (2)) of each coating system based on the merit function.

2. Adapt the thickness of the layers of a system using genetic operators. Evaluate the objective value of the offspring (systems).

3. Select N solutions with the lowest values of merit function from these systems.

4. Execute step 2 and step 3 repeatedly until the terminal criteria are satisfied.

3 Family Competition Evolutionary Algorithm (FCEA)

In this section, we present the details of the FCEA for optical thin-film designs. The FCEA is a multi-operator approach that integrates decreasing-based Gaussian mutation M_{dg}, self-adaptive Cauchy mutation M_c, and self-adaptive Gaussian mutation M_g. Performance depends heavily on the same factor *step size* that decides the perturbation size of a mutation operator. The family competition and adaptive rules are designed to incorporate these three mutations to balance the global search and local search.

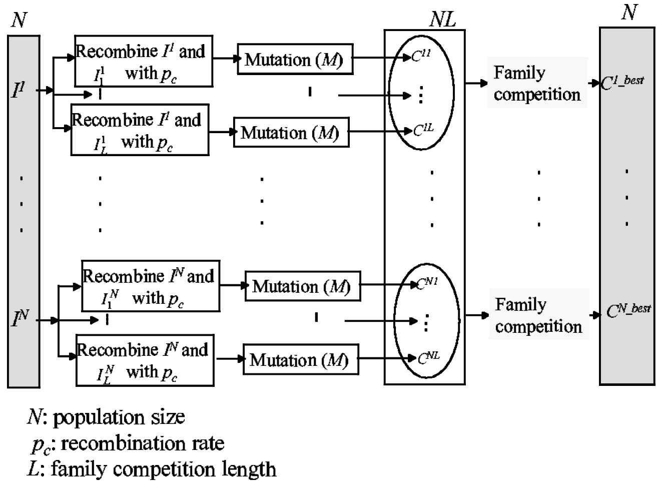


Figure 3: The main steps of the family competition.

The basic steps of the FCEA are (see Figure 2): N solutions are randomly generated as the initial population. Then the FCEA enters the main evolutionary loop, in which each generation consists of three nearly identical procedures. Each procedure is realized by doing recombination, mutation, family competition, and selection. These three procedures differ only in the mutation used: decreasing-based Gaussian mutation, self-adaptive Cauchy mutation, or self-adaptive Gaussian mutation. Hence we call the procedure "FC_adaptive," and we describe it later in detail. Note that the input of an FC_adaptive procedure is N solutions. Then the output is a new quasi-population with N solutions that is the input of the next FC_adaptive procedure.

The FC_adaptive procedure (Figure 2(b)) employs three parameters to generate a new quasi-population: the parent population P with N solutions, mutation operator M, and family competition length L. The main procedures are the family competition (Figure 3) and the adaptive rules. During the family competition procedure, each individual I_1 sequentially becomes the "family father." With a probability p_c, this family father and another solution I_1^1 randomly chosen from the rest of the parent population are used as the parents in a recombination operation. Then the new offspring (or the family father if the recombination is not conducted) is operated on by mutation to generate an offspring C^{11}. For each family father, this procedure is repeated L times. Finally, L solutions $(C^{11}, \ldots, C^{1L})$ are produced but only the C^{1_best} with the best value of merit function survives. The merit function is defined as Equation (2). Since we create L solutions from the same family father and perform selection, this family competition strategy is similar to $(1, \lambda)$ selection, i.e., the family father is not included. We feel this is a good way to avoid premature convergence but also to keep the spirit of local searches, based on the results of our previous study (Yang and Kao, 2000b).

Two adaptive rules are implemented to adapt step size when self-adaptive mutation is used in the FC_adaptive procedure. The adaptive rules are not applied when decreasing-based mutation is used. The FCEA adjusts the step sizes while mutations are applied, however, such updates may be insufficient. According to dynamic evolutionary information, adaptive rules are designed to decrease the step sizes of self-adaptive mutations or to grow the step sizes of decreasing-based mutations in order to create the relationship of mutations after the family competition procedure. The mutation and recombination operators will be described in Subsections 3.1 and 3.2. Subsection 3.3 will introduce the adaptive rules.

The FC_adaptive procedure is as follows:

{*Parameters:* P is the working population, M is the applied mutation (M_{dg}, M_g, or M_c), and L denotes the family competition length (L_a or L_d).}
1. Let C be an empty set ($C = \emptyset$).
2. **for** each solution a, called *family father*, in the population
   2.1 **for** $l = 1$ to L {*Family Competition*}
   - Generate a offspring c by using a recombination (c=recombination(a, b)) with probability p_c or by copying the *family father* a to c ($c = a$) with probability $1 - p_c$.
   - Generate a offspring c_l by mutating c as follows:

$$c_l = \begin{cases} M_{dg}(c) & \text{if } M \text{ is } M_{dg}; \{\textit{decreasing-based Gaussian mutation}\} \\ M_g(c) & \text{if } M \text{ is } M_g; \{\textit{self-adaptive Gaussian mutation}\} \\ M_c(c) & \text{if } M \text{ is } M_c. \{\textit{self-adaptive Cauchy mutation}\} \end{cases}$$

   endfor
   2.2 Select the one (c^{best}) with the lowest objective value from $c_1, \cdots, c_L$. {*family competition*}
   2.3 Apply **adaptive rules** only if M is a self-adaptive mutation operator (M_c or M_g)
   - Apply **A-decrease-rule** to decrease the step sizes (ψ or v) of M_c or M_g if the objective value of the *family father* a is better than c^{best}, That is, $w_j^a = 0.97 w_j^a$.
   - Apply **D-increase-rule** to increase the step size (σ) of M_{dg} if the objective value of the *family father* a is worse than c^{best}. That is, $\sigma_j^{best} = \max(\sigma_j^{best}, \beta v_{mean}^{best})$.

   2.4 Add the c^{best} into the set C.

endfor

Return the set C with N solutions.

