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Importance sampling of products from illumination and BRDF using spherical radial basis functions

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Abstract In this paper, a new approach for the importance sampling of products from a complex high dynamic range (HDR) environment map and measured bidirectional reflectance distribution function (BRDF) data using spherical radial basis functions (SRBFs) is presented. In the pre-process, a complex HDR environment map and measured BRDF data are transformed into a scattered SRBF representation by using a non-uniform and non-negative SRBF fitting algorithm. An initial guess is determined for the fitting operation. In the run-time rendering process, after the product of the two SRBFs is evaluated, this is used to guide the number of samples. The sampling is done by mixing samples from the various "product" SRBFs using multiple importance sampling. Hence, the proposed approach efficiently renders images with multiple HDR environment maps and measured BRDFs.

Keywords Illumination · Environment map · Bidirectional reflectance distribution function · Importance sampling · Spherical radial basis function

1 Introduction

To improve the quality of realistic images, more and more research has been focusing on efficient rendering with image-based illumination and measured bidirectional reflectance distribution function (BRDF) data. The main reason for this interest is image-based illumination captures complex real-world lighting and measured BRDF data contain real-world material property. Monte Carlo-based approaches [29, 30] are often used to incorporate high dynamic range (HDR) environment maps with complex BRDF models. However, there is a major problem with the Monte Carlo-based approaches. When tracing the rays in the scene, the tracing path of each ray has to be selected according to the product distribution of the illumination and BRDF. It would waste a lot of samples if they are generated randomly or uniformly, because only few sampling paths reach the right intensity. On the other hand, if samples are generated against the high energy direction of the product distribution, it would achieve low variance and increase the efficiency of rendering. Therefore, sampling the importance of products from the illumination and BRDF is critical for efficient realistic image rendering.

Lately, many researchers have transformed environment maps and the original measured BRDF data into other representation forms, such as wavelets [3, 4, 14, 17], factored representations [15] and so on. Then they would analyze the original data to sample probability distributions of sampling directions. They would generate sampling directions according to the probability distribution found in the specified representation. In the proposed approach, the environment map and the measured BRDF data are represented by using spherical radial basis functions (SRBFs) [7, 18, 28].

The SRBFs are special radial basis functions (RBFs) defined on the unit sphere. Because of their intrinsic potential, SRBFs are more suitable for representing the spherical data, such as an environment map and BRDF data. SRBFs are used in the proposed approach as the basis functions with the following benefits:

- Since SRBFs are defined in the spherical domain, the illumination as well as the BRDF can be directly fitted to the data without re-parameterization. Therefore, any inaccuracy because of the re-parameterization process is avoided.
- High-frequency signals can be handled efficiently because of the spatial localization property of the SRBFs.
- Since SRBFs are circularly axis-symmetric and rotation-invariant functions, it is simple to rotate SRBF functions.
- The convolution of two SRBF kernels in some situations has a simple mathematical form which makes it possible to evaluate the integral for probability estimation without extra processes to construct the probability density function (PDF).
- The approximated results are accurate enough to represent most features of the original data and the probability can be directly estimated for importance sampling with the simple form of the convolution by choosing the appropriate SRBF kernels.

In this paper, the scattered SRBFs are used to represent the HDR environment map and the BRDF data. With useful SRBF properties, the resulting representation can be easily applied in the Monte Carlo-based importance sampling technique. In the pre-process, a complex HDR environment map and measured BRDF data are transformed into a scattered SRBF representation by using a non-uniform and non-negative SRBF fitting algorithm. An initial guess is introduced for the fitting operation. In the run-time process, after the product of the two SRBFs is computed, this is used to guide the number of samples. The sampling is done by mixing samples from the various "product" SRBFs using multiple importance sampling [29, 30]. This is different from the sampling from the "true" product distribution such as is done in the wavelet importance sampling [3] or bidirectional sampling approaches [2]. Figure 1 gives an overview of the proposed approach.

The rest of the paper is organized as follows: Sect. 2 reviews the related works. In Sect. 3, the SRBFs are described. Sections 4 and 5 present the off-line SRBF fitting process and the run-time rendering process, respectively. The results are given in Sect. 6. Finally, conclusions are discussed in Sect. 7.

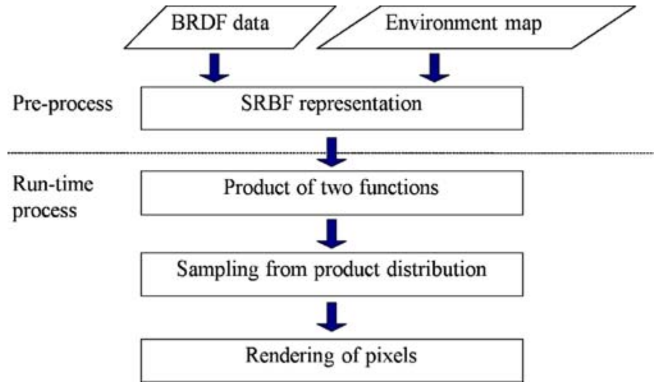

Fig. 1. Overview of proposed approach

2 Related works

For distant illumination, the rendering equation [10] is

$$B_x(\eta_o) = \int_{\Omega} L(\eta_i)\rho_x(\eta_i, \eta_o)(\eta_i \cdot n_x)V_x(\eta_i)\,d\omega(\eta_i), \tag{1}$$

where $B_x(\eta_o)$ denotes the outgoing radiance from a point x in direction η_o, $L(\eta_i)$ is the incident radiance in direction η_i, $\rho_x(\eta_i, \eta_o)$ denotes the BRDF, $V_x(\eta_i)$ is the visibility function, and n_x is the surface normal at x. To evaluate the integral of the rendering equation, it is common to use Monte Carlo-based approaches. They solve the integrals by computing the average of random samples of the integrand, accumulating these values and taking the average. Importance sampling is a variance reduction technique of Monte Carlo-based approaches.

