Antijam Capability Analysis of RS-Coded Slow Frequency-Hopped Systems

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***Abstract*—The application of Reed–Solomon codes in slow frequency-hopped systems has been extensively studied. Earlier investigations assumed an infinite interleaving length and considered partial-band noise jammers only. This paper extends previous efforts by analyzing the effect of finite interleaving length and the impact of band multitone jammers. We also explain why two-threshold (2T) erasure-insertion methods (EIM) are needed and examine their performance. Numerical results are presented to compare the effectiveness of EIM's and jammer types and to study the relationships among the hop rate, the interleaver size, and the code rate. The use of 2T EIM's necessitates the estimation of several additional channel and signal parameters. Simple and effective estimation algorithms are provided as well.**

***Index Terms*—Errors-and-erasures decoding, frequency-hop, jamming, RS code.**

I. INTRODUCTION

A FAST frequency-hopped (FFH) system employs both frequency and time diversity and enjoys the advantage of having a "coding gain." On the other hand, to acquire a satisfactory antijam (AJ) capability, a slow frequency-hopped (SFH) system usually has to add an extra mechanism of protection—that is where forward error-control (FEC) coding comes into play. Stark [1] compared the performance among repetition codes, convolutional codes, and Reed–Solomon (RS) codes for SFH systems. M-ary frequency-shift keying (MFSK) or differential phase-shift keying (DPSK) are two practical modulation schemes that most frequently go with RS-coded FH signals. An excellent review of the application of RS codes to SFH/MFSK systems can be found in [10].

Side information, which offers the information about the received symbol's reliability, can help increase the error-correcting capability of a given code. Stark [1] showed that the use of binary side information about the presence of a jammer to determine whether a received symbol should be erased can enhance the error-control capability of RS codes. Hagenauer and Lutz [4] used channel state and erasure information derived from received waveform's amplitude to improve the performance of a mobile satellite system. In [6], test symbols were utilized as a reference of symbol reliability in various concatenated, coded SFH systems. Viterbi introduced the ratio threshold test (RTT) as a symbol reliability measure [8]. RTT was later used as an erasure-insertion method (EIM) for errors-and-erasures (EE) decoding of RS codes in the presence of partial-band jamming [10]. Bayesian methods for erasure-insertion was investigated by Baum and Pursely [9]. Reference [11] examines various design issues pertaining to the use of concatenated coding in SFH/binary frequency-shift keying packet radio networks.

Interleaving is needed to randomize burst errors and to increase the effectiveness of FEC codes. Most investigators assume perfect interleaving in their analysis. We take the effect of finite interleaving size into account and consider both partial-band noise jamming and band multitone jamming. Two EIM's, namely, Viterbi's RTT and Bayesian method are examined in this paper. Besides, we propose two-threshold (2T) EIM's to enhance the EE decoder's performance. As will be shown, the 2T-RTT can achieve performance very close to that of the much more complicated Bayesian method.

The rest of this paper is organized as follows. Section II gives a general description of the RS-coded SFH/MFSK system and the jamming models to be studied. Related system and jammer parameters are also defined. Section III presents our analysis of the codeword-error probability (CEP), taking finite interleaving length effect into account. Then Section IV argues why a 2T EIM is preferred, suggests how the corresponding optimal thresholds can be found when a single-pass EE decoder is used and derives the conditional probabilities needed in computing CEP for different jamming threats. In addition to partial-band noise jammers (PBNJ), which have been considered by earlier investigations on RS-coded SFH systems, we also deal with band multitone jammers (BMTJ). Section V presents a Bayesian EIM against BMTJ and discusses 2T extensions for both RTT and Bayesian methods. Numerical results and related discussion are given in Section VI. Finally, Section VII summarizes our major results, and the appendix presents channel and signal parameter estimation algorithms that are needed for both one-threshold (1T) and 2T EIM's.

II. SYSTEM DESCRIPTION AND DEFINITIONS

Shown in Fig. 1 is a block diagram of an RS-coded SFH/MFSK system. A binary data sequence of rate $R_b = 1/T_b$ bits/s is first converted to an M-ary symbol sequence with a symbol rate $R_s = R_b/\log_2 M = 1/T_s$ information symbols/s and then sent to the (N, k) extended RS encoder whose output rate is R_s/r coded symbols/s, where $r = k/N$ is the code rate.

The output sequence of the encoder is symbol-interleaved. The interleaver can be a block interleaver or a convolutional

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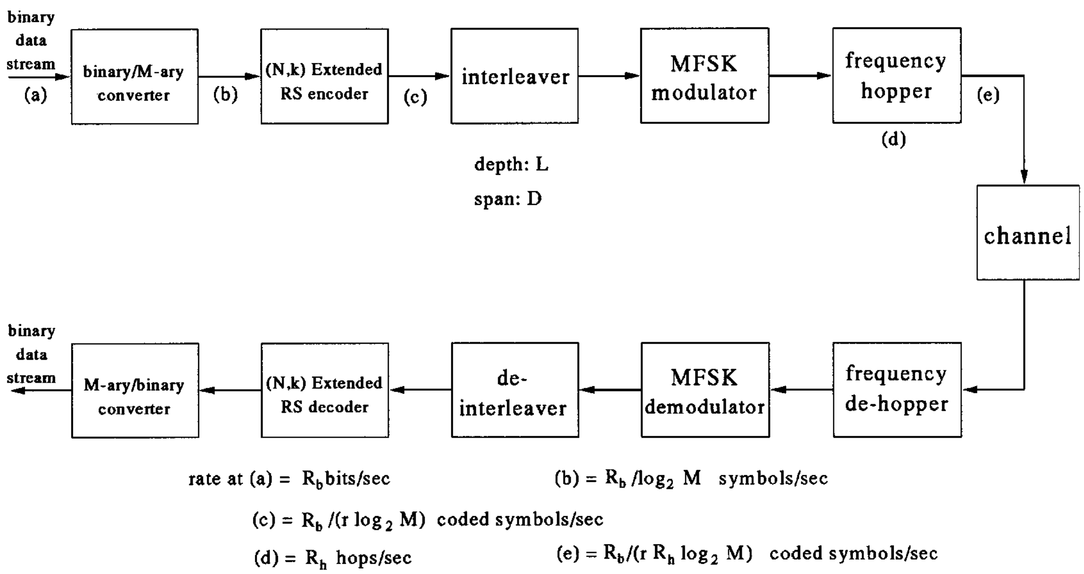

Fig. 1. Block diagram of an RS-coded SFH/MFSK system.

one. To avoid the "mismatch" between the coded symbol size (i.e., codeword length N) and the modulated signal dimension (M), we shall assume $M = N$ throughout our discussion. Hence, when MFSK modulation is used, each element in the coded symbol field is represented by a different MFSK tone. The associated MFSK signal hops at R_h hops/s according to a preassigned pattern. The received waveform is dehopped, despreaded, and noncoherently detected before being deinterleaved and decoded. Such an FH system assumes that the hopper can only hop into uniformly and nonoverlapped spaced bands. There are certainly other ways to arrange the candidate signal bands.

Consider a block symbol interleaver with depth L (number of columns) and span D (number of rows), where D is equal to an integer multiple of the codeword length N, i.e., $D = lN$ for some l. Only $l = 1$ is considered in this paper. Generalization to an arbitrary l is straightforward. Code symbols are written into the interleaver by columns and read out by rows. Consequently, two consecutive input code symbols are separated by $L - 1$ symbols at the output. At the receiving end, the deinterleaver simply performs the inverse operation where demodulated symbols are written into the deinterleaver as rows and read out as columns. Convolutional interleavers can achieve the same effect with only half of the storage requirement. Since both interleavers can accomplish the same performance, we will limit our discussion to block interleavers only. A bank of M energy detectors is used by the receiver to noncoherently detect the dehopped MFSK signal. It is assumed that the desired dehopped M-ary band for the candidate MFSK tones occupies M continuous channels. This M-ary band is called the signal baseband. The channel used by the transmitted signal is called the signal or the message channel and the other $M-1$ channels are referred to as noise channels.

