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The speed-controlled interpolator for machining parametric curves

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Abstract

Modern CNC systems are designed with the function of machining arbitrary parametric curves to save massive data communication between CAD/CAM and CNC systems and improve their machining quality. Although available CNC interpolators for parametric curves generally achieve contouring position accuracy, the specified feedrate, which dominates the quality of the machining processes, is not guaranteed during motion. Recently, some approximation results concerning motion speed have been reported [Shpitalni M, Koren Y, Lo CC. Computer-Aided Design 1994;26(11):832–838; Bedi S, Ali I, Quan N. Trans ASME J Engng Industry 1993;115:329–336; Chou JJ, Yang DCH. Trans. ASME J Engng Industry 1991;113:305–310; Chou JJ, Yang DCH. Trans ASME J Engng Industry 1992;114:15–22]. In this paper, by deriving a compensatory parameter, the proposed interpolation algorithm has significantly improved curve speed accuracy. In real applications, the proposed algorithm results in: (1) constant speed; and (2) specified acceleration and deceleration (ACC/DEC) to meet the feedrate commands. The motion accuracy and feasibility of the present interpolator have been verified with a provided non-uniform rational B-spline (NURBS) example.

Keywords: Parametric curve; CNC interpolators; Speed accuracy; NURBS

1. Introduction

In modern CAD/CAM systems, profiles for parts like dies, vanes, aircraft models, and car models are usually represented in parametric forms. As conventional CNC machines only provide linear and circular arc interpolators, the CAD/CAM systems have to segment a curve into a huge number of small, linearized segments and send them to CNC systems. Such linearized-segmented contours processed on traditional CNC systems are undesirable in real applications as follows:

- the transmission error between CAD/CAM and CNC systems for a huge number of data may easily occur, i.e. lost data and noise perturbation;
- the discontinuity of segmentation deteriorates surface accuracy;
- the motion speed becomes unsmooth because of the linearization of the curve in each segment, especially in acceleration and deceleration.

As the generated curves or profiles may be in a parametric form, only parametric curve information is required to be efficiently transferred among CAD/CAM/CNC systems as shown in Fig. 1. Shpitalni et al. [1] proposed the curve segments transfer between CAD and CNC systems and Bedi et al. [2] proposed the B-spline curve and B-spline surface interpolation algorithm to obtain both accurate curves and gouge free surface. Huang and Yang [3] developed a generalized interpolation algorithm for different parametric curves with improved speed fluctuation. Moreover, Yang and Kong [4] studied both linear and parametric interpolators for machining processes.

In these CNC systems, parametric curves are profiles in different formats like the Bezier curve, B-spline, cubic spline, and NURBS (non-uniform rational B-spline). The general parameter iteration method used is

$$u_{i+1} = u_i + \Delta(u_i)$$

where u_i is the present parameter, u_{i+1} is the next parameter, and $\Delta(u_i)$ is the incremental value. The interpolated points are calculated by substituting u_i into the corresponding mathematical model to recover the originally designed curves. As the cutter moves straight between contiguous interpolated points, two position errors may occur as: (a) radial error; and (b) chord error [5] during motion for a parametric curve as shown in Fig. 2.

The radial error is the perpendicular distance between the interpolated points and the parametric curve. Basically, the radial error is caused by the rounding error of computer

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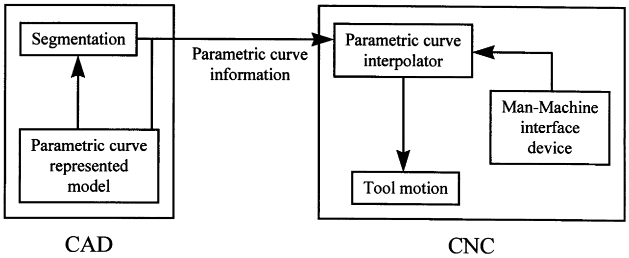

Fig. 1. The machining systems with parameters transmission.

systems. With the rapid development of microprocessors with higher precision, the radial error is no longer a major concern in present applications. In addition, the chord error is the maximum distance between the secant $\overline{CD}$ and the secant arc $\widehat{AB}$. A small curvature radius with a fast feedrate command may cause the chord error. Thus, an adaptive feedrate is required to keep the chord error within an acceptable range.