2.1 BRDF importance sampling

Importance sampling of BRDFs is a technique to reduce the image variance in physically based rendering. The concept is to find the distribution based on the representation of BRDFs. Simple analytical models such as diffuse, Phong or generalized cosine models can be sampled analytically.

Shirley [24] demonstrated how to sample the traditional Phong BRDF model efficiently. Lafortune and Willems [13] also presented importance sampling schemes for the modified Phong model. Ward [31] showed the stochastic sampling method for the BRDF models composed of elliptical Gaussian kernels. Lafortune et al. [12] used multiple cosine-lobes for representing the BRDF.

They used a non-linear fitting algorithm to fit sums of cosine-lobes to an analytical model or to actual measurements. Although this representation is simple and can be applied for the Monte Carlo importance sampling, it is hard to approximate the complex BRDF by using their fitting process. McCool and Harwood [16] generalized a k-D tree representation of probability distributions to support generation of samples from conditional distributions. Lalonde [14] used wavelets to represent the BRDF and proposed an importance sampling scheme for measured BRDFs. Matusik et al. [17] also used a wavelets representation of BRDF and presented a numerical sampling method based on wavelets analysis. Lawrence et al. [15] demonstrated an importance sampling method based on a factored representation. They reparameterized the BRDF by using a half-angle [23] and then the non-negative matrix factorization (NMF) twice to decompose the BRDF data for efficient importance sampling.

2.2 Environment map importance sampling

Environment map importance sampling is another technique for increasing the efficiency of ray tracing-based algorithms, together with complex lighting captured in a HDR environment map.

In some previous works, the environment maps were transformed into finite basis functions, such as wavelets [19], spherical harmonics [21, 22, 25] and steerable functions [26]. Some researchers used importance sampling techniques to distribute samples according to the energy distribution in the environment map [1, 6, 11, 20]. The importance sampling is often implemented based on clustering algorithms or hierarchical tiling schemes. Similarly, such an approach performs poorly for highly specular surfaces since samples chosen do not take the specular lobe into account.

2.3 Sampling from product distributions

More recently, several researchers have worked on this problem by drawing samples from the product distribution of the illumination and the BRDF. These approaches produce high quality images with a small number of samples.

Burke et al. [2] introduced a technique which is called bidirectional sampling. They considered both energy of incident illumination and the surface BRDF in the sampling process. Two Monte Carlo algorithms for sampling from the product distribution are presented. One is based on reflection sampling and the other is based on sampling-importance re-sampling (SIR). Clarberg et al. [3] presented a technique for importance sampling from products of the illumination and the BRDF using a hierarchical wavelet representation. Their method is very efficient for measured BRDF data but requires significant precomputation for environment maps. Talbot et al. [27] presented an importance resampling algorithm to generate more equally weighted samples for Monte Carlo integration. Cline et al. [5] proposed an importance sampling algorithm to generate samples based on the product of an environment map and a BRDF. It performs well for scenes with complex BRDFs and environment maps. Ghosh et al. [8] presented a sequential sampling algorithm for dynamic environment map illumination. While exploiting temporal coherence, it samples from the product of illumination and BRDF.

3 Spherical radial basis functions

An SRBF is recognized as an axis-symmetric reproducing kernel function defined on S^m, the unit sphere embedded in R^{m+1}. The kernel function only depends on the spherical distance between unit vectors. Let η and ξ be two points on S^m and $\theta(\eta, \xi)$ be the geodesic distance between η and ξ on S^m, which is the arc length of the great circle joining η and ξ. Since SRBF kernel functions are dependent on θ, SRBFs can be expressed in the expansions of Legendre polynomials as

$$G(\cos\theta) = G(\eta\cdot\xi) = \sum_{l=0}^{\infty} G_l P_l(\eta\cdot\xi), \tag{2}$$

where $P_l(\eta\cdot\xi)$ is Legendre polynomials of degree l and Legendre coefficients G_ls of Legendre polynomials satisfy

$$G_l \geq 0 \quad \text{and} \quad \sum_{l=0}^{\infty} G_l < \infty.$$

Since SRBFs have expansions of Legendre polynomials, there is a useful property based on the orthogonal property of Legendre polynomials in $[-1, 1]$ called spherical singular integral [7, 18, 28] by

$$\begin{aligned}(G *_m H)(\xi_g\cdot\xi_h) &= \int_{S^m} G(\eta\cdot\xi_g)H(\eta\cdot\xi_h)\,\mathrm{d}\omega(\eta) \\ &= \sum_{l=0}^{\infty} G_l H_l \frac{\omega_m}{d_{m,l}} P_l(\xi_g\cdot\xi_h),\end{aligned} \tag{3}$$

where ω_m is the total surface area of S^m, $d_{m,l}$ is the dimension of the space of order-l spherical harmonics on S^m, and $\mathrm{d}\omega$ denotes the differential surface element on S^m.