The first class of jammers considered is the PBNJ who distributes its total power P_J evenly over a continuous spectrum of bandwidth W_J. Let W_{ss} be the total hopping bandwidth, then $\rho = W_J/W_{ss} \le 1$ is the fraction of band jammed. Within the jammed band, the transmitted signal is corrupted by an equivalent additive white Gaussian noise whose power spectral density (PSD) level N_T is equal to $N_J/\rho + N_0$, where $N_J = P_J/W_{ss}$; otherwise, the PSD level is $N_T = N_0$. If channel 1 is the message channel, then the corresponding energy detector output R_1 is a noncentral chi-square random variable whose probability density function (pdf) is given by

$$f_{R_1}(x) = \frac{1}{2\sigma^2}\, e^{-(s^2+x)/2\sigma^2} I_0\left(\frac{\sqrt{x}s}{\sigma^2}\right), \qquad x \ge 0 \tag{1}$$

where s is the signal amplitude, σ^2 is the noise variance and, $I_0(\cdot)$ is the modified Bessel function of the first kind of order zero. The energy detector output for a noise channel, on the other hand, is central chi-square distributed, i.e.,

$$f_{R_i}(x) = \frac{1}{2\sigma^2}\, e^{-(x/2\sigma^2)}, \qquad i = 2, \cdots, M. \tag{2}$$

This is a result of our assumption that if one channel is jammed by a PBNJ, the entire hopped M-ary signal band is jammed as well. Another class of jammers to be considered is the BMTJ [5], which produces $\tilde{Q}$ equal power continuous-wave tones and places $\tilde{m}$ jamming tones in some randomly selected M-ary signal bands. The reason why a BMTJ can do this is the assumption that it knows how the communicators partition the total hopping band into disjoint M-ary subbands. If the number of orthogonal tones within the hopping band is $N_t = W_{ss}/R_s$, then the fraction of tones jammed is $\rho = \tilde{Q}/N_t$. Define $\mu = \Pr\{\text{the dehopped signal band is jammed}\}$, and we have

$$\mu = \tilde{Q}M/(\tilde{m}N_t) = \rho M/\tilde{m}. \tag{3}$$

Only the worst-case $\tilde{m} = 1$ [5] is considered in this paper. Note that the above equation uses an assumption mentioned before: the candidate message bands are uniformly and nonoverlapped spaced. If we allow the hopper to have a frequency step smaller than a signal band or even smaller than a channel such that the candidate bands are overlapped, then the worst-case ($\tilde{m} = 1$) BMTJ will become less effective.

If the M-ary band containing the signal is not jammed, then R_1 is noncentral chi-square distributed and its pdf is the same as (1) with $\sigma^2 = N_0/2$. The other outputs are independent with a common pdf given by (2) where $\sigma^2 = N_0/2$. When an M-ary band suffers from the worst-case BMTJ, the jamming tone is either in the message channel or in one of the noise channels. If the message channel is jammed, then the pdf of R_1 can be expressed as [7]

$$f_{R_1}(x) = \frac{1}{2}\int_0^\infty \rho J_0(s\rho)J_0(I\rho)\, e^{-(\sigma^2\rho^2/2)} J_0(\sqrt{x}\rho)\, d\rho, \quad x \geq 0 \tag{4}$$

where I is the jammer amplitude. In case the jamming tone is in, say the second (noise) channel, then the outputs of the signal channel and the jammed noise channel are noncentral chi-square random variables, while those of the other noise channels are independently, identically distributed central chi-square random variables.

III. Decoder Performance Analysis

Recall that an RS code can correct any combination of t erroneous symbols and e erased symbols as long as $e + 2t < d_{\min}$, where $d_{\min}$ is the minimum distance of the code used. Let us assume that among t incorrectly detected symbols, t_1 of them come from jammed symbols, and the remaining $t_0 = t - t_1$ symbol errors are caused by thermal noise alone. Similarly, we divide e erasures into those caused by jamming (e_1) and those resulted from thermal noise only ($e_0 = e - e_1$). In investigating the effect of a block interleaver, we assume that the hop duration is equal to that of multiple rows of the interleaver. Hence, there will be several hops in one interleaving block of depth L and span D, and at the interleaver output, symbols of several adjacent rows (or columns) will be in the same hop. Again, we want to emphasize that in general, D can be chosen to be equal to an integer multiple of M, but only the case $D = M$ is considered in this paper. Extension to the more general case is straightforward. Let r be the code rate, $K_I = DL$ be the interleaving size, H be the number of hops per $K_I T_s$ seconds, and assume J of H hops are jammed. If one hop consists of M symbols from S rows, the number of jammed symbols in one codeword is JS and the remaining $M - JS$ symbols are free of jamming. The numbers of jammed and unjammed erasures and errors e_1, e_0, t_1, and t_0 must satisfy the inequalities $0 \leq e_1 + t_1 \leq JS$ and $0 \leq e_0 + t_0 \leq M - JS$. Furthermore, the hop rate R_h is related to R_b, L, and S by $R_b/R_h = r(LS)\log_2 M$ bits/hop. The corresponding relation between the bit and hop signal energy $E_b = s/R_b$, $E_h = s/R_h$ can easily be derived accordingly.

With the above definitions and assumptions, we can write the average CEP P_w as

$$P_w = \sum_{J=0}^{H} P_{w|J} P(J) \tag{5}$$

where $P_{w|J}$ is the CEP given that J out of H hops per interleaving block is jammed. Assuming a random hopping pattern, we can express the probability of the latter event $P(J)$ as

$$P(J) = \binom{H}{J} \rho^J (1-\rho)^{H-J}. \tag{6}$$

Substituting (6) into (5), we obtain

$$P_w = \sum_{J=0}^{H} \binom{H}{J} \rho^J (1-\rho)^{H-J} P_{w|J}. \tag{7}$$

In case BMTJ ($\tilde{m} = 1$) instead of PBNJ is present, the associated P_w has the same expression with ρ replaced by μ defined in (3). For EE decoding, $P_{w|J} = \Pr\{2t + e \geq d_{\min}\}$, and for errors-only (EO) decoding, $P_{w|J} = \Pr\{2t \geq d_{\min}\}$.

The following definitions are needed.

- $P_T(t)$ $\Pr\{t$ symbols of a codeword are incorrectly detected but not erased$\}$,
- $P_E(e)$ $\Pr\{e$ erasures in one codeword $\}$,
- $P_{e|j}$ $\Pr\{$ symbol erased | symbol jammed $\}$,
- $P_{e|u}$ $\Pr\{$ symbol erased | symbol unjammed $\}$,
- $P_{s|j}$ $\Pr\{$ symbol incorrectly detected | symbol jammed but not erased $\}$,
- $P_{s|u}$ $\Pr\{$ symbol incorrectly detected | symbol unjammed and not erased $\}$.

It is worth noting that the condition—symbol erased or not erased—becomes irrelevant if we set the erasure-insertion threshold τ or θ (see the next two paragraphs for details) to one, corresponding to the case when the EO decoder is used. For this case, we have, with $\tau = 1$ or $\theta = 1$

$$P_{w|J} = \sum_{t=\lceil d_{\min}/2 \rceil}^{M} \sum_{t_1=0}^{\min\{JS,\, t\}} \binom{JS}{t_1} P_{s|j}^{t_1} (1 - P_{s|j})^{JS-t_1} \cdot \binom{M-JS}{t_0} P_{s|u}^{t_0} (1 - P_{s|u})^{M-JS-t_0} \tag{8}$$

while for EE decoding

$$P_{w|J} = \sum_{e=0}^{d_{\min}-1} P_E(e) \sum_{t=\lceil d_{\min}-e/2 \rceil}^{M-e} P_T(t) + \sum_{e=d_{\min}}^{M} P_E(e) \tag{9}$$

where

$$P_T(t) = \sum_{t_1=0}^{t_U} \binom{JS - e_1}{t_1} P_{s|j}^{t_1} (1 - P_{s|j})^{JS-e_1-t_1} \cdot \binom{M-JS-e_0}{t_0} P_{s|u}^{t_0} (1 - P_{s|u})^{M-JS-e_0-t_0} \tag{10}$$

$e_0 = e - e_1$, $t_0 = t - t_1$, and $t_U = \min(JS - e_1,\, t)$. When $H = M$, $S = 1$, each symbol of a codeword belongs to a different hop, and the performance of the decoder is equivalent to that of the ideal (perfect) interleaving case ($H = \infty$). This can also be shown by substituting the above condition into (6) and (7) [or (8)] and comparing the resulting expression with that of the ideal interleaving case.