However, cutting speed dominates the quality of the machining process. To achieve the specified feedrate for parametric curves is usually difficult. The undesirable motion speed may deteriorate the quality of the machined surface. Several researchers have developed different interpolation algorithms for parametric curves to improve motion speed accuracy. Bedi et al. [2] set $\Delta(u_i)$as a constant to form the uniform interpolation algorithm which is the simplest method and its chord error and curve speed however are not guaranteed. According to the analysis of CNC machine kinematics and cutter path geometry, an improved interpolation algorithm in position, velocity, and acceleration was proposed by Chou and Yang [6] if the CNC machine kinematics model is known exactly. Further, Houng and Yang [3] developed the cubic spline parametric curve interpolator by using the Euler method. Shpitalni et al. [1] derived the same interpolation algorithm by using Taylor's expansion. Lo and Chung [7] proposed the contouring error compensation interpolation algorithm which contains real-time contouring error calculations and a simplified parameter iteration method to achieve satisfactory motion accuracy. In acceleration and deceleration, Kim [8] obtained a simple method for parametric curves while its position and speed errors are significant.

The present speed-controlled interpolation algorithm is proposed by deriving a suitable compensatory parameter for the first-order approximation [1,6,8] to obtain desirable motion speed. Then, the proposed interpolator is applied to the constant-speed mode and the acceleration/deceleration mode to achieve constant feedrate and the specified speed profiles, respectively. Thus, the present CNC interpolators result in stable motion and smoothly changed speed to avoid mechanical shock or vibration in machining processes. The proposed speed-controlled interpolators have also been successfully applied to a NURBS command on a personal computer to achieve high motion precision.

2. Parametric curve formulation

Suppose $C(u)$ is the parametric curve representation function and the time function u is the curve parameter as

$$u(t_i) = u_i \qquad \text{and} \qquad u(t_{i+1}) = u_{i+1}$$

By using Taylor's expansion, the approximation up to the second derivative is

$$u_{i+1} = u_i + \left.\frac{\mathrm{d}u}{\mathrm{d}t}\right|_{t=t_i}(t_{i+1}-t_i) + \frac{1}{2}\left.\frac{d^2u}{\mathrm{d}t^2}\right|_{t=t_i}\cdot(t_{i+1}-t_i)^2 + \mathrm{HOT} \tag{1}$$

As the curve speed $V(u_i)$ can be represented as

$$V(u_i) = \left\|\frac{\mathrm{d}C(u)}{\mathrm{d}t}\right\|_{u=u_i} = \left\|\frac{\mathrm{d}C(u)}{\mathrm{d}u}\right\|_{u=u_i}\left.\frac{\mathrm{d}u}{\mathrm{d}t}\right|_{t=t_i}$$

the first derivative of u with t is obtained as

$$\left.\frac{\mathrm{d}u}{\mathrm{d}t}\right|_{t=t_i} = \frac{V(u_i)}{\left\|\frac{\mathrm{d}C(u)}{\mathrm{d}u}\right\|_{u=u_i}} \tag{2}$$

By taking the derivative of Eq. (2), the second derivative of

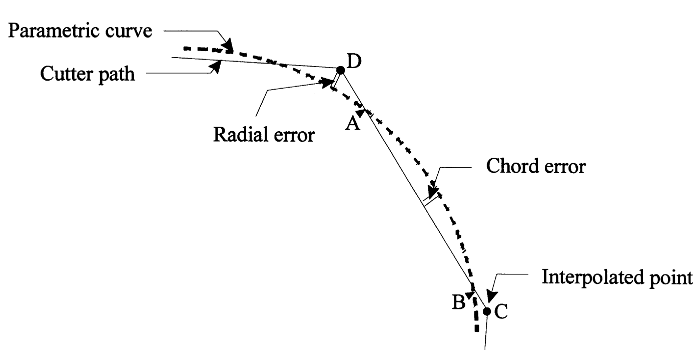

Fig. 2. The radial error and the chord error.

u with t is

$$\left.\frac{d^2u}{dt^2}\right|_{t=t_i} = \frac{-1}{\left\|\frac{dC(u)}{du}\right\|^2_{u=u_i}}\left[V(u_i)\cdot\frac{d\left(\left\|\frac{dC(u)}{du}\right\|\right)\Big|_{u=u_i}}{dt}\right] \tag{3}$$

where

$$\frac{d\left(\left\|\frac{dC(u)}{du}\right\|\right)\Big|_{u=u_i}}{dt} = \frac{d\left(\left\|\frac{dC(u)}{du}\right\|\right)\Big|_{u=u_i}}{du}\cdot\left.\frac{du}{dt}\right|_{t=t_i}$$

$$= \frac{d\left(\left\|\frac{dC(u)}{du}\right\|\right)\Big|_{u=u_i}}{du}\cdot\frac{V(u_i)}{\left\|\frac{dC(u)}{du}\right\|_{u=u_i}} \tag{4}$$