After the FC_adaptive procedure, there are N parents and N children left. Based on different stages, we employ various means of obtaining a new quasi-population with N individuals. In both Gaussian and Cauchy self-adaptive mutation procedures, in each pair of family father and its child, the individual with a better merit function value survives. This procedure is called *family selection* and is similar to $(1 + 1)$ selection. On the other hand, *population selection* chooses the best N individuals from all N parents and N children. With a probability P_{ps}, the FCEA applies population selection to speed up the convergence when the decreasing-based Gaussian mutation is used. For the probability (1-P_{ps}), family selection is still considered. In order to reduce the ill effects of greediness on population selection, the initial P_{ps} is set to 0, but it is changed to 0.2 when the mean step size of self-adaptive Gaussian mutation is larger than that of decreasing-based Gaussian mutation.

The FCEA procedure is described as follows:

1. Set the initial step sizes (σ, v, and ψ), family competition lengths (L_d and L_a), and crossover probabilities (p_{cD} and p_{cA}). Let $g = 1$.
2. Randomly generate an initial population $P(g)$ with N solutions. Each solution is represented as $(M^i, I^i, x^i, \sigma^i, v^i, \psi^i), \forall i \in \{1, 2, \cdots, N\}$.
3. Evaluate the fitness of each solution in the population $P(g)$.
4. **repeat**
   4.1 {*Decreasing-based Gaussian mutation* (M_{dg})}
   - Generate a children set C_T with N solutions by calling FC_Adaptive with parameters: $P(g)$, M_{dg}, and L_d. That is, $P_1(g)$ =FC_Adaptive($P(g)$, M_{dg}, L_d).
   - Select the best N solutions as a new quasi-population $P_1(g)$ by population selection from the union set $\{P(g) \cup C_T\}$ with probability P_{ps} or by family selection with $1 - P_{ps}$.

   4.2 {*Self-adaptive Cauchy mutation* (M_c)}: Generate a new children set C_T by calling FC_Adaptive with parameters: $P_1(g)$, M_c, and L_a. Apply family selection to select a new quasi-population $P_2(g)$ with N solutions from the union set $\{P_1(g) \cup C_T\}$.
   4.3 {*Self-adaptive Gaussian mutation* (M_g)}: Generate a new children set C_T by calling FC_Adaptive with parameters: $P_2(g)$, M_g, and L_a. Apply family selection to select a new quasi-population P_{next} with N solutions from the union set $\{P_2(g) \cup C_T\}$.

4.4 Let $g = g + 1$ and $P(g) = P_{next}$.
until (termination criteria are met)
Output the best solution and the value of merit function.

Regarding chromosome representation, we present each solution (Figure 4) of a population as $(M, I, x, \sigma, v, \psi)$, where M is the number of layers of a coating system. The indicator I represents the structure of the refractive indices because only one pair of materials with η_l and η_h is used. The refractive index of the first layer is η_l when I is 0 and is η_h when I is 1. The vector x is the thickness vector of a coating system to be optimized; σ, v, and ψ are the step-size vectors of decreasing-based mutation, self-adaptive Gaussian mutation, and self-adaptive Cauchy mutation, respectively. In other words, each solution x is associated with three parameters for step-size control. The number of element of each vector x, σ, v, and ψ is M. The initial value M is randomly chosen from $[M_l, M_h]$, where M_l and M_h are the numbers of the lower bound and upper bound layers, respectively. Initially, I is randomly set to 1 or 0. The initial value of each entry of x is randomly chosen over a feasible region; σ, v, and ψ are set to 0.04, 0.01, and 0.01.

In the remainder of this section, we explain each component of the FC_adaptive procedure: recombination operators, mutation operations, and rules for adapting step sizes (σ, v, and ψ). For an easy description of the operators, we use $a = (M^a, I^a, x^a, \sigma^a, v^a, \psi^a)$ to represent the family father and $b = (M^b, I^b, x^b, \sigma^b, v^b, \psi^b)$ as another parent (only for the recombination operator). The offspring $c = (M^c, I^c, x^c, \sigma^c, v^c, \psi^c)$ is generated by a genetic operation. We also use x_j^d to denote the thickness of jth layer of a solution d, $\forall j \in \{1, \ldots, M\}$.

3.1 Recombination Operators

The advantages and disadvantages of recombination for a particular objective function cannot be fully accessed in advance (Bäck et al., 1997). Therefore, we have implemented two simple recombination operators to generate offspring: modified discrete recombination and intermediate recombination (Bäck, 1996). Here, we mention again that recombination operators are activated only with a probability p_c. The optimizing solution x and a step size (σ, v, or ψ) are recombined in a recombination operator.

Modified Discrete Recombination The original discrete recombination (Bäck, 1996) generates a child that inherits genes from two parents with equal probability. Here the two parents of the recombination operator are the family father and another randomly selected solution. Our experience indicates that the FCEA can be more robust if the child inherits genes from the family father with a higher probability (Yang and Kao, 2000b). Therefore, we modified the operator as follows:

$$x_j^c = \begin{cases} x_j^a \text{ with probability } 0.8 \\ x_j^b \text{ with probability } 0.2. \end{cases} \tag{6}$$

For a family father, applying this operator in the family competition is viewed as a local search procedure because this operator is designed to preserve the relationship between a child and its family father.

Intermediate Recombination We define intermediate recombination as:

$$x_j^c = x_j^a + 0.5(x_j^b - x_j^a) \text{ and} \tag{7}$$

$$w_j^c = w_j^a + 0.5(w_j^b - w_j^a), \tag{8}$$

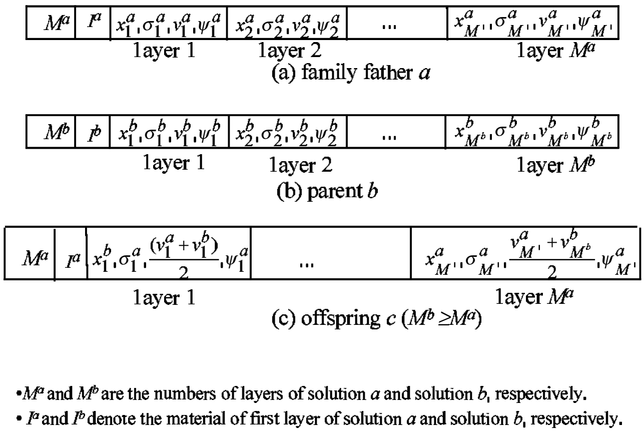


Figure 4: The chromosome representation and recombination operators: Family parent (a) and the other parent (b) in the self-adaptive Gaussian mutation procedure used a recombination operator to generate an offspring (c). The x and v are updated by modified discrete recombination and intermediate recombination, respectively; σ and ψ are unchanged.

where w is v, σ, or ψ based on the mutation operator applied in the family competition. We follow the evolution strategies work of Bäck and Schwefel (1993) to employ only intermediate recombination on step-size vectors σ, v, and ψ.