On the other hand, the probability $P_E(e)$ depends on the EIM used. We shall consider two such schemes. The first scheme, borrowed from Viterbi's RTT [8] and was used in [9], computes the ratio between the largest and the second largest outputs of the energy detector bank and compares it with a threshold τ. An erasure is inserted when this ratio is smaller than τ. Let $R_1, R_2, \cdots, R_M$ be the outputs of the energy detector bank. Define $Z_1 = \min_i\{R_i\}$, $Z_2 = \min_i\{\{R_i\}\backslash Z_1\} \cdots$, and $Z_k =$ the kth smallest among R_i's. That is, $\{Z_1, Z_2, \cdots, Z_M\}$ is a permutation of $\{R_1, R_2, \cdots, R_M\}$, which resulted from arranging the latter in ascending order of magnitude. Then a symbol is erased if $Z_{M-1}/Z_M > \tau$.

The second scheme is derived from Bayesian decision theory [9] and can be described as follows. Let s_i be the ith code symbol alphabet, $\boldsymbol{Y} = (Y_1, Y_2, \cdots, Y_M)$ be the M-ary envelope detector outputs, $f(\boldsymbol{y}|s_i)$, the conditional pdf of $\boldsymbol{Y}$ given that s_i was sent, and π_i, the *a priori* probability of sending s_i. The Bayesian method decides that a received symbol should be erased if

$$\frac{\max_j \pi_j f(\boldsymbol{y}|s_j)}{\sum_{i=0}^{n-1} \pi_i f(\boldsymbol{y}|s_i)} \leq (1-\theta) \tag{11}$$

where $0 < \theta < 1$. Reference [9] considered PBNJ only; the extension to the BMTJ case is presented in Section IV.

According to the above description, no matter whether the RTT or Bayesian method is used, we have

$$P_E(e) = \sum_{e_1=0}^{\min\{JS,\, e\}} \binom{JS}{e_1} P_{e|j}^{e_1}(1-P_{e|j})^{JS-e_1} \cdot \binom{M-JS}{e_0} P_{e|u}^{e_0}(1-P_{e|u})^{M-JS-e_0}. \tag{12}$$

IV. Erasure Insertion Analysis

The above analysis indicates that to calculate the CEP, one has to evaluate the conditional probabilities $P_{e|u}, P_{e|j}, P_{s|u}$, and $P_{s|j}$. They are functions of the channel condition (jammer type) and the EIM.

A. Why Two Thresholds?

Both RTT and Bayesian methods involve a threshold comparison operation. Previous investigations did not consider channel state information (jammed or unjammed) and use only one threshold no matter whether the received symbol is jammed or not. Since the decoder's performance is a function of the channel state, if we use different thresholds for different channel states, the resulting performance should be improved.

Although the CEP is a very complicated function of the EIM performance, as presented in the last section, an ideal single-pass EIM should erase those symbols that have been incorrectly demodulated and leave the other symbols intact. In practice, we want to maximize the following two probabilities:

$$\begin{aligned} P_r[\text{symbol not erased}|\text{symbol correctly detected}] &\triangleq P_r[d_1|H_1] \\ P_r[\text{symbol erased}|\text{symbol incorrectly detected}] &\triangleq P_r[d_0|H_0]. \end{aligned} \tag{13}$$

Let Z be the erasure-decision variable, and then such a design goal can be achieved by the likelihood ratio test (LRT)

$$\frac{p_Z(z|H_1)}{p_Z(z|H_0)} \underset{d_0}{\overset{d_1}{\gtrless}} \frac{p(H_0)}{p(H_1)} \tag{14}$$

or equivalently

$$\frac{p(z,\, H_1)}{p(z,\, H_0)} \underset{d_0}{\overset{d_1}{\gtrless}} 1 \tag{15}$$

where $p_Z(z|H_i)$ is the conditional pdf of Z, given $H_i, p(z,\, H_i)$ is the corresponding joint pdf, while $p(H_i)$ is the *a priori* probability of H_i. It can be shown that for RTT in PBNJ

$$p(z,\, H_1) = \sum_{n=0}^{M-1} n \binom{M-1}{n} (-1)^{n+1} \cdot \left[\frac{1}{(zn+1)^2} + \frac{\beta}{(zn+1)^3}\right] \exp\left(-\frac{zn}{zn+1}\beta\right) \tag{16}$$

and

$$\begin{aligned} p(z, H_0) = \sum_{n=0}^{M-2} \binom{M-1}{n+1} (-1)^n & \cdot \left\{\frac{-n}{(zn+1)^2} \frac{z(n+1)}{z(n+1)+1} + \frac{1}{zn+1} \frac{n+1}{[z(n+1)+1]^2} \right. \\ & \left. + \frac{1}{zn+1} \frac{z(n+1)}{z(n+1)+1} \frac{\beta}{[z(n+1)+1]^2}\right\} \\ & \cdot \exp\left(-\frac{zn+1}{z(n+1)+1}\beta\right) \end{aligned} \tag{17}$$

where $\beta \stackrel{\text{def}}{=} s^2/(2\sigma^2) = (E_s/N_T)$, E_s being the signal energy per coded symbol.

Fig. 2 depicts the joint pdf's for the above test when a PBNJ is present. Obviously, the optimal thresholds derived from the LRT for the jammed and unjammed channel states are far from each other. Hence, a single threshold EIM is not appropriate for this case. We can use a similar likelihood ratio to determine the optimal thresholds for Bayesian erasure method, but closed-form expressions for the corresponding joint (or conditional) pdf's cannot be found. It is worth pointing out that, as examples in Section VI will show, there are cases when the two optimal thresholds derived from the LRT are so close that a single threshold for both channel states is good enough.

B. Noise Jamming

Subsequent analysis assumes that channel 1 is the message channel. We shall consider RTT only; for the Bayesian method, we have been unable to find closed-form expressions for the conditional probabilities that are needed to evaluate CEP. For PBNJ, the following four equations are known [9, Appendix]:

$$\begin{aligned} P_{s|u} &= \frac{P_{s,\,\overline{e}|u}}{1-P_{e|u}} \\ &= \frac{1}{1-P_{e|u}} \sum_{n=0}^{M-2} \binom{M-1}{n+1} \frac{(-1)^n \tau(n+1)}{\tau(n+1)+1} \cdot \frac{e^{-((\tau n+1)\beta/\tau(n+1)+1)}}{\tau n+1} \end{aligned} \tag{18}$$

$$\begin{aligned} P_{e|u} &= 1 - P_{s,\,\overline{e}|u} - P_{c,\,\overline{e}|u} \\ &= \sum_{n=0}^{M-2} \binom{M-1}{n+1} (-1)^n \frac{e^{-(\tau(n+1)/\tau(n+1)+1)\beta}}{\tau(n+1)+1} \cdot \left[1 - \frac{\tau(n+1)}{\tau n+1} e^{(-(1-\tau)\beta/\tau(n+1)+1)}\right] \end{aligned} \tag{19}$$

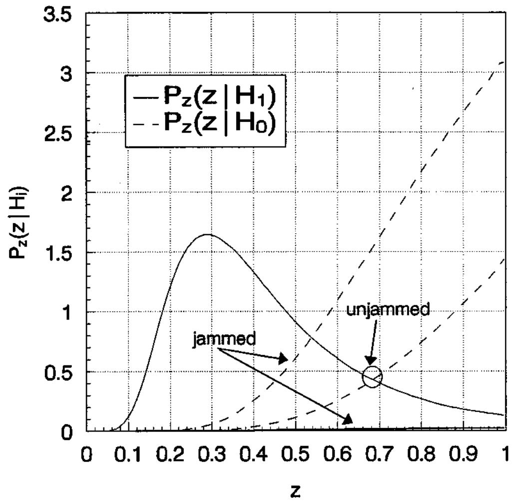

Fig. 2. Conditional pdf functions for the erasure-decision variable of RTT.

$$\begin{aligned}P_{c,\bar{e}|u} &\overset{\text{def}}{=} \Pr\left\{R_1 = Z_M,\, \frac{Z_{M-1}}{Z_M} < \tau | N_T = N_0\right\}\\ &= \sum_{n=0}^{M-1} \binom{M-1}{n} (-1)^n \frac{e^{-(\tau n/\tau n+1)\beta}}{\tau n+1} \qquad (20)\end{aligned}$$

$$\begin{aligned}P_{s,\bar{e}|u} &\overset{\text{def}}{=} \Pr\left\{R_1 \neq Z_M,\, \frac{Z_{M-1}}{Z_M} < \tau | N_T = N_0\right\}\\ &= \sum_{n=0}^{M-2} \binom{M-1}{n+1} (-1)^n \frac{\tau(n+1)}{\tau(n+1)+1}\\ &\quad \cdot \frac{e^{-(\tau n+1/\tau(n+1)+1)\beta}}{\tau n+1}. \qquad (21)\end{aligned}$$

The above conditional probabilities are derived under the assumption that the symbol of concern is not jammed. The corresponding jammed conditional probabilities can be obtained by the substitution $N_T = N_0 + \rho^{-1} N_J$.