One example of SRBFs is the Gaussian SRBF kernel. The definition of Gaussian SRBF kernel is

$$G^{\mathrm{Gau}}(\eta\cdot\xi;\lambda) = e^{-\lambda} e^{\lambda(\eta\cdot\xi)}, \quad \lambda > 0, \tag{4}$$

where λ denotes the parameter called bandwidth and controls the coverage of the SRBF. The Gaussian SRBF kernel is adopted as the kernel function for the following reasons. The convolution of two Gaussian SRBF kernels has a mathematically simple form with small m [28]. The convolution of two Gaussian SRBFs can be written as

$$G^{\text{Gau}} *_m H^{\text{Gau}}(\xi_g \cdot \xi_h; \lambda_g, \lambda_h) = e^{-(\lambda_g+\lambda_h)} \omega_m \Gamma\left(\frac{m+1}{2}\right) I_{\frac{m-1}{2}}(\|r\|) \left(\frac{2}{\|r\|}\right)^{\frac{m-1}{2}}, \quad (5)$$

where $r = \lambda_g \xi_g + \lambda_h \xi_h$, Γ is the Gamma function, I_p is the modified Bessel functions of the first kind of order p, and $\| \cdot \|$ is the Euclidean norm.

Given a set of K distinct points $\{\xi_1, \xi_2, \ldots, \xi_K\}$ on S^m, which is called the set of SRBF centers, and another set of real numbers $\{\lambda_1, \lambda_2, \ldots, \lambda_K\}$, which is called the set of SRBF bandwidth parameters, a spherical function $F(\eta)$ can be represented in SRBF expansions as

$$F(\eta) = \sum_{k=1}^{K} F_k G(\eta \cdot \xi_k; \lambda_k), \quad (6)$$

where F_k is the SRBF coefficient.

Distribution of the SRBFs' centers on the sphere affects the compression efficiency significantly. If uniform SRBFs are used to represent the data with sparse distribution, it would waste lots of basis kernels on the region without data. On the other hand, using scattered SRBFs, i.e., adapting the center, bandwidth and coefficient of each basis, the SRBF kernels can be located based on the data distribution on the sphere. Therefore, scattered SRBFs can capture the features of the original data with much fewer bases than those used in uniform SRBFs.

4 Off-line SRBF fitting process

In the pre-process, scattered SRBFs are used to represent the HDR environment maps and the measured BRDF data. The non-uniform and non-negative SRBF fitting algorithm is introduced to transform the HDR environment maps and the measured BRDF data into scattered SRBFs.

Given a desired number of SRBFs n_l, there are three sets of parameters that are to be optimized: the set of SRBF coefficients $L = \{L_1, L_2, \ldots, L_{n_l}\}$, the set of SRBF centers $\Xi = \{\xi_1, \xi_2, \ldots, \xi_{n_l}\}$ on S^2 and the set of bandwidth parameters $\Lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_{n_l}\}$ in R. The objective is to minimize the square error between the original data and the approximated data by

$$\{L, \Xi, \Lambda\} = \underset{\{L,\Xi,\Lambda\}}{\arg\min} \int_{S^2} |D(\eta_{\text{i}}) - \tilde{D}(\eta_{\text{i}})|^2 \, \mathrm{d}\omega(\eta_{\text{i}})$$

$$\tilde{D}(\eta_{\text{i}}) = \sum_{k=1}^{n_l} L_k G(\eta_{\text{i}} \cdot \xi_k; \lambda_k), \quad (7)$$

where $D(\eta_{\text{i}})$ is the original data and $\tilde{D}(\eta_{\text{i}})$ is the approximated data.

The proposed approach modifies the previous approach [28] to solve the optimization problem. The coefficients of all bases are constrained to be positive since the coefficients are needed in estimating the probability distribution. Therefore, the L-BFGS-B solver [32] is used to optimize the coefficients.

The non-uniform and non-negative SRBF fitting algorithm consists of the following main steps:

1. Use the L-BFGS-B solver to optimize the set of centers from a given initial guess or the results of the previous iteration.
2. Use the L-BFGS-B solver to optimize the set of bandwidth parameters and the set of coefficients respectively.
3. The process is terminated if the difference of squared errors between current and previous iteration are less than a threshold E, or the count of iterations exceeds a user-defined threshold T.

This process is one kind of non-linear optimization. The fitting results are highly dependent on the initial guess, i.e. the initial guess would dominate the accuracy of the representation. The proposed approach applies the previous initial guess approach [28] to determinate an initial guess for a fitting operation. After the fitting process, the HDR environment map with SRBFs is represented as

$$L(\eta_{\text{i}}) \approx \sum_{j=1}^{M} F_j^{\text{illu}} G(\eta_{\text{i}} \cdot \xi_j; \lambda_j), \quad (8)$$

where $L(\eta_{\text{i}})$ is the incident radiance in direction η_{i}, G is the SRBF kernel function, ξ_j is the center of basis on unit sphere, λ_j is the bandwidth of the basis, F_j^{illu} is the basis coefficient for the illumination, and M is the number of the SRBFs for illumination.

Similarly, the measured BRDF data is represented in scattered SRBFs for each fixed outgoing direction as

$$\rho_x(\eta_{\text{i}}, \eta_{\text{o}}) \approx \sum_{k=1}^{N} F_k^{\text{brdf}} G(\eta_{\text{i}} \cdot \xi_k; \lambda_k), \quad (9)$$

where $\rho_x(\eta_{\text{i}}, \eta_{\text{o}})$ is the original BRDF data, G is the SRBF kernel function, ξ_k is the basis center on unit sphere, λ_k is the basis bandwidth, F_k^{brdf} is the basis coefficient for the BRDF, and N is the number of SRBFs for the BRDF.