Figure 4 shows a recombination example in the self-adaptive Gaussian mutation procedure. The offspring c is generated from the family father a and another parent b by applying modified discrete recombination for thickness x and intermediate recombination for step size v. In other words, the σ and ψ remained unchanged in the self-adaptive Gaussian mutation procedure. Note that the values of M and I of the offspring c were directly inherited from the family father.

3.2 Mutation Operators

Mutations are main operators of the FCEA. After recombination, a mutation operator is applied to the family father or the new offspring generated by a recombination. In the FCEA, the mutation is performed independently on each vector element of the selected individual by adding a random value with expectation zero:

$$x'_j = x_j + wD(\cdot), \tag{9}$$

where x_j is the thickness of the jth layer of x, x'_j is the jth variable of x' mutated from x, $D(\cdot)$ is a random variable, and w is the step size. In this paper, $D(\cdot)$ is evaluated as $N(0,1)$ or $C(1)$ if the mutations are Gaussian mutation or Cauchy mutation, respectively.

Self-Adaptive Gaussian Mutation We adapted Schwefel's (1981) proposal to use self-adaptive Gaussian mutation. The mutation is accomplished by first mutating the

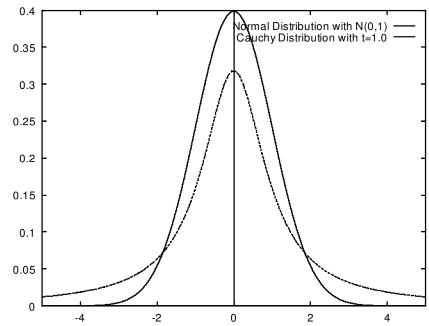


Figure 5: Density functions of Gaussian and Cauchy distributions.

step size v_j and then the thickness x_j:

$$v_j^c = v_j^a \exp[\tau' N(0,1) + \tau N_j(0,1)], \tag{10}$$
$$x_j^c = x_j^a + v_j^c N_j(0,1), \tag{11}$$

where $N(0,1)$ is the standard normal distribution. $N_j(0,1)$ is a new value with distribution $N(0,1)$ that must be regenerated for each index j. For the FCEA, we adopted Bäck and Schwefel (1993) by setting τ and τ' as $(\sqrt{2n})^{-1}$ and $(\sqrt{2\sqrt{n}})^{-1}$, respectively.

Self-Adaptive Cauchy Mutation The self-adaptive Cauchy mutation (Yao and Liu, 1997) works as follows:

$$\psi_j^c = \psi_j^a \exp[\tau' N(0,1) + \tau N_j(0,1)], \tag{12}$$
$$x_j^c = x_j^a + \psi_j^c C_j(t). \tag{13}$$

In our experiments, t is 1. Note that self-adaptive Cauchy mutation is similar to self-adaptive Gaussian mutation except that Equation (11) is replaced by Equation (13).

Decreasing-Based Gaussian Mutations Our decreasing-based Gaussian mutation uses the step-size vector σ with a fixed decreasing rate $\gamma = 0.97$ as follows:

$$\sigma^c = \gamma \sigma^a \tag{14}$$
$$x_j^c = x_j^a + \sigma^c N_j(0,1) \tag{15}$$

Previous results (Yang and Kao, 2000b) demonstrated that self-adaptive mutations converge faster than decreasing-based mutations but, for rugged functions, self-adaptive mutations can be more easily trapped into local optima than decreasing-based mutations.

Figure 5 compares density functions of Gaussian distribution ($N(0,1)$) and Cauchy distributions ($C(1)$). Clearly Cauchy mutation is able to make a larger perturbation than Gaussian mutation. This implies that Cauchy mutation has a higher probability of escaping from local optima than Gaussian mutation does. For decreasing mutation, it is like searching for a better child in a hypersphere centered at the parent. However, for self-adaptive mutation, the search space becomes a hyperellipse. Figure 6 illustrates this difference by using two-dimensional contour plots. Therefore, children are searched in two different types of regions. For these reasons, we use these three types of mutations.

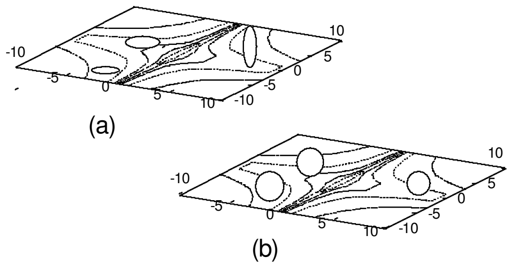


Figure 6: (a) and (b) show the difference in search space between self-adaptive (ellipses) and decreasing-based mutations (circles) on the two-dimensional contour plots.

3.3 Adaptive Rules

The performance of Gaussian and Cauchy mutations is largely influenced by the step sizes. The FCEA adjusts the step sizes while mutations are applied (e.g., Equations (10), (12), and (14)). However, such updates have insufficiently considered the performance of the whole family. Therefore, after the family competition, some additional rules are implemented. Note that the adaptive rules are applied in the procedures of self-adaptive mutations but not of decreasing-based mutations.

1. **A-decrease-rule** Immediately after self-adaptive mutations, if objective values of all offspring are greater than or equal to that of the family father, we decrease the step-size vectors v (Gaussian) or ψ (Cauchy) of the parent:

$$w_j^a = 0.97 w_j^a, \tag{16}$$

   where w^a is the step size vector of the parent. In other words, if there is no improvement after self-adaptive mutations, we may propose a more conservative approach. That is, smaller step size tends to make better improvement in the next iteration. This is inspired from the 1/5-success rule of (1+λ)-ES (Bäck, 1996).

