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Theory and Methodology

An algorithm for generalized fuzzy binary linear programming problems

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Abstract

Fuzzy binary linear programming (FBLP) problems are very essential in many fields such as assignment and assembly line balancing problems in operational research, multiple projects, locations, and candidates selection cases in management science, as well as representing and reasoning with prepositional knowledge in artificial intelligence. Although FBLP problems play a significant role in human decision environment, not very much research has focused on FBLP problems. This work first proposes a simple means of expressing a triangular fuzzy number as a linear function with an absolute term. A method of linearizing absolute terms is also presented. The developed goal programming (GP) model weighted by decision-makers' (DMs) preference aims to optimize the expected objective function and minimize the sum of possible membership functions' deviations. After presented a novel way of linearizing product terms, the solution algorithm is proposed to generate a crisp trade-off promising solution that is also an optimal solution in a certain sense. Three examples, equipment purchasing choice, investment project selection, and assigning clients to project leaders, illustrate that the proposed algorithm can effectively solve generalized FBLP problems.

Keywords: Fuzzy mathematical programming; Binary linear programming

1. Introduction

Binary linear programming (BLP) problems play a prominent role in representing and reasoning with prepositional knowledge in artificial intelligence [3,9,16,21], assignment and assembly line balancing problems in operational research [5,6,17,30,31], as well as multiple materials, projects, locations, and candidates selection decisions in management science [12,13,19,22]. The common difficulty in solving a BLP problem is the uncertainty related to its decision environment and the lack of an objective measure to the

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related criterion and relationships that guide the evaluation environment. Employing the fuzzy set theory is one of the most successful ways to solve this difficulty [4,7,29,33,34].

A decision-maker (DM) may feel more natural in specifying vague (possible) values than precise values since she/he frequently knows only approximations of exact numbers. A BLP problem with fuzzy coefficients and constraints named as a fuzzy binary linear programming (FBLP) problem can be formulated as follows:

FBLP problem:

$$\begin{aligned} &\text{Maximize} \quad \tilde{c}_1 x_1 + \tilde{c}_2 x_2 + \cdots + \tilde{c}_n x_n \\ &\text{subject to} \quad \sum_{i=1}^{n} \tilde{a}_{ij} x_j \leqslant \tilde{b}_j, \end{aligned} \tag{1.1}$$

where x_i are zero–one variables, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$, $\tilde{c}_i$ the fuzzy coefficients in the objective function, x_i express zero–one decision variables, $\tilde{a}_{ij}$ represent the fuzzy coefficient with respect to x_i in the jth constraint, and $\tilde{b}_j$ denote the fuzzy number in the right-hand side of the jth constraint, $i = 1, 2, \ldots, n$, and $j = 1, 2, \ldots, m$.

The papers by Mohanty [4], Carlsson [7], Herrera et al. [10,11], Bitran [14], Liang et al. [15], Maeda [18], Castro et al. [20], Teng et al. [23], Delgado et al. [25], Wang et al. [26], Abboud et al. [27], Dias [28], and Chen [32] give an overview of FBLP techniques and a list of references. Survey by Kandal [1], Kaufmann et al. [2] and Lai et al. [33,34] are also excellent texts for these techniques. However, most of the methods proposed to solve FBLP problems either use α-cut techniques that require iterative processes or utilize arithmetic operations that require tedious computation. Moreover, conventional methods can only solely treat either fuzzy coefficients in the objective function or fuzzy numbers in the right-hand side of constraints.

This study proposes an algorithm that can simultaneously solve a BLP problem with fuzzy coefficients in the objective function, fuzzy coefficients in the constraint matrix, and fuzzy numbers in the right-hand side of constraints. The presented model also enables a DM to solve a FBLP problem in a more natural and direct way as a DM can directly obtain the a crisp promising solution merely after finishing a linear programming (LP) computation. Consequently, the proposed method is a worthwhile alternative to existing methods from a practical point of view.

2. Interpreting a fuzzy value

At first, a clear and simple means of representing a triangular membership function without limiting the symmetric triangle form is introduced.

Proposition 1. *Let $\mu(c_i)$ be a triangle membership function of a fuzzy value c_i, as depicted in Fig.* 1, *where $c_{i,k}$ $(k = 1, 2, 3)$ are, respectively, the possible lowest number, middle number, and highest number, $s_{i,k}$ $(k = 1, 2)$ are the slopes of line segments between $c_{i,k}$ and $c_{i,k+1}$, and*

$$s_{i,k} = \frac{\mu(c_{i,k+1}) - \mu(c_{i,k})}{c_{i,k+1} - c_{i,k}} \quad \textit{for } k = 1, 2.$$

$\mu(c_i)$ can then be expressed below:

$$\mu(c_i) = \mu(c_{i,1}) + s_{i,1}(c_i - c_{i,1}) + \frac{s_{i,2} - s_{i,1}}{2}(|c_i - c_{i,2}| + c_i - c_{i,2}), \tag{2.1}$$

where $|\cdot|$ is the absolute value of '·'

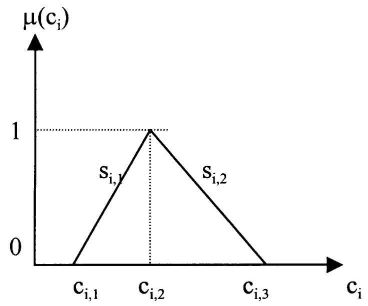

Fig. 1. A triangle membership function.

Proof. This proposition can be examined as follows:
(i) If $c_i \leqslant c_{i,2}$, then

$$\mu(c_i) = \mu(c_{i,1}) + s_{i,1}(c_i - c_{i,1}) + \frac{s_{i,2} - s_{i,1}}{2}(|c_i - c_{i,2}| + c_i - c_{i,2}) = \mu(c_{i,1}) + s_{i,1}(c_i - c_{i,1}).$$

(ii) If $c_{i,2} \leqslant c_i \leqslant c_{i,3}$, then

$$\begin{aligned}\mu(c_i) &= \mu(c_{i,1}) + s_{i,1}(c_i - c_{i,1}) + \frac{s_{i,2} - s_{i,1}}{2}(|c_i - c_{i,2}| + c_i - c_{i,2}) \\ &= \mu(c_{i,1}) + s_{i,1}(c_i - c_{i,1}) + (s_{i,2} - s_{i,1})(c_i - c_{i,2}) = \mu(c_{i,1}) + s_{i,1}(c_{i,2} - c_{i,1}) + s_{i,2}(c_i - c_{i,2}).\end{aligned}$$

This proposition is then proven. □

Example 1.

$$\begin{aligned}&\text{Maximize} \quad \mu(c_1) \\ &\text{subject to} \quad c_1 \in F \ (F \text{ is a feasible set}), \ c_1 \geqslant 0,\end{aligned}$$

where c_1 is a fuzzy value depicted in Fig. 2(a).