5 Run-time rendering process

The flowchart of the run-time rendering process is shown in Fig. 2. When the view ray hits the object in the scene, the product of the environment map and the BRDF represented in SRBFs is first calculated. Next, the number of

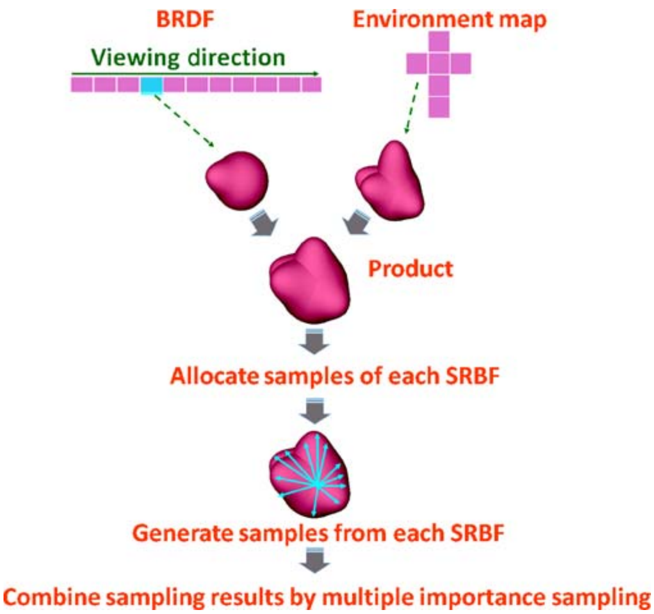

Fig. 2. Run-time rendering process

the samples of each SRBF is determined according to its integral. Then samples are generated from each SRBF and the results are combined by the multiple importance sampling technique [29, 30].

5.1 Product of illumination and BRDF

As mentioned before, taking the Gaussian SRBF as the kernel function has some benefits. One is that it is easy to calculate the product of the two Gaussian SRBFs. Ignoring the normalized term, the product of the two Gaussian SRBFs is

$$
\begin{aligned}
&F_3 e^{\lambda_3(\eta\cdot\xi_3)} = e^{-\lambda_1} e^{\lambda_1(\eta\cdot\xi_1)} \cdot e^{-\lambda_2} e^{\lambda_2(\eta\cdot\xi_2)}\\
&F_3 = e^{-(\lambda_1+\lambda_2)}\\
&\lambda_3 = |\lambda_1\xi_1 + \lambda_2\xi_2|\\
&\xi_3 = \frac{\lambda_1\xi_1 + \lambda_2\xi_2}{\lambda_3},
\end{aligned} \tag{10}
$$

where F_3 is the coefficient, λ_3 is the bandwidth, and ξ_3 is the center of the product result. The product of the illumination and the BRDF is defined as follows:

$$
\begin{aligned}
&L(\eta_{\rm i})\rho_x(\eta_{\rm i}, \eta_{\rm o})\\
&\quad\approx \sum_{j=1}^{M} F_j^{\rm illu} G(\eta_{\rm i}\cdot\xi_j; \lambda_j) \sum_{k=1}^{N} F_k^{\rm brdf} G(\eta_{\rm i}\cdot\xi_k; \lambda_k).
\end{aligned} \tag{11}
$$

After calculating the product of the two SRBFs, the number of basis functions becomes $M \times N$. If all the basis functions are used to generate the samples, computation cost will be high. Therefore, the basis functions with large coefficients are reserved and the basis functions with small coefficients are pruned. Since most of the energy is distributed in a few basis functions with large coefficients, a good approximation for original data is obtained even with only keeping the n largest coefficients.

5.2 Multiple importance sampling

After calculating the product of the environment map and the BRDF, it is desirable to generate rays distributed according to the density of the product. When the integral of the incident illumination for a fixed outgoing direction $\eta_{\rm o}$ located at x with normal n_x is evaluated, the Monte Carlo estimator for the integral can be written as

$$
\begin{aligned}
B_x(\eta_{\rm o}) &= \int_{S^2} L(\eta_{\rm i})\rho_x(\eta_{\rm i}, \eta_{\rm o})(\eta_{\rm i}\cdot n_x)V_x(\eta_{\rm i})\,{\rm d}\omega(\eta_{\rm i})\\
&\approx \frac{1}{n}\sum_{s=1}^{n}\left[\frac{L(\eta_{\rm i})\rho_x(\eta_{\rm i}, \eta_{\rm o})}{\gamma(\eta_s|\eta_{\rm o})}\right](\eta_s\cdot n_x)V_x(\eta_s),
\end{aligned} \tag{12}
$$

where $\gamma(\eta_s|\eta_{\rm o})$ is the probability of generating sample direction η_s assuming that $\eta_{\rm o}$ is fixed.

However, it is expensive to construct a single PDF $\gamma(\eta_s|\eta_{\rm o})$ that follows the shape of the complex product of the illumination and the BRDF. A technique for importance sampling [29, 30], multiple importance sampling, is adopted. The combination of several potentially good estimators makes the Monte Carlo integration a more robust technique. The estimators calculated with different PDFs can have different qualities in different regions of the integration domain. It makes a weighted-average of all estimators where the weights depend on the sampling positions. If the integral of $f(x)$ is evaluated as

$$\int_{\Omega} f(x)\,{\rm d}x,$$

and there are n different estimators, the combined estimator is given by

$$F = \sum_{i=1}^{n}\frac{1}{n_i}\sum_{j=1}^{n_i} w_i(X_{i,j})\frac{f(X_{i,j})}{p_i(X_{i,j})}, \tag{13}$$

where p_i is the PDF for each estimator, n_i denotes the number of samples from p_i, $X_{i,j}$ are the samples from p_i for $j = 1, 2, \ldots, n_i$, and all samples are assumed to be independent. Also, w_i is the weighting function and satisfies the following two conditions:

$$\sum_{i=1}^{n} w_i(x) = 1, \quad w_i(x) = 0 \quad \text{whenever } p_i(x) = 0. \tag{14}$$

Then, the expected value of the combined estimator F would be equal to the integral of $f(x)$.