2. **D-increase-rule** It is difficult to decide the rate γ of decreasing-based mutations. Unlike self-adaptive mutations that adjust step sizes automatically, its step size goes to zero as the number of iterations increases. Therefore, it is essential to employ a rule that can enlarge the step size in some situations. The step size of the decreasing-based mutation should not be too small, when compared to step sizes of self-adaptive mutations. Here, we propose to increase σ if either of the two self-adaptive mutations generates better offspring. That is, after a self-adaptive mutation, if the best child with step size v is better than its family father, the step size of the decreasing-based mutation is updated as follows:

$$\sigma_j^c = \max(\sigma_j^c, \beta v_{mean}^c), \tag{17}$$

   where v_{mean}^c is the mean value of the vector v; and β is 0.2 in our experiments.

Table 1: Parameters of the FCEA and notation used in this paper.

parameter name	the value and notation of parameter
recombination probability (p_c)	p_{cD} =0.8 (decreasing-based Gaussian mutation (M_{dg})) p_{cA} =0.2 (two self-adaptive mutations (M_g and M_c))
family competition length	L_d = 6 (decreasing-based Gaussian mutation) L_a = 6 (two self-adaptive mutations)
step sizes	$v_i = \psi_i = 0.01, \sigma_i = 4v_i$
population size (N)	50
other notation	M: number of layers; L_T: total family competition length ($2L_a + L_d$); MF: value of merit function; $\sum \eta d$: total thickness of a solution

Table 2: Some solutions of the FCEA for infrared antireflection coatings over the region $7.7 \leq \lambda \leq 12.3\ \mu m$ on a $\eta_s = 4.0$ substrate based on the refractive index pair 2.2 and 4.2.

	S-20	S-27	S-33	S-40	S-44	S-51	S-61	S-71
M	15	16	17	23	27	27	34	36
$\sum \eta d(\mu\text{m})$	20.34	27.04	33.96	40.17	44.98	51.19	61.7	71.15
MF(%)	0.855	0.697	0.614	0.577	0.553	0.522	0.509	0.494

4 Experimental Results

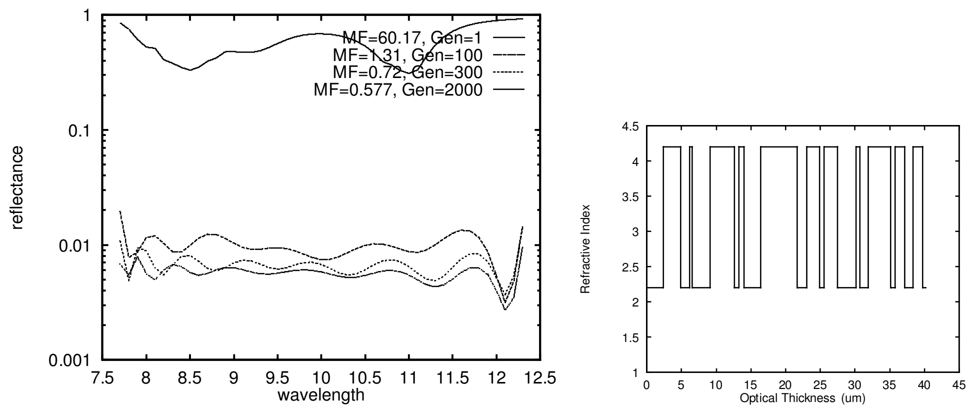
In this section, we present the numerical results for the synthesis of three different optical coatings to illustrate the proposed method. We would like to mention again that all the materials were assumed to be nonabsorbing and nondispersive with normal incidence. Table 1 indicates the setting of the FCEA parameters, such as initial step sizes, family competition lengths, and recombination probabilities. They are used for synthesis problems defined in this work. L_d, σ, and p_{cD} are the parameters for decreasing-based mutation; L_a, v, ψ, and p_{cA} are for self-adaptive mutations. The population size is 50. These parameters are decided after the experiments have been conducted on some optical coating problems (Aguilera et al., 1988; Li and Dobrowolski, 1992; Sullivan and Dobrowolski, 1996) with various values.

4.1 Infrared Antireflection Coating

The first design problem is the synthesis of a wide-band antireflection (AR) coating for germanium in the infrared. At least 60 different solutions were published for this AR coating, including non-evolutionary approaches (Aguilera et al., 1988; Dobrowolski and Kemp, 1990; Dobrowolski, 1997; Druessel and Grantham, 1993) and evolutionary approaches (Bäck and Schütz, 1995; Schutz and Sprave, 1996), which are mixed-integer algorithms. In contrast, our proposed approach is a real-valued optimizer.

The target design was the reflectance R specified to zero at 0.1 μm wavelength increments between 7.7 and 12.3 μm; with W being defined in Equation (2) as 47. The incident medium is air ($\eta_m = 1$), and the substrate refractive index is $\eta_s = 4.0$. The high- and low-index coating materials were Ge (η_h = 4.2) and ZnS (η_l = 2.2).

The initial number of layers was randomly chosen from 15 to 40. The initial thickness of each layer was uniformly selected from the region from 0.2 to 1.0 μm. The FCEA executed a total of 100 times, and the maximum number of generations was 2000. Table 2 shows several best solutions obtained by our FCEA on different total optical thickness. Figure 7(a) shows the spectral profiles of the series of intermediate solutions of the solution S-40 shown in Table 2. Initially, the value of merit function



(a) Intermediate performance

(b) The refractive-index profile of the solution (MF=0.577% and $\sum \eta d = 40.17\mu$m)

Figure 7: Series of intermediate solutions and refractive-index profiles of a solution generated by the FCEA for the wide-band AR coating on a $\eta_s = 4.0$ substrate.

is 60.17%. The values are reduced to 1.31%, 0.72%, and 0.577% when the numbers of generations are 100, 300, and 2000, respectively. The number of layers and the total thickness of the final solution (0.577%) are 23 and 40.17 μm, respectively. Figure 7(b) shows the refractive-index profile of the final solution.

Some solutions from the previous research and one predicted curve by Willy (1993) are shown in Figure 8. One curve obtained by Dobrowolski et al. (1996) and believed to correspond to optimum solutions was also plotted. Figure 8 shows that several, but by no means all, of these published solutions lie close to the believable optimum curve. Nevertheless, a number of these solutions are relatively far from this curve. This implies the importance of using good starting designs or using good design techniques. Figure 8 shows that the solutions of the FCEA are very close to the believable optimum curve. They are better than the solutions obtained by parallel evolution strategy (Schutz and Sprave, 1996) when the total thickness is larger than 35 μm. The FCEA is also competitive with well-known approaches, such as damped least squares, modified gradient, Hook and Jeeves search (Aguilera et al., 1988), genetic algorithm (Martin et al., 1995), and evolution strategy (Bäck and Schütz, 1995).

