15 September 1999

Optics Communications

Optics Communications 168 (1999) 417–422

www.elsevier.com/locate/optcom

The self-defocusing of a diffraction beam in a photorefractive crystal and its application in a Fourier plane multiple beam splitter

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Received 27 April 1999; accepted 9 July 1999

Abstract

We propose using a diffraction beam to induce a self-defocusing lens and applying it in a Fourier plane multiple beam splitter (FPMBS). The self-induced lens array in the FPMBS can increase the splitting ratio, which is generated by inserting a photorefractive crystal at the Rayleigh distance of the diffraction beam from each pixel. © 1999 Elsevier Science B.V. All rights reserved.

PACS: 42.65.Hw; 42.65.Jx; 42.79.Fm; 42.79.Kr
Keywords: Photorefractive effect; Self-defocusing; Fourier plane multiple beam splitter; Liquid crystal display

1. Introduction

The self-focusing or self-defocusing effect of a Gaussian beam in a photorefractive crystal has been studied in many papers [1]. The phenomenon based on the photorefractive effect can generate a self-focusing lens with lower power requirement compared with the conventional nonlinear effect. Thus it has the potential to apply this effect in a practical system which necessitates changing the beam parameters. In general, the beam considered in previous work such as Ref. [1] is an ideal propagation mode, i.e., Gaussian mode. However, because of the finite aperture in a practical system, the Gaussian beam will change to the propagation mode of the diffraction. Furthermore, the beam in many systems, which is generated from an aperture illuminated by a multi-mode laser or a uniform light source, is also a diffraction wave. Thus it is essential to consider the possibility of the self-focusing effect induced by a diffraction beam. In this report, we will show that the diffraction beam generated by readout of a slit could be used as an input beam to induce a self-defocusing lens in a $Bi_{12}SiO_{20}$ (BSO) photorefractive crystal. Then, this lens was used as a typical component for an optical computing system — a Fourier plane multiple beam splitter (FPMBS) [2]. The original component could duplicate the input pattern in the Fourier plane for the purpose of the following

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PII: S0030-4018(99)00385-5

parallel processing. Thus the important criterion is the uniformity of all diffraction orders. We can use a lens array to increase the uniformity efficiently. The FPMBS using the self-defocusing lens has the advantage of the dynamic splitting ratio and self-alignment. Therefore, we will show the feasibility of applying the self-defocusing effect in an FPMBS. The property of the lens in the FPMBS was explored by numerical calculation and demonstrated by the experiments.

2. The self-focusing lens induced by a diffraction beam

From the theory of the self-focusing in the photorefractive crystal, the intensity distribution of the illuminating beam will determine the index profile of the self-focusing lens [1]. There are severe changes on the intensity profile of a diffraction beam from the aperture to the various distances [3]. If we use the diffraction light as an illuminating beam, the evolution of profile along the propagation direction must be surveyed. Briefly, the intensity of the diffraction beam with many small peaks will transit to that with less peaks after propagation. After some distance, the number of peaks located in the central region is almost reduced to only one and the profile changes like a sinc function, i.e., the beam is diffracted in the Fraunhofer region. The distance of the transition is called the Rayleigh distance. For a given aperture, the Rayleigh distance can be written as

$$R = \frac{a^2}{\lambda}, \tag{1}$$

where a is the aperture dimension and λ is the wavelength. To get the lens-like profile, the peak of the profile of the input beam must be dominated by only one. This can be obtained after the Rayleigh distance. In addition, to get a larger focusing power, the position with a larger intensity gradient of light must be selected to generate a larger gradient of the refractive index. Beyond the Rayleigh distance, the decreased intensity gradient will decrease the focusing power. Therefore, it is the optimum input position near the Rayleigh distance for getting the lens with the highest focusing power. To discuss the lens properties, we consider the one-dimensional case of the diffraction for simplicity. The diffraction beam is generated from a slit which is illuminated by a uniform light. The configuration is shown in Fig. 2. The intensity distribution of the light illuminating on the crystal can be written as

