Eur. Phys. J. C 13, 695–697 (2000)
Digital Object Identifier (DOI) 10.1007/s100520000175

THE EUROPEAN PHYSICAL JOURNAL C

Chan–Paton soliton gauge states of the compactified open string

Jen-Chi Lee[a]

Department of Electrophysics, National Chiao-Tung University, Hsinchu, Taiwan, 30050, R.O.C.

Received: 14 May 1999 / Published online: 17 March 2000 – © Springer-Verlag 2000

Abstract. We study the mechanism of the enhanced gauge symmetry of the bosonic open string compactified on a torus by analyzing the zero-norm soliton (non-zero winding of the Wilson line) gauge states in the spectrum. Unlike the closed string case, we find that the soliton gauge state exists only at massive levels. These soliton gauge states correspond to the existence of enhanced massive gauge symmetries with transformation parameters containing both Einstein and Yang–Mills indices. In the T-dual picture, these symmetries exist only at some discrete values of compactified radii when N D-branes are coincident.

1 Introduction

The discovery of the D-brane as an R–R charge carrier [1] and its applications to various string dualities has made it clear that the open string is essential in the study of string theory. Historically, Yang–Mills gauge symmetry was incorporated into string theory through different mechanisms for closed and open strings. For the closed string, it was built into the theory through compactification of the string coordinates or, more generally, by adding an internal Kac–Moody conformal field theory with the appropriate central charge[2]. For the open string instead, a Yang–Mills degree of freedom was built into the theory through the Chan–Paton effect [3] by adding charges at the end points of string.

In the previous paper [4], we related the closed string Kac–Moody gauge symmetry to the existence of massless *zero-norm* soliton gauge states (SGS) in the spectrum of the torus compactification. This program was then extended to the massive states. The existence of the massive SGS thus implies that there is an infinite enhanced gauge symmetry of compactified closed string theory. In this paper, we will study the SGS of compactified *open* string theory. Unlike the closed string case, we find that the SGS exists only at massive levels. This SGS corresponds to the enhanced massive symmetries with transformation parameters containing both Einstein and Yang–Mills indices. This is reminiscent of the symmetry of the closed massive heterotic string modes discovered previously [5]. In the T-dual picture, these SGS implies the existence of enhanced massive gauge symmetry at some *discrete* values of the compactified radii when N D-branes are coincident.

This paper is organized as follows. In Sect. 2, we discuss the uncompactified open string. We first derive both the massless and massive Chan–Paton zero-norm gauge states. The corresponding gauge symmetries and Ward identities are then derived. In the massive case, we get a mixed Einstein–Yang–Mills type symmetry, which is similar to the one we derived in the closed heterotic string theory. Section 3 is devoted to the compactified open string case. Massive SGS, which is responsible for the enhancement of massive gauge symmetry, is shown to exist at any higher massive levels of the spectrum. A brief discussion is given in Sect. 4.

[a] E-mail: jcclee@cc.nctu.edu.tw

2 Chan–Paton gauge states

In this section, we discuss the (zero-norm) gauge state of the uncompactified open string with a Chan–Paton factor and its implication for the on-shell symmetry and for the Ward identity. For simplicity, we consider the oriented $U(N)$ case. The vertex operators of massless gauge state are

$$\theta^a \lambda^a_{ij} k \cdot \partial x \mathrm{e}^{\mathrm{i}kx} \tag{1}$$

where $\lambda \in U(N)$, $i \in N$, $j \in \overline{N}$ and $a \in$ the adjoint representation of $U(N)$. The on-shell conformal deformation and the $U(N)$ gauge symmetry to lowest order in the weak background field approximation are ($\Box\theta^a = 0, \Box \equiv \partial_\mu \partial^\mu$)

$$\delta T = \lambda^a_{ij} \partial_\mu \theta^a \partial x^\mu \tag{2}$$

and

$$\delta A^a_\mu = \partial_\mu \theta^a \tag{3}$$

with T the energy momentum tensor and A^a_μ the massless gauge field.