C. Tone Jamming

For this case, $P_{e|u}$ and $P_{s|u}$ are the same as the corresponding conditional probabilities—(19) and (18)—in PBNJ. Similarly, $P_{e|j}$ and $P_{s|j}$ can be calculated from

$$P_{e|j} = 1 - P_{c,\bar{e}|j} - P_{s,\bar{e}|j} \qquad (22)$$

and

$$P_{s|j} = \frac{P_{s,\bar{e}|j}}{1 - P_{e|j}} \qquad (23)$$

where $P_{c,\bar{e}|j}$ and $P_{s,\bar{e}|j}$ are the same conditional probabilities as $P_{c,\bar{e}|u}$ and $P_{s,\bar{e}|u}$, except that the unjammed condition is replaced by the BMTJ-jammed condition. It is straightforward to see

$$\begin{aligned}P_{s,\bar{e}|j} &= (M-1)\Pr\{R_i < \tau R_2 \quad \forall i \neq 2 |\ \text{signal band is jammed}\}\\ &= (M-1)\,\{\Pr[\text{ the first channel is jammed}|\ \text{signal band is jammed}]\\ &\quad \cdot \Pr[R_i < \tau R_2 \quad \forall i \neq 2 |\ \text{the first channel is jammed}]\\ &\quad + \Pr[\text{ the second channel is jammed}|\ \text{signal band is jammed}]\\ &\quad \cdot \Pr[R_i < \tau R_2 \quad \forall i \neq 2 |\ \text{the second channel is jammed}]\\ &\quad + \Pr[\text{the } i\text{th channel is jammed}, \quad i \neq 1, 2 |\ \text{signal band is jammed}]\\ &\quad \cdot \Pr[R_i < \tau R_2 \quad \forall i \neq 2 |\ \text{the } i\text{th channel is jammed}, \quad i \neq 1, 2]\}\\ &\overset{\text{def}}{=} (M-1)\left[\frac{1}{M} P_{s_1 j} + \frac{1}{M} P_{s_2 j} + \frac{M-2}{M} P_{s_3 j}\right]. \qquad (24)\end{aligned}$$

Using (4) and applying [13, eqs. (6.631.4) and (6.633.2)], we obtain after some manipulation

$$\begin{aligned}P_{s_1 j} = \sum_{n=0}^{M-2} &\binom{M-2}{n} \frac{(-1)^n \tau}{(\tau n+1)(\tau n+\tau+1)}\\ &\cdot \exp\left(-\frac{\tau n+1}{\tau n+\tau+1} \frac{s^2+I^2}{2\sigma^2}\right)\\ &\cdot I_0\left(\frac{\tau n+1}{\tau n+\tau+1} \frac{sI}{\sigma^2}\right) \qquad (25)\end{aligned}$$

$$\begin{aligned}P_{s_2 j} = \sum_{n=0}^{M-2} &\binom{M-2}{n} \frac{2(-1)^n e^{-(I^2/2\sigma^2)}}{\tau n+1}\\ &\cdot \int_0^\infty y \left[1 - Q\left(\frac{s}{\sigma}, \sqrt{\frac{2\tau}{\tau n+1}}\, y\right)\right]\\ &\cdot I_0\left(\sqrt{\frac{2}{\tau n+1}}\, \frac{I}{\sigma}\, y\right) e^{-y^2}\, dy \qquad (26)\end{aligned}$$

and

$$\begin{aligned}P_{s_3 j} = \sum_{n=0}^{M-3} &\binom{M-3}{n} \frac{2(-1)^n}{\tau n+1} \int_0^\infty y\, e^{-y^2}\\ &\cdot \left[1 - Q\left(\frac{s}{\sigma}, \sqrt{\frac{2\tau}{\tau n+1}}\, y\right)\right]\\ &\cdot \left[1 - Q\left(\frac{I}{\sigma}, \sqrt{\frac{2\tau}{\tau n+1}}\, y\right)\right] dy \qquad (27)\end{aligned}$$

where

$$Q(a, b) \triangleq \int_b^\infty \exp\left(-\frac{a^2+x^2}{2}\right) I_0(ax) x\, dx. \qquad (28)$$

The conditional probability

$$\begin{aligned}P_{c,\bar{e}|j} &= \Pr\{R_1 = Z_M,\, Z_{M-1}/Z_M < \tau |\ \text{signal band is jammed}\}\\ &= \Pr\{R_i < \tau R_1 \quad \forall i \neq 1 |\ \text{signal band is jammed}\}\end{aligned}$$

can be computed from

$$
\begin{aligned}
P_{c,\bar{e}|j} =& \Pr\{\text{the first channel is jammed}|\text{signal band is jammed}\} \\
&\cdot \Pr\{R_i < \tau R_1 \quad \forall\, i \neq 1|\text{the first channel is jammed}\} \\
&+ \Pr\{\text{the } i\text{th channel is jammed}|\text{signal band is jammed}\} \\
&\cdot \Pr\{R_i < \tau R_1 \quad \forall\, i \neq 1|\text{the } i\text{th channel is jammed}\} \\
\stackrel{\text{def}}{=}& \frac{1}{M} P_{c_1 j} + \frac{M-1}{M} P_{c_2 j}
\end{aligned} \tag{29}
$$

where we can show

$$
P_{c_1 j} = \sum_{n=0}^{M-1} \binom{M-1}{n} \frac{(-1)^n}{\tau n + 1} \cdot \exp\left(-\frac{\tau n}{\tau n + 1}\, \frac{s^2 + I^2}{2\sigma^2}\right) I_0\left(\frac{\tau n}{\tau n + 1}\, \frac{sI}{\sigma^2}\right) \tag{30}
$$

and

$$
\begin{aligned}
P_{c_2 j} = \sum_{n=0}^{M-2} \binom{M-2}{n} \frac{2(-1)^n\, e^{-(s^2/2\sigma^2)}}{\tau n + 1} &\cdot \int_0^\infty y \left[1 - Q\left(\frac{I}{\sigma}, \sqrt{\frac{2\tau}{\tau n + 1}}\, y\right)\right] \\
&\cdot I_0\left(\sqrt{\frac{2}{\tau n + 1}}\, \frac{s}{\sigma}\, y\right) e^{-y^2}\, dy.
\end{aligned} \tag{31}
$$

V. Related Design Issues

A. Bayesian Erasure-Insertion in BMTJ

Equation (11) indicates that in order to derive the Bayesian erasure-insertion rule in BMTJ, we need to obtain the corresponding conditional pdf's. It can easily be seen that in the absence of jamming, the conditional pdf becomes

$$
f_N(\boldsymbol{y}|s_j) = \frac{\prod_{i=1}^{M} y_i}{(\sigma^2)^M} \exp\left(-\frac{s^2 + \sum_{i=1}^{M} y_i^2}{2\sigma^2}\right) I_0\left(\frac{y_j s}{\sigma^2}\right). \tag{32}
$$

When a BMTJ is present, the conditional pdf becomes $f_J(\boldsymbol{y}|s_j) = \sum_{k=1}^{M} (1/M) f_J(\boldsymbol{y}|s_j, k)$, where $f_J(\boldsymbol{y}|s_j, k)$ is the conditional pdf of $\boldsymbol{Y} = \boldsymbol{y}$ given that s_i was sent and the kth channel is jammed. It is straightforward to show that if $j \neq k$

$$
f_J(\boldsymbol{y}|s_j, k) = \frac{\prod_{i=1}^{M} y_i}{(\sigma^2)^M} \exp\left(-\frac{s^2 + I^2 + \sum_{i=1}^{M} y_i^2}{2\sigma^2}\right) \cdot I_0\left(\frac{y_k I}{\sigma^2}\right) I_0\left(\frac{y_j s}{\sigma^2}\right) \tag{33}
$$

and if $j = k$

$$
f_J(\boldsymbol{y}|s_j, k) = \frac{\prod_{i=1}^{M} y_i}{(\sigma^2)^M} \exp\left(-\frac{\sum_{\substack{i=1 \\ i\neq j}}^{M} y_i^2}{2\sigma^2}\right) \cdot \int_0^\infty \sigma^2 \rho J_0(s\rho) J_0(I\rho) J_0(y_j\rho) e^{-(\sigma^2\rho^2/2)}\, d\rho. \tag{34}
$$