$$I(z) = |u(z)|^2$$
$$= \left| \int_{-a/2}^{a/2} \exp\left[j\frac{k}{2R}(z_i - z)^2 \right] \mathrm{d}z_i \right|^2, \tag{2}$$

where u is the electric field. a is the width of slit, R is the Rayleigh distance from Eq. (1), k is the wave vector, and z and z_i are the axes perpendicular to the direction of the propagation on the input plane and the crystal, respectively. If the thickness of the crystal is much smaller than the Rayleigh distance, then the diffraction and lens effect on the intensity distribution will be neglected in the crystal. This case is called thin lens approximation and the illuminating light can be assumed to be constant through the crystal. The illuminating light will induce the spatial charge in the crystal and the distribution of the electric field from this charge

$$E_z(z) = \frac{I_0}{I_0 + I(z)} E_0, \tag{3}$$

where I_0 is the intensity of the additional uniform bias light and E_0 is the externally applied electric field. The electric field will induce an index profile by the Pockel effect. For maximum change of index, the I_0 is set equal to the peak value of $I(z)$. The index change of the crystal can be written as

$$\Delta n(z) = \frac{1}{2} n_0^3 r_{41} E_z(z), \tag{4}$$

where n_0 is the index of the crystal and r_{41} is the electro-optical coefficient of the BSO crystal. Finally, the transmission function of the crystal can be written as

$$t(z) = \exp[-jk\Delta n(z) d], \tag{5}$$

where d is the thickness of the crystal. We can normalize the refractive index change to fractional one which is relative to the index change by the bias electric field. Fig. 1 shows the intensity distribution of input light and induced fractional index change. The distribution of the index change in the region of the central lobe of the diffraction beam can be found

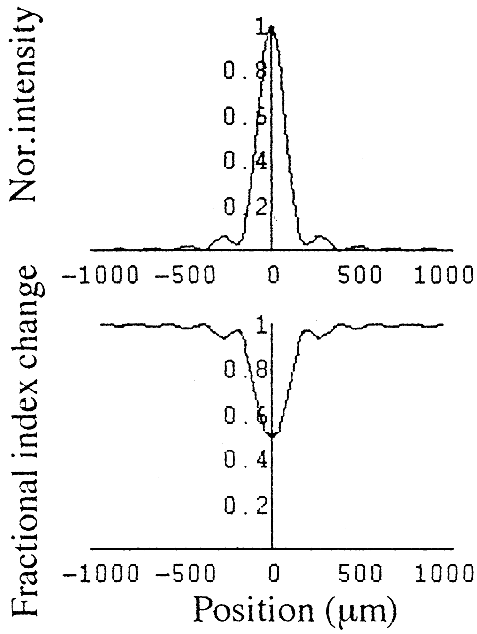

Fig. 1. The intensity distribution of input light and induced fractional index change relative to that induced by applied electric field.

as lens-like. If the applied DC bias field is negative relative to the principle axis of the crystal, the fractional index change is negative. Thus, the central lobe of the input light which contains the most beam power will be defocused by this index profile.

3. Increasing the splitting ratio of an FPMBS

As an example of an FPMBS in the two-dimension case, a pixelated spatial light modulator is an input device in a Fourier transform system. The system will generate a two-dimensional spectrum array of the input function in the Fourier plane [4]. The spatial light modulator can be a liquid crystal display (LCD). The diffraction beam generated from each pixel of the LCD has the potential to generate self-induced lens in a nonlinear media. To discuss the improvement of the system, we consider the one-dimensional case for simplicity. The FPMBS consists of a slit array in the input plane and a Fourier transform lens, as shown in Fig. 3. The slit array illuminated by a plane wave will generate an array of the spectrum of the slit in the Fourier plane. This is caused by the fact that each slit can diffract the light and then superpose all the light in the Fourier plane. If the slit array works as the spatial light modulator, then the input function can be written as

$$u_{\mathrm{i}}(z_{\mathrm{i}}) = \sum_m \mathrm{rect}\left(\frac{z_{\mathrm{i}} - mb}{a}\right) f_{\mathrm{i}}(z_{\mathrm{i}}), \tag{6}$$