One can verify the corresponding Ward identity by calculating e.g., 1-vector and 3-tachyons four point correlators. The amplitude is calculated to be

$$T^{abcd}_\mu = \int \prod_{i=1}^{4} \mathrm{d}x_i \langle \mathrm{e}^{\mathrm{i}k_1 x_1} \partial x_\mu \mathrm{e}^{\mathrm{i}k_2 x_2} \mathrm{e}^{\mathrm{i}k_3 x_3} \mathrm{e}^{\mathrm{i}k_4 x_4} \rangle$$

$$\times \mathrm{Tr}\left(\lambda^a\lambda^b\lambda^c\lambda^d\right) = \frac{\Gamma\left(-\frac{s}{2}-1\right)\Gamma\left(-\frac{t}{2}-1\right)}{\Gamma\left(\frac{\mu}{2}+1\right)} \times \left[k_{3\mu}\left(\frac{s}{2}+1\right) - k_{1\mu}\left(\frac{t}{2}+1\right)\right] \times \mathrm{Tr}\left(\lambda^a\lambda^b\lambda^c\lambda^d\right). \quad (4)$$

In (4) , s, t and u are the usual Mandelstam variables. One can then verify the Ward identity

$$\theta^b k_2^\mu T_\mu^{abcd} = 0. \quad (5)$$

We now discuss the massive gauge states. The vertex operator of the type I massive vector gauge state is

$$\theta_\mu^a \lambda_{ij}^a \left[k\cdot\partial x\partial x^\mu + \partial^2 x^\mu\right] \mathrm{e}^{\mathrm{i}kx}. \quad (6)$$

We note that the gauge state polarization contains both Einstein and Yang–Mills indices. This is very similar to the 10D closed heterotic string case [5]. The only difference is that in the heterotic string one could have more than one Yang–Mills index. The on-shell conformal deformation and the mixed Einstein–Yang–Mills type symmetry to lowest order in the weak field approximation are $\left((\Box-2)\,\theta_\mu^a = \partial\cdot\theta^a = 0\right)$

$$\delta T = \lambda_{ij}^a \partial_{(\mu}\theta_{\nu)}^a \partial x^\mu \partial x^\nu + \lambda_{ij}^a \theta_\mu^a \partial^2 x^\mu \quad (7)$$

and

$$\delta M_{\mu\nu}^a = \partial_\mu\theta_\nu^a + \partial_\nu\theta_\mu^a. \quad (8)$$

One can also derive the corresponding massive Ward identity by calculating the decay rate of one massive state to three tachyons. The most general amplitude is calculated to be

$$A^{abcd} = \varepsilon^a\varepsilon^c\varepsilon^d\left(\varepsilon_{\mu\nu}^b T^{\mu\nu} + \varepsilon_\mu^b T^\mu\right)\mathrm{Tr}\left(\lambda^a\lambda^b\lambda^c\lambda^d\right) \quad (9)$$

where

$$T^{\mu\nu} = \frac{\Gamma\left(-\frac{s}{2}-1\right)\Gamma\left(-\frac{t}{2}-1\right)}{\Gamma\left(\frac{\mu}{2}+2\right)} \times \left\{\frac{s}{2}\left(\frac{s}{2}+1\right)k_3^\mu k_3^\nu + \frac{t}{2}\left(\frac{t}{2}+1\right)k_1^\mu k_1^\nu - 2\left(\frac{s}{2}+1\right)\left(\frac{t}{2}+1\right)k_1^{(\mu}k_3^{\nu)}\right\} \quad (10)$$

and

$$T^\mu = \frac{\Gamma\left(-\frac{s}{2}-1\right)\Gamma\left(-\frac{t}{2}-1\right)}{\Gamma\left(\frac{\mu}{2}+2\right)} \times \left\{-k_3^\mu\frac{s}{2}\left(\frac{s}{2}+1\right) - k_1^\mu\frac{t}{2}\left(\frac{t}{2}+1\right)\right\}. \quad (11)$$

