Feedback Median Filter for Robust Preprocessing of Glint Noise

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Glint noise may arise in a target tracking system. The non-Gaussian behavior of glint noise can severely degrade the tracking performance. Measurement preprocessing at the front-end of the tracker is an effective method to reduce glint noise. The preprocessor proposed by Hewer, Martin, and Zeh, which used the computationally intensive M-estimator, may not be suitable for practical implementation. An alternative method employing the median filter is studied here. The median filter is well known for its simplicity and robustness. However, the efficiency of the median filter can be seriously degraded if input samples are not identically distributed. This is what we may encounter in the tracking problem. A feedback median filter is then proposed to overcome this impediment without substantially increasing complexity. Simulations show that the new preprocessor can greatly improve tracking performance in the glint noise environment.

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I. INTRODUCTION

The Kalman filter is widely applied in target tracking problems. It is known to be the linear optimal filter in the white Gaussian noise environment. In some radar applications [1—9], the measurement noise may deviate from the Gaussian assumption. For instance, complex targets can cause irregular electromagnetic wave reflection. This phenomenon varies the target center in a radar and gives rise to outliers in angle tracking, known as "target glint." The glint noise has a long-tailed distribution [1, 2] and can severely degrade the Kalman filter performance. Kalman filtering with non-Gaussian noise has been a difficult problem. In 1975, Masreliez [10, 11] introduced a score function based scheme. While this approach looks promising, he encountered some implementation problems. Wu [4, 5] developed an efficient method to approximate the score function and applied it to target tracking problems. He also incorporated the Masreliez filter into the interacting multiple model (IMM) algorithm and obtained a nonlinear IMM algorithm [6]. Daeipour and Bar-shalom [9] characterized glint noise as a mixture of two Gaussian components and used two different models to represent the noise arising from these two Gaussian components. By doing so, they were able to apply the original IMM tracking algorithm.

When the radar pulse repetition rate is higher than the requisite tracking rate, a tracking system can provide more measurement data than that it can process. In this case, there is a simple approach to deal with glint noise: we can preprocess a batch of measurements and then forward the results to the Kalman filter. One intuitive thought to perform preprocessing may be the use of sample averaging. It can be easily shown that this simple operation is optimal when the target is still and the measurement noise is Gaussian. Wang and Varshney [12, 13] used the maximum likelihood (ML) estimation as the preprocessing algorithm to enhance tracking performance. They considered the case where the target has a constant velocity and the measurement noise is Gaussian. They found that the optimal estimate is also the averaging operation. When the measurement noise is non-Gaussian, averaging is not optimal anymore. Hewer, Martin, and Zeh [1] proposed to use the robust M-estimator as the preprocessing scheme. They showed that the Kalman filter performance can be greatly enhanced. Although the robust algorithm is effective, it requires intensive computations. In addition, this approach assumed that the target position is constant in the preprocessing batch, which may not hold in all situations.

An alternative technique using the median filter is studied here. Due to its simplicity and good properties, the median filter is widely used in image processing [14–17]. There are three distinct properties not shared by the averaging filter. First, it can preserve abrupt changes of signals. Second, it can remove impulsive noise. Third, it has high filtering efficiency for long tailed distributed noise. Since the distribution of glint noise is long-tailed and the target may maneuver, the median filter is then a good choice for preprocessing. Some other order statistic filters such as the alpha-trimmed mean filter and the modified trimmed mean filter [18, 21], can also be used [8]. However, the median filter is preferable due to its simple structure. Also, some fast median algorithms have been developed [22–25] making efficient implementation possible. The basic assumption for using the M-estimator and the median filter is that measurements are independent and identically distributed (IID). When this assumption is violated, the filtering efficiency is reduced. In the tracking problem, this will occur when the target velocity is high and the measurement noise level is relatively low. We have theoretically analyzed the output variance of the median filter and verified that for a fixed noise variance, the output variance will increase as the target velocity increases. To remedy this problem, a new structure named the feedback median filter is proposed. This structure is shown to be robust and does not substantially increase complexity. Simulations show that the proposed structure can greatly improve tracking accuracy.

We briefly describe the median filter and its useful properties in Section II. Incorporating the median filter to preprocessing is shown in Section III. In Section IV, we give some analytic results for evaluating the median filter performance. Simulation results are shown in Section V and the conclusion is drawn in Section VI.

II. MEDIAN FILTER

Let $\{X_1, X_2, \ldots, X_n\}$ be n random variables. We arrange the observations in ascending order such that

$$X_{(1)} \le X_{(2)} \le \cdots \le X_{(n)}. \tag{1}$$

The random variable $X_{(i)}$, $i = 1, 2, \ldots, n$ is called the ith order statistic of the n random variables. For convenience, we assume that $n = 2m-1$ is an odd number. Let $X^n = \{X_1, X_2, \ldots, X_n\}$ be the collection of these random variables. The output of the median filter is defined as

$$\mathrm{median}(X^n) = X_{(m)}. \tag{2}$$

As we can see, the median filter is a nonlinear filter. Thus, its behavior is much different from that of the averaging filter. In what follows, we describe some important properties of the median filter. These properties are used in the later development of our algorithm.