where rect is the function of each slit, m is the number for pixel, b is the period of the slit array, and f_{i} is the input function. For the same peak value of $I(z)$ in Eq. (3), the f_{i} must be an image with binary or phase modulation. After the Fourier transform, the output field can be written as

$$U(z_{\mathrm{o}}) = \sum_n \mathrm{sinc}\left(\frac{na}{b}\right) F\left(\frac{z_{\mathrm{o}}}{\lambda f} - \frac{n}{b}\right), \tag{7}$$

where the sinc function is the Fourier transform of each slit function, f is the focal length, n is the diffraction order, and F is the spectrum of the input pattern f_{i}. The output of the FPMBS will be an array of the spectrum of the input pattern multiplied by a factor. From Eq. (7), the relative intensity of each spectrum is determined by the width of the slit a with respective to the period b, i.e., a/b, which is the fill factor. We can define the splitting ratio of the FPMBS as the number of the spectrum with the intensity above the defined threshold to describe the uniformity of the array. The smaller fill factor will generate the larger splitting ratio. However, to directly reduce the fill factor will cause low energy efficiency. Another method for obtaining the larger splitting ratio is to reduce the effect width of the slit,

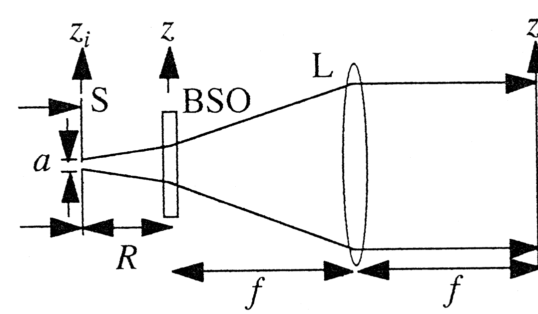

Fig. 2. The configuration of the self-defocusing lens and the illustrating system. S is a slit with width a. A BSO crystal is placed at the Rayleigh distance R. f is the focal length of Fourier lens L.

i.e., to broaden the width of the spectrum pattern of each slit. This method will not suffer from the low energy efficiency compared with the previous one. It is our goal to utilize the self-defocusing lens induced by the diffraction beam from each slit to reduce the effect width of the slit. Thus the transmission function of the media is like a lens array which can self-align all slits simultaneously. If the width of the slit is small enough compared with the dimension of the input, then the slit function will transfer to a Fraunhofer diffraction and the input function will generate a geometrical projection in the position of the Rayleigh distance. Thus we can insert a crystal into the Rayleigh distance to modify the configuration of the original FPMBS. The modulated crystal will therefore work as a new input device. To satisfying the assumption, the condition derived from Eq. (1) and Ref. [5] is

$$B \gg \frac{a}{\sqrt{2}}, \tag{8}$$

where B is the dimension of the input function. In addition, if

$$a + \frac{d\lambda}{a} < \frac{b}{2}, \tag{9}$$

then we will neglect the interaction between the slits through the crystal. To satisfy this condition and get the largest energy efficiency, the width of the slit

$$a = \frac{b}{4} + \sqrt{\frac{b^2}{16} + d\lambda}\ .$$

Here we will only consider the case satisfying Eq. (9). Thus we can analyze the self-defocusing lens

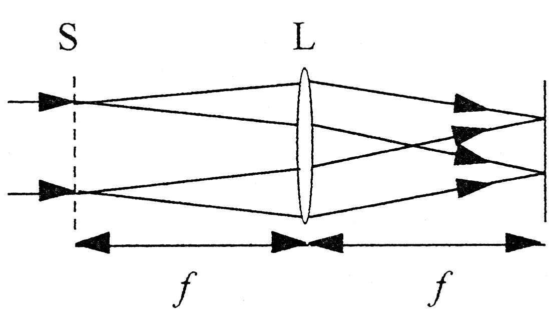

Fig. 3. The multiple Fourier plane beam splitter. The slit array mask S illuminated by a plane wave is Fourier transformed by lens L. The output in the Fourier plane is spectrum array. f is the focal length.