By using Proposition 1, $\mu(c_1)$ can be expressed as follows:

$$\begin{aligned}\mu(c_1) &= 0.125(c_1 - 47) + \frac{-0.125 - 0.125}{2}(|c_1 - 55| + c_1 - 55) \\ &= 0.125(c_1 - 47) - \frac{0.25}{2}(|c_1 - 55| + c_1 - 55).\end{aligned} \tag{2.2}$$

The following proposition is then presented to linearize the absolute term.

Proposition 2. *Consider a problem expressed below*:

PP1:

$$\begin{aligned}&\text{Maximize} \quad Z = -(|f(X) - g| + f(x) - g) \\ &\textit{subject to} \quad X \in F \ (F \textit{ is a feasible set}), \ g \textit{ is a given non-negative constant}.\end{aligned}$$

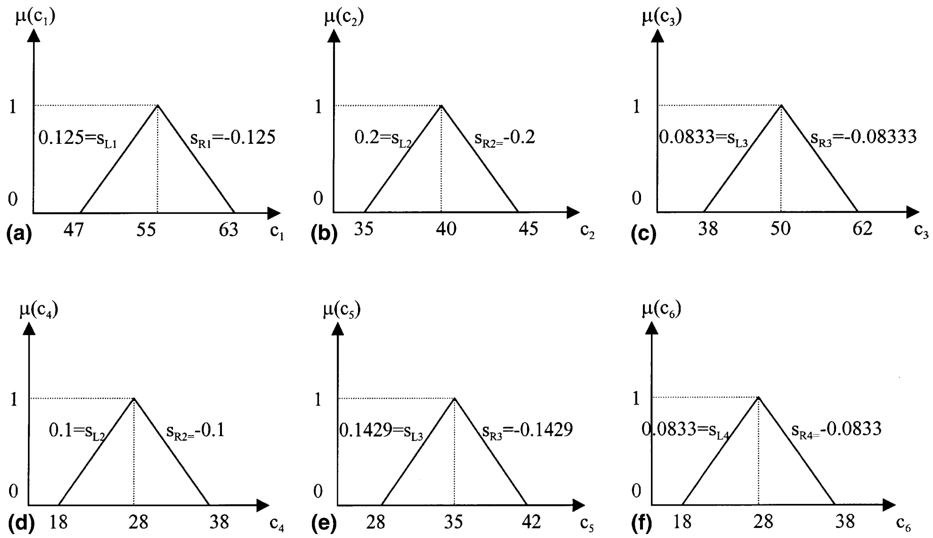

Fig. 2. (a) Membership function $\mu(c_1)$. (b) Membership function $\mu(c_2)$. (c) Membership function $\mu(c_3)$. (d) Membership function $\mu(c_4)$. (e) Membership function $\mu(c_5)$. (f) Membership function $\mu(c_6)$.

can be linearized as PP2 below:

PP2:

$$\begin{aligned} &\text{Maximize} \quad ZZ = -2(f(X) - g + d) \\ &\textit{subject to} \quad f(X) - g + d \geqslant 0, \;\; d \geqslant 0, \;\; X \in F. \end{aligned}$$

Proof. This proposition can be verified as follows:
(i) *Case 1*: $f(x) - g \geqslant 0$. At the optimal solution d will be forced as $d = 0$, which results in $ZZ = -2(f(x) - g) = Z$.
(ii) *Case 2*: $f(x) - g < 0$. At the optimal solution d will be forced as $d = g - f(x)$, which results in $ZZ = 0 = Z$.
This proposition is then finished. □

Based on Proposition 2, expression (2.2) in Example 1 can be linearized as follows:

$$\begin{aligned} &\text{Maximize} \quad \mu(c_1) = 0.125(c_1 - 47) - 0.25(c_1 - 55 + d) = -0.125c_1 - 0.25d + 7.875 \\ &\text{subject to} \quad c_1 + d \geqslant 55, \;\; c_1 \in F \;(F \text{ is a feasible set}), \;\; c_1, \;\; d \geqslant 0. \end{aligned}$$

By running on the LINDO [24], the obtained solution set is ($d_1 = 0, c_1 = 55$, and $\mu(c_1) = 1$).

3. Fuzzy coefficients in the objective function

Encountered changeable future, a DM typically has difficulty in evaluating an R&D project, financial investment, delivery route or transportation construction. This section discusses how to solve a BLP problem with fuzzy coefficients in the objective function.

Example 2 (*Taken from* [11]). A computer science department wishes to purchase equipment for some computer rooms. Each room will have different equipment to create a diverse variety of workstations. Six different types of proposals are received and a study is performed based on students that will use the equipment. The cost of each classroom is given in millions of pesetas and each number of students is given as a percentage of total students, where the percentages are uncertain as displayed in Table 1.

The goal is to purchase equipment for the classrooms to maximize their use by the number of students. It is only possible to buy equipment for three classrooms since only 32 million pesetas are available. In addition, it is necessary to have at least one of type A, B, or C, and another of type C, E, or F. Thus, this problem's constraints are precise but the objective is imprecise. The problem is formulated as

$$\text{Maximize} \quad \tilde{c}_1x_1 + \tilde{c}_2x_2 + \tilde{c}_3x_3 + \tilde{c}_4x_4 + \tilde{c}_5x_5 + \tilde{c}_6x_6 \tag{3.1}$$

$$\text{subject to} \quad 14x_1 + 11x_2 + 17x_3 + 7x_4 + 13x_5 + 10x_6 \leqslant 32, \tag{3.2}$$

$$x_1 + x_2 + x_3 \geqslant 1, \quad x_3 + x_5 + x_6 \geqslant 1, \tag{3.3}$$

$$x_1, x_2, x_3, x_4, x_5, \text{ and } x_6 \text{ are zero–one variables,} \tag{3.4}$$

where the fuzzy numbers $\tilde{c}_1, \tilde{c}_2, \tilde{c}_3, \tilde{c}_4, \tilde{c}_5$ and $\tilde{c}_6$ are depicted in Figs. 2(a), (b), (c), (d), (e) and (f), respectively.

Using Proposition 1 to express the membership functions of $\tilde{c}_1, \tilde{c}_2, \tilde{c}_3, \tilde{c}_4, \tilde{c}_5$, and $\tilde{c}_6$ as

$$\mu(c_1) = 0.125(c_1 - 47) - (0.25/2)(|c_1 - 53| + c_1 - 53),$$
$$\mu(c_2) = 0.2(c_2 - 35) - (0.4/2)(|c_2 - 40| + c_2 - 40),$$
$$\mu(c_3) = 0.0833(c_3 - 38) - (0.1667/2)(|c_3 - 50| + c_3 - 50),$$
$$\mu(c_4) = 0.1(c_4 - 18) - (0.2/2)(|c_4 - 28| + c_4 - 28),$$
$$\mu(c_5) = 0.1429(c_5 - 28) - (0.2857/2)(|c_5 - 35| + c_5 - 35),$$
$$\mu(c_6) = 0.0833(c_6 - 31) - (0.1667/2)(|c_6 - 43| + c_6 - 43),$$

respectively.