This technique is applied to the importance sampling of product distribution, and Eq. 12 would become:

$$B_x(\eta_o) = \int_{S^2} L(\eta_i)\rho_x(\eta_i, \eta_o)(\eta_i \cdot n_x)V_x(\eta_i)\,d\omega(\eta_i)$$
$$\approx \sum_{i=1}^{n} \frac{1}{n_i} \sum_{j=1}^{n_i} \left[\frac{n_i p_i(X_{i,j})}{\sum_k n_k p_k(X_{i,j})}\right] \times \left[\frac{L(X_{i,j})\rho_x(X_{i,j}, \eta_o)}{p_i(X_{i,j})}\right](X_{i,j} \cdot n_x)V_x(X_{i,j}), \tag{15}$$

where n is the number of SRBFs, n_i denotes the number of samples from each SRBF, and p_i is the PDF calculated from each SRBF kernel function for $i = 1, \ldots, n$. $X_{i,j}$ is the sample of each SRBF kernel function for $j = 1, \ldots, n_i$.

When computing Eq. 15 in a run-time rendering process, the number of samples, n_i, should be taken from each SRBF kernel and the sampling directions distributed according to each SRBF kernel, $X_{i,j}$, should be generated.

5.3 Sampling algorithm

Intuitively, each scattered SRBF covers a part of the entire product region. Although there would be two or more overlapping SRBFs in the same region, the intensity of multiple sampling directions within a pixel can still be gathered using the multiple importance sampling approach.

The product of the environment map and the BRDF for a given viewing direction in the run-time rendering process is first calculated. Then, recall that SRBF is defined on the unit sphere, and its integral is easy to calculate by using the spherical singular integral property of SRBF. The integral of each SRBF can be taken as its total energy gathered from all directions. Therefore, it is straightforward to allocate samples according to the ratio of the integral of each SRBF to the sum of all SRBFs' integrals. Furthermore, more samples in the SRBFs with a higher level of energy are allocated. Thus, the energy by squaring the influence of the integral is emphasized. The probability of choosing the SRBF l for each sample is given by

$$P(l) = \frac{I_l}{\sum_{i=1}^{n} I_i}, \tag{16}$$

where n is the number of SRBFs and I_i is the integral of SRBF i. A 1D cumulative distribution function (CDF) over l from these probabilities is calculated. In the dispatching process, a uniform variable in $[0, 1]$ is initially generated and this variable with a random number for each sample is jittered. Then, the CDF is traversed to determine where the sample should be taken from.

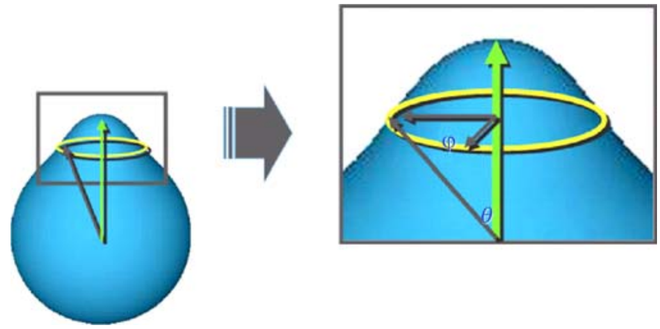

Fig. 3. Elevation angle and azimuth angle defined against an SRBF

Next, a sample direction in each SRBF kernel is generated by sequentially selecting the elevation angle θ and the azimuth angle φ, as shown in Fig. 3. For the elevation angle θ, the metropolis random walk algorithm [29, 30] is used to generate samples with a desired density. It should be noted that the SRBF kernel itself only describes 1D density, while the desired density of sampling is the density over the sphere. Therefore, when the elevation angle θ is selected by this approach, the influence of the circumference around the center of the SRBF kernel, in particular, should be considered. For the azimuth angle φ, each SRBF kernel is symmetric against the vector that is defined by its center and the origin of the unit sphere. Therefore, the distribution of the azimuth angle φ is uniform, and a random number in $[0, 2\pi]$ can simply be generated to evaluate φ.

6 Results

All results are generated on an AMD Athlon64 FX-60 PC with NVIDIA GeForce 7900 GTX.

6.1 Fitting errors

The fitting performances of the Lafortune model [12] and the proposed SRBF representation are compared. The Lafortune model, which combines multiple generalized cosine-lobes, can be written as

$$\rho(u, v) = \sum_i (C_{x,i}u_x v_x + C_{y,i}u_y v_y + C_{z,i}u_z v_z)^{n_i}, \tag{17}$$

where u is the incident direction and v is the outgoing direction. In this fitting process, a non-linear optimization technique was applied to determine the parameters $C_{x,i}$, $C_{y,i}$, $C_{z,i}$ and n_i. The objective was to minimize the mean-square error of the reflectance functions multiplied by the cosines of the incidence angles with the normal. Because the model was fitted depending on the outgoing and incident directions at the same time, it was not easy to make an initial guess. Consequently, it is sometimes hard to fit this model to some complex BRDF data.

Table 1 displays the fitting errors of the Lafortune model and the scattered SRBFs with Gaussian SRBF kernels. When fitting the Lafortune model, the initial guesses were randomly generated and the fitting process was executed numerous times (100 times for each BRDF). In the experiments, although the number of lobes for the Lafortune model was added, it remained difficult to improve the fitting performance. The results of the proposed approach are compared with the best-fit Lafortune models (with three lobes).