PROPERTY 1 (Scale and Translation Invariance) *For any input random sequence $x^n = \{x_1, x_2, \ldots, x_n\}$ where $n = 2m-1$, we have*

$$\mathrm{median}(c \cdot x^n + d\{1\}) = c \cdot \mathrm{median}(x^n) + d = c \cdot x_{(m)} + d. \tag{3}$$

A special case is that x^n is a linear trend signal. From Property 1, we can easily see that in this case, the output signal equals to the input signal. This is called the linear trend preservation property. Proof of Property 1 can be found in [21].

PROPERTY 2 (Filtering Efficiency) [19, 20] *Let $x^n = \{x_i\}$ and $x_i = c + v_i$, where c is a desired constant signal, and v_i is zero-mean white noise. If v_i has a Laplacian distribution, then* $\mathrm{median}(x^n)$ *is the optimal estimate of c in the ML sense.*

This property reveals that the median filter is suitable for filtering long-tailed distributed noise. In many cases, images are subject to impulse-like noise. This is another reason why the median filter is widely used in image processing. The distribution of glint noise is known to be long tailed. Median filtering is then adequate in the tracking problem. Hereinafter, we consider the output distribution of the median filter. Assume that $X_1, X_2, \ldots, X_n$ are IID with probability density function (pdf) $f(x)$ and cumulative distribution function (cdf) $F(x)$, and $Y_1, Y_2, \ldots, Y_n$ are their order statistics. The joint pdf of $Y_1, Y_2, \ldots, Y_n$ is found to be [27]

$$\begin{aligned} f(y_1, y_2, \ldots, y_n) &= n!\, f(y_1) f(y_2) \ldots f(y_n), \qquad a \le Y_1 \le Y_2 \le \cdots \le Y_n \le b \\ &= 0, \qquad \text{elsewhere} \end{aligned} \tag{4}$$

where a and b are two constants bounding X_i. Let Y_m be the median of $\{Y_1, Y_2, \ldots, Y_n\}$. The pdf of Y_m can be obtained

$$\begin{aligned} g_m(y) &= \int_{y_{n-1}}^{b} \int_{y_{n-2}}^{b} \cdots \int_{y_m}^{b} \int_{a}^{y_m} \cdots \int_{a}^{y_3} \int_{a}^{y_2} \\ &\quad \times f(y_1, y_2, \ldots, y_n)\, dy_1 \ldots dy_{m-1}\, dy_{m+1} \ldots dy_n \\ &= n \binom{n-1}{m-1} F(y)^{m-1} [1 - F(y)]^{n-m} f(y), \\ &\qquad a \le y \le b. \end{aligned} \tag{5}$$

The cdf of Y_m is

$$\begin{aligned} G_m(y) &= \Pr\{Y_m \le y\} \\ &= \Pr\{\text{at least } m \text{ samples of the } X_i \text{ are less than or equal to } y\} \\ &= \sum_{i=m}^{n} \binom{n}{i} F^i(y) [1 - F(y)]^{n-i}. \end{aligned} \tag{6}$$

For the IID random variables, the moments and product moments of order statistics can be expressed in terms of elementary functions for $n \leq 5$ [28]. By repeated integration by parts, many higher moments can be obtained.

If $X_1, X_2, \ldots, X_n$ are independent but not IID, then the cdf of Y_m is given by [29]

$$G_m(y) = \sum_{i=m}^{n} \sum_{S_i} \prod_{l=1}^{i} F_{j_l}(y) \prod_{l=i+1}^{n} [1 - F_{j_l}(y)] \qquad (7)$$

where $f_i(x)$ and $F_i(x)$ are the pdf and cdf of X_i, respectively, and the summation S_i extends over all permutations $(j_1, j_2, \ldots, j_n)$ of $1, 2, \ldots, n$ for which $j_1 < j_2 < \cdots < j_i$ and $j_{i+1} < j_{i+2} < \cdots < j_n$. For example, using (7) to evaluate the cdf of the median of three samples $\{X_1, X_2, X_3\}$, we have

$$G_2(y) = F_1(y)F_2(y)[1 - F_3(y)] + F_1(y)F_3(y)[1 - F_2(y)] + F_2(y)F_3(y)[1 - F_1(y)] + F_1(y)F_2(y)F_3(y). \qquad (8)$$

Previous results for calculating moments of the median output for IID random variables cannot be applied here. Although we have another approach to deal with this problem for some low moments [30], a more direct method is to use numerical integration. The mean μ_{Y_m} and the variance $\sigma^2_{Y_m}$ of Y_m, by definition, are given as follows:

$$\mu_{Y_m} = \int_{-\infty}^{\infty} y g_m(y)\, dy \qquad (9)$$

and

$$\sigma^2_{Y_m} = \int_{-\infty}^{\infty} y^2 g_m(y)\, dy - \mu^2_{Y_m} \qquad (10)$$

respectively. The mean of the median output of zero-mean IID Gaussian random variables is shown to be zero [29]. The same result can be obtained for some zero-mean non-IID random variables, which is described below.

