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Fuzzy ARIMA model for forecasting the foreign exchange market

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Abstract

Considering the time-series ARIMA(p, d, q) model and fuzzy regression model, this paper develops a fuzzy ARIMA (FARIMA) model and applies it to forecasting the exchange rate of NT dollars to US dollars. This model includes interval models with interval parameters and the possibility distribution of future values is provided by FARIMA. This model makes it possible for decision makers to forecast the best- and worst-possible situations based on fewer observations than the ARIMA model. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: ARIMA; Foreign exchange market; Fuzzy regression; Fuzzy ARIMA; Time series

1. Introduction

Since it has been suggested by Box–Jenkins [1] that the time-series ARIMA model has enjoyed fruitful applications in forecasting social, economic, engineering, foreign exchange, and stock problems. It assumes that the future values of a time series have a clear and definite functional relationship with current, past values and white noise. This model has the advantage of accurate forecasting in a short time period; it also has the limitation that at least 50 and preferably 100 observations or more should be used. In addition, this model uses the concept of measurement error to deal with the differences between estimators and observations, but these data are precise values that do not include measurement errors.

Tanaka et al. [9–11] have suggested fuzzy regression to solve the fuzzy environment and to avoid a modeling error. This model is basically an interval prediction model with the disadvantage that the prediction interval can be very wide if some extreme values are present.

Song and Chissom [6–8] presented the definition of fuzzy time series and outlined its modeling by means of fuzzy relational equations and approximate reasoning. Chen [2] presented a fuzzy time-series method based on the concept of Song and Chissom. An application of fuzzy regression to fuzzy time-series analysis was

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found by Watada [12], but this model did not include the concept of the Box–Jenkins model. In this paper, based upon the works of time-series ARIMA(p, d, q) model and fuzzy regression model, we combine the advantages of two methods to develop the fuzzy ARIMA model.

In order to show the applicability and effectiveness of our proposed method in practical application, we conduct an illustration for forecasting the foreign exchange market. In the results, we found that the proposed method makes good forecasts in several situations for which FARIMA appears to be the most appropriate tool. The situations are listed as follows:

- (i) To provide the decision makers the best- and worst-possible situations.
- (ii) The required number of observations is less than the ARIMA model requires, which is at least 50 and preferably more than 100 observations.

The structure of this paper is organized as follows: Concepts of time-series ARIMA and fuzzy regression are reviewed in Section 2. In Section 3, the FARIMA model is formulated and proposed. The FARIMA model is applied to forecasting the foreign exchange rate of NT dollars to US dollars in Section 4 and finally the conclusions are discussed.

2. ARIMA model and fuzzy regression model review

A time-series $\{Z_t\}$ is generated by an ARIMA(p, d, q) process with mean μ of the Box–Jenkins model [1]

$$\text{if } \varphi(B)(1 - B)^d(Z_t - \mu) = \theta(B)a_t, \quad (1)$$

where $\varphi(B) = 1 - \varphi_1 B - \varphi_2 B^2 - \dots - \varphi_p B^p$, $\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \dots - \theta_q B^q$ are polynomials in B of degree p and q , B is the backward shift operator, p, d, q are integers, Z_t denotes the observed value of time-series data, $t = 1, 2, \dots, k$, and time-series data are the observations.

The ARIMA model formulation includes four steps:

1. Identification of the ARIMA(p, d, q) structure. Use autocorrelation function (ACF) and partial autocorrelation function (PACF) to develop the rough function.
2. Estimation of the unknown model parameter.
3. Diagnostic checks are applied with the object of uncovering possible lack of fit and diagnosing the cause.
4. Forecasting from the selection model.

It is assumed that a_t are independent and identically distributed as normal random variables with mean 0 and variance σ^2 , and the roots of $\varphi(Z) = 0$ and $\theta(Z) = 0$ all lie outside the unit circle. If possible, at least 50 and preferably 100 observations or more should be used. In the real world, however, the environment is uncertain and changes rapidly, we usually must forecast future situations using little data in a short span of time, and it is hard to verify that the data is a normal distribution. So this assumption has limitations. This model uses the concept of measurement error to deal with the difference between estimators and observations, but these data are precise values and do not include measurement errors. It is the same as the basic concept of the fuzzy regression model as suggested by Tanaka et al. [11].