In (9) ε^a etc. are the polarization of the tachyons and $\left(\varepsilon_{\mu\nu}^b, \varepsilon_\mu^b\right)$ is the polarization of the massive state. The above amplitude satisfies the following Ward identity:

$$k_{(\mu}\theta_{\nu)}^a T^{\mu\nu} + \theta_\mu^a T^\mu = 0. \quad (12)$$

A similar consideration can be applied to the following type II massive scalar gauge state

$$\left[\frac{1}{2}\alpha_{-1}\cdot\alpha_{-1} + \frac{5}{2}k\cdot\alpha_{-2} + \frac{3}{2}\left(k\cdot\alpha_{-1}\right)^2\right] \mid k, l=0, i, j\rangle, \quad (13)$$

which corresponds to a *massive* $U(N)$ symmetry.

3 Chan–Paton soliton gauge state on $R^{25}\otimes T^1$

In this section, we discuss soliton gauge states on a torus compactification of the bosonic open string. As is well known, the massless $U(N)$ gauge symmetry will be broken in general after compactification unless N D-branes, in the T-dual picture, are coincident. We will see that when D-branes are coincident, one has enhancement of (unwinding) zero-norm gauge states and the massless $U(N)$ symmetry will be recovered. These zero-norm gauge states can be considered as charges or symmetry parameters of an $U(N)$ group.

In the discussion of open string compactification, one needs to turn on the Wilson line or nonzero background gauge field in the compact direction. This will affect the momentum in the compact direction, and the Virasoro operators become

$$L_0 = \frac{1}{2}\left(\frac{2\pi l - \theta_j + \theta_i}{2\pi R}\right)^2 + \frac{1}{2}(k^\mu)^2 + \sum_{n=1}^{\infty}\left(\alpha_{-n}^\mu\alpha_n^\mu + \alpha_{-n}^{25}\alpha_n^{25}\right), \quad (14)$$

$$L_m = \frac{1}{2}\sum_{-\infty}^{\infty} \vec{\alpha}_{m-n}\cdot\vec{\alpha}_n\ . \quad (15)$$

Note that in (15), $\alpha_0^{25} \equiv p^{25}$ which also appears in the first term in (14). k is the 25D momentum. θ_i, R are the gauge and space-time moduli, respectively, and l is the winding number in the compact direction. The spectra of type I and type II zero-norm gauge states become [4]

$$M^2 = \left(\frac{2\pi l - \theta_j + \theta_i}{2\pi R}\right)^2 + 2I, \quad (16)$$

and

$$M^2 = \left(\frac{2\pi l - \theta_j + \theta_i}{2\pi R}\right)^2 + 2(I+1) \quad (17)$$

where $I = \sum_{n=1}^{\infty}(\alpha_{-n}^\mu\alpha_n^\mu + \alpha_{-n}^{25}\alpha_n^{25})$.

For the massless case $I = l = 0$, one gets N^2 massless solution from (16)

$$k_\mu\alpha_{-1}^\mu \mid k, l=0, i, j\rangle \quad (18)$$

if all θ_i are equal, or in the T-dual picture when N D-branes are coincident. These N^2 massless gauge states correspond to the charges of the massless $U(N)$ gauge symmetry. There is no type II massless solution in (17).

We are now ready to discuss the interesting massive case. For $M^2 = 2$ and general moduli (R, θ_i),