PROPERTY 3 *Let* $\{X_i,\ i = 1, 2, \ldots, n\}$ *be n IID random variables with the same zero-mean symmetric pdf, and* $\{\alpha_j,\ j = 1, 2, \ldots, n\}$ *be a monotonic and antisymmetric sequence, i.e.,* $\alpha_1 \leq \alpha_2 \leq \cdots \leq \alpha_n$, $\alpha_{(n+1)/2} = 0$, *and* $\alpha_j = -\alpha_{n-j+1}$, $j = 1, 2, \ldots, (n-1)/2$. *Construct a new sequence* $Z_i = X_i + \alpha_i$, $i = 1, 2, \ldots, n$. *Then, the mean of the* median$(\{Z_i\})$ *is zero. Proof of this property is straightforward and thus omitted. An application of this property is presented in the following section.*

In this paragraph, we discuss the computational complexity of median filtering. As we can see, if the sample size is large, the ranking operations in median filtering can be complicated. Fortunately, a fast algorithm called the threshold decomposition has been developed [18–21]. This algorithm applies to discrete-valued signals and no ranking operations are required. With this method, an m-level signal can be decomposed into the sum of m-1 binary signals, each

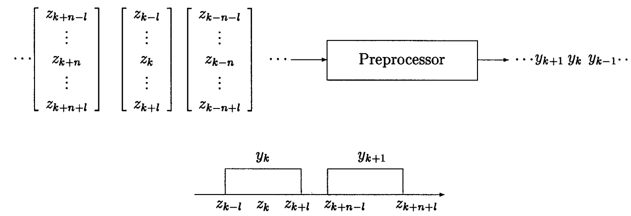

Fig. 1. Batch preprocessing.

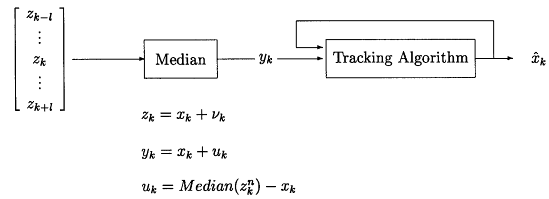

Fig. 2. Direct structure for median preprocessing.

of the binary signals is median filtered separately, and then the results are linearly combined to form the output signal. Since median filtering of binary samples is found by counting the number of 1s of the samples, the implementation of these filters become much simpler. If the sample value is continuous, we then perform quantization. The number of the quantization level becomes a tradeoff between hardware complexity and filtering performance.

III. TARGET TRACKING WITH MEDIAN PREPROCESSING

Let the input to the preprocessor be divided into successive batches, the batch centered at the measurement z_k be denoted as $z_k^n = \{z_{k-l}, \ldots, z_k, \ldots, z_{k+l}\}$ where $n = 2l + 1$, and the output be denoted as y_k. The next input batch is then $z_{k+n}^n = \{z_{k+n-l}, \ldots, z_{k+n}, \ldots, z_{k+n+l}\}$ and the corresponding output is y_{k+1}. Since the preprocessed data are nonoverlapped, if input noise is white, the output noise will also be white. This white assumption is essential for Kalman filtering. The preprocessing scheme is shown in Fig. 1.

Assume that the target dynamics and natural spherical observation model have been decoupled into independent channels in their corresponding axes. Consider a one-dimensional tracker, and assume that only position measurements are available. The target position and the noisy position measurement can be described by the following equations:

$$x_{k+1} = x_k + v_k T + \tfrac{1}{2} a_k T^2 \qquad (11)$$

$$z_k = x_k + \nu_k \qquad (12)$$

where T is the sampling period, v_k and a_k are the velocity and acceleration of the target, respectively, z_k is the measurement, and ν_k is white zero-mean measurement noise. An intuitive structure to apply median filtering is shown in Fig. 2. We call this the median filtering with the direct structure or the direct median filtering. From Property 1, the output y_k can be written by

$$y_k = \mathrm{median}(z_k^n)$$

$$= x_k + \mathrm{median}\left(\begin{bmatrix} (x_{k-l} - x_k) + \nu_{k-l} \\ \vdots \\ (x_{k-1} - x_k) + \nu_{k-1} \\ 0 + \nu_k \\ (x_{k+1} - x_k) + \nu_{k+1} \\ \vdots \\ (x_{k+l} - x_k) + \nu_{k+l} \end{bmatrix}\right) \tag{13}$$

$$= x_k + u_k. \tag{14}$$

Consider the situation that the target has a constant velocity or that $v_k \gg a_k T$. The third term in the right-hand side of (11) can be ignored and the process u_k becomes

$$u_k \approx \mathrm{median}\left(\begin{bmatrix} -l\Delta_1 + \nu_{k-l} \\ \vdots \\ -\Delta_1 + \nu_{k-1} \\ 0 + \nu_k \\ \Delta_1 + \nu_{k+1} \\ \vdots \\ l\Delta_1 + \nu_{k+l} \end{bmatrix}\right) \tag{15}$$

where

$$\Delta_1 = v_k T. \tag{16}$$

Here Δ_1 in (15) cannot be moved out from the median operation because the linear trend preservation in Property 1 can only hold for deterministic signals. From Property 3, we know that $E\{u_k\} = 0$. If $\Delta_1 = 0$ and ν_k is a white Gaussian process with variance σ_ν^2, the variance of u_k was found to be [31]

$$E\{u_k^2\} \approx \pi\sigma_\nu^2 / 2(n + \pi/2 - 1). \tag{17}$$

If $\Delta_1 \neq 0$, which is the practical case, no theoretical results are available. As we show in the next section, $E\{u_k^2\}$ is larger than that in (17). The larger the Δ_1, the larger $E\{u_k^2\}$ we obtain. In other words, if the target velocity is higher, the performance of the median filter is poorer. For non-Gaussian measurement noise, the same observation can be obtained.