Following Proposition 2, we then have

$$\mu(c_1) = -0.125c_1 - 0.25d_1 + 7.875, \quad c_1 + d_1 \geqslant 55, \tag{3.5}$$

$$\mu(c_2) = -0.2c_2 - 0.4d_2 + 9, \quad c_2 + d_2 \geqslant 40, \tag{3.6}$$

$$\mu(c_3) = -0.0833c_3 - 0.1667d_3 + 5.1696, \quad c_3 + d_3 \geqslant 50, \tag{3.7}$$

$$\mu(c_4) = -0.1c_4 - 0.2d_4 + 3.8, \quad c_4 + d_4 \geqslant 28, \tag{3.8}$$

$$\mu(c_5) = -0.14286c_5 - 0.2857d_5 + 6, \quad c_5 + d_5 \geqslant 35, \tag{3.9}$$

$$\mu(c_6) = -0.0833c_6 - 0.1667d_6 + 4.58344, \quad c_6 + d_6 \geqslant 43. \tag{3.10}$$

Table 1
The cost and possible number of students for each type of class

Type of class	Cost	Percentage of students
A	14	55 (8% tolerance)
B	1	40 (5% tolerance)
C	17	50 (12% tolerance)
D	7	28 (10% tolerance)
E	13	35 (7% tolerance)
F	10	43 (12% tolerance)

Since the faculty and students in Example 1 have desired to maximize the objective function (3.1), the goal can be considered to maximize $c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4 + c_5x_5 + c_6x_6$ and $\mu(c_1) + \mu(c_2) + \mu(c_3) + \mu(c_4) + \mu(c_5) + \mu(c_6)$ concurrently. Accordingly, the optimization problems (3.1)–(3.4) can be reformulated as follows:

$$\text{Maximize} \quad c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4 + c_5x_5 + c_6x_6, \tag{3.11}$$
$$\text{Maximize} \quad \mu(c_1) + \mu(c_2) + \mu(c_3) + \mu(c_4) + \mu(c_5) + \mu(c_6), \tag{3.12}$$
$$\text{subject to} \quad (3.2)\text{–}(3.10).$$

Since the problem was encountered in incorporating (3.11) and (3.12), the following definition and proposition are considered.

Definition 1. Let $C(X)$ be an objective function. $X = (x_1, x_2, \ldots, x_n)$ expresses an n-vector objective function and $(X, \mu_1, \mu_2, \ldots, \mu_n)$ denotes a system of membership functions of fuzzy values. Since an optimization problem in a fuzzy environment is a system of membership functions together with an objective function, the optimization problem of considering both an objective function and a system of membership functions simultaneously becomes a trade-off problem.

Proposition 3. *An optimal solution for the model*

PP3:

$$\text{Maximize} \quad \left(\sum_{i=1}^{n} c_i x_i, \ \sum_{i=1}^{n} \mu(c_i) \right)$$
$$\text{subject to} \quad c_i \in F \ (F \text{ is a feasible set}), \quad c_i \geqslant 0,$$

is the solution that maximizes the following model:

PP4:

$$\text{Maximize} \quad \sum_{i=1}^{n} c_i x_i - \sum_{i=1}^{n} (w_i^+ \delta_i^+ + w_i^- \delta_i^-)$$
$$\text{subject to} \quad \mu_i(c_i) - \delta_i^+ + \delta_i^- = 1, \quad c_i \in F \ (F \text{ is a feasible set}), \quad c_i \geqslant 0,$$

where $w_i^+ = |1/s_{\mathrm{L}i}|$ and $w_i^- = |1/s_{\mathrm{R}i}|$ are the inverse of slopes as depicted in Fig. 3.

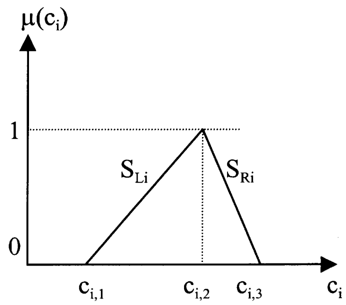

Fig. 3. Membership function $\mu(c_i)$.

Proof. Due to the complexity in solving problems with multiple and non-commensurable goals a single solution capable of optimizing all the goals generally does not exist, the concurrent optimization of the objective function and membership functions is naturally considered as a trade-off problem. Since the range of each membership function is from 0 to 1 and the unit change in $c_i x_i$ results in proportionally change in left-hand side or right-hand side line segment of $\mu(c_i)$ guided by $|1/s_{Li}|$ or $|1/s_{Ri}|$, then a solution for the model is

$$\left\{ \text{Maximize} \left(\sum_{i=1}^{n} c_i x_i, \sum_{i=1}^{n} \mu(c_i) \right) \right\}$$

is any point which

$$\text{Maximize} \left(\sum_{i=1}^{n} c_i x_i - \sum_{i=1}^{n} \left(w_i^+ \delta_i^+ + w_i^- \delta_i^- \right) \right).$$

Take $c_1 x_1$ in (3.11) as an instance. Since the range of membership function $\mu(c_1)$ is from 0 to 1 and the range of $c_1 x_1$ is between 47 and 63,

$$w_1^+ = \left| \frac{1}{s_{L,1}} \right| = \left| \frac{1}{0.125} \right| = 8$$

and

$$w_1^- = \left| \frac{1}{s_{R,1}} \right| = \left| \frac{1}{-0.125} \right| = 8,$$

as depicted in Fig. 2(a). By utilizing adequate weights to regulate the behavior between $c_i x_i$ and $\mu(c_i)$, this proposition is completed. □

Consequently, the models (3.11) and (3.12) can be reformulated as the following goal programming (GP) model:

Model (1):

$$\begin{aligned} &\text{Maximize} && \sum_{i=1}^{6} (c_i x_i) - \sum_{i=1}^{6} (w_i^+ \delta_i^+ + w_i^- \delta_i^-) \\ &\text{subject to} && (3.2)\text{–}(3.10), \ \mu(c_i) - \delta_i^+ + \delta_i^- = 1, \ i = 1, 2, \ldots, 6, \end{aligned} \tag{3.13}$$

where the weights w_i^+ and w_i^- are equal to $|1/s_{Li}|$ and $|1/s_{Ri}|$, respectively, as indicated in Figs. 2(a)–(e).