Table 1. Comparison of fitting errors

	Lafortune model	Scattered SRBFs
Paint blue	32%	14%
Garnet red	7.6%	4%
Krylon blue	10%	5.9%
Cayman	19%	5.9%

6.2 Rendering

The rendering results of a complex HDR environment map and several BRDF data are demonstrated. The proposed approach is compared to SRBF-based BRDF importance sampling.

Figures 4–6 show the comparisons between the proposed approach with a varying number of samples and BRDF importance sampling. The Buddha model in ‘Grace Cathedral’ is rendered with different BRDF data. From Figs. 4–6, the materials are ‘Garnet Red’, ‘Krylon Blue’, and ‘Cayman’ measured by Cornell University. The sampling results a–e are from the SRBF product with a varying number of samples. And f is the rendered results using SRBFs-based BRDF importance sampling. Performing

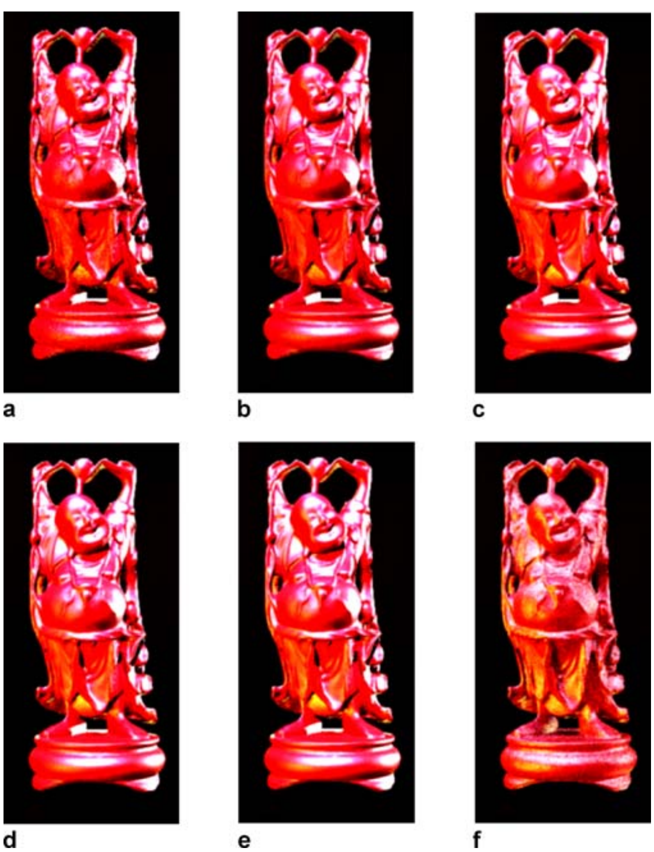

Fig. 4a–f. Sampling results with material ‘Garnet red’. **a** 10 samples, **b** 40 samples, **c** 60 samples, **d** 80 samples, **e** 100 samples and **f** 100 samples (BRDF)

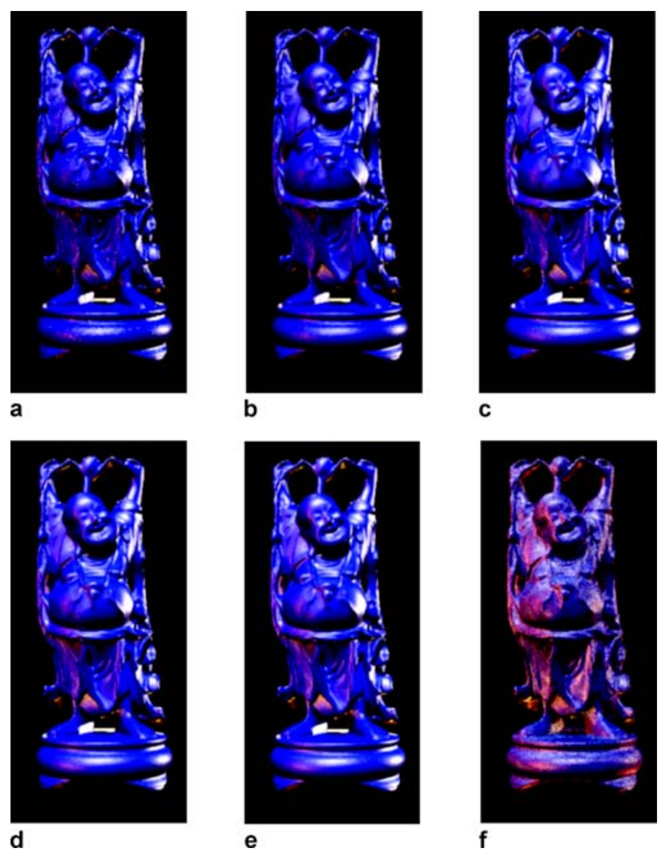

Fig. 5a–f. Sampling results with material ‘Krylon blue’. **a** 10 samples, **b** 40 samples, **c** 60 samples, **d** 80 samples, **e** 100 samples and **f** 100 samples (BRDF)