The increased variance of u_k degrades the performance of the Kalman tracking algorithm. Here we propose a new structure, as shown in Fig. 3, to overcome this problem. The idea is to suppress Δ_1 by some estimated value $\hat{\Delta}_1$. From (16), we see that Δ_1 is directly related to the velocity v_k. Thus, we can subtract v_k from its predicted value, denoted as $\overline{v}_k$, obtained from the Kalman tracking algorithm. This makes the process u_k become

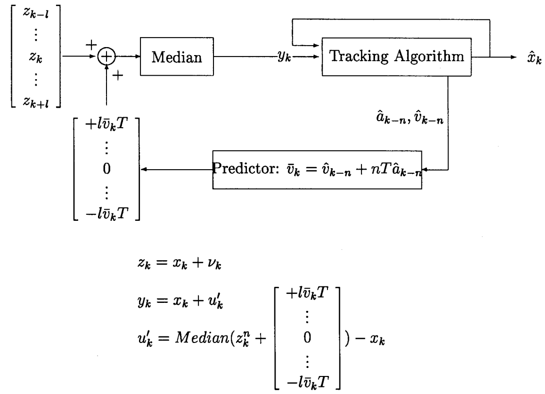

Fig. 3. Feedback structure for median preprocessing.

$$u'_k \approx \mathrm{median}\left(\begin{bmatrix} -l\Delta_2 + \nu_{k-l} \\ \vdots \\ -\Delta_2 + \nu_{k-1} \\ 0 + \nu_k \\ \Delta_2 + \nu_{k+1} \\ \vdots \\ l\Delta_2 + \nu_{k+l} \end{bmatrix}\right) \tag{18}$$

where

$$\Delta_2 = (v_k - \overline{v}_k)T. \tag{19}$$

The velocity prediction $\overline{v}_k$ is calculated by

$$\overline{v}_k = \hat{v}_{k-n} + nT\hat{a}_{k-n} \tag{20}$$

where $\hat{v}_{k-n}$ and $\hat{a}_{k-n}$ correspond to the estimates of the Kalman tracking algorithm in the previous batch. When the predicted velocity approaches the actual velocity, the variance of u'_k will be reduced to that in (17) for Gaussian noise. This condition can be approximately achieved when the tracking algorithm is performing well. In general condition, we can expect that $v_k - \overline{v}_k < v_k$ such that $\Delta_2 < \Delta_1$. Hence, the tracking performance of the new structure will be better than the direct structure in Fig. 2. Note that Δ_1 is deterministic for a constant velocity target, but Δ_2 is random. We call the new structure median filtering with a feedback structure, or feedback median filtering.

IV. VARIANCE ANALYSIS

In this section, we analyze the variances of u_k and u'_k in the glint noise environment (including Gaussian noise as a special case). Before we do this, we first have to find a model for glint noise. There are many models describing the long-tailed nature of glint noise distribution. A simple model that is frequently used is called the scaled-contaminated model (SCM). This model is commonly used to accommodate outliers in statistical practice [32, 33] and also used to describe glint noise in [1, 6, 9]. The SCM is expressed by a mixture of two Gaussian components. One Gaussian component is with large variance and small occurring probability while the other is with small variance and large occurring probability. Let ν_k be glint noise described by the SCM. Then,

$$N_\nu(0,\sigma_0^2,\sigma_1^2) = \epsilon N(0,\sigma_0^2) + (1-\epsilon)N(0,\sigma_1^2) \quad (21)$$

where $\sigma_1^2 \ll \sigma_0^2$ and $\epsilon < 1$ is a small positive value. Note that when $\epsilon = 0$, the SCM is reduced to the Gaussian distribution.

We first consider the variance of u_k (i.e., the results of the direct median preprocessing). For notational consistency, we denote the cdf of u_k as $G_m(y)$ and the pdf as $g_m(y)$. Thus, the variance of u_k can be obtained by (7) and (10). Here, we use numerical integration to solve (10). The required distribution functions $F_i(y)$ in (7) are the cdfs of the vector entries inside the median operation in (15). Their pdf's are given by

$$f_i(y) = \frac{\epsilon}{\sqrt{2\pi\sigma_0^2}} \exp\left\{-\frac{[y-(i-l-1)\Delta_1]^2}{2\sigma_0^2}\right\} + \frac{1-\epsilon}{\sqrt{2\pi\sigma_1^2}} \exp\left\{-\frac{[y-(i-l-1)\Delta_1]^2}{2\sigma_1^2}\right\}, \quad i = 1,2,\ldots,n. \quad (22)$$