Given the above equations, the Bayesian erasure-insertion test (11) becomes

$$
\frac{(1-\mu) \max_j f_N(\boldsymbol{y}|s_j) + \mu \cdot \max_j \sum_{k=1}^{M} \frac{1}{M} f_J(\boldsymbol{y}|s_j, I_k)}{(1-\mu) \sum_{i=1}^{M} f_N(\boldsymbol{y}|s_i) + \mu \cdot \sum_{i=1}^{M} \sum_{k=1}^{M} \frac{1}{M} f_J(\boldsymbol{y}|s_i, I_k)} \leq (1-\theta). \tag{35}
$$

B. 2T Methods

As discussed before, a 2T system is often needed to obtain optimal decoder performance. However, 2T systems require more than the knowledge of the channel state. More specifically, besides E_b/N_0, we need to have an estimate of E_b/N_T when a PBNJ is present, and if the receiver is jammed by a BMTJ, the signal and the interferer strength (s, I) and the noise power σ^2 are needed. Methods for generating these estimates are given in the appendix. Given these estimates, the 2T-RTT will erase a received symbol if $Z_{M-1}/Z_M > \tau$ where τ is a function of E_b/N_T or (s, I, σ^2). The analysis presented in the last section can be used to evaluate the performance of both 1 and 2T-RTT systems. When a 2T-Bayesian method is used to combat PBNJ, a received symbol is erased if

$$
\frac{\max_j I_0(y_j s/\sigma_J^2)}{\sum_{i=1}^{M} I_0(y_i s/\sigma_J^2)} \leq (1-\theta_j) \text{ and a jammer is present} \tag{36}
$$

or if

$$
\frac{\max_j I_0(y_j s/\sigma_N^2)}{\sum_{i=1}^{M} I_0(y_i s/\sigma_N^2)} \leq (1-\theta_n) \text{ and no jammer is present} \tag{37}
$$

where $\theta_n \neq \theta_j$. When BMTJ is present, (36) should be replaced by

$$
\frac{\max_j \sum_{k=1}^{M} f_J(\boldsymbol{y}|s_j, I_k)}{\sum_{i=1}^{M} \sum_{k=1}^{M} f_J(\boldsymbol{y}|s_i, I_k)} \leq (1-\theta_j). \tag{38}
$$

VI. Numerical Results and Discussions

The numerical results presented in this section are based on the following system parameter values: $D = M = 32$ and a hopping rate of 80 coded symbols/hop. Given a total jamming power P_J, a jammer can choose a suitable value for ρ or μ such that the resulting CEP is maximized. On the other hand, either RTT or Bayesian method needs threshold values which can be optimized to yield a minimum worst-case CEP. Since the decoder performance depends on E_b/N_T, E_b/N_J, or s/I, we need information about these parameters to find an optimal threshold. Algorithms for estimating these channel and signal parameters are given in the appendix. Numerical examples presented below assume that all estimations are perfect. To compute CEP in PBNJ, we have invoked (7), (9), (10), (12), (18), and (19). In addition to these equations, (22)–(27) and (29)–(31) are also used in computing CEP in BMTJ.

Fig. 3 shows the worst-case CEP performance for the (32, 18) extended RS code when the 1T-RTT EE decoder is used. We have shown performance with fixed threshold values 0.65, 0.71, and the optimal threshold, which depending on E_b/N_J, is within the interval (0.59, 0.81). Evidently, 1T-RTT is not very sensitive to the threshold value as long as the latter is within a certain range of the optimal value. Those curves labeled with "suboptimal threshold" are obtained by using the optimal threshold associated with a known E_b/N_0, assuming no jamming (i.e., $E_b/N_J = \infty$). Their performance is very close to that when the true optimal threshold is used. The advantage of using an EE decoder can be seen from both Figs. 3 and 4 where the worst-case CEP performance of both the EO decoder and the EE decoder with RTT are depicted. Each PBNJ curve shown in Fig. 4 seems to converge to a plateau which is due to a finite E_b/N_0 value (6 dB in this case). The effect of finite-length interleaving, or equivalently, the hopping rate, is examined in Figs 3–5. For a fixed interleaving length, the increase of H leads to a smaller S, while increasing the hopping rate (smaller symbols/hop) with a fixed interleaving length yields a larger H. It is worth mentioning that in a real system, the hopping rate is usually kept constant. If jamming is severe, the data rate is reduced but the hopping rate is unchanged. With the same amount of interleaver, the interleaving depth can be increased as the data rate is decreased. These figures indicate the following. 1) EE decoding is preferred only if the interleaving size is large enough; 2) BMTJ is a more effective jammer against MFSK signals. We also notice that the degradation due to finite interleaving length becomes more significant as E_b/N_J increases. Furthermore, the EE decoding gain is a decreasing function of CEP and is larger when a PBNJ is present. The performance improvement obtained by using the Bayesian method can be found in Fig. 5. The improvement is more impressive when used against BMTJ; it is also an increasing function of H.

The effect of code rate for the EE-RTT decoder is shown in Fig. 6. A lower rate code has a better error-correcting capability but yields a smaller symbol energy E_s if E_b/N_0 is fixed. This is similar to noncoherent FFH systems for which there exists an optimal diversity order that achieves the best balance between the diversity gain and the noncoherent combining loss.

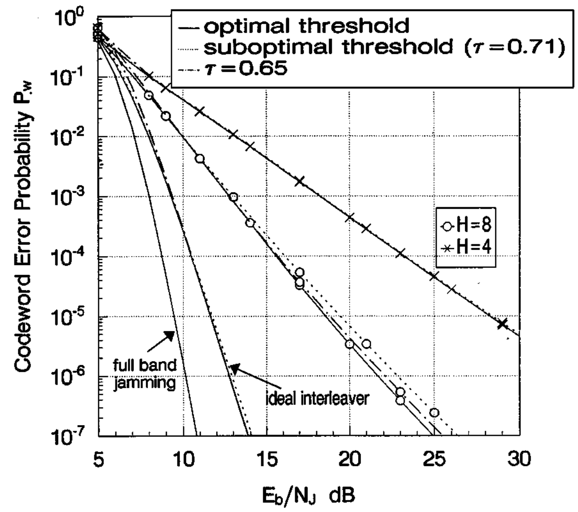

Fig. 3. The influence of the threshold value on the CEP performance of 1T-RTT against PBNJ. Performance under full-band jamming is also shown to demonstrate the effectiveness of the PBNJ.

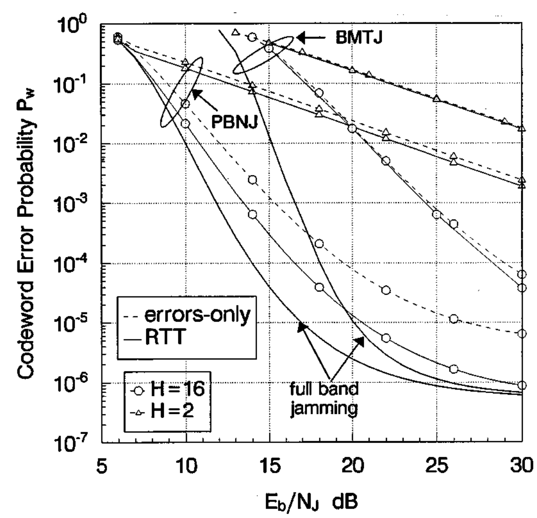

Fig. 4. CEP performance of the EO and the 1T EE-RTT decoders against PBNJ and BMTJ. Performance under full-band jamming is also given.

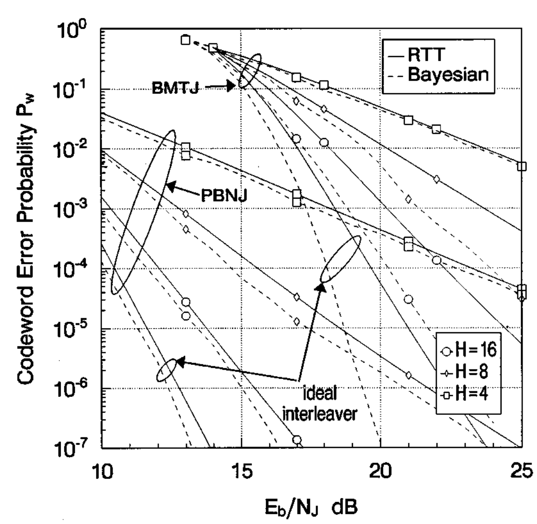

Fig. 5. Worst-case performance comparison between two EIM's against PBNJ and BMTJ. The effect of finite interleaver can be seen as well.