By substituting Eq. (4) into Eq. (3), the second derivative of u is rewritten as

$$\left.\frac{d^2u}{dt^2}\right|_{t=t_i} = -\frac{V^2(u_i)}{\left\|\frac{dC(u)}{du}\right\|^3_{u=u_i}}\cdot\frac{d\left(\left\|\frac{dC(u)}{du}\right\|\right)\Big|_{u=u_i}}{du} \tag{5}$$

where

$$\frac{d\left(\left\|\frac{dC(u)}{du}\right\|\right)\Big|_{u=u_i}}{du} = \frac{\frac{dC(u)}{du}\cdot\frac{d^2C(u)}{du^2}}{\left\|\frac{dC(u)}{du}\right\|_{u=u_i}} \tag{6}$$

By substituting Eq. (6) into Eq. (5), the second derivative of u becomes

$$\left.\frac{d^2u}{dt^2}\right|_{t=t_i} = -\frac{V^2(u_i)\cdot\left(\frac{dC(u)}{du}\cdot\frac{d^2C(u)}{du^2}\right)}{\left\|\frac{dC(u)}{du}\right\|^4_{u=u_i}} \tag{7}$$

Let the sampling time in interpolation be T_s seconds and

$t_{i+1} - t_i = T_s$

The first- and second-order approximation interpolation algorithms are obtained by substituting Eqs. (2) and (7) into Eq. (1), respectively. By neglecting the higher order term, the interpolation algorithms in Eq. (1) can be processed as follows:

The first-order approximation interpolation algorithm [1,6,9]

$$u_{i+1} = u_i + \frac{V(u_i)T_s}{\left\|\frac{dC(u)}{du}\right\|_{u=u_i}} \tag{8}$$

The second-order approximation interpolation algorithm [6,9]

$$u_{i+1} = u_i + \frac{V(u_i)\cdot T_s}{\left\|\frac{dC(u)}{du}\right\|_{u=u_i}} - \frac{V^2(u_i)\cdot T_s^2\cdot\left(\frac{dC(u)}{du}\cdot\frac{d^2C(u)}{du^2}\right)\Big|_{u=u_i}}{2\cdot\left\|\frac{dC(u)}{du}\right\|^4_{u=u_i}} \tag{9}$$

where $V(u_i)$ can be either the feedrate command, the specified speed profiles of ACC/DEC, or any desired speed in a general machining process.

3. The speed-controlled interpolation algorithm

3.1. The compensatory parameter

In Eqs. (8) and (9), the first-order and the second-order interpolation algorithms are approximated results by neglecting the higher order term as in Eq. (1). Therefore, the approximation error for those methods is unavoidable and an interpolation algorithm concerning the curve speed is proposed in this paper. The present interpolation algorithm is based on the first-order approximation interpolation algorithm with a compensatory value $\epsilon(u_i)$ as

$$u_{i+1} = u'_{i+1} + \epsilon(u_i)$$

where $\epsilon(u_i)$ is the compensatory value and

$$u'_{i+1} = u_i + \frac{V(u_i)\cdot T_s}{\left\|\frac{dC(u)}{du}\right\|_{u=u_i}}$$

To precisely calculate the compensatory value $\epsilon(u_i)$, Taylor's expansion for the curve and its speed must be concerned. Suppose

$$C(u) = \begin{bmatrix} C_x(u) \\ C_y(u) \end{bmatrix}$$

is the parametric curve representation function for the X and Y axes, the interpolated points $C_x(u_{i+1})$ and $C_y(u_{i+1})$ are approximated as follows:

$$C_x(u_{i+1}) = C_x(u'_{i+1}) + \frac{dC_x(u'_{i+1})}{du}\epsilon(u_i) \tag{10}$$

$$C_y(u_{i+1}) = C_y(u'_{i+1}) + \frac{dC_y(u'_{i+1})}{du}\epsilon(u_i) \tag{11}$$

There are two approximation techniques used in deriving the interpolation algorithm. The first one is the Taylor's expansion of the parameter u with respect to the time t to obtain the first- or second-order approximation interpolation algorithms [1,6,9], as in Eqs. (8) and(9). The second one is the Taylor's expansion of curve C with respect to the