Using the definition of the error function

$$\mathrm{erf}(y) \triangleq \frac{2}{\sqrt{\pi}} \int_0^y e^{-t^2}\, dt \quad (23)$$

and the relation

$$\int_{-\infty}^{y} \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(t-\mu)^2}{2\sigma^2}\right] dt = \frac{1}{2}\left[1 + \mathrm{erf}\left(\frac{y-\mu}{\sqrt{2}\sigma}\right)\right] \quad (24)$$

we can express $F_i(y)$ by

$$F_i(y) = \frac{\epsilon}{2}\left[1 + \mathrm{erf}\left(\frac{y-(i-l-1)\Delta_1}{\sqrt{2}\sigma_0}\right)\right] + \frac{1-\epsilon}{2}\left[1 + \mathrm{erf}\left(\frac{y-(i-l-1)\Delta_1}{\sqrt{2}\sigma_1}\right)\right], \quad i = 1,2,\ldots,n. \quad (25)$$

Apparently, the median output variance σ_u^2 is a function of Δ_1. The relations between the median output standard deviation (STD) and Δ_1 are plotted in Figs. 4 and 5 for Gaussian noise and glint noise, respectively. In these figures, the mean filtering corresponds to measurement averaging. The x-axis is Δ_1 and the y-axis is the output STD. The batch size is five. In Fig. 4, the STD of Gaussian noise is $100m$. In Fig. 5, the SCM parameters are $\sigma_1 = 100m$, $\sigma_0 = 7\sigma_1$, and $\epsilon = 0.1$. As we can see from Fig. 4, the output STD of mean filtering is not affected by the target velocity and it is smaller than that of median filtering. This is expected since the mean filter is the optimal filter for Gaussian noise. Note that the STD of the median output increases as Δ_1 increases. Finally, the output becomes the middle sample in the batch and the STD becomes the noise STD. For glint noise, the median filter has much smaller output STD than the mean filter for small Δ_1. However, as Δ_1 increases, the median output STD increases. This behavior is similar to that for Gaussian noise. When $\Delta_1 > 150m$, the output STD of the median filter is larger than that of the mean filter. These results indicate that the direct median filtering is inadequate.

We now examine the performance of feedback median filtering. We have noted that Δ_2 is a random variable and correlated to outputs of the tracking

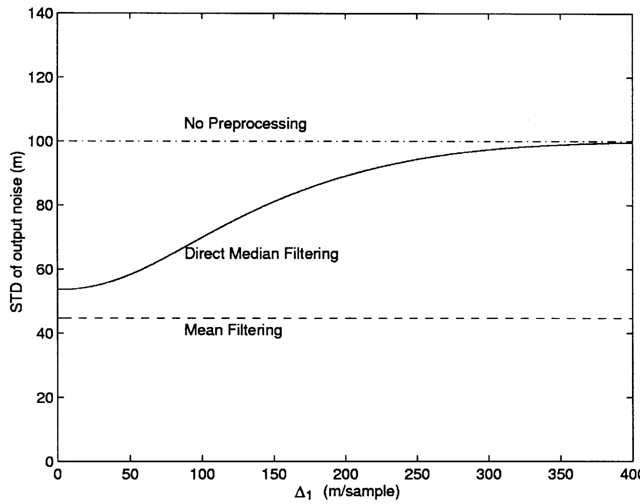

Fig. 4. Output STD of direct median filering with Gaussian noise.

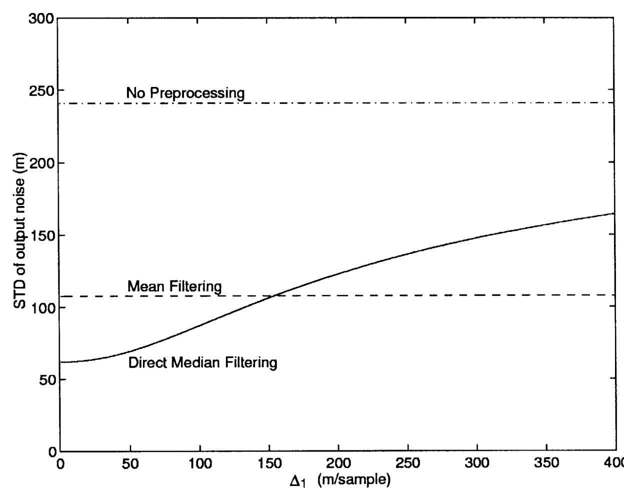

Fig. 5. Output STD of direct median filtering with glint noise.