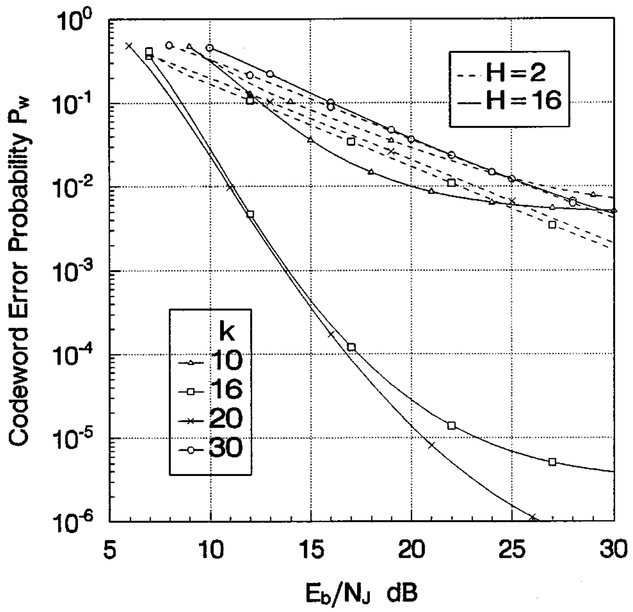

Fig. 6. The effect of the code rate on the worst-case CEP performance.

Fig. 6 and other numerical investigation [12] indicate that when the given E_b/N_0 is larger than 4 dB but less than 10 dB, the corresponding optimal code rate lies somewhere between 0.5 and 0.625. Figs. 7 and 8 compare the CEP and bit-error probability (BEP) performance of four EIM's, 1T-RTT, 2T-RTT, 1T-Bayesian, and 2T-Bayesian, in PBNJ when $E_b/N_0 = 7$ dB. A closed-form formula relating CEP and BEP cannot be found. If we assume that all codewords are equiprobable and all decoding errors are equally likely, then the identity [14, p. 262]

$$\text{BEP} = \frac{M}{2M-1}\,\text{CEP} \tag{39}$$

according to our simulation results [12], is a quite accurate approximation no matter which EIM is used. Due to space limitation, only one set of decoded BEP curves is shown in Fig. 8.

At lower E_b/N_J's, 1T-Bayesian and 2T-Bayesian give almost the same performance and outperform the other two schemes. The performance of 2T-RTT is very close to that of Bayesian methods and is superior to that of 1T-RTT. For high E_b/N_J values, however, all erasure schemes yield similar performance, as erasure scarcely exists. The reason for the performance difference between 2T-RTT and 1T-RTT can easily be found from Fig. 2 and related discussion in Section IV: at low E_b/N_J's the optimal thresholds for the two states are far apart, while at high E_b/N_J's they are much closer. For Bayesian methods, we find that the corresponding Bayesian ratio values computed from (11) and (36) or (35) and (38) are often very close and are dominated by the values contributed by the correct conditional pdf's. As our simulation assumes perfect channel state detection and parameter estimations, there is almost no performance difference between 1T- or 2T-Bayesian methods.

Fig. 9 shows the CEP performance of four EIM's in BMTJ for $\mu = 0.2$ and 0.5. Similar to the PBNJ case, two Bayesian methods give the best and almost identical performance, while 2T-RTT is far superior to 1T-RTT at low and medium E_b/N_J's.

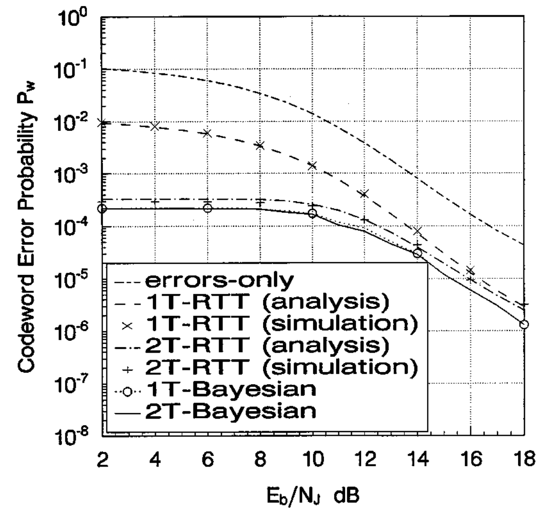

Fig. 7. CEP performance comparison of four EIM's against PBNJ.

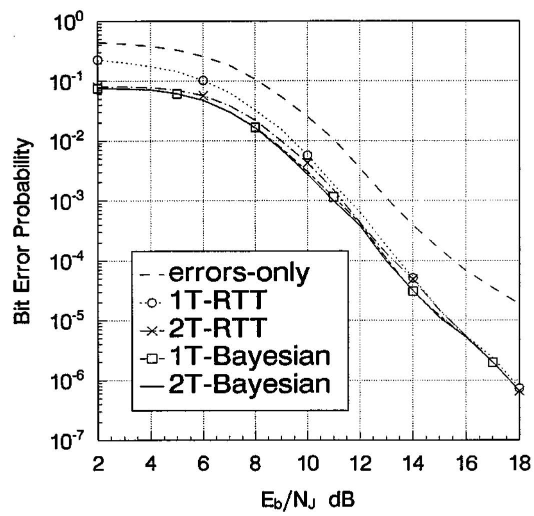

Fig. 8. BEP performance of four EIM's against PBNJ.

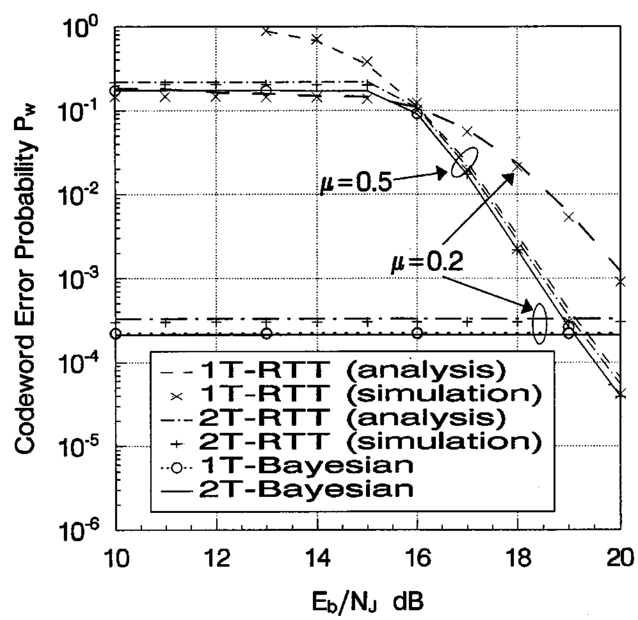

Fig. 9. CEP performance comparison of four EIM's against BMTJ.

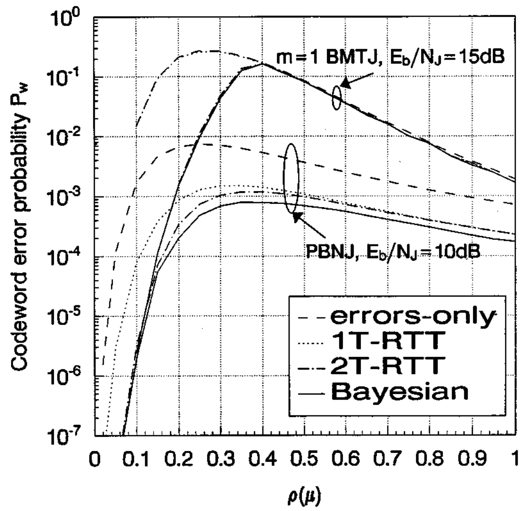

Fig. 10. CEP performance of various EIM's as a function of ρ.