The basic concept of the fuzzy theory of fuzzy regression is that the residuals between estimators and observations are not produced by measurement errors, but rather by the parameter uncertainty in the model, and the possibility distribution is used to deal with real observations.

The following is a generalized model of fuzzy linear regression:

$$Y = \beta_0 + \beta_1 x_1 + \dots + \beta_n x_n = \sum_{i=1}^n \beta_i x_i = \mathbf{x}' \boldsymbol{\beta}, \quad (2)$$

where \mathbf{x} is the vector of independent variables, superscript $'$ denotes the transposition operation, n is the number of variables and β_i represents fuzzy sets representing the i th parameter of the model.

Instead of using crisp, fuzzy parameter β_i in the form of L -type fuzzy numbers of Dubois and Prade [3], $(\alpha_i, c_i)_L$, possibility distribution is

$$\mu_{\beta_i}(\beta_i) = L\{(\alpha_i - \beta_i)/c\}, \tag{3}$$

where L is a function type. Fuzzy parameters in the form of triangular fuzzy numbers are used:

$$\mu_{\beta_i}(\beta_i) = \begin{cases} 1 - \frac{|\alpha_i - \beta_i|}{c_i}, & \alpha_i - c_i \leq \beta_i \leq \alpha_i + c_i, \\ 0 & \text{otherwise,} \end{cases} \tag{4}$$

where $\mu_{\beta_i}(\beta_i)$ is the membership function of the fuzzy set represented by parameter β_i , α_i is the center of the fuzzy number and c_i is the width or spread around the center of the fuzzy number.

Through the extension principle, the membership function of the fuzzy number $y_t = \mathbf{x}'_t \beta$ can be defined by using pyramidal fuzzy parameter β as follows:

$$\mu_y(y_t) = \begin{cases} 1 - \frac{|y_t - \mathbf{x}'_t \alpha|}{\mathbf{c}' |\mathbf{x}_t|} & \text{for } \mathbf{x}_t \neq \mathbf{0}, \\ 1 & \text{for } \mathbf{x}_t = \mathbf{0}, y_t = 0, \\ 0 & \text{for } \mathbf{x}_t = \mathbf{0}, y_t \neq 0, \end{cases} \tag{5}$$

where α and c denote vectors of model values and spreads for all model parameters, respectively, t is the number of observations, $t = 1, 2, \dots, k$.

Finally, the method uses the criterion of minimizing the total vagueness, S , defined as the sum of individual spreads of the fuzzy parameters of the model.

$$\text{Minimize } S = \sum_{t=1}^k \mathbf{c}' |\mathbf{x}_t|. \tag{6}$$

At the same time, this approach takes into account the condition that the membership value of each observation y_t is greater than an imposed threshold, h level, $h \in [0, 1]$. This criterion data simply expresses the fact that the fuzzy output of the model should be over all the data points y_1, y_2, \dots, y_k to a certain h -level. A choice of the h -level value influences the widths c of the fuzzy parameters:

$$\mu_y(y_t) \geq h \quad \text{for } t = 1, 2, \dots, k. \tag{7}$$

The index t refers to the number of nonfuzzy data used in constructing the model. Then the problem of finding the fuzzy regression parameters was formulated by Tanaka et al. [10] as a linear programming problem:

$$\begin{aligned} \text{Minimize } & S = \sum_{t=1}^k \mathbf{c}' |\mathbf{x}_t| \\ \text{subject to } & \mathbf{x}'_t \alpha + (1 - h) \mathbf{c}' |\mathbf{x}_t| \geq y_t, \quad t = 1, 2, \dots, k, \\ & \mathbf{x}'_t \alpha - (1 - h) \mathbf{c}' |\mathbf{x}_t| \leq y_t, \quad t = 1, 2, \dots, k, \\ & c \geq 0, \end{aligned} \tag{8}$$

where $\alpha' = (\alpha_1, \alpha_2, \dots, \alpha_n)$ and $c' = (c_1, c_2, \dots, c_n)$ are vectors of unknown variables and S is the total vagueness as previously defined.