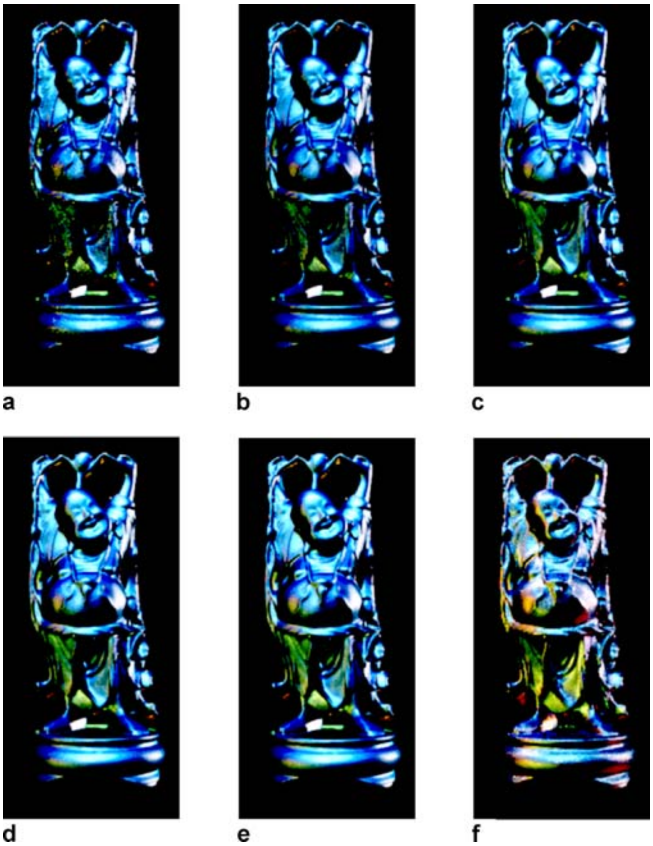

Fig. 6a–f. Sampling results with material 'Cayman'. **a** 10 samples, **b** 40 samples, **c** 60 samples, **d** 80 samples, **e** 100 samples and **f** 100 samples (BRDF)

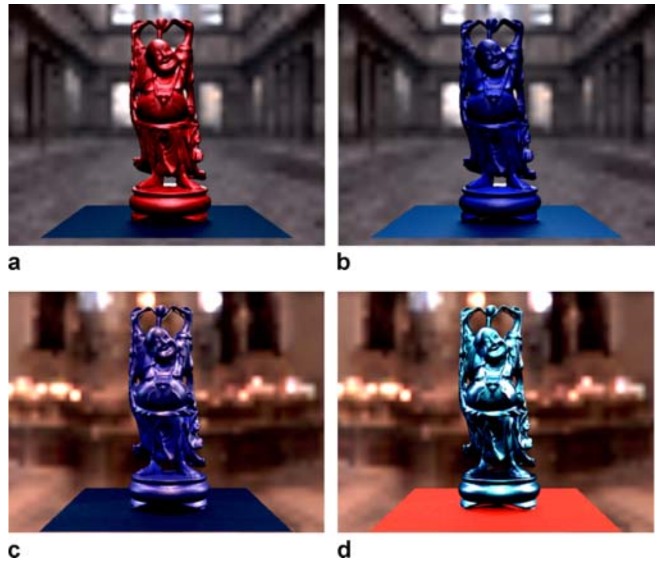

Fig. 7. a a 'Garnet red' Buddha and a 'Cayman' plane in 'Uffizi Gallery' HDR environment, **b** a 'Krylon blue' Buddha and a 'Cayman' plane in 'Uffizi Gallery', **c** a 'Krylon blue' Buddha and a 'Cayman' plane in 'St. Peter's Basilica' and **d** a 'Cayman' Buddha and a 'Garnet red' plane in 'St. Peter's Basilica'

the SRBFs product sampling in the run-time process obviously adds some overhead. In the current implementation, there was a 40% increase in computation time over the SRBF-based BRDF importance sampling for an equal number of samples. However, the quality of the rendered images with the proposed method is much better than with the SRBF-based BRDF importance sampling.

A complex scene with two complex HDR environment maps and three different measured BRDF data with 60 samples per pixel are given in Fig. 7. Each environment map with 100 Gaussian kernels and each BRDF measurement in the scattered SRBFs is represented with five Gaussian kernels. After computing the products, 100 Gaussian kernels are reserved to generate samples.

Compared with previous methods, the SRBF representation can significantly decrease pre-computed data storage. For example, wavelet importance sampling presented by Clarberg et al. [3] requires significant precomputation for environment maps when rotating lighting environment. In order to get a smooth result, they must bilinearly interpolate between the four nearest wavelets in the environment map. Since an SRBF is a rotation-invariant function, rotating functions in the SRBF representation is as straightforward as rotating the centers of the SRBF. Correct SRBFs can simply be obtained by one rotation operation.

The proposed approach achieves low variance in non-occluded regions. However, the resulting images still have noise in partially occluded regions as the proposed approach does not take visibility into account during the sampling process. Ghosh and Heidrich [9] presented an approach to address the noise in partial shadow regions. On the other hand, the major computation cost for ray tracing is the visibility testing.

7 Conclusions

A new approach for importance sampling of products from illumination and BRDF has been presented. The proposed approach efficiently renders images with multiple HDR environment maps and measured BRDFs. The main contributions of this paper are as follows: (a) in the off-line SRBF fitting process, an initial guess is determined for a fitting operation; (b) in the run-time rendering process, after the product of the two SRBFs is computed, this is used to guide the number of samples. The proposed approach samples each SRBF of a subset of SRBFs and combines the samples' values by multiple importance sampling.

In the future, an algorithm to generate samples smarter will be developed based on some heuristic approaches.