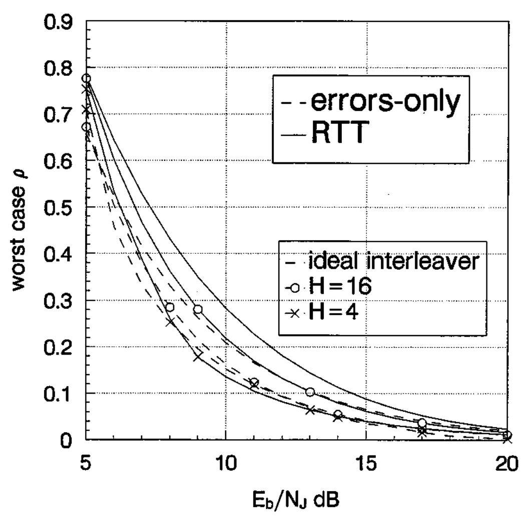

Fig. 11. Worst-case ρ for EO and 1T-RTT decoders against PBNJ.

At high E_b/N_J's, all erasure schemes fail to achieve significant EE decoding gain. As we have learned from Fig. 4, EE decoding using 1T-RTT gives only negligible performance gain in BMTJ. Using two thresholds does make RTT more useful in this case. To validate our analysis, simulated performance of 1T-RTT and 2T-RTT is also given in Figs. 7 and 9. The simulated performance of 1T-RTT and 2T-RTT matches those predicted by our analysis. Fig. 10 shows the CEP performance as a function of ρ against PBNJ and BMTJ, respectively. The (32, 18) extended RS code is used. When a BMTJ is present, 2T-RTT and Bayesian method render similar performance for all ρ. But this is true for PBNJ only if $\rho < 0.15$. 1T-RTT does not provide any decoding gain over the EO decoder when $\rho < 0.25$ and a BMTJ is present. For both jammers, the performance of 1T-RTT and 2T-RTT converge as ρ becomes greater than 0.5. Similar performance trends are observed for some other cases [12]. Fig. 11 depicts the worst-case ρ as a function of E_b/N_J. The behavior shown in Fig. 11 is similar to other uncoded cases: when E_b/N_J is small, the jammer has enough power to spread its power over a larger portion of the communication band but if E_b/N_J is large, the jammer has to concentrate its power over a much smaller band.

VII. Conclusions

This paper examines various design issues of SFH/MFSK systems that use an RS code and a block interleaver. These systems are designed to operate in the presence of PBNJ or BMTJ. The capability to combat both types of jammers is enhanced by using a single-pass EE decoder. We compare the effectiveness of the two jammers and analyze the influence of the interleaving length and the hopping rate. Other important issues discussed are the selections of the EIM and the code rate. The study of the influence of finite interleaving length enables us to carry out tradeoffs between interleaving length and the CEP performance. It is concluded that the 2T EE decoder does offer noticeable performance improvement over the EO decoder when E_b/N_J is not too high and the interleaving length, or equivalently, the hopping rate, is large enough.

Four EIM's (1T-RTT, 2T-RTT, 1T-Bayesian, and 2T-Bayesian) for supporting an EE decoder are investigated. We found that in most cases of interest, the performance of 2T-RTT is very close to that of 1T-Bayesian, while the latter is almost the same as that of 2T-Bayesian. The performance of 1T-RTT is not as impressive as that of 2T-RTT, especially when E_b/N_J is small. Because RTT is much easier to implement than the Bayesian methods, 2T-RTT is clearly the most appropriate EIM among the four.

We also use our CEP analysis to evaluate the effect of the code rate and find an optimal range of code rates. For the two classes of jammers—BMTJ and PBNJ—under investigation, the former, since it possesses more information about the communication signal, is clearly a more effective jammer. Finally, for completeness, we present in the appendix simple and effective algorithms for estimating several channel and signal parameters that are needed in deciding the optimal erasure threshold.

Appendix
Channel and Signal Parameters Estimations

As mentioned in the main text, the threshold level used in an erasure insertion decision is a function of some channel and signal parameters. A maximum-likelihood (ML) or maximum *a posteriori* estimate based on the energy detector bank output $\{R_i\}$ has to compute some conditional pdf of $\{R_i\}$. Unfortunately, the associated conditional pdf is not available because we do not know which channel is the message channel. This problem can be solved by using a training sequence. For example, the transmitter can insert known MFSK symbols at the beginning of every hop. But for reliable estimation of the channel and signal parameters, we may need several hundreds or even more than a thousand training symbols. Another possible solution can be obtained by applying the so-called generalized ML approach. The resulting receiver would have to perform joint detection and estimation; the structure is complicated and is different from the simple engergy detector bank discussed in this paper. Hence, we shall use an alternative approach that does not need extra training symbols and will not entail complicated algorithms.

A. PBNJ

When a PBNJ is present, the parameter of interest is E_b/N_T (or E_s/N_T), which is a function of the signal amplitude s and the noise variance σ^2. Consider the sum of the energy detector outputs $R = R_1 + R_2 + \cdots + R_M$. The pdf of R is given by

$$f_R(r) = \frac{1}{2\sigma^2}\left(\frac{r}{s^2}\right)^{(M-1)/2} e^{-((s^2+r)/2\sigma^2)} I_{M-1}\left(\sqrt{r}\frac{s}{\sigma^2}\right), \quad r \geq 0. \tag{A.1}$$

Obviously, the statistic of R is independent of the fact that the kth channel is the message channel and is a function of both s and σ^2. ML estimates of s and σ^2 based on (A.1) can be obtained, but the resulting estimates have to compute special functions like $I_{M-2}(x)$ and $I_M(x)$. To alleviate this difficulty and provide simple, efficient estimates, we invoke the method of moments, which calls for the evaluation of moments of R. The first two moments of R are given by [14]

$$E(R) = 2M\sigma^2 + s^2 \tag{A.2}$$
$$E(R^2) = 4M\sigma^4 + 4\sigma^2 s^2 + (2M\sigma^2 + s^2)^2 \tag{A.3}$$
$$\sigma_r^2 = 4M\sigma^4 + 4\sigma^2 s^2. \tag{A.4}$$

Let r_i be the ith sample of R. The weak law of large number says that the time average of the kth moment $\overline{R^k} \stackrel{\text{def}}{=} n^{-1}\sum_{i=1}^{n} r_i^k$, if it exists, converges in probability to $E(R^k)$, i.e., $\overline{R^k} \xrightarrow{p} E(R^k)$. It is easy to see

$$\sqrt{\overline{R}^2 - M(\overline{R^2} - \overline{R}^2)} \stackrel{\text{def}}{=} \overline{s^2} \xrightarrow{p} s^2 \tag{A.5}$$

$$\frac{\overline{R} - \overline{s^2}}{2M} \stackrel{\text{def}}{=} \overline{\sigma^2} \xrightarrow{p} \sigma^2 \tag{A.6}$$

where, as defined above, $\overline{R}$ and $\overline{R^2}$ are the time average of R and R^2, respectively. The weak convergence of the above two equations follow from the facts that $\overline{s^2}$ and $\overline{\sigma^2}$ are continuous functions of $\overline{R}$ and $\overline{R^2}$. These two equations imply that a reasonable estimate for E_b/N_T, $(\widehat{E_b/N_T})$, is given by

$$\left(\widehat{\frac{E_b}{N_T}}\right) \stackrel{\text{def}}{=} \beta = \frac{\overline{s^2}}{2\log_2 M\overline{\sigma^2}}. \tag{A.7}$$

Fig. 12 shows the mean and standard deviation [i.e., root mean squared (rms)] estimation error of the above estimator as a function of E_b/N_J and the number of samples used. It is clear that 100 samples are enough to render an rms error smaller than 3 dB.