The first term in (3.13) is the expected objective value the DM naturally aims to maximize while the second term is the sum the possible fuzzy value deviations the DM desires to minimize.

Proposition 4 is presented to linearize the product term $c_i x_i$ in (3.13).

Proposition 4. *Consider the following program*:

PP3:

$$\begin{aligned} &\text{Maximize} && cx \\ &\text{subject to} && x \in 0\text{–}1 \text{ variable}, \ c \geqslant 0 \end{aligned}$$

is equivalent to

PP4:

$$\begin{aligned} \text{Maximize} \quad & y = cx \\ \text{subject to} \quad & y \leqslant c + M(1-x), && (3.14) \\ & y \leqslant Mx, && (3.15) \end{aligned}$$

where x is an zero–one variable, *M a big value*, *and* $y, c \geqslant 0$.

This proposition can be examined as follows:
(i) if at optimal solution $x = 1$, then (3.14) and (3.15) result in $y = c$;
(ii) if at optimal solution $x = 0$, then (3.14) and (3.15) result in $y = 0$.
This proposition is proven.
Therefore, Model (1) can be linearized accordingly:

$$\begin{aligned} \text{Maximize} \quad & y_1 + y_2 + y_3 + y_4 + y_5 + y_6 - 8\delta_1^+ - 8\delta_1^- - 5\delta_2^+ - 5\delta_2^- - 12\delta_3^+ - 12\delta_3^- - 10\delta_4^+ \\ & - 10\delta_4^- - 7\delta_5^+ - 7\delta_5^- - 12\delta_6^+ - 12\delta_6^- && (3.16) \\ \text{subject to} \quad & (3.2)\text{–}(3.10) && (3.17) \\ & \mu(c_i) - \delta_i^+ + \delta_i^- = 1, \ \ i = 1, 2, \ldots, 6, && (3.18) \\ & y_1 \leqslant c_1 + M(1-x_1), \ \ y_1 \leqslant Mx_1, \ \ y_2 \leqslant c_2 + M(1-x_2), \ \ y_2 \leqslant Mx_2, && (3.19) \\ & y_3 \leqslant c_3 + M(1-x_3), \ \ y_3 \leqslant Mx_3, \ \ y_4 \leqslant c_4 + M(1-x_4), \ \ y_4 \leqslant Mx_4, && (3.20) \\ & y_5 \leqslant c_5 + M(1-x_5), \ \ y_5 \leqslant Mx_5, \ \ y_6 \leqslant c_6 + M(1-x_6), \ \ y_6 \leqslant Mx_6, && (3.21) \end{aligned}$$

where M is a big number and x_1, x_2, x_3, x_4, x_5, and x_6 are zero–one variables.

Solving Example 2 by the LINDO [24], the found solution set is ($x_1 = 1, x_2 = 0, x_3 = 0, x_4 = 1, x_5 = 0$, and $x_6 = 1$), which is the same as found in Herrera et al. [11].

4. Fuzzy coefficients in the objective function and the constraint matrix as well as fuzzy values in the constraints' right-hand side

This section addresses how to concurrently solve a BLP problem with fuzzy coefficients in the objective function and the constraint matrix as well as fuzzy values in the constraints' right-hand side. Consider the following example:

Example 3 (*Modified from* [16]). The board of directors of a large manufacturing firm is considering the investment project illustrated in the following table. The board wishes to maximize the total expected return and investment around the available annual budget. Five projects are being considered for execution over the next three years while the expected return for each project naturally is uncertain. The return, available funds and required yearly investments (in millions of dollars) are displayed in Table 2.

In Table 2, $r_1 = (18.5, 20, 23)$, $r_2 = (38, 40, 41.5)$, $r_3 = (19, 20, 21.5)$, $r_4 = (13.8, 15, 16.3)$, $r_5 = (28.2, 30, 34.5)$, $b_1 = b_2 = b_3 = (22, 25, 27)$, $a_{13} = (6.5, 8, 10)$, $a_{23} = (9, 10, 11)$, $a_{33} = (2.5, 3, 4)$, $a_{43} = (1.5, 2, 2.5)$, and $a_{53} = (9, 10, 11)$ are depicted in Figs. 4(a), (b), (c), (d), (e), (f), (g), (h), (i), (j), (k), (l), and (m), respectively, where $(18.5, 20, 23)$ represents a fuzzy number which the possible smallest amount is 18.5 million, the possible center amount is 20 million, and the possible largest amount is 23 million.

Table 2
Available investment information

Project	Investments for Year 1	Year 2	Year 3	Returns
1	6	2	(7, 8, 9)	(18.5, 20, 23)
2	5	8	(9, 10, 11)	(38, 40, 41.5)
3	3	10	(2.5, 3, 3.5)	(19, 20, 21.5)
4	7	5	(1.5, 2, 2.5)	(13.8, 15, 16.3)
5	9	7	(9, 10, 11)	(28.2, 30, 34.5)
Available funds	(22, 25, 27)	(22, 25, 27)	(22, 25, 27)	

The decision problem can be formalized as

$$\text{Maximize} \quad \tilde{r}_1x_1 + \tilde{r}_2x_2 + \tilde{r}_3x_3 + \tilde{r}_4x_4 + \tilde{r}_5x_5$$

$$\text{subject to} \quad 6x_1 + 5x_2 + 3x_3 + 7x_4 + 9x_5 \leqq \tilde{b}_1, \tag{4.1}$$

$$2x_1 + 8x_2 + 10x_3 + 5x_4 + 7x_5 \leqq \tilde{b}_2, \tag{4.2}$$

$$\tilde{a}_{13}x_1 + \tilde{a}_{23}x_2 + \tilde{a}_{33}x_3 + \tilde{a}_{43}x_4 + \tilde{a}_{53}x_5 \leqq \tilde{b}_3, \tag{4.3}$$

$$x_1, x_2, x_3, x_4, x_5 \in \text{0–1 variables}, \tag{4.4}$$

where the binary variable x_i represent the ith project, $i = 1, \ldots, 5$.