algorithm. As a consequence, assessing the variance of u'_k requires some more work. First, we utilize the following equalities

$$E\{u'_k\} = E_{\Delta_2}\{E_\nu\{u'_k \mid \Delta_2\}\} \tag{26}$$

$$E\{u'^2_k\} = E_{\Delta_2}\{E_\nu\{u'^2_k \mid \Delta_2\}\}. \tag{27}$$

The significance of (26) and (27) is that it allows us to evaluate the variance of u'_k using previous results. As we discussed above, it is simple to see that $E_\nu\{u'_k \mid \Delta_2\}$ is zero. The variance of u'_k conditioned on Δ_2 is then $E_\nu\{u'^2_k \mid \Delta_2\}$. Due to the conditioning operation, Δ_2 can be treated as a constant. Thus, (18) is identical to (15), and (22) and (25) can be directly applied by substituting Δ_1 with Δ_2. Now, if we assume that the distribution of Δ_2 is known, the second expectation operation in (26) and (27) can be performed. Let the STD of Δ_2 be ρ. Then, we have $E\{u'_k\} = 0$ and

$$\sigma^2_{u'}(\rho) = \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} y^2 g_m(y;\Delta_2)dy\right] f(\Delta_2;\rho)d\Delta_2$$

$$= \int_{-\infty}^{\infty} \sigma^2_u(\Delta_2) f(\Delta_2;\rho)d\Delta_2 \tag{28}$$

where $g_m(y;\Delta_2)$ is the pdf of u'_k conditioned on Δ_2, and $f(\Delta_2;\rho)$ is the pdf of Δ_2 with STD ρ. From (18), we see that if the variance of Δ_2 is small compared with that of noise, u'_k is just the median of noise samples $\{\nu_{k-l},\ldots,\nu_{k+l}\}$. As outliers are removed by the median filtering, it is then reasonable to assume that u'_k is a Gaussian process. Notice that u'_k is the new measurement noise to the Kalman filter. Since the Kalman filter yields linear unbiased state estimates, zero-mean Gaussian measurement noise will result in zero-mean Gaussian estimation errors. Hence one can approximate Δ_2 by a zero-mean Gaussian random variable. In practice, ρ can be estimated from the corresponding entry of the covariance matrix in the Kalman filter. We elaborate on this in the next section.

In Figs. 6 and 7, the relations of $\sigma_{u'}$ and ρ are plotted for Gaussian noise and glint noise, respectively. For comparison purposes, we also plot the relations of σ_u and Δ_1 in the same figures. Note that Δ_1 and ρ have the same unit. We first note that $\sigma_{u'}$ increases as ρ increases. Thus, ρ plays the same role as Δ_1 in the direct structure. The other important observation is that $\sigma_{u'}(\rho)$ is less than $\sigma_u(\Delta_1)$ for the same ρ and Δ_1. As we mentioned, ρ is usually less than Δ_1 for the same inputs. This indicates that σ'_u may be much smaller than σ_u. Also note that due to the feedback structure, u'_k is not white anymore. Since Δ_2 is obtained after preprocessing and Kalman filtering, ρ can be much smaller than σ^2_ν. Thus, we can consider that the non-white effect is small.

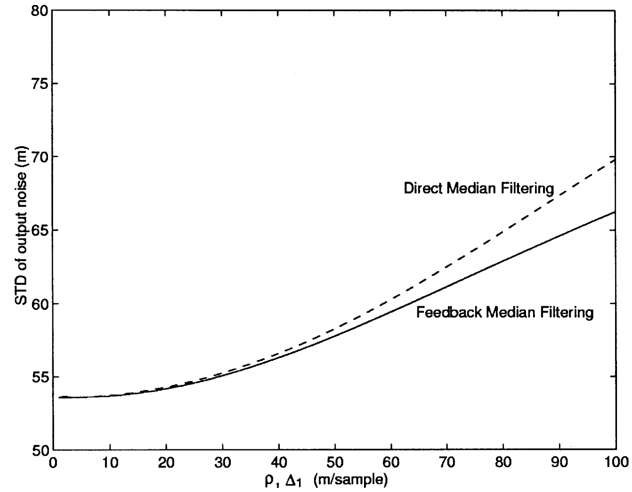

Fig. 6. Output STD of feedback median filtering with Gaussian noise (output STD of direct median filtering is shown for comparison).

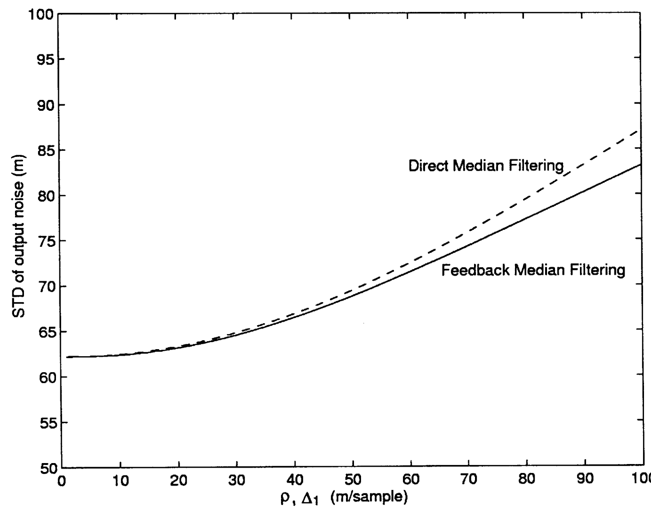

Fig. 7. Output STD of direct median filtering with glint noise (output STD of direct median filtering is shown for comparison).