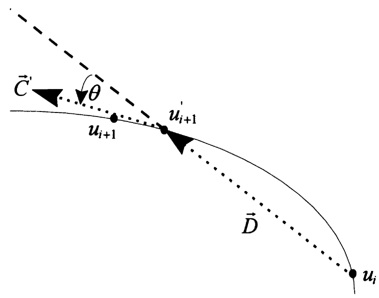

Fig. 3. Geometrical representation of parameters.

$$DX = C_x(u'_{i+1}) - C_x(u_i)$$

$$DY = C_y(u'_{i+1}) - C_y(u_i)$$

$$X'(u'_{i+1}) = \frac{\mathrm{d}C_x(u'_{i+1})}{\mathrm{d}u}$$

$$Y'(u'_{i+1}) = \frac{\mathrm{d}C_y(u'_{i+1})}{\mathrm{d}u}$$

The compensatory value $\epsilon(u_i)$ can be directly obtained as

$$\epsilon_{1,2}(u_i) = \frac{-Z \pm \sqrt{Z^2 - 4UW}}{2U}$$
$$= \frac{-\left[DX\cdot X'(u'_{i+1}) + DY\cdot Y'(u'_{i+1})\right] \pm \sqrt{[X'(u'_{i+1})^2 + Y'(u'_{i+1})^2](V(u_i)T_s)^2 - [DY\cdot X'(u'_{i+1}) - DX\cdot Y'(u'_{i+1})]^2}}{X'(u'_{i+1})^2 + Y'(u'_{i+1})^2} \quad (13)$$

parameter u as in Eqs. (10) and (11). By comparing Eqs. (1), (8) and (9) with Eqs. (10) and (11), the higher-order term of the Taylor's expansion in the first approximation is estimated as the compensatory value $\epsilon(u_i)$ by applying the second approximation. Eqs. (10) and (11) implies that the difference between position

$$\begin{bmatrix} C_x(u_{i+1}) \\ C_y(u_{i+1}) \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} C_x(u'_{i+1}) \\ C_y(u'_{i+1}) \end{bmatrix}$$

is determined by the slope of curve with the adjustment gain $\epsilon(u_i)$.

Although $V(u_i)$ is the instantaneous velocity at $u = u_i$ in the interpolation algorithm, it is usually assigned as the constant feedrate command F in real interpolation applications. The following equation is provided to accurately achieve a linear motion from

$$\begin{bmatrix} C_x(u_i) \\ C_y(u_i) \end{bmatrix} \quad \text{to} \quad \begin{bmatrix} C_x(u_{i+1}) \\ C_y(u_{i+1}) \end{bmatrix}$$

with the desired speed $V(u_i)$:

$$\frac{\sqrt{[C_x(u_{i+1}) - C_x(u_i)]^2 + [C_y(u_{i+1}) - C_y(u_i)]^2}}{T_s} = V(u_i) \quad (12)$$

then, a quadratic equation for the compensatory parameter is derived as

$$U\epsilon^2 + Z\epsilon + W = 0$$

where

$$U = X'(u'_{i+1})^2 + Y'(u'_{i+1})^2$$

$$Z = 2[DX\cdot X'(u_{i+1}) + DY\cdot Y'(u_{i+1})]$$

$$W = DX^2 + DY^2 - (V(u_i)T_s)^2$$

3.2. Selection of compensatory parameters

As the two values in Eq. (13) are the roots of a quadratic equation, characteristics of roots have to be discussed in real applications. Define two vectors as

$$\vec{D} = \begin{bmatrix} DX \\ DY \end{bmatrix}$$

$$\vec{C}' = \begin{bmatrix} X'(u'_{i+1}) \\ Y'(u'_{i+1}) \end{bmatrix}$$

Eq. (13) can be rewritten as

$$\epsilon_{1,2}(u_i) = \frac{-(\vec{D}\cdot\vec{C}') \pm \sqrt{\|\vec{C}'\|^2(V(u_i)T_s)^2 - |\vec{C}'\times\vec{D}|^2}}{\|\vec{C}'\|^2} \quad (14)$$

where the geometrical relationship between vectors $\vec{D}$ and $\vec{C}'$ correspond to parameters among which the parameters u_i, u'_{i+1}, u_{i+1} are shown in Fig. 3. θ is the angle between the difference vector $\vec{D}$ and the differential vector $\vec{C}'$, and

$$\|\vec{C}'\|^2\cdot V(u_i)T_s)^2 - |\vec{C}'\times\vec{D}|^2 = \|\vec{C}'\|^2 T_s^2\left[V^2(u_i) - \frac{|\vec{C}'\times\vec{D}|^2}{\|\vec{C}'\|^2 T_s^2}\right]$$

$$= \|\vec{C}'\|^2 T_s^2\left[V^2(u_i) - \left|\frac{\vec{C}'}{\|\vec{C}'\|}\times\frac{\vec{D}}{T_s}\right|^2\right]$$

$$= \|\vec{C}'\|^2 T_s^2\left[V^2(u_i) - \left\|\frac{\vec{D}}{T_s}\right\|^2 \sin^2\theta\right]$$