References

1. Agarwal, S., Ramamoorthi, R., Belongie, S., Jensen, H.W.: Structured importance sampling of environment maps. ACM Trans. Graph. **22**(3), 605–612 (2003)
2. Burke, D., Ghosh, A., Heidrich, W.: Bidirectional importance sampling for direct illumination. In: Proceedings of Eurographics Symposium on Rendering, pp. 147–156. Eurographics Association, Konstanz (2005)
3. Clarberg, P., Jarosz, W., Akenine-Möller, T., Jenson, H.W.: Wavelet importance sampling: efficiently evaluating products of complex functions. ACM Trans. Graph. **24**(3), 1166–1175 (2005)
4. Claustres, L., Paulin, M., Boucher, Y.: BRDF measurement modeling using wavelets for efficient path tracing. Comput. Graph. Forum **22**(4), 701–716 (2003)
5. Cline, D., Egbert, P.K., Talbot, J.F., Cardon, D.L.: Two stage importance sampling for direct lighting. In: Proceedings of Eurographics Symposium on Rendering, pp. 103–113. Eurographics Association, Nicosia, Cyprus (2006)
6. Cohen, J., Debevec, P.: LightGen, HDRShop plugin, 2001
7. Freeden, W., Gervens, T., Schreiner, M.: Constructive Approximation on the Sphere. Oxford University Press, Oxford (1998)
8. Ghosh, A., Doucet, A., Heidrich, W.: Sequential sampling for dynamic environment map illumination. In: Proceedings of Eurographics Symposium on Rendering, pp. 115–126. Eurographics Association, Nicosia, Cyprus (2006)
9. Ghosh, A., Heidrich, W.: Correlated visibility sampling for direct illumination. Visual Comput. **22**(9–11), 693–701 (2006)
10. Kajiya, J.T.: The rendering equation. In: Proceedings of SIGGRAPH 86, pp. 143–150. ACM Press, Dallas, TX (1986)
11. Kollig, T., Keller, A.: Efficient illumination by high dynamic range images. In: Proceedings of Eurographics Symposium on Rendering, pp. 45–50. Eurographics Association, Leuven, Belgium (2003)
12. Lafortune, E.P., Foo, S.C., Torrance, K.E., Greenberg, D.P.: Non-linear approximation of reflectance functions. In: Proceedings of SIGGRAPH 97, pp. 117–126. ACM Press, Los Angeles, CA (1997)
13. Lafortune, E.P., Willems, Y.D.: Using the modified Phong reflectance model for physically based rendering. Technical Report RP-CW-197. Department of Computing Science, K.U. Leuven (1994)
14. Lalonde, P.: Representations and uses of light distribution functions. Ph.D. Dissertation, University of British Columbia (1997)
15. Lawrence, J., Rusinkiewicz, S., Ramamoorthi, R.: Efficient BRDF importance sampling using a factored representation. ACM Trans. Graph. **23**(3), 496–505 (2004)
16. McCool, M.D., Harwood, P.K.: Probability trees. In: Proceedings of Graphics Interface '97, pp. 37–46. Canadian Information Processing Society, Kelowna, British Columbia, Canada (1997)
17. Matusik, W., Pfister, H., Brand, M., McMillan, L.: A data-driven reflectance model. ACM Trans. Graph. **22**(3), 759–769 (2003)
18. Narcowich, F.J., Ward, J.D.: Nonstationary wavelets on the m-sphere for scattered data. Appl. Comput. Harmonic Anal. **3**(4), 324–336 (1996)
19. Ng, R., Ramamoorthi, R., Hanrahan, P.: All-frequency shadows using non-linear wavelet lighting approximation. ACM Trans. Graph. **22**(3), 376–381 (2003)
20. Ostromoukhov, V., Donohue, C., Jodoin, P.M.: Fast hierarchical importance sampling with blue noise properties. ACM Trans. Graph. **23**(3), 488–495 (2004)
21. Ramamoorthi, R., Hanrahan, P.: An efficient representation for irradiance environment maps. In: Proceedings of SIGGRAPH 01, pp. 497–500. ACM Press, Los Angeles, CA (2001)
22. Ramamoorthi, R., Hanrahan, P.: Frequency space environment map rendering. In: Proceedings of SIGGRAPH 02, pp. 517–526. ACM Press, San Antonio, TX (2002)
23. Rusinkiewicz, S.M.: A new change of variables for efficient BRDF representation. In: Proceedings of Eurographics Workshop on Rendering, pp. 11–22. Eurographics Association, Vienna (1998)
24. Shirley, P.S.: Physically based lighting calculations for computer graphics. Ph.D. Dissertation, University of Illinois at Urbana-Champaign (1991)
25. Sloan, P., Kautz, J., Snyder, J.: Precomputed radiance transfer for real-time rendering in dynamic, low-frequency environments. In: Proceedings of SIGGRAPH 02, pp. 527–536. ACM Press, San Antonio, TX (2002)
26. Subr, K., Arvo, J.: Steerable stratified importance sampling. In: Proceedings of IEEE/EG Symposium on Interactive Ray Tracing, pp. 133–140. IEEE, Ulm University (2007)
27. Talbot, J.F., Cline, D., Egbert, P.: Importance resampling for global illumination. In: Proceedings of Eurographics Symposium on Rendering, pp. 139–146. Eurographics Association, Konstanz (2005)
28. Tsai, Y.T., Shih, Z.C.: All-frequency precomputed radiance transfer using spherical radial basis functions and clustered tensor approximation. ACM Trans. Graph. **25**(3), 967–976 (2006)
29. Veach, E.: Robust Monte Carlo methods for light transport Simulation. Ph.D. Dissertation, Stanford University (1997)
30. Veach, E., Guibas, L.: Optimally combining sampling techniques for Monte Carlo rendering. In: Proceedings of SIGGRAPH 95, pp. 419–428. ACM Press, Los Angeles, CA (1995)
31. Ward, G.J.: Measuring and modeling anisotropic reflection. In: Proceedings of SIGGRAPH 92, pp. 265–272. ACM Press, Chicago, IL (1992)
32. Zhu, C., Byrd, R.H., Lu, P., Nocedal, J.: Algorithm 778: L-BFGS-B: Fortran subroutines for large scale bound-constrained optimization. ACM Trans. Math. Softw. **23**(4), 550–560 (1997)

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