V. SIMULATION RESULTS

In this section, we carry out some simulations to evaluate the performance of the proposed algorithm. We only consider a 1-D tracking problem where the system models are described in (11) and (12). Let the total tracking interval be 150 s. The initial target velocity is 120 m/s. Maneuvering occurs from the 60th second to 80th second with the constant acceleration 20 m/s^2. After maneuvering, the velocity changes from 120 m/s to 540 m/s. From (16), we find that $\Delta_1 = 36$ before maneuvering and $\Delta_1 = 162$ after maneuvering.

The IMM algorithm is used to track the maneuvering target. The IMM algorithm consists of a two-order model for nonmaneuvering and a third-order model for maneuvering [34]. The state-space models can be written as

$$\begin{bmatrix} x \\ v \end{bmatrix}_{k+1} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ v \end{bmatrix}_k + \begin{bmatrix} \frac{1}{2}T^2 \\ T \end{bmatrix} w_k \tag{29}$$

$$\begin{bmatrix} x \\ v \\ a \end{bmatrix}_{k+1} = \begin{bmatrix} 1 & T & \frac{1}{2}T^2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ v \\ a \end{bmatrix}_k + \begin{bmatrix} \frac{1}{6}T^3 \\ \frac{1}{2}T^2 \\ T \end{bmatrix} w_k^m. \tag{30}$$

The process noise variances are chosen as $E[w_k w_k] = 0\ (\mathrm{m/s^2})^2$ and $E[w_k^m w_k^m] = 20^2\ (\mathrm{m/s^2})^2$. The Markovian transition probability matrix in our two-model IMM algorithm is set as follows:

$$[p_{ij}] = \begin{bmatrix} 0.95 & 0.05 \\ 0.05 & 0.95 \end{bmatrix}. \tag{31}$$

As (31) shows, the transition probability of the same model between two sample batches is 95%. A larger p_{12} yields less tracking error when the target is maneuvering, but renders a higher error penalty when the target is nonmaneuvering.

Another parameter yet to be determined is the output STD of the preprocessor. For direct median filtering, we assume that the actual target velocity is known and find the corresponding output STD from Figs. 4 and 5. Note that this is just for comparison purposes since we cannot know the actual target velocity in practice. For feedback median filtering, we use the STD of the velocity estimation error in the Kalman filter to approximate ρ and find the output variance from Figs. 6 and 7.

We compare the tracking performances of preprocessors using mean filtering, direct median filtering, and feedback median filtering. Two sets of experiments are conducted to demonstrate the effectiveness of our feedback median filtering for Gaussian noise and glint noise. The performance is compared using the root mean squares error (RMSE) criterion, which is defined as follows:

$$\mathrm{RMSE}(k) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\mathbf{x}_k - \hat{\mathbf{x}}_k^i)^2}, \quad k = 1,2,\ldots,100; \quad N = 500 \tag{32}$$

where $\hat{\mathbf{x}}_k^i$ denotes the state estimate of the ith Monte Carlo run for the kth sample. We carry out 500 Monte Carlo runs and report the averaged RMSE.

In the first set of experiments, we let the measurement sampling period be 0.3 s and the batch size be five. We add white Gaussian noise with STD = $100m$ to the tracking trajectory. The tracking results are shown in Fig. 8. We can see that the performance of the median filter with the direct structure is seriously degraded when the target velocity is high. This is consistent with the results

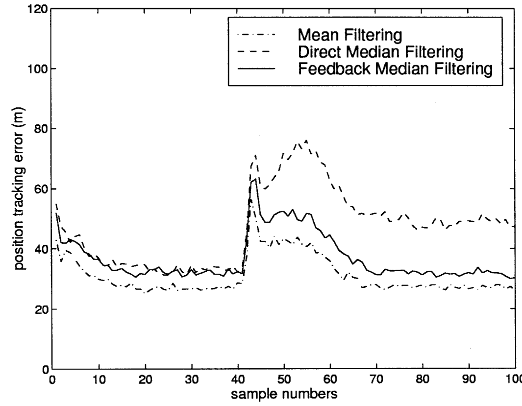

Fig. 8. RMSE comparison of position tracking with Gaussian noise.

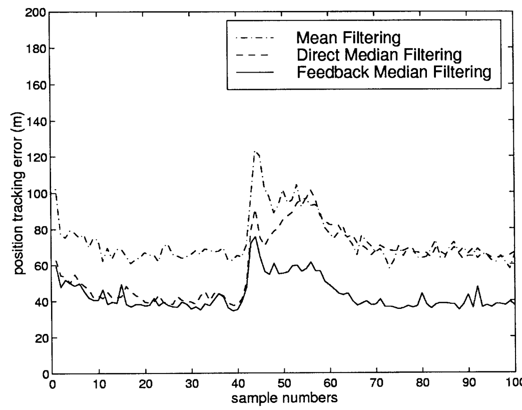

Fig. 9. RMSE comparison of position tracking with glint noise.

in Fig. 4. Since the velocity increases constantly during target maneuvering, the tracking behavior in this period becomes peculiar. The tracking error first increases due to the suddenly target maneuvering. Then the number of observations increases, the error decreases. However, as the maneuvering continues, the target velocity becomes higher and higher and the median filter performance becomes worse and worse. The error increases again. The median preprocessor with the feedback structure has compensated for the degradation due to velocity. As described, the mean filter is optimal in this case. However, the performance difference between the mean filtering and the feedback median filtering is small.