There are some observations according to Figure 8.

1. Willy's (1993) predicated optimal values were overestimated when the total thickness was lower than 30 μm. The observation was consistent with the previous findings (Schutz and Sprave, 1996; Dobrowolski et al., 1996).

2. In Schutz and Sprave (1996), the authors described Willy's predicated curve as the best estimate when the total thickness is beyond 30 μm. The solutions obtained by the FCEA are better than Willy's predicated values. Therefore, we claim that the solutions obtained in Dobrowolski et al. (1996) correspond to the optimum solutions for this AR coating.

3. Figure 9 shows that the clusters of layers are evident in our solutions. That is, the

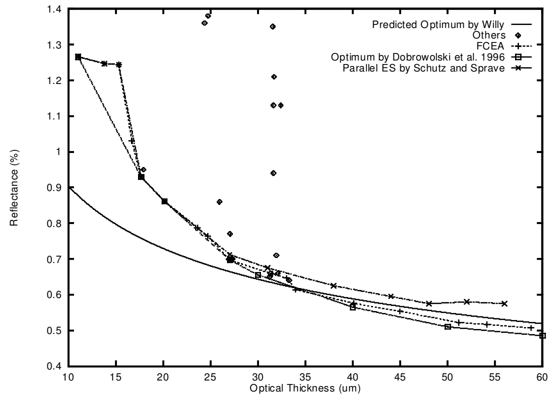
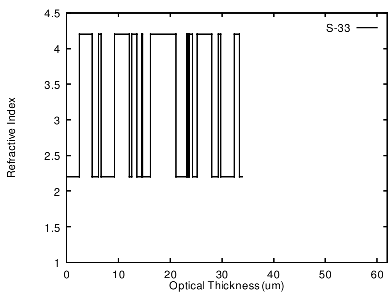
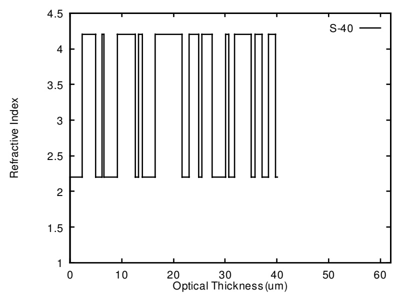


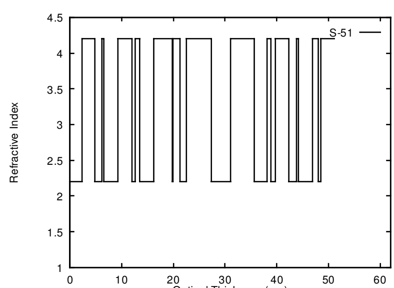
Figure 8: Comparison the FCEA with Parallel ES and over 20 approaches on the antireflection coating problem.



(a) $\sum \eta d = 33.96\mu$m



(b) $\sum \eta d = 40.17\mu$m



(c) $\sum \eta d = 51.19\mu$m

Figure 9: Several refractive-index profiles of solutions obtained by the FCEA with different optical overall thickness for the wide-band AR coating.

Table 3: Construction parameters of several solutions obtained by the FCEA for the antireflection coating problem, the filter with 0.0, 0.5, and 1.0 transmission regions, and the CIE$_{\bar{x}_\lambda}$ filter for colorimetry.

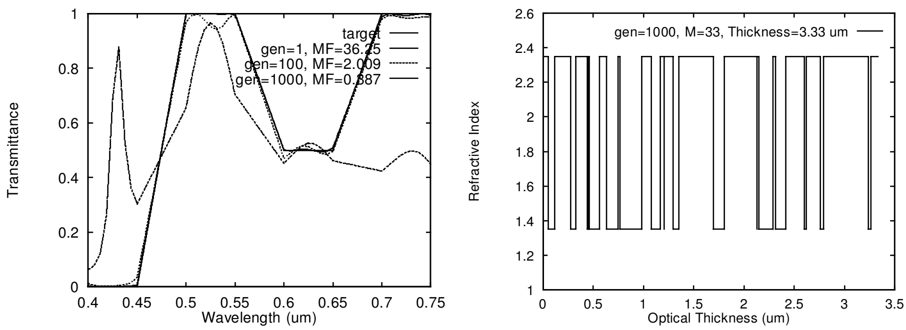
	Antireflection				0.0-0.5-1.0 filter			CIE$_{\bar{x}_\lambda}$	
Layer	η	$\eta d(\mu\text{m})$	$\eta d(\mu\text{m})$	$\eta d(\mu\text{m})$	η	$\eta d(\mu\text{m})$	$\eta d(\mu\text{m})$	η	$\eta d(\mu\text{m})$
Subs(η_s)	4.0				1.52			1.52	
1	2.2	0.5024	0.5506	0.4156	2.35	0.0912	0.0678	1.468	0.0750
2	4.2	1.0047	0.9791	1.4223	1.35	0.0633	0.0276	2.323	0.3289
3	2.2	2.6823	2.5962	1.1770	2.35	0.0168	0.4476	1.468	0.1159
4	4.2	0.3341	0.4992	1.4257	1.35	0.0050	0.0316	2.323	0.1029
5	2.2	1.5401	1.3068	0.6412	2.35	0.0411	0.1366	1.468	0.3522
6	4.2	2.9295	2.8681	3.2112	1.35	0.0031	0.0238	2.323	0.1259
7	2.2	0.5613	0.7713	1.1923	2.35	0.0066	0.1849	1.468	0.1196
8	4.2	1.0228	0.6784	0.5698	1.35	0.1315	0.0980	2.323	0.2100
9	2.2	2.6903	0.2762	2.6681	2.35	0.0048	0.0288	1.468	0.2643
10	4.2	1.2822	0.2100	1.9116	1.35	0.0546	0.1391	2.323	0.1580
11	2.2	0.3334	2.1329	0.6723	2.35	0.0898	0.0179	1.468	0.1662
12	4.2	2.9046	4.9398	1.7747	1.35	0.0466	0.0013	2.323	0.1373
13	2.2	2.6785	1.5095	1.4158	2.35	0.0260	0.3210	1.468	0.0972
14	4.2	0.4880	0.2100	5.2383	1.35	0.0256	0.1084	2.323	0.2529
15	2.2	1.1391	0.8416	2.4343	2.35	0.0610	0.3487	1.468	0.2610
16	2.2	2.5555	1.0764	0.7082	1.35	0.1523	0.0577	2.323	0.1142
17	4.2	2.3929	0.4154	0.6236	2.35	0.0876	0.0844	1.468	0.1311
18	2.2		2.8695	3.4998	1.35	0.0783	0.0049	2.323	0.2189
19	4.2		2.6401	2.5964	2.35	0.1230	0.0358	1.468	0.1291
20	2.2		0.4476	0.4224	1.35	0.1045	0.0911	2.323	0.2850
21	4.2		1.2131	1.2308	2.35	0.3301	0.0952	1.468	0.0980
22	2.2		2.5381	2.5281	1.35	0.0939	0.2132	2.323	0.3049
23	4.2		2.3905	2.3872	2.35	0.1212	0.0190	1.468	0.0789
24					1.35	0.1382	0.1143	2.323	0.1092
25					2.35	0.0397	0.0694	1.468	0.0385
26					1.35	0.1223	0.1065	2.323	0.1311
27					2.35	0.0051	0.0023	1.468	0.0739
28					1.35	0.0019	0.0111	2.323	0.1331
29					2.35	0.0862	0.1196	1.468	0.0628
30					1.35	0.0013	0.0477	2.323	0.0763
31					2.35	0.2120	0.1602	1.468	0.0731
32					1.35	0.0169	0.0646	2.323	0.3066
33					2.35		0.0503		
medium (η_m)	1.0				1.0			1.52	
$\sum \eta d(\mu\text{m})$		27.04	33.96	40.17		1.96	3.33		5.13
MF(%)		0.697	0.614	0.577		1.724	0.387		0.427