As $\|\vec{C}'\|^2 > 0$ and $T_s^2 > 0$, $\{\|\vec{C}'\|^2(V(u_i)T_s)^2 - |\vec{C}'\times\vec{D}|^2\}$ has the same sign as

$$\left\{\left[V^2(u_i) - \left\|\frac{\vec{D}}{T_s}\right\|^2 \sin^2\theta\right]\right\}$$

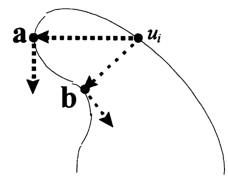

Fig. 4. Illustrative conditions (ii) and (iii).

The solutions of Eq. (13) can be in the following three categories:

(i) $\epsilon_{1,2}(u_i)$ are two different real numbers if

$$\left[V^2(u_i) > \left(\frac{\|\vec{D}\|}{T_s}\right)^2 \sin^2\theta\right]$$

(ii) $\epsilon_{1,2}(u_i)$ are the same real numbers if

$$\left[V^2(u_i) = \left(\frac{\|\vec{D}\|}{T_s}\right)^2 \sin^2\theta\right]$$

(iii) $\epsilon_{1,2}(u_i)$ are a pair of complex conjugate numbers if

$$\left[V^2(u_i) < \left(\frac{\|\vec{D}\|}{T_s}\right)^2 \sin^2\theta\right]$$

Compared with $(\|\vec{D}\|/T_s)$ and the desired speed $V(u_i)$, we conclude that the sign of

$$\left\{\left[V^2(u_i) - \left\|\frac{\vec{D}}{T_s}\right\|^2 \cdot \sin^2\theta\right]\right\}$$

is dominated by angle θ. Conditions (ii) and (iii), which may produce the same real roots or complex conjugate roots of the quadratic equation, are shown in Fig. 4. As the vector $\vec{D}$ and the differential vector $\vec{C}'$ are almost perpendicular and parameter u_{i+1} is near points a or b as shown in Fig. 4, $\sin^2\theta \cong 1$ and the conditions of

$$\left[V^2(u_i) \le \left(\frac{\|\vec{D}\|}{T_s}\right)^2 \sin^2\theta\right]$$

may occur. In physical meaning, the multiple real roots and the complex conjugate roots exist where the curvature is relatively large. In general applications, the curvature should be small to achieve precise interpolation. Thus, conditions (ii) and (iii) are not allowed in real applications and only the two different real roots as in condition (i) are concerned in the present algorithm.

According to Eq. (14)

$$\epsilon_{1,2}(u_i) = \frac{-\left(\vec{D}\cdot\frac{\vec{C}'}{\|\vec{C}'\|}\right) \pm \sqrt{(V(u_i)\cdot T_s)^2 - \left|\frac{\vec{C}'}{\|\vec{C}'\|}\times\vec{D}\right|^2}}{\|\vec{C}'\|}$$

$$= \frac{-\left(\|\vec{D}\|\cos\theta\right) \pm \sqrt{(V(u_i)T_s)^2 - \|\vec{D}\|^2\sin^2\theta}}{\|\vec{C}'\|}$$

$$= \frac{-(\|\vec{D}\|\cos\theta) \pm \sqrt{(V(u_i)T_s)^2 - \|\vec{D}\|^2 + \|\vec{D}\|^2\cos^2\theta}}{\|\vec{C}'\|}$$

Let

$$(V(u_i)T_s)^2 - \|\vec{D}\|^2 = \mu$$

By applying Taylor's expansion, roots of the quadratic equation have the approximated values in simple forms as

$$\epsilon_1 \cong \frac{\mu}{2\cdot\|\vec{C}'\|\,\|\vec{D}\|\cos\theta}$$

$$\epsilon_2 \cong \frac{-2\,\|\vec{D}\|\cos\theta}{\|\vec{C}'\|}$$

As μ is small, the first root is near zero and the other root is negative and relatively large. To achieve reliable compensation and forward motion during the interpolation process, the small compensatory parameter is preferable as

$$\epsilon(u_i) = \frac{-[DX\cdot X'(u'_{i+1}) + DY\cdot Y'(u'_{i+1})] + \sqrt{[X'(u'_{i+1})^2 + Y'(u'_{i+1})^2](V(u_i)T_s)^2 - [DY\cdot X'(u'_{i+1}) - DX\cdot Y'(u'_{i+1})]^2}}{[X'(u'_{i+1})^2 + Y'(u'_{i+1})^2]}$$