Referring to Propositions 1 and 2, the membership functions depicted in Figs. 4(a)–(m) can be represented by

$$\mu(r_1) = -0.3333r_1 - d_1 + 7.66605, \quad r_1 + d_1 \geqslant 20, \tag{4.5}$$

$$\mu(r_2) = -0.6667r_2 - 1.1667d_2 + 27.6668, \quad r_2 + d_2 \geqslant 40, \tag{4.6}$$

$$\mu(r_3) = -0.6667r_3 - 1.6667d_3 + 14.334, \quad r_3 + d_3 \geqslant 20, \tag{4.7}$$

$$\mu(r_4) = -0.7692r_4 - 1.6026d_4 + 12.5385, \quad r_4 + d_4 \geqslant 15, \tag{4.8}$$

$$\mu(r_5) = -0.2222r_5 - 0.7778d_5 + 7.6667, \quad r_5 + d_5 \geqslant 30, \tag{4.9}$$

$$\mu(b_1) = -0.49997b_1 - 0.8333d_6 + 13.4992, \quad b_1 + d_6 \geqslant 25, \tag{4.10}$$

$$\mu(b_2) = -0.49997b_2 - 0.8333d_7 + 13.4992, \quad b_2 + d_7 \geqslant 25, \tag{4.11}$$

$$\mu(b_3) = -0.49997b_3 - 0.8333d_8 + 13.4992, \quad b_3 + d_8 \geqslant 25, \tag{4.12}$$

$$\mu(a_{13}) = -a_{13} - 2d_9 + 9, \quad a_{13} + d_9 \geqslant 8, \tag{4.13}$$

$$\mu(a_{23}) = -a_{23} - 2d_{10} + 11, \quad a_{23} + d_{10} \geqslant 10, \tag{4.14}$$

$$\mu(a_{33}) = -2a_{33} - 4d_{11} + 7, \quad a_{33} + d_{11} \geqslant 3, \tag{4.15}$$

$$\mu(a_{43}) = -2a_{43} - 4d_{12} + 5, \quad a_{43} + d_{12} \geqslant 2, \tag{4.16}$$

$$\mu(a_{53}) = -a_{53} - 2d_{13} + 11, \quad a_{53} + d_{13} \geqslant 10. \tag{4.17}$$

This decision problem can then be expressed as

$$\text{Maximize} \quad \sum_{i=1}^{I}(r_ix_i), \tag{4.18}$$

$$\text{Maximize} \quad \sum_{i=1}^{I}\mu(r_i), \tag{4.19}$$

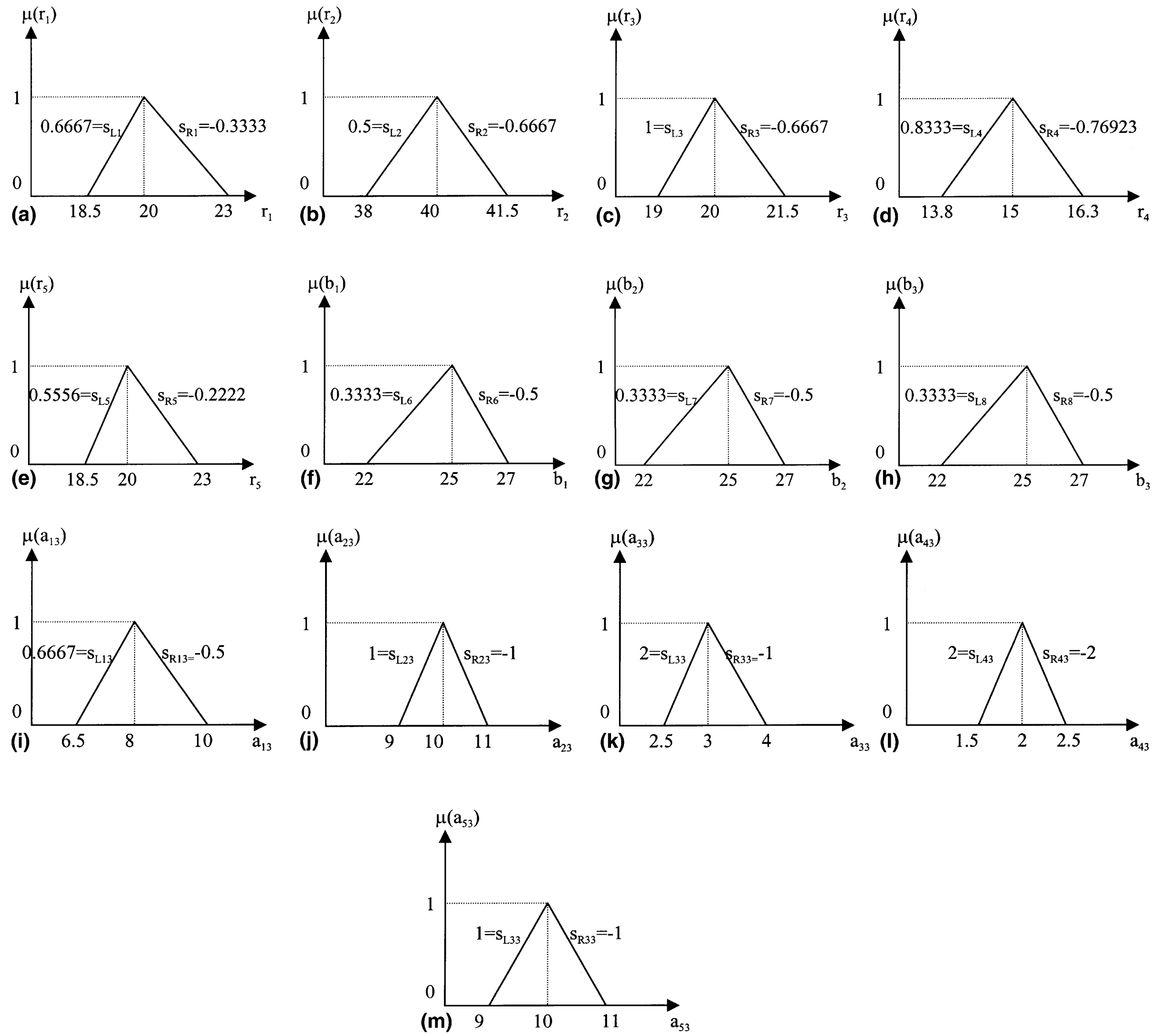

Fig. 4. (a) Membership function $\mu(r_1)$. (b) Membership function $\mu(r_2)$. (c) Membership function $\mu(r_3)$. (d) Membership function $\mu(r_4)$. (e) Membership function $\mu(r_5)$. (f) Membership function $\mu(b_1)$. (g) Membership function $\mu(b_2)$. (h) Membership function $\mu(b_3)$. (i) Membership function $\mu(a_{13})$. (j) Membership function $\mu(a_{23})$. (k) Membership function $\mu(a_{33})$. (l) Membership function $\mu(a_{43})$. (m) Membership function $\mu(a_{53})$.

$$\text{Maximize} \quad \sum_{j=1}^{J} \mu(b_j), \tag{4.20}$$

$$\text{Maximize} \quad \sum_{i=1}^{I} \mu(a_{iJ}), \tag{4.21}$$

$$\text{subject to} \quad 6x_1 + 5x_2 + 3x_3 + 7x_4 + 9x_5 \leqq b_1, \tag{4.22}$$

$$2x_1 + 8x_2 + 10x_3 + 5x_4 + 7x_5 \leqq b_2, \tag{4.23}$$

$$a_{13}x_1 + a_{23}x_2 + a_{33}x_3 + a_{43}x_4 + a_{53}x_5 \leqq b_3, \tag{4.24}$$

$$x_1, x_2, x_3, x_4, x_5 \text{ are 0–1 variables}, \tag{4.25}$$

where $I = 5, J = 3$, the binary variable x_i represents the ith project.