refractive-index profiles of these three solutions have similar structures from the first layer to the fifth. This observation was also discussed for normal-incidence antireflection coatings (Dobrowolski et al., 1996).

4. The solution quality will be improved when the number of layers becomes larger and the total thickness greater. We would like to stress that the cost of coatings increases with the number of layers.

5. We observed the "Bermuda Triangle," i.e., the great difference between the empirical and analytical estimate (Schutz and Sprave, 1996), in the region from 11 μm to 17 μm. The evidence of clusters of layers may be used to explain this observation.

The construction parameters of the three best solutions obtained by our FCEA are given in Table 3 for this AR coating.



(a) Transmittance

(b) Refractive index

Figure 10: Series of intermediate performance and the refractive-index profiles of our FCEA for the filter with 0.0, 0.5, and 1.0 transmission region $0.4 \leq \lambda \leq 0.75$ μm on a $\eta_s = 1.52$ substrate based on the refractive index pair 1.35 and 2.35.

4.2 Filter with 0.0, 0.5, and 1.0 Transmission Regions

The second example concerns the synthesis of a filter in the region 0.4-0.75 μm. The desired performance is

$$
\begin{aligned}
0.4 \leq \lambda \leq 0.45 &\quad : \quad T = 0.0 \text{ (reflector)},\\
0.5 \leq \lambda \leq 0.55 &\quad : \quad T = 1.0 \text{ (antireflector)},\\
0.6 \leq \lambda \leq 0.65 &\quad : \quad T = 0.5 \text{ (beam splitter)},\\
0.7 \leq \lambda \leq 0.75 &\quad : \quad T = 1.0 \text{ (antireflector)}.
\end{aligned}
$$

It was believed that it would be difficult to design such a coating system for this problem without using synthesis approaches because the starting design is not easily obtainable. Such multiwavelength specifications might be required for laser and electro-optical systems. This problem was used for comparison of synthesis methods (Dobrowolski, 1986; Li and Dobrowolski, 1992). Following these works, we only used two coating materials whose refractive indices are $\eta_l = 1.35$ and $\eta_h = 2.35$, and the substrate and medium indices are $\eta_s = 1.52$ and $\eta_m = 1.0$, respectively. The merit function was defined at 36 points. That is, each region has 9 points.

The initial number of layers was randomly chosen from 25 to 35. The initial thickness of each layer was uniformly selected from the region from 0.01 to 0.1 μm. The maximum number of generations was 1000.

Figure 10(a) shows a series of intermediate solutions of a solution obtained by the FCEA. The value of merit function is 36.25% when the number of generations is 1. The objective values become 2.009% and 0.504% when the number of generations is 100 and 300, respectively. The objective value of the final solution is 0.387% after the FCEA exhausted 1000 generations. The number of layers is 33 and the thickness is 3.33 μm of the final solution. Figure 10(b) shows the refractive-index profile of the final solution.

Table 4 shows the comparison of the FCEA with well-known synthesis methods (Li and Dobrowolski, 1992), such as gradual-evolution and inverse-Fourier transform methods; and refinement methods (Li and Dobrowolski, 1992), such as dumped-least-square and Hooke and Jeeves methods, on the filter with 0.0, 0.5, and 1.0 transmission

Table 4: Comparison of the FCEA with synthesis methods (Li and Dobrowolski, 1992) and refinement methods (Li and Dobrowolski, 1992) on filter with 0.0, 0.5, and 1.0 transmission regions.

	FCEA			synthesis methods				refinement methods		
				Gradual-Evolution	Minus-Filter	Flip-Flop	Inverse-Fourier Transformation	Dumped-Least Square	Golden-Selection	Hooke Jeeves
M	32	33	32	9	37	34	43	35	35	35
$\sum \eta d(\mu\text{m})$	1.96	3.33	2.805	2.350	4.550	1.99	2.37	1.975	2.671	3.434
MF(%)	1.72	0.387	0.587	5.10	2.13	2.56	2.20	1.86	3.81	4.970

Table 5: Comparison of the FCEA with the needle method on CIE$_{\bar{x}_\lambda}$ filter for the tristimulus colorimeters.