As the present speed-controlled interpolation algorithm incorporates the first approximation interpolation algorithm and a suitable compensatory value which corrects the curve speed error, the obtained curve speed almost equals the specified speed $V(u_i)$ during the interpolating process. The roots condition

$$\{\|\vec{C}'\|^2(V(u_i)T_s)^2 - |\vec{C}'\times\vec{D}|^2\}$$

can be examined before calculating the compensatory value. When the undesirable condition may occur, the compensatory value is set to be zero to avoid the complex conjugate roots.

In real machining processes, the present interpolator achieves (1) a constant speed and (2) specified ACC/DEC. The constant speed mode keeps the curve speed almost the same as the given constant feedrate command during the machining process. The ACC/DEC mode makes the curve speed in smooth profiles with the specified speed for machining parametric curves.

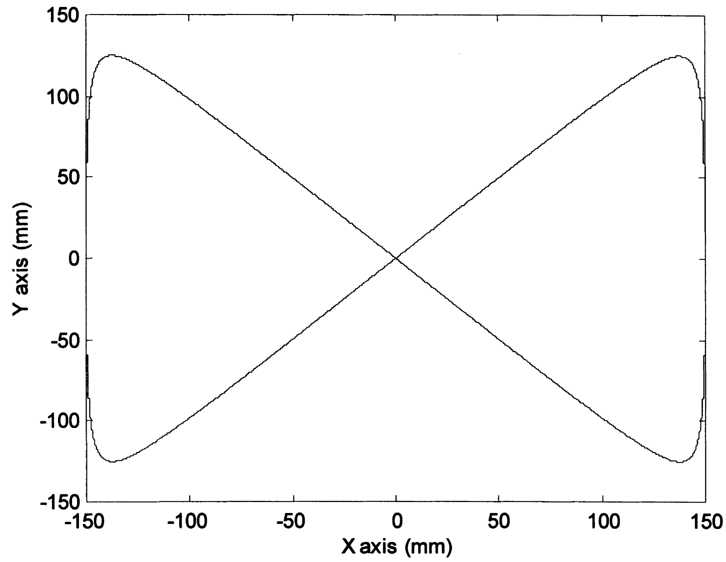

Fig. 5. The example of NURBS.

4. Applications

4.1. The example of NURBS

In this simulation, the interpolator is written by Turbo C 2.0 and is executed on a personal computer with both 80 and 200 MHz CPU. The present interpolator is applied to a NURBS [10] parametric curve with two degrees as shown in Fig. 5.

The control points, weight vector, and knot vector of NURBS are assigned as follows:

- The ordinal control points are

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} -150 \\ -150 \end{bmatrix}, \quad \begin{bmatrix} -150 \\ 150 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 150 \\ -150 \end{bmatrix}, \quad \begin{bmatrix} 150 \\ 150 \end{bmatrix}, \quad \text{and} \quad \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{(mm)}.$$

- The weight vector is $W = [1 \quad 25 \quad 25 \quad 1 \quad 25 \quad 25 \quad 1]$.
- The knot vector is $U = [0 \;\; 0 \;\; 0 \;\; \frac{1}{4} \;\; \frac{1}{2} \;\; \frac{1}{2} \;\; \frac{3}{4} \;\; 1 \;\; 1 \;\; 1]$.

The interpolating processes are as follows:

- the sampling time in interpolation is $T_s = 0.002$ s and
- the feedrate command is $F = 200$ mm/s $= 12$ m/min.

In many recent applications, the machining is in a high speed like high-speed machining, e.g. high-speed milling [11–14], machining by linear motor [15,16], and laser machining [17]. In this example, the provided weight vector which results in sharp corners is used to exam the speed deviation of different interpolation algorithms under the feedrate command $F = 200$ mm/s $= 12$ m/min.

4.2. The constant-speed mode

The curve speed fluctuations for different interpolation algorithms are compared as below: (a) the uniform, (b) the first-order approximation [1,6,9] (c) the second-order approximation [6,9] and (d) the proposed speed-controlled mode. Simulation results for