Based on Definition 1 and Proposition 3, we have

Model (2):

$$\begin{aligned} \text{Maximize} \quad & \sum_{i=1}^{I}(r_i x_i) - \sum_{i=1}^{I}\left(w_i^+\delta_i^+ + w_i^-\delta_i^-\right)\sum_{j=1}^{J}\lambda_j\left(w_{I+j}^+\delta_{I+j}^+ + w_{I+j}^+\delta_{I+j}^-\right) \\ & - \sum_{i=1}^{I}\beta_{iJ}\left(w_{J3}^+\delta_{iJ}^+ + w_{iJ}^-\delta_{J3}^-\right) \end{aligned} \tag{4.26}$$

$$\text{subject to} \quad (4.6)\text{–}(4.18) \text{ and } (4.23)\text{–}(4.26), \tag{4.27}$$

$$\mu(r_i) - \delta_i^+ + \delta_i^- = 1, \quad i = 1, 2, \ldots, I, \tag{4.28}$$

$$\mu(b_j) - \delta_{I+j}^+ + \delta_{I+j}^- = 1, \quad j = 1, 2, \ldots, J, \tag{4.29}$$

$$\mu(a_{iJ}) - \delta_{iJ}^+ + \delta_{iJ}^- = 1, \quad i = 1, 2, \ldots, I, \tag{4.30}$$

$$I = 5, \quad J = 3, \tag{4.31}$$

where the weights w_i^+ and w_i^- are equal to inverses of slopes $|1/s_{Li}|$ and $|1/s_{Ri}|$, respectively, as shown in Figs. 4(a)–(h). The weights w_{i3}^+ and w_{i3}^- equal the slopes $|1/s_{Li3}|$ and $|1/s_{Ri3}|$, respectively, as displayed in Figs. 4(i)–(m).

The first term in (4.27) is the total expected objective value that the DM would like to maximize. The second, third, and fourth terms in (4.27) are variances of possibilities of the coefficients in the expected objective function, the right-hand side values, and the decision variables' coefficients in the constraint matrix, respectively. λ_j and β_{iJ} are trade-off weights adjusting among the expected goal, the possible right-hand side values and the possible coefficients of decision variables in the Jth constraint.

After employing Proposition 4 to linearize the product terms $r_i x_i$ in (4.27) and $a_{ij}x_i$ in (4.25), Example 3 is reformulated as

$$\begin{aligned} \text{Maximize} \quad & y_1 + y_2 + y_3 + y_4 + y_5 - 1.5\delta_1^+ - 3\delta_1^- - 2\delta_2^+ - 1.5\delta_2^- - 1\delta_3^+ - 1.5\delta_3^- - 1.2\delta_4^+ - 1.3\delta_4^- \\ & - 1.8\delta_5^+ - 4.5\delta_5^- - 3\lambda_1\delta_6^+ - 2\lambda_1\delta_6^- - 3\lambda_2\delta_7^+ - 2\lambda_2\delta_7^- - 3\lambda_3\delta_8^+ - 2\lambda_3\delta_8^- - \beta_{13}\delta_{13}^+ - \beta_{13}\delta_{13}^- \\ & - \beta_{23}\delta_{23}^+ - \beta_{23}\delta_{23}^- - 0.5\beta_{33}\delta_{33}^+ - 0.5\beta_{33}\delta_{33}^- - 0.5\beta_{43}\delta_{43}^+ - 0.5\beta_{43}\delta_{43}^- - \beta_{53}\delta_{53}^+ - \beta_{53}\delta_{53}^- \\ \text{subject to} \quad & 6x_1 + 5x_2 + 3x_3 + 7x_4 + 9x_5 \leqslant b_1, \ \ 2x_1 + 8x_2 + 10x_3 + 5x_4 + 7x_5 \leqslant b_2, \\ & y_{13} + y_{23} + y_{33} + y_{43} + y_{53} \leqslant b_3, \ \ (4.23)\text{–}(4.26), \ \ (4.28)\text{–}(4.31), \\ & y_1 \leqslant r_1 + M(1 - x_1), \ \ y_1 \leqslant Mx_1, \ \ y_2 \leqslant r_2 + M(1 - x_2), \ \ y_2 \leqslant Mx_2, \\ & y_3 \leqslant r_3 + M(1 - x_3), \ \ y_3 \leqslant Mx_3, \ \ y_4 \leqslant r_4 + M(1 - x_4), \ \ y_4 \leqslant Mx_4, \\ & y_5 \leqslant r_5 + M(1 - x_5), \ \ y_5 \leqslant Mx_5, \ \ y_{13} \leqslant a_{13} + M(1 - x_1), \ \ y_{13} \leqslant Mx_1, \\ & y_{23} \leqslant a_{23} + M(1 - x_2), \ \ y_{23} \leqslant Mx_2, \ \ y_{33} \leqslant a_{33} + M(1 - x_3), \ \ y_{33} \leqslant Mx_3, \\ & y_{43} \leqslant a_{43} + M(1 - x_4), \ \ y_{43} \leqslant Mx_4, \ \ y_{53} \leqslant a_{53} + M(1 - x_5), \ \ y_{53} \leqslant Mx_5, \end{aligned}$$

where M is a big value, x_i the 0–1 variables, $y_i, r_i, a_{i3} \geqslant 0$ and $i = 1, 2, \ldots, 5$.

Assume that all λ_j and β_{i3} $(j = 1, 2, 3; i = 1, 2, \ldots, 5)$ are equal to 1. After computing on the LINDO [24], the obtained solution set is $(x_1, x_2, x_3, x_4, x_5, b_1, b_2, b_3, a_{13}, a_{23}, a_{33}, a_{43}, a_{53}) = (1, 1, 1, 0, 1, 25, 27, 25, 8, 10, 3, 2, 10)$.

5. Solution algorithm

Proposition 4 in PP3 is proposed for linearize the product term cx in the maximization problem. For the minimization problem, Proposition 5 is introduced as follows:

Proposition 5. *Consider the following program*:

PP5:

$$\begin{aligned} &\text{Minimize} \quad cx \\ &\text{subject to} \quad x \in 0\text{–}1 \text{ variable}, \quad c \geqslant 0, \end{aligned}$$

is equivalent to

PP6:

$$\begin{aligned} &\text{Minimize} \quad y = cx \\ &\text{subject to} \quad y \geqslant c + M(x-1), \qquad (5.1) \\ &\qquad\qquad\quad\; y \geqslant 0, \qquad (5.2) \end{aligned}$$

where M is a big value.