	FCEA		Needle Method
number of layers	32	34	31
total thickness ($\sum \eta d$)(μm)	5.13	4.8	3.982
MF(%)	0.427	0.588	0.62

regions. The FCEA obtains the best solutions among these comparative approaches. The solution quality also depended on the total thickness. For example, the values of merit function are 1.72% and 0.316% for the FCEA when the total thickness are 1.96 μm and 3.51 μm, respectively. The construction parameters of these two solutions of our FCEA are given in Table 3 for this filter.

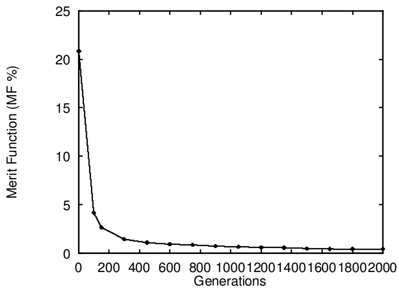
4.3 Tristimulus Filter

The objective of the third example is to produce a filter that matches the CIE$_{\bar{x}_\lambda}$ curve for the standard observer in the 380-780 nm spectral region (Sullivan and Dobrowolski, 1996). This filter is used in tristimulus colorimeters. The target curve is the solid line shown in Figure 11(b). The final designs should consist of only two coating materials SiO_2 and Nb_2O_5 whose refractive indices are $\eta_l = 1.468$ and $\eta_h = 2.323$, respectively. Both substrate and medium are assumed to be glass whose index is $\eta_s = \eta_m = 1.52$. The merit function was defined at 41 equispaced points on the interesting wavelength scale.

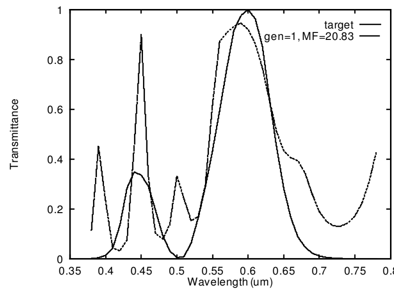
The initial number of layers was randomly chosen from 25 to 35. The initial thickness of each layer was uniformly selected from the region from 0.01 to 0.1 μm, and the maximum number of generations was 2000.

Figure 11(a) shows the convergence curve of value of the merit function of the FCEA on the CIE$_{\bar{x}_\lambda}$ filter. A series of intermediate solutions of a solution obtained by the FCEA are illustrated in Figure 11(b)–(e). The dash curves are the designed results and the solid curve is the target design. The value of the initial merit function is 20.83%. The values of solutions are reduced to 4.17% and 1.44% when the numbers of generations are at 100 and 300, respectively. The final solution quality is 0.427% after the FCEA exhausted 2000 generations. The number of layers is 32, and the total thickness is 5.13 μm of the final solution. Figure 11(f) shows the refractive-index profile of the final solution.

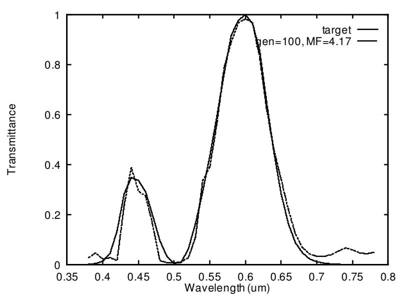
Table 5 shows that the FCEA is very comparative with the needle method (Sullivan and Dobrowolski, 1996), which is a very powerful synthesis method. The last column in Table 3 shows the construction parameters of the best solution obtained by the FCEA for the CIE$_{\bar{x}_\lambda}$ filter.



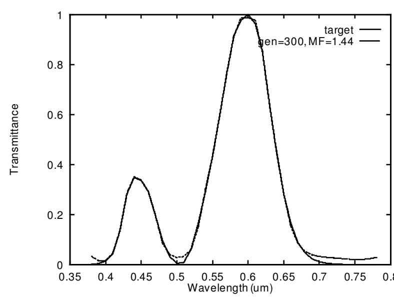
(a) Convergence of MF value



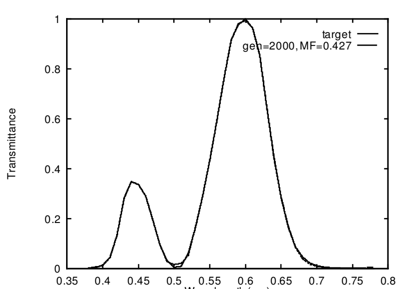
(b) Transmittance (gen=1)



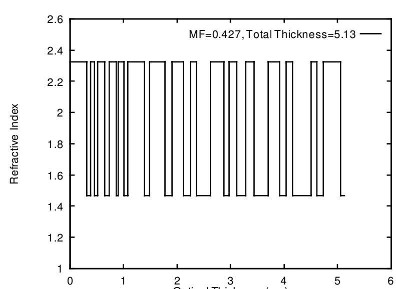
(c) Transmittance (gen=100)



(d) Transmittance (gen=300)

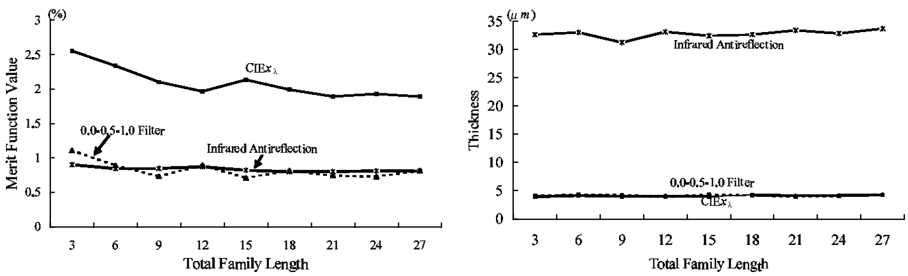


(e) Transmittance (gen=2000)



(f) Refractive index(gen=2000)

Figure 11: Series of intermediate performance and the refractive-index profiles of our FCEA for the CIE$_{\bar{x}_\lambda}$ filter for the tristimulus colorimeters in the region 380-780 nm.



(a) length versus merit function

(b) length versus total thickness

Figure 12: The relationship between the merit function and the total thickness of solutions obtained by the FCEA on different total family competition lengths (L_T) for the design of thin-film optical coatings. Each problem is tested on 30 independent runs for each length, and the maximum number of generations is 500.