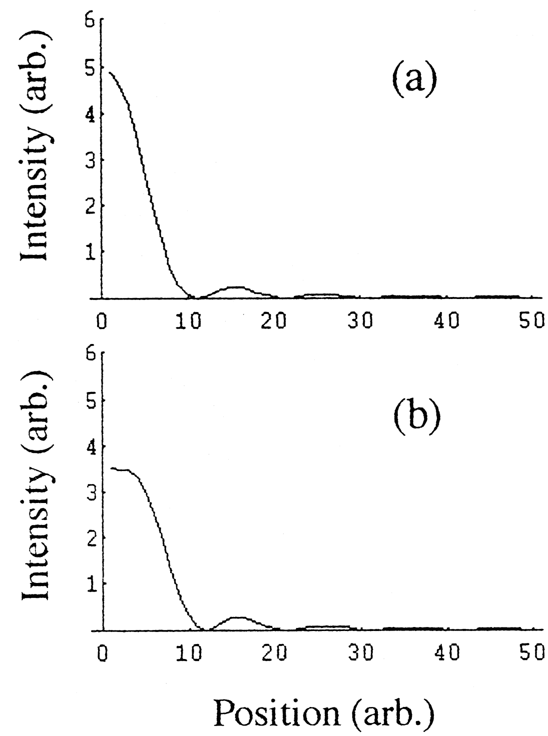

Fig. 4. The spectrum variation in the positive axis. The central position of the spectrum is at coordinate 1. (a) is the original spectrum intensity; and (b) is the spectrum intensity with the applied electric field 12 kV/cm.

generated by a single slit to illustrate the properties of the lens array. The configuration of the illustrating system is also represented as Fig. 2. S is a slit with width a. A BSO crystal is placed at the Rayleigh distance R. f is the focal length of the Fourier lens L. If R is much smaller than f, then the original configuration (in Fig. 3) will not change and just insert a crystal in front of spatial light modulator. Thus, from Eq. (2) and Eq. (5), the intensity distribution of the spectrum of one slit in the Fourier plane can be written as

$$P(z_o) = \mathscr{F}\{u(z)t(z)\}, \tag{10}$$

where $\mathscr{F}$ is Fourier transform operator. Comparing with Eq. (7), the output of the complete system can be written as

$$U(z_o) = \sum_n P\left(\frac{na}{b}\right) F\left(\frac{z_o}{\lambda f} - \frac{n}{b}\right). \tag{11}$$

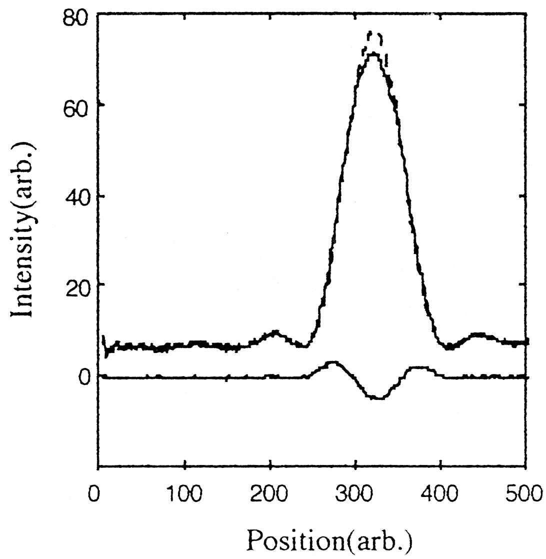

Fig. 5. The results of the experimental demonstration. The dotted curve is the original spectrum. The solid curve is the spectrum with self-defocusing effect. The third curve below the two curves is the difference between them.