Proof.
(i) if at optimal solution $x = 1$, then (5.1) and (5.2) force $y = c$;
(ii) if at optimal solution $x = 0$, then (5.1) and (5.2) force $y = 0$.
This proposition is finished.

Thus, the solution algorithm for the general FBLP problems in (1.1) entails the following steps:
Step 1. Utilizing Proposition 1 to express each membership function as follows:

$$\mu(c_i) = \mu(c_{i,1}) + s_{i,1}(c_i - c_{i,1}) + \frac{s_{i,2} - s_{i,1}}{2}(|c_i - c_{i,2}| + c_i - c_{i,2}).$$

Step 2. Employing Proposition 2 to linearize the absolute terms accordingly:

$$\mu(c_i) = \mu(c_{i,1}) + s_{i,1}(c_i - c_{i,1}) + (s_{i,2} - s_{i,1})(c_1 - c_{i,2} + d_i), \quad \mu(c_i) - c_{i,2} + d_i \geqslant 0,$$

where $s_{i,2} < s_{i,1}$.
Step 3. Using Definition 1 and Proposition 3 formulate the problem as Model (1) or (2).
Step 4. Proposition 4 is used to linearize the product terms in a maximization problem, while Proposition 5 is used to linearize the product terms in a minimization problem.
Step 5. The linear mixed zero–one program is solved by the LP package.

6. Numerical example

The assignment problem arises in a variety of decision making situations: Typical assignment problems involve assigning jobs to machines, agents to tasks, sales personnel to sales territories, contracts to bidders, and so on. Now consider the following problem:

Table 3
Completion time for each project leader working with each client

Project leader	Client 1	Client 2	Client 3
Terry	(8.5, 10, 11)	(14, 15, 16)	(7, 9, 11)
Carle	(8, 9, 10)	(16, 18, 19)	(4, 5, 5.5)
McClymonds	(5, 6, 7)	(13, 14, 15)	(2.5, 3, 3.5)

Example 4 (*Slightly modified from* [8]). A marketing research company has just received requests for market research studies from three new clients. The company faces the task of assigning a project leader to each client. The management realizes that the time required to complete each study largely depend on the experience and ability of the assigned project leader. With three project leaders and three clients, nine assignment alternatives are possible. Table 3 summarizes the alternatives and the estimated project completion times in days. The $(8.5, 10, 11)$ represents a triangle fuzzy number with a most optimistic completion time of 8.5 days, a most possible completion time of 10 days, and a most pessimistic completion time of 11 days.

Let $c_{11} = (8.5, 10, 11)$, $c_{12} = (14, 15, 16)$, $c_{13} = (7, 9, 11)$, $c_{21} = (8, 9, 10)$, $c_{22} = (16, 18, 19)$, $c_{23} = (4, 5, 5.5)$, $c_{31} = (5, 6, 7)$, $c_{32} = (13, 14, 15)$, and $c_{33} = (2.5, 3, 3.5)$. Initially, this assignment problem can be written as follows:

$$\begin{aligned} \text{Minimize} \quad & \tilde{c}_{11}x_{11} + \tilde{c}_{12}x_{12} + \tilde{c}_{13}x_{13} + \tilde{c}_{21}x_{21} + \tilde{c}_{22}x_{22} + \tilde{c}_{23}x_{23} + \tilde{c}_{31}x_{31} + \tilde{c}_{32}x_{32} + \tilde{c}_{33}x_{33} \\ \text{subject to} \quad & x_{11} + x_{12} + x_{13} \leqslant 1, \;\; x_{21} + x_{22} + x_{23} \leqslant 1, \;\; x_{31} + x_{32} + x_{33} \leqslant 1, \\ & x_{11} + x_{21} + x_{31} = 1, \;\; x_{12} + x_{22} + x_{32} = 1, \;\; x_{13} + x_{23} + x_{33} = 1, \end{aligned}$$

where $x_{ij}, \; i = 1, 2, 3, \; j = 1, 2, 3$ are zero–one variables.

Following the solution algorithm, we have:

Step 1. Utilizing Proposition 1 to express each membership function as follows:

$$\begin{aligned} &\mu(c_{11}) = 0.6667(c_{11} - 8.5) - 0.8333(|c_{11} - 10| + c_{11} - 10), \\ &\mu(c_{12}) = (c_{12} - 14) - (|c_{12} - 15| + c_{12} - 15), \\ &\mu(c_{13}) = 0.5(c_{13} - 7) - 0.5(|c_{13} - 9| + c_{13} - 9), \\ &\mu(c_{21}) = (c_{21} - 8) - (|c_{21} - 9| + c_{21} - 9), \\ &\mu(c_{22}) = 0.5(c_{22} - 16) - 0.75(|c_{22} - 18| + c_{22} - 18), \\ &\mu(c_{23}) = (c_{23} - 4) - 1.5(|c_{23} - 5| + c_{23} - 5), \\ &\mu(c_{31}) = (c_{31} - 5) - (|c_{31} - 6| + c_{31} - 6), \\ &\mu(c_{32}) = (c_{32} - 13) - (|c_{32} - 14| + c_{32} - 14), \\ &\mu(c_{33}) = 2(c_{33} - 2.5) - 2(|c_{33} - 3| + c_{33} - 3). \end{aligned}$$

Step 2. Employing Proposition 2 to linearize the absolute terms as:

$$\begin{aligned} &\mu(c_{11}) = -c_{11} - 1.6667d_{11} + 11, \quad c_{11} - 10 + d_{11} \geqslant 0, \\ &\mu(c_{12}) = -c_{12} - 2d_{12} + 16, \quad c_{12} - 15 + d_{12} \geqslant 0, \\ &\mu(c_{13}) = -0.5c_{13} - d_{13} + 6.5, \quad c_{13} - 9 + d_{13} \geqslant 0, \\ &\mu(c_{21}) = -c_{21} - 2d_{21} + 10, \quad c_{21} - 9 + d_{21} \geqslant 0, \\ &\mu(c_{22}) = -c_{22} - 1.5d_{22} + 19, \quad c_{22} - 18 + d_{22} \geqslant 0, \end{aligned}$$

$$\mu(c_{23}) = -2c_{23} - 3d_{23} + 11, \quad c_{23} - 5 + d_{23} \geqslant 0,$$
$$\mu(c_{31}) = -c_{31} - 2d_{31} + 7, \quad c_{31} - 6 + d_{31} \geqslant 0,$$
$$\mu(c_{32}) = -c_{32} - 2d_{32} + 15, \quad c_{32} - 14 + d_{32} \geqslant 0,$$
$$\mu(c_{33}) = -2c_{33} - 4d_{33} + 7, \quad c_{33} - 3 + d_{33} \geqslant 0.$$