1. $I = 1, l = 0$, One gets two gauge states solutions from (16):
$$[(\varepsilon \cdot \alpha_{-1})(k \cdot \alpha_{-1}) + \varepsilon \cdot \alpha_{-2}] \mid k, l = 0, i, i\rangle, \quad \varepsilon \cdot k = 0 \tag{19}$$
and
$$\left(k \cdot \alpha_{-1}\alpha_{-1}^{25} + \alpha_{-2}^{25}\right) \mid k, l = 0, i, i\rangle. \tag{20}$$
If all θ_i are equal, the (i, i) is enhanced to (i, j). Equation (20) implies a massive $U(N)$ symmetry with transformation parameter θ^a. Equation (19) implies a massive Einstein–Yang–Mills type symmetry with transformation parameter θ_μ^a.
2. $I = 0, (2\pi l - \theta_j + \theta_i)/(2\pi R) = \pm(2)^{1/2}$, one gets the solution from (16):
$$\left(k \cdot \alpha_{-1} \pm \sqrt{2}\alpha_{-1}^{25}\right) \mid k, l, i, j\rangle. \tag{21}$$
Now since $|\theta_i - \theta_j| < 2\pi$, for any given R, there is at most one solution of $(|l|, |\theta_i - \theta_j|)$. One is tempted to consider the case
$$\left(k \cdot \alpha_{-1} \pm \sqrt{2}\alpha_{-1}^{25}\right) \mid k, l = \pm\sqrt{2}R, i, i\rangle. \tag{22}$$
That means in the moduli $\left(R = (2)^{1/2}n, \theta_i\right)$ with $n \in Z^+$, one has *soliton* gauge states which imply a *massive* $U(1)^N$ symmetry. If all θ_i are equal, the (i, i) is enhanced to (i, j). Equation (22) implies a massive $U(N)$ symmetry at the *discrete* values of the moduli points $R = (2)^{1/2}n$. For example, in the T-dual picture, for $R = (2)^{1/2}$, $l = \pm 2$, and if all D-branes are coincident, we have an enhanced massive $U(N)$ symmetry. This phenomenon is very different from the massless case, where one gets enhanced $U(N)$ symmetry at *any* radius R when N D-branes are coincident.

We would like to point out that a similar Einstein–Yang–Mills type symmetry was discovered before in the closed heterotic string theory. There, however, one could have more than one Yang–Mills indices on the transformation parameters.

For the type II states with $M^2 = 2$ in (17), $I = l = 0$. One gets one more $U(N)$ gauge state
$$\left[\frac{1}{2}\alpha_{-1} \cdot \alpha_{-1} + \frac{1}{2}\alpha_{-1}^{25}\alpha_{-1}^{25} + \frac{5}{2}k \cdot \alpha_{-2} + \frac{3}{2}(k \cdot \alpha_{-1})^2\right] \mid k, l = 0, i, j\rangle \tag{23}$$
if all θ_i are equal.

For the general mass level, choosing $I = 0$ and i, j in (15), we have $l/R = \pm M$. For, let us take, say, $R = (2)^{1/2}$ and $l = \pm(2)^{1/2}M$, which implies
$$M^2 = 2n^2, \quad n = 0, 1, 2, \cdots \tag{24}$$
So we have Chan–Paton soliton gauge states at any higher massive level of the spectrum. A similar result was found in the closed string case.

4 Conclusion

The zero-norm gauge state solution in the old covariant quantization of string theory is closely related to the BRST cohomology of the theory. Physically, this corresponds to the charges of the symmetries [6]. It is believed that all space-time symmetry of string theory, including closed or open and compactified or uncompactified ones, are due to the existence of a (soliton) gauge state in the spectrum. A similar consideration can be applied to the R–R charges and D-branes. Presumably, there are no R–R zero-norm gauge states as charges of R–R gauge fields in the type II string spectrum. How D-branes carry the zero-norm gauge state charges to emit R–R fields is an interesting question to study.

Acknowledgements. This research is supported by National Science Council of Taiwan, R.O.C., under grant number NSC 88-2112-M-009-011.

References

1. J. Polchinski, Phys. Rev. Lett. **75**, 4724 (1995); Phys. Rev. D **50**, 6041 (1994); J. Dai, R.G. Leigh, J. Polchinski, Mod. Phys. Lett. A **4**, 2073 (1989); R.G. Leigh, Mod. Phys. Lett. A **4**, 2767 (1989)
2. P. Goddard, D. Olive, in Advanced Series in Mathematical Physics (World Scientific 1988)
3. J. Paton, Chan Hong-Mo, Nucl. Phys. B **10**, 519 (1969)
4. J.C. Lee, Eur. Phys. J. C (1999), to appear
5. J.C. Lee, Phys. Lett. B **337**, 69 (1994)
6. Tze-Dan Chung, J.C. Lee, Phys. Lett. B **350**, 22 (1995); Z. Phys. C **75**, 555 (1997)