5 Investigation of the FCEA

In this section, we discuss several characteristics of the FCEA using numerical experiments. The main idea of the FCEA is that the mutation operators are able to cooperate with each other by applying the adaptive rules and family competition. We will first describe the parameters of the FCEA and then discuss the effectiveness of using multiple mutations.

5.1 Parameters of the FCEA

We have chosen the parameters listed in Table 1, based on the following observations:

1. Since the family length is a critical factor of the FCEA, in Figure 12, we have tested the performance of different total family competition lengths (L_T). Figure 12(a) shows the relationship between L_T and the value of merit function, while Figure 12(b) reflects that between L_T and the total thickness of solutions. It can be seen that the solution quality improved when the family competition length increased. The total thickness of solutions is almost independent of L_T. Note that the convergent speed of the FCEA will slow when the family competition length is larger. Therefore, we set both L_a and L_d to 6.

2. We have implemented our FCEA on the design of optical coatings with recombination probability p_c between 0.0 to 1.0. We noticed that the performance of the FCEA was insensitive to these recombination probabilities when $p_c > 0.2$. Based on experimental results, we set p_{cD} and p_{cA} to 0.8 and 0.2, respectively.

3. The step sizes v and ψ of self-adaptive mutations are set to 0.01. Decreasing-based with a large initial step size ($\sigma = 4v$) is a global search strategy in the FCEA. The FCEA is less sensitive with the initial step sizes than evolution strategies on the design of optical coatings, because the FCEA applies both self-adaptive (Equation 12 and Equation 10) and A-decrease-rule (Equation 16) mechanisms to adjust v_i and ψ_i according to experimental results.

Table 6: Comparison of various approaches of the FCEA.

Problems	Methods	Average MF (%)	Average number of layers	Average of Total thickness	Best MF (%)	Worst MF (%)
Antireflection	FCEA	0.824	23.30	35.911	0.658	1.079
	FCEA$_{ncr}$	0.990	23.13	35.145	0.685	1.531
	$M_{dg}+M_c$	0.851	23.44	36.442	0.617	1.160
	$M_{dg}+M_g$	0.840	23.50	35.097	0.743	1.129
	M_c	0.824	23.73	35.809	0.643	1.269
	M_g	0.845	23.16	34.355	0.686	1.140
	M_{dg}	0.813	22.83	33.444	0.675	1.129
0-0.5-1.0 Filter	FCEA	0.613	30.77	3.406	0.316	1.478
	FCEA$_{ncr}$	1.645	30.33	3.168	0.649	2.665
	$M_{dg}+M_c$	0.674	30.71	3.739	0.271	1.585
	$M_{dg}+M_g$	0.625	30.83	3.242	0.368	1.535
	M_c	0.769	30.00	3.634	0.403	1.490
	M_g	0.608	31.70	3.040	0.335	1.167
	M_{dg}	0.973	30.83	3.239	0.557	1.615
CIE$_{\bar{x}_\lambda}$	FCEA	1.892	30.80	4.569	1.182	3.459
	FCEA$_{ncr}$	2.741	31.13	4.129	1.451	5.310
	$M_{dg}+M_c$	2.144	30.80	4.598	0.810	3.101
	$M_{dg}+M_g$	1.676	30.20	3.971	0.642	3.018
	M_c	2.741	31.67	5.107	1.121	4.399
	M_g	2.128	31.50	3.725	1.362	3.768
	M_{dg}	2.070	31.89	4.195	1.157	2.779

5.2 The Effectiveness of Multiple Operators and Family Competition

Using multiple mutations in each generation is one of the main features of the FCEA. Using numerical experiments, we will demonstrate that these three FC_adaptive procedures with different mutations cooperate with one another and possess good local and global properties.

We have compared seven different uses of mutation operators in Table 6. Each use combines some of the three operators applied in the FCEA: decreasing-based Gaussian mutation M_{dg}, self-adaptive Cauchy mutation M_c, and self-adaptive Gaussian mutation M_g. For example, the M_c approach uses only the FC_adaptive procedure with self-adaptive Cauchy mutation; the $M_{dg}+M_c$ approach integrates two FC_adaptive procedures with decreasing Gaussian mutation and self-adaptive Cauchy mutation; and the FCEA is an approach integrating M_{dg}, M_c, and M_g. The FCEA$_{ncr}$ approach is a special case of FCEA without adaptive rules, that is, without A-decrease-rule (Equation (16)) and D-increase-rule (Equation (17)). Except for FCEA$_{ncr}$, the others use adaptive rules. To enable a fair comparison, we have set the total length of family competition L_T of all seven approaches at the same value. For example, if $L_d = L_a = 6$ in the FCEA, $L_T = 18$ for one-operator approaches (M_{dg}, M_c, and M_g) and $L_d = L_a = 9$ for two-operator approaches ($M_{dg}+M_c$ and $M_{dg}+M_g$). The maximum number of generations is set to 500.

The following observations are obtained from this experiment:

1. Table 6 shows that one-operator approaches (M_{dg}, M_c, and M_g) have different performance.

2. Generally, these six approaches employing adaptive rules have little difference in performance on these test systems. However, the FCEA performs more robustly than the others in terms of the solution quality.

3. Family competition is a useful strategy. M_g is better than evolution strategies on

the AR coating. We have also obtained the same results on function optimization (Yang and Kao, 2000b). We think the reason is that M_g applies adaptive rules and family competition.

4. The control of step sizes is important and the adaptive rules (A-decrease-rule and D-increase-rule) are useful for the FCEA according to the comparisons of FCEA$_{ncr}$ and FCEA.

6 Conclusions

This study presents the FCEA as a stable synthesis approach for optical thin-film designs. From our experience, it is suggested that a global optimization method should consist of both global and local search strategies. For our FCEA, the decreasing-based mutation with large initial step sizes is the global search strategy; the self-adaptive mutations with family competition procedure and replacement selection are local search strategies. Based on the family competition and adaptive rules, these mutation operators can closely cooperate with one another. Experiments of three well-known optical coating problems verify that the proposed approach is very comparative with evolutionary algorithms and traditional approaches. We believe that the flexibility and robustness of the FCEA make it an effective synthesis method of optical thin-film designs.

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