Step 3. Using Definition 1 and Proposition 3 formulate the problem below:

$$\text{Minimize} \quad \sum_{i=1}^{3}\sum_{j=1}^{3} c_{ij}x_{ij} - \sum_{i=1}^{3}\sum_{j=1}^{3}\left(w_{ij}^{+}\delta_{ij}^{+} + w_{ij}^{-}\delta_{ij}^{-}\right)$$

$$\text{subject to} \quad x_{11}+x_{12}+x_{13} \leqslant 1,\ x_{21}+x_{22}+x_{23} \leqslant 1,\ x_{31}+x_{32}+x_{33} \leqslant 1, \tag{6.1}$$
$$x_{11}+x_{21}+x_{31} = 1,\ x_{12}+x_{22}+x_{32} = 1,\ x_{13}+x_{23}+x_{33} = 1, \tag{6.2}$$
$$\mu(c_{11}) - \delta_{11}^{+} + \delta_{11}^{-} = 1,\ \mu(c_{11}) = -c_{11} - 1.6667d_{11} + 11,\ c_{11} - 10 + d_{11} \geqslant 0, \tag{6.3}$$
$$\mu(c_{12}) - \delta_{12}^{+} + \delta_{12}^{-} = 1,\ \mu(c_{12}) = -c_{12} - 2d_{12} + 16,\ c_{12} - 15 + d_{12} \geqslant 0, \tag{6.4}$$
$$\mu(c_{13}) - \delta_{13}^{+} + \delta_{13}^{-} = 1,\ \mu(c_{13}) = -0.5c_{13} - d_{13} + 6.5,\ c_{13} - 9 + d_{13} \geqslant 0, \tag{6.5}$$
$$\mu(c_{21}) - \delta_{21}^{+} + \delta_{21}^{-} = 1,\ \mu(c_{21}) = -c_{21} - 2d_{21} + 10,\ c_{21} - 9 + d_{21} \geqslant 0, \tag{6.6}$$
$$\mu(c_{22}) - \delta_{22}^{+} + \delta_{22}^{-} = 1,\ \mu(c_{22}) = -c_{22} - 1.5d_{22} + 19,\ c_{22} - 18 + d_{22} \geqslant 0, \tag{6.7}$$
$$\mu(c_{23}) - \delta_{23}^{+} + \delta_{23}^{-} = 1,\ \mu(c_{23}) = -2c_{23} - 3d_{23} + 11,\ c_{23} - 5 + d_{23} \geqslant 0, \tag{6.8}$$
$$\mu(c_{31}) - \delta_{31}^{+} + \delta_{31}^{-} = 1,\ \mu(c_{31}) = -c_{31} - 2d_{31} + 7,\ c_{31} - 6 + d_{31} \geqslant 0, \tag{6.9}$$
$$\mu(c_{32}) - \delta_{32}^{+} + \delta_{32}^{-} = 1,\ \mu(c_{32}) = -c_{32} - 2d_{32} + 15,\ c_{32} - 14 + d_{32} \geqslant 0, \tag{6.10}$$
$$\mu(c_{33}) - \delta_{33}^{+} + \delta_{33}^{-} = 1,\ \mu(c_{33}) = -2c_{33} - 4d_{33} + 7,\ c_{33} - 3 + d_{33} \geqslant 0, \tag{6.11}$$
$$w_{11}^{+} = 1.5,\ w_{11}^{-} = 1,\ w_{12}^{+} = 1,\ w_{12}^{-} = 1,\ w_{13}^{+} = 2,\ w_{13}^{-} = 2, \tag{6.12}$$
$$w_{21}^{+} = 1,\ w_{21}^{-} = 1,\ w_{22}^{+} = 2,\ w_{22}^{-} = 1,\ w_{23}^{+} = 1,\ w_{23}^{-} = 0.5, \tag{6.13}$$
$$w_{31}^{+} = 1,\ w_{31}^{-} = 1,\ w_{32}^{+} = 1,\ w_{32}^{-} = 1,\ w_{33}^{+} = 0.5,\ w_{33}^{-} = 0.5, \tag{6.14}$$
$$x_{ij},\ i = 1,2,3,\ j = 1,2,3 \text{ are zero–one variables.} \tag{6.15}$$

Step 4. Since the management desires to minimize the objective function, Proposition 5 is used to linearize the product terms $c_{ij}x_{ij}$. Then, we have

$$\begin{aligned} &\text{Minimize} \quad \sum_{i=1}^{3}\sum_{j=1}^{3} y_{ij} - \sum_{i=1}^{3}\sum_{j=1}^{3}\left(w_{ij}^{+}\delta_{ij}^{+} + w_{ij}^{-}\delta_{ij}^{-}\right) \\ &\text{subject to} \quad (6.1)\text{–}(6.15),\ y_{ij} \geqslant c_{ij} + M(x_{ij} - 1),\ y_{ij} \geqslant 0,\ i = 1,2,3,\ j = 1,2,3, \end{aligned} \tag{6.16}$$

where M is a large number.

Step 5. Employing the package LINDO [24] to solve model (6.16).

The obtained solution tells that Terry is assigned to client 2 ($x_{12} = 1$), Carle to client 3 ($x_{23} = 1$), and McClymonds to client 1 ($x_{31} = 1$).

Notably, a distinguished feature of this assignment problem is that it does not have any technology constraint matrix in its attempt to minimize the objective function. Examples 2–4 have successfully demonstrated the proposed algorithm can solve generalized FBLP problems in (1.1).

7. Concluding remarks

BLP problems normally are unable to have precise coefficients in the objective function and the constraint matrix as well as the right-hand side limits of constrains. Some examples include: "We want a return around or larger than b_1 dollars", "we would like to invest substantially less than or near b_2 dollars", "the production rate is estimated to be almost b_3 (pcs/minute)", "the profit rate will be approximately b_4 (\$/month) for the first three months", and "the estimated completion time is around b_5 days for the second project". Therefore, the major difficulty in solving FBLP problems is how to treat vague numbers.

This work has proposed a clear and simple way to express a widespread triangular fuzzy number followed by an absolute term linearizing technique. Then a trade-off GP model was built to optimize the objective function and minimize the sum of possible membership function deviations. As the proposed algorithm can concurrently treat a BLP problem with fuzzy coefficients in the objective function, fuzzy coefficients in the constraint matrix, and fuzzy numbers in the constraints' right-hand sides. Hence, the proposed method is a worthwhile alternative to existing methods from a practical point of view.

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