B. BMTJ

When the jammer is a BMTJ, the parameters of interest are (s, I, σ^2), or equivalently, $(s/I,\ E_b/N_0)$; see (4). Following the approach used in the PBNJ case, we have to find a parameter

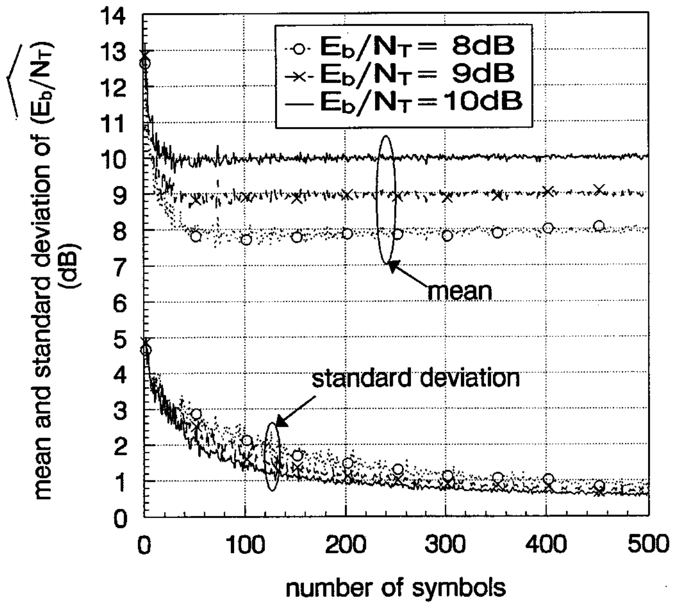

Fig. 12. Mean and rms error of the E_b/N_T estimator (A.7).

that is a function of the above parameters but is independent of the information about which channel is jammed and which channel bears the transmitted message. The parameter used in the previous case, the sum of the energy detector output R, is not a good candidate in this case, since the corresponding pdf does not render a closed-form expression. Moreover, the pdf and the moments of R depend on whether the jamming tone is in the message channel or in a noise channel. Therefore, we consider the new parameter defined by

$$Y = Y_i^2 + Y_q^2 \tag{A.8}$$

where

$$Y_i = R_{1i} + R_{2i} + \cdots + R_{Mi} \tag{A.9}$$
$$Y_q = R_{1q} + R_{2q} + \cdots + R_{Mq}. \tag{A.10}$$

R_{ni} and R_{nq} are the in-phase and the quadrature-phase components of the nth channel output. It follows that the pdf of Y is given by

$$f_Y(y) = \frac{1}{2}\int_0^\infty zJ_0(sz)J_0(Iz)\,e^{-(\sigma^2 z^2/2)}J_0(\sqrt{y}z)\,dz, \quad y \geq 0 \tag{A.11}$$

where $\sigma^2 = MN_0$. Furthermore

$$E(Y) = s^2 + I^2 + 2M\sigma^2, \qquad \text{if a BMTJ is present} \tag{A.12}$$

$$E(Y) = s^2 + 2M\sigma^2, \qquad \text{otherwise.} \tag{A.13}$$

and

$$E(Y^2) = s^4 + 4s^2I^2 + I^4 + 8M\sigma^2(s^2 + I^2 + M\sigma^2), \quad \text{if a BMTJ is present} \tag{A.14}$$

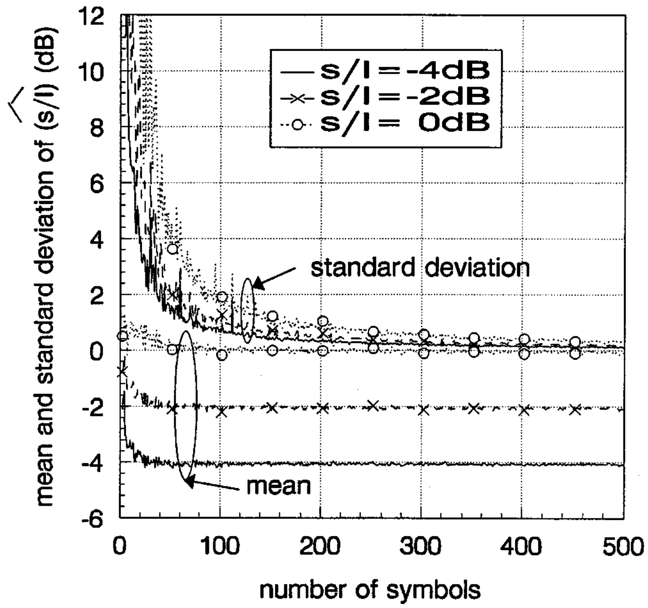

Fig. 13. Means and rms errors of the S/I estimator (A.20).

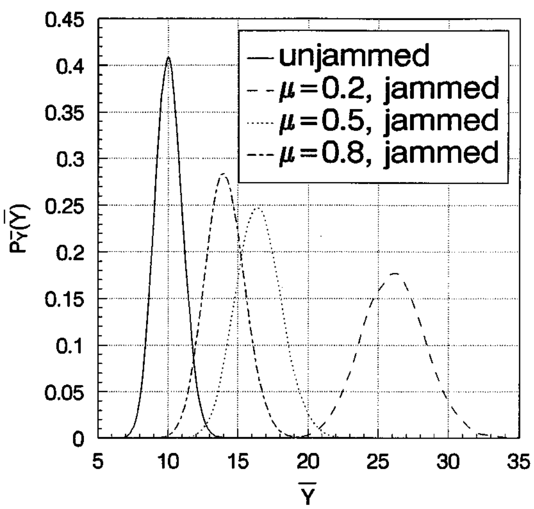

Fig. 14. PDF's of $\overline{Y}$.

$$E(Y^2) = s^4 + 8M\sigma^2(s^2 + M\sigma^2), \qquad \text{otherwise.} \tag{A.15}$$

Equivalently

$$\sigma_Y^2 = 2s^2I^2 + 4M\sigma^2(s^2 + I^2 + M\sigma^2), \quad \text{if a BMTJ is present} \tag{A.16}$$

$$\sigma_Y^2 = 4M\sigma^2(s^2 + M\sigma^2), \qquad \text{otherwise.} \tag{A.17}$$

The above results immediately lead to

$$\overline{Y}|_{\text{jammed}} - \overline{Y}|_{\text{unjammed}} \stackrel{def}{=} \overline{I^2} \xrightarrow{p} I^2. \tag{A.18}$$

Estimations for s and σ can be obtained by using samples of R from the unjammed hops; see (A.5) and (A.6). We can also use samples of unjammed Y and apply the method of moments to (A.13) and (A.15). An alternative approach, assuming estimate of I has been obtained, is based on (A.12), (A.14), and

$$\sqrt{\overline{Y}^2 - M(\overline{Y^2} - \overline{Y}^2) - (\overline{I^2})^2} \stackrel{\text{def}}{=} \widehat{s^2} \xrightarrow{p} s^2$$

$$\frac{\overline{Y} - \overline{I^2} - \widehat{s^2}}{2M} \stackrel{def}{=} \widehat{\sigma^2} \xrightarrow{p} \sigma^2 \tag{A.19}$$

where $\overline{Y}$ and $\overline{Y^2}$ are the time averages of Y and Y^2, respectively. Equations (A.18) and (A.19) suggest that

$$\widehat{\left(\frac{s}{I}\right)} = \sqrt{\widehat{s^2}/\widehat{I^2}} \tag{A.20}$$

$$\widehat{\left(\frac{E_b}{N_0}\right)} = \frac{\widehat{s^2}}{2\,\log_2 M\widehat{\sigma^2}}. \tag{A.21}$$

The performance of the estimator (A.20) as a function of the number of symbols and s/I is shown in Fig. 13. Like the E_b/N_T estimator (A.7), a hundred symbols are good enough to guarantee an rms estimation error smaller than 3 dB.

The above estimators (A.20) and (A.21) assume that the channel state information—whether a jammer is present or not—is given. If the strength of the tone jammer is not too weak, (A.12) suggests that $\overline{Y}$ is a good channel state indicator. We can classify the jammed and the unjammed states by computing the log likelihood ratio associated with $\overline{Y}$, which is equivalent to the test

$$\begin{aligned} &\overline{Y} > \gamma \rightarrow \text{jammed} \\ &\overline{Y} \le \gamma \rightarrow \text{not jammed.} \end{aligned} \tag{A.22}$$

Shown in Fig. 14 are the pdf's of $\overline{Y}$ for both jammed and unjammed cases. Obviously, a threshold γ that separates $p(\overline{Y}|\text{jammed})$ and $p(\overline{Y}|\text{unjammed})$ can easily be found, and the associated classification rule (A.22) will render only small decision error. On the other hand, if I^2 is small, i.e., E_b/N_J is large, the *distance* between the above two pdf's is small, and it is more difficult to separate the two channel states. But in this case (high E_b/N_J), 1T systems perform just as well 2T systems whence we can use the threshold derived from the E_b/N_T estimate alone without compromising the system performance.

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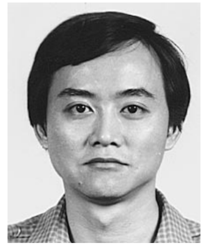

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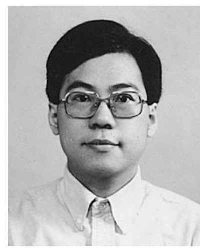

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