Then, we examine the tracking results with glint noise. We set the parameters of the SCM as $\sigma_1 = 100m$, $\sigma_0 = 7\sigma_1$, and $\epsilon = 0.1$. Here, we assume that these parameters are known to the tracker. If these parameters are unknown, we can identify them in advance, using algorithms such as those in [7]. In Fig. 9, we compare tracking performance for all preprocessing schemes. As we can see, the mean filter is inadequate here. The median-based filters perform better. Before maneuvering, the performance of the feedback median filter is similar to that of the

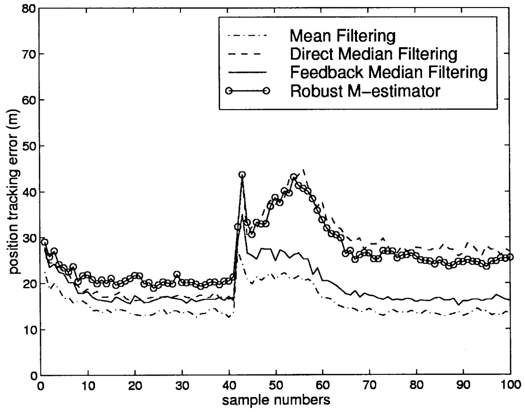

Fig. 10. RMSE comparison of position tracking with Gaussian noise.

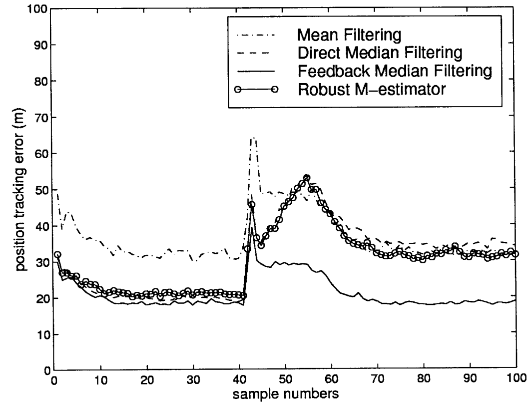

Fig. 11. RMSE comparison of position tracking with glint noise.

direct median filter. However, the performance of the feedback median filter is much better when the target velocity is high.

In the second set of experiments, we let the sampling period be 0.0714 s and the batch size be 21. Note that the time span for a batch is the same as that in the first set of experiments. For the direct median filtering, the output STD functions in Figs. 4 and 5 are not valid anymore since the batch size is different now. To understand the batch size effect, we carry out simulations finding the relationship between the output STD and the batch size (the time span remains the same). We have the following observations. First, the output STD decreases as the batch size increases. This is not surprising since we have more data in a batch. Second, the output STD increases as the target velocity increases no matter what block size is used. This indicates that direct median filtering will always suffer in the non-IID problems. Only when the time span of the batch is small will this problem be alleviated.

In this set of experiments, we add the robust M-estimator preprocessing algorithm used in [1] for additional comparison. Being an iterative algorithm, the M-estimator requires intensive computations. As we mentioned, it is not suitable for practical implementation. The parameter c in [1] is chosen as 3 in our simulations. Fig. 10 shows tracking results for Gaussian noise and Fig. 11 for glint noise. From these figures, we can see that performance of the robust M-estimator is similar to that of the direct median filtering scheme. As the direct median filtering, the M-estimator is strongly affected by the target velocity. In contract, feedback median filtering is not sensitive to the target velocity. Based on these results, we conclude that while the feedback median preprocessing can provide best tracking performance for glint noise, it also has similar performance to the mean filter for Gaussian noise. Thus, this is a robust preprocessing scheme for maneuvering target tracking.

VI. CONCLUSION

In this paper, we suggest the median filter as a preprocessor in place of the complicated robust M-estimator proposed by Hewer, Martin, and Zeh [1]. Due to its superior capabilities for preserving abrupt signal changes and rejecting impulse-like noise, the median filter is suitable for the tracking application. The simple structure is another advantage facilitating real-time implementation. When the target velocity is high, direct application of the median filter does not yield good results. We have proposed a feedback scheme that can solve this problem without substantially increasing complexity. The effect of non-IID inputs is also theoretically analyzed. Simulations show that our scheme is robust and suitable for real-time implementation. It can provide satisfactory results either in the Gaussian noise or in the glint noise environment.

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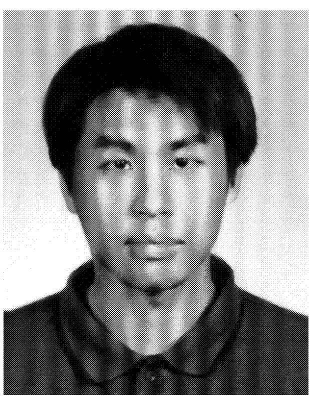

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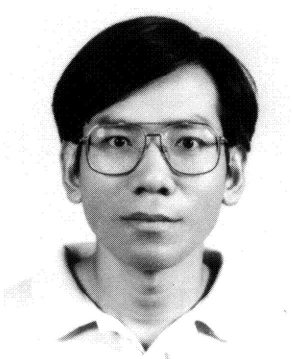

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