Watada [12] suggested a fuzzy time-series analysis, which is formulated by the possibility regression model but does not include the concept of the Box–Jenkins [1] model; also in this model, the weight of the objective

function does not contain criteria which maybe somewhat subjective. Those limitations are derived to formulate the fuzzy ARIMA model by using the criterion of the fuzzy regression model and to improve the limitations in the Watada model.

3. Model formulation

The ARIMA model is a precise forecasting model for short time periods, but the limitation of a large amount of historical data (at least 50 and preferably 100 or more) is required. However, in our society today, due to factors of uncertainty from the integral environment and rapid development of new technology, we usually have to forecast future situations using little data in a short span of time. The historical data must be less than what the ARIMA model requires which limits its application. The fuzzy regression model is an interval forecasting model suitable for the condition of little attainable historical data. In order to make the model include all possible conditions, the spread is wide when data includes a significant difference or bias. The purpose of this paper is to combine the advantages of the fuzzy regression and ARIMA models to formulate the fuzzy ARIMA model and to fulfill the limitations of fuzzy regression and the ARIMA model. The parameter of ARIMA(p, d, q), $\varphi_1, \dots, \varphi_p$ and $\theta_1, \dots, \theta_q$ are crisp. Instead of using crisp, fuzzy parameters, $\tilde{\varphi}_1, \dots, \tilde{\varphi}_p$ and $\tilde{\theta}_1, \dots, \tilde{\theta}_q$, in the form of triangular fuzzy numbers are used. By using the fuzzy parameters, the requirement of historical data would be reduced (a_t should be obtained from observation value, with the result that a_t will be crisp). In addition, this study adapts the methodology formulated by Ishibuchi and Tanaka [4] for the condition which includes a wide spread of the forecasted interval.

A fuzzy ARIMA(p, d, q) model is described by a fuzzy function with a fuzzy parameter:

$$\tilde{\Phi}_p(B)W_t = \tilde{\theta}_q(B)a_t, \quad (9)$$

$$W_t = (1 - B)^d(Z_t - \mu), \quad (10)$$

$$\tilde{W}_t = \tilde{\varphi}_1 W_{t-1} + \tilde{\varphi}_2 W_{t-2} + \dots + \tilde{\varphi}_p W_{t-p} + a_t - \tilde{\theta}_1 a_{t-1} - \tilde{\theta}_2 a_{t-2} - \dots - \tilde{\theta}_q a_{t-q}, \quad (11)$$

where Z_t are observations, $\tilde{\varphi}_1, \dots, \tilde{\varphi}_p$ and $\tilde{\theta}_1, \dots, \tilde{\theta}_q$, are fuzzy numbers. Eq. (11) is modified as

$$\tilde{W}_t = \tilde{\beta}_1 W_{t-1} + \tilde{\beta}_2 W_{t-2} + \dots + \tilde{\beta}_p W_{t-p} + a_t - \tilde{\beta}_{p+1} a_{t-1} - \tilde{\beta}_{p+2} a_{t-2} - \dots - \tilde{\beta}_{p+q} a_{t-q}. \quad (12)$$

Fuzzy parameters in the form of triangular fuzzy numbers are used:

$$\mu_{\tilde{\beta}_i}(\beta_i) = \begin{cases} 1 - \frac{|\beta_i - \alpha_i|}{c_i} & \text{if } \alpha_i - c_i \leq \beta_i \leq \alpha_i + c_i, \\ 0 & \text{otherwise,} \end{cases} \quad (13)$$

where $\mu_{\tilde{\beta}_i}(\beta_i)$ is the membership function of the fuzzy set that represents parameter β_i , α_i is the center of the fuzzy number, and c_i is the width or spread around the center of the fuzzy number.

Using fuzzy parameters β_i in the form of triangular fuzzy numbers and applying the extension principle, it becomes clear [10] that the membership of W in Eq. (12) is given as

$$\mu_{\tilde{w}}(W_t) = \begin{cases} 1 - \frac{|W_t - \sum_{i=1}^p \alpha_i W_{t-i} - a_t + \sum_{i=p+1}^{p+q} \alpha_i a_{t+p-i}|}{\sum_{i=1}^p c_i |W_{t-i}| + \sum_{i=p+1}^{p+q} c_i |a_{t+p-i}|} & \text{for } W_t \neq 0, a_t \neq 0, \\ 0 & \text{otherwise.} \end{cases} \quad (14)$$

Simultaneously, Z_t represents the t th observation, and h -level is the threshold value representing the degree to which the model should be satisfied by all the data points Z_1, Z_2, \dots, Z_k . A choice of the h value influences

the widths c of the fuzzy parameters:

$$Z_Z(Z_t) \geq h \quad \text{for } t = 1, 2, \dots, k. \tag{15}$$

The index t refers to the number of nonfuzzy data used for constructing the model. On the other hand, the fuzziness S included in the model is defined by

$$S = \sum_{i=1}^p \sum_{t=1}^k c_i |\varphi_{ii}| |W_{t-i}| + \sum_{i=p+1}^{p+q} \sum_{t=1}^k c_i |\rho_{i-p}| |a_{t+p-i}|, \tag{16}$$

where ρ_{i-p} is the autocorrelation coefficient of time lag $i - p$, φ_{ii} is the partial autocorrelation coefficient of time lag i .

The weight of c_i depends on the relation of time lag i and the present observation, where the p of AR(p) is derived by PACF and the q of MA(q) is derived by ACF. Next, the problem of finding the fuzzy ARIMA parameters was formulated as a linear programming problem:

$$\begin{aligned} \text{Minimize} \quad & S = \sum_{i=1}^p \sum_{t=1}^k c_i |\varphi_{ii}| |W_{t-i}| + \sum_{i=p+1}^{p+q} \sum_{t=1}^k c_i |\rho_{i-p}| |a_{t+p-i}| \\ \text{subject to} \quad & \sum_{i=1}^p \alpha_i W_{t-i} + a_t - \sum_{i=p+1}^{p+q} \alpha_i a_{t+p-i} + (1-h) \left(\sum_{i=1}^p c_i |W_{t-i}| + \sum_{i=p+1}^{p+q} c_i |a_{t+p-i}| \right) \geq W_t, \\ & t = 1, 2, \dots, k, \\ & \sum_{i=1}^p \alpha_i W_{t-i} + a_t - \sum_{i=p+1}^{p+q} \alpha_i a_{t+p-i} - (1-h) \left(\sum_{i=1}^p c_i |W_{t-i}| + \sum_{i=p+1}^{p+q} c_i |a_{t+p-i}| \right) \leq W_t, \\ & t = 1, 2, \dots, k, \\ & c_i \geq 0 \quad \text{for all } i = 1, 2, \dots, p+q. \end{aligned} \tag{17}$$

The procedure of fuzzy ARIMA is as follows:

Phase I: Fitting the ARIMA(p, d, q) by using the available information of observations, i.e., input data is considered nonfuzzy. The result of phase I that the optimum solution of the parameter, $\alpha^* = (\alpha_1^*, \alpha_2^*, \dots, \alpha_{p+q}^*)$ and the residuals a_t (is white noise), is used as one of the input data sets in phase II (the concept is derived by Savic and Pedrycz [5]).

Phase II: Determining the minimal fuzziness by using the same criterion as in Eq. (17) and $\alpha^* = (\alpha_1^*, \alpha_2^*, \dots, \alpha_{p+q}^*)$. The number of constraint functions are the same as the number of observations (the concept derived by Savic and Pedrycz [4]). The fuzzy ARIMA model is

$$\tilde{W}_t = \langle \alpha_1, c_1 \rangle W_{t-1} + \dots + \langle \alpha_p, c_p \rangle W_{t-p} + a_t - \langle \alpha_{p+1}, c_{p+1} \rangle a_{t-1} - \dots - \langle \alpha_{p+q}, c_{p+q} \rangle a_{t-q}, \tag{18}$$

where $W_t = (1 - B)^d (Z_t - \mu)$, α_i is the center of the fuzzy number, and c_i is the width or spread around the center of the fuzzy number.

Phase III. We delete the data around the model's upper bound and lower bound when the fuzzy ARIMA model has outliers with wide spread.

In order to make the model include all possible conditions, fuzzy ARIMA c_j has a wide spread when the data set includes a significant difference or outlying case. Ishibuchi and Tanaka [4] suggest deleting the data around the model's upper and lower boundaries, and then reformulating the fuzzy regression model. The fuzzy ARIMA model uses the same method.

4. Application to forecast exchange rate of NT dollars to US dollars

In order to demonstrate the appropriateness and effectiveness of the proposed method, consider the following application of forecasting the exchange rate of NT dollars (NTD, Taiwan dollars) to US dollars (USD).

The characteristics of the domestic foreign exchange market in Taiwan are as follows:

1. Currently, the exchange rate policy made by the government does not encourage the NTD to be international, and some restrictions still exist for foreign commercial banks to have their NTD accounts here mainly for national economic concerns.
2. USD is typically used when foreign currency exchange occurs between banks and their customers.
3. Different kinds of exchanges include the following:
 - (a) Spot transaction: including USD to NTD and to the others.
 - (b) Forward transaction: only USD to NTD is permitted.
 - (c) Swap transaction: only USD to NTD is permitted.

Because the US is the largest country of international trade for Taiwan, and NTD is not an international currency, forecasting the exchange rate between USD and NTD is very important for international trade in Taiwan. The resource data shown in Fig. 1 is the asking price of NTD/USD spot exchange rate between the bank and customers provided by The First Commercial Bank in Taiwan. It consists of 40 observations from 1 August 1996 to 16 September 1996.

4.1. The forecasts

Applying the fuzzy ARIMA method, we use the first 30 observations to formulate the model and the next 10 observations to evaluate the performance of the model.

Phase I: fitting ARIMA(p, d, q) model: Using the Scientific Computing Associates (SCA) package software, the best-fitted model is ARIMA(2, 0, 0) and the values of residuals are white noise. The results are plotted in Fig. 2 and the model is

$$Z_t = 28.093 + 0.499Z_{t-1} - 0.519Z_{t-2} + a_t. \quad (19)$$

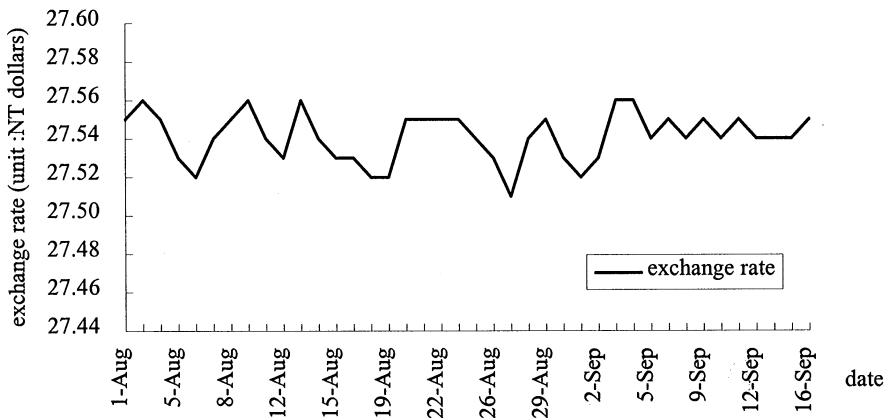


Fig. 1. Exchange rate of NTD and USD (1 August–16 September 1996).

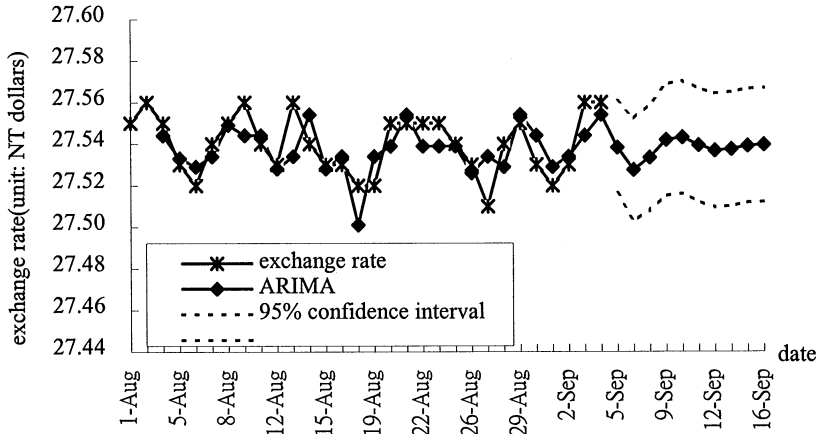


Fig. 2. Results of ARIMA.

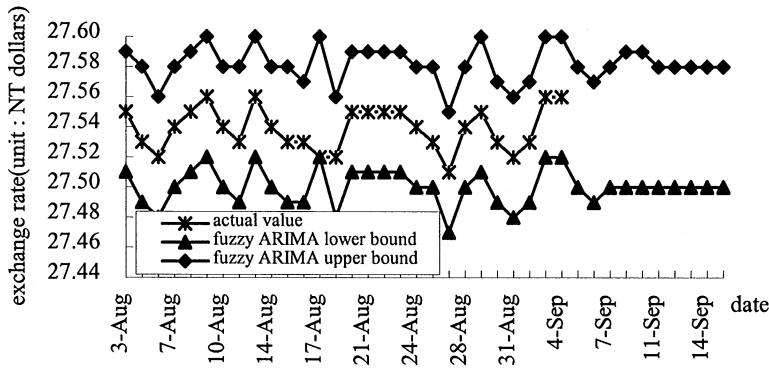


Fig. 3. Results of fuzzy ARIMA (30 historical data).

Phase II: determining the minimal fuzziness: Setting $(\alpha_0, \alpha_1, \alpha_2) = (21.4952, 0.2195, 0.2297)$, the fuzzy parameters obtained by using Eq. (17) (with $h=0$) are shown in Eq. (20). These results are plotted in Fig. 3.

$$\tilde{Z}_t = 28.0932 + \langle 0.499, 0.0015 \rangle Z_{t-1} + \langle -0.519, 0 \rangle Z_{t-2} + a_t. \tag{20}$$

The fuzzy ARIMA method provides possible intervals. From Fig. 3, we know the actual values located in the fuzzy intervals but the thread of fuzzy intervals are too wide, especially when the macro-economic environment is smooth. We use the method of Ishibuchi and Tanaka [4] to resolve this problem and provide a narrower interval for the decision maker.

Phase III: It is known from the above results that the observation of 17 August is located at the upper bound (outlier), so the LP constrained equation that is produced by this observation is deleted and renews phase II, let $h=0$ then we get the model that is in Eq. (21). The results are plotted in Fig. 4 and shown in Table 1.

$$\tilde{Z}_t = 28.093 + \langle 0.499, 0.0004 \rangle Z_{t-1} + \langle -0.519, 0 \rangle Z_{t-2} + a_t. \tag{21}$$

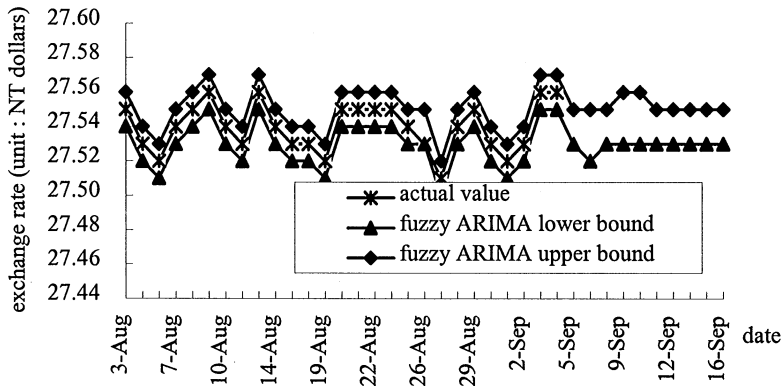


Fig. 4. Result of fuzzy ARIMA (delete August 17).

Table 1
Results of fuzzy ARIMA model (delete 17 August)

Data	Actual value	ARIMA predicted value	Fuzzy ARIMA lower bound	Fuzzy ARIMA upper bound	Data	Actual value	ARIMA predicted value	Fuzzy ARIMA lower bound	Fuzzy ARIMA upper bound
1-Aug	27.55	—	—	—	20-Aug	27.55	27.54	27.54	27.56
2-Aug	27.56	—	—	—	21-Aug	27.55	27.55	27.54	27.56
3-Aug	27.55	27.54	27.54	27.56	22-Aug	27.55	27.54	27.54	27.56
5-Aug	27.53	27.53	27.52	27.54	23-Aug	27.55	27.54	27.54	27.56
6-Aug	27.52	27.53	27.51	27.53	24-Aug	27.54	27.54	27.53	27.55
7-Aug	27.54	27.53	27.53	27.55	26-Aug	27.53	27.53	27.53	27.55
8-Aug	27.55	27.55	27.54	27.56	27-Aug	27.51	27.53	27.5	27.52
9-Aug	27.56	27.54	27.55	27.57	28-Aug	27.54	27.53	27.53	27.55
10-Aug	27.54	27.54	27.53	27.55	29-Aug	27.55	27.55	27.54	27.56
12-Aug	27.53	27.53	27.52	27.54	30-Aug	27.53	27.54	27.52	27.54
13-Aug	27.56	27.53	27.55	27.57	31-Aug	27.52	27.53	27.51	27.53
14-Aug	27.54	27.55	27.53	27.55	2-Sep	27.53	27.53	27.52	27.54
15-Aug	27.53	27.53	27.52	27.54	3-Sep	27.56	27.54	27.55	27.57
16-Aug	27.53	27.53	27.52	27.54	4-Sep	27.56	27.55	27.55	27.57
19-Aug	27.52	27.53	27.51	27.53					

Using the revised fuzzy ARIMA model, we forecast the future value of the exchange rate of the next 10 transaction days whose results are shown in Table 2. The results of the prediction are very good, and the fuzzy intervals are narrower than the results before the model was revised and 95% of the confidence interval of ARIMA. In short, it will help the customers of the bank to understand that the possible interval of exchange rate in the macro-economic environment is stable.

We use the same data to formulate Chen’s fuzzy time series [2] and Watada’s fuzzy time series [12], too. The results are plotted in Figs. 5 and 6.

Table 2
Results of predictions of exchange rate from 5–16 September 1996

Date	Actual value	Fuzzy ARIMA lower bound	Fuzzy ARIMA upper bound
5-Sep	27.54	27.53	27.55
6-Sep	27.55	27.52	27.55
7-Sep	27.54	27.53	27.55
9-Sep	27.55	27.53	27.56
10-Sep	27.54	27.53	27.56
11-Sep	27.55	27.53	27.55
12-Sep	27.54	27.53	27.55
13-Sep	27.54	27.53	27.55
14-Sep	27.54	27.53	27.55
16-Sep	27.55	27.53	27.55

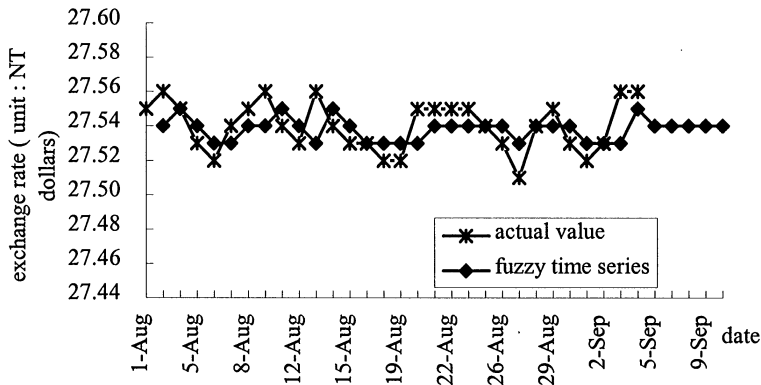


Fig. 5. Results of Chen's fuzzy time series.

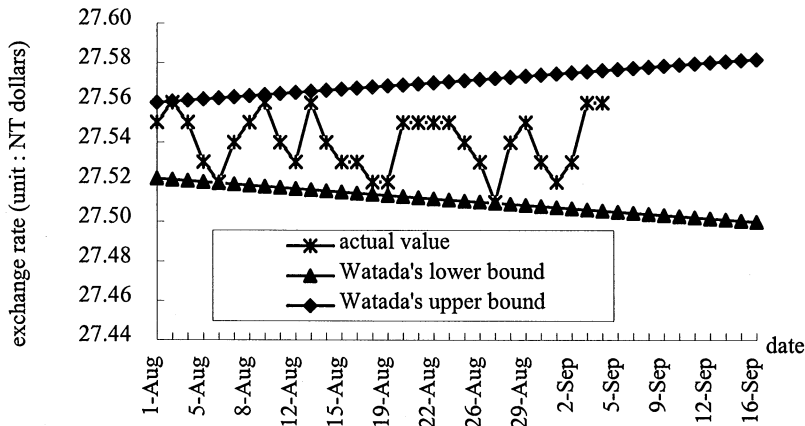


Fig. 6. Results of Watada's fuzzy time series.

Table 3
Comparison of four kinds of time-series methods

ARIMA	Fuzzy ARIMA	Chen fuzzy time series	Watada fuzzy time series
The relationship of input and output is a previous function.	The relationship of input and output is a fuzzy function.	The relationship of input and output is a fuzzy relation.	The relationship of input and output is a fuzzy function.
Provide confidence interval.	Provide possibility interval.	Point estimation.	Provide possibility interval.
At least 50 and preferably 100 observations or more.	Less than ARIMA.	Less than ARIMA.	Less than ARIMA.

4.2. Discussions and analysis

Based on the empirical results of this example, we find that the predictive capability of the fuzzy ARIMA is rather encouraging and the possible interval of fuzzy ARIMA is narrower than 95% of the confidence interval of ARIMA. ARIMA has the tendency to increase in the confidence interval. However, fuzzy ARIMA does not have the situation of ARIMA. This evidence shows that the performance of fuzzy ARIMA is better than ARIMA.

Chen's fuzzy time series [2] is the method of point estimation, and fuzzified historical data must lose some information. However, the fuzzy ARIMA and Watada's time series [12] are interval forecasting model and more information would be obtained.

Fig. 6 shows that Watada's time series has the tendency to increase in the forecasting interval. However, it has not happened in the fuzzy ARIMA model.

Though the basic concept of ARIMA is used to formulate FARIMA, the output of fuzzy ARIMA is fuzziness to release the assumption of white noise (a_t). This makes fuzzy ARIMA require fewer observations than ARIMA. There are several situations for which fuzzy ARIMA appears to be the most appropriate tool. The situations are as follows:

- (i) Fuzzy ARIMA can provide the decision makers the best- and worst-possible situations and can detect the outliers of the historical data.
- (ii) The required observations are less than that required by the ARIMA model which is preferably more than 100.

A comparison of four kinds of time-series methods is shown in Table 3.

5. Conclusions

In this paper, based on the basic concepts of the ARIMA model and Tanaka fuzzy regression, we propose a new method (i.e. fuzzy ARIMA) and apply it to forecasting the foreign exchange rate of NTD to USD for showing the appropriateness and effectiveness of our proposed method. From the example, we can see that the proposed method not only can make good forecasts but also provides the decision makers with the best- and worst-possible situations. The performance of fuzzy ARIMA is better than ARIMA, Chen's fuzzy time series and Watada's fuzzy time series.

Though the basic concept of ARIMA is used to formulate the model, the output of fuzzy ARIMA is fuzziness to release the assumption of white noise (a_t). This makes fuzzy ARIMA require fewer observations than ARIMA. The result of the example shows that the fuzzy ARIMA is more satisfactory than ARIMA. There are several situations for which fuzzy ARIMA appears to be the most appropriate tool. The situations

are as follows:

- (i) To provide the decision makers the best- and worst-possible situations.
- (ii) The required number of observations is less than the ARIMA model required (prefer more than 100).

Fuzzy ARIMA is established by the concept of ARIMA, which assumes that the present value is the linear combination of past value and white noise. White noise is an estimated value, so we must use a two-step method to formulate the fuzzy ARIMA model which can be discussed as another issue.

While the result of ARIMA(p, d, q) shows lack of fit with the data from step 1, we are still encouraged by the fuzzy ARIMA model. Because the result of fuzzy ARIMA is a possibility distribution which can be discussed as another issue.

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