To show the effect of the BSO crystal, firstly, we calculated the variation of the spectrum of one slit caused by the self-defocusing lens. As an example of the illustrating system, the width of the slit (S) is 200 μm and the wavelength is 0.6328 μm. The position of the photorefractive BSO crystal is at the Rayleigh distance ($R = 63$ mm). The thickness d, the refractive index n_0, and the electro-optic coefficient r_{41} of the crystal are 4 mm, 2.54, and 3.6×10^{-12} m/V, respectively. Since the thickness of the crystal is much smaller than the Rayleigh distance, the thin crystal approximation can be applied. An additional uniform beam (I_0) illuminating the entire crystal has the same intensity as the peak of the diffraction beam. The spectrum variation in the positive axis is shown as Fig. 4. Fig. 4(a) is the original spectrum intensity and Fig. 4(b) is the spectrum intensity with the applied electric field 12 kV/cm. Comparing these two figures, it can be seen that the peak intensity of the central lobe has decreased and the width of the first zero has increased. Secondly, the illustrating system was demonstrated experimentally. The intensity profile of the spectrum is measured by CCD. This has the benefit of rejecting the spatial noise by averaging all scan lines. The results are shown in Fig. 5. The dotted curve is the intensity of the original spectrum. The solid curve is the intensity of the spectrum with self-defocusing effect by turn on the applying voltage of the crystal. The curve shown below is the difference between them. The central part of the spectrum has decreased and the two sides of it have increased. Evidently, the width of the spectrum of each slit can be increased. To numerically demonstrate the output of the complete system, we calculate the intensity of the diffraction orders from 50 slits with a period of three times of

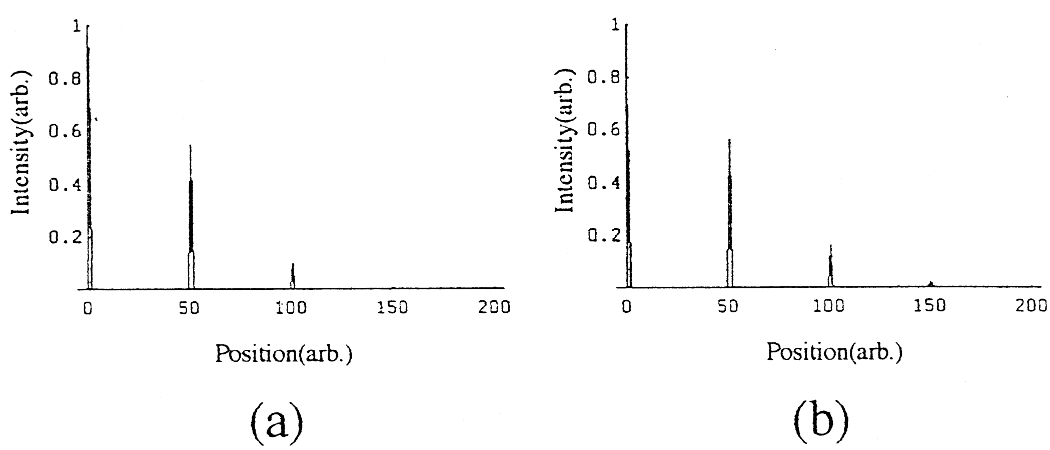

Fig. 6. The intensity of the diffraction orders from 50 slits with the period of 3 times the aperture width. The central position of the spectrum is at coordinate 1. (a) is without self-defocusing effect; and (b) is with self-defocusing effect.

the aperture width. From Eq. (9), the minimum aperture size is 69 μm. From the result as shown in Fig. 6, it can be seen that the power distributed in the zero order will transfer to the first and the second order. Fig. 6(a) is the intensity distribution without self-defocusing effect. Fig. 6(b) is the intensity distribution with the self-defocusing effect.

4. Conclusions

We used a diffraction beam to induce the self-defocusing lens and shown the potential of using it in an FPMBS. From the numerical and experimental demonstrations, the width of the spectrum of one slit in the SLM can be increased by simply inserting a photorefractive crystal at the Rayleigh distance. If the dimension of the input function is large enough and the fill factor is small enough, we can deduce that the novel FPMBS will duplicate the input function with a larger splitting ratio. Because of the requirement of the reference beam, the modulation of the input function must be binary or phase only. The use of the self-induced lens array has the advantage of avoiding an alignment process. From our analyses, the splitting ratio can be increased by a thicker crystal or other crystal with a higher electro-optical coefficient. However, the thin lens approximation in our analyses must be modified.

Acknowledgements

This study was supported by the National Science Council, R.O.C. under contract number: NSC-88-2215-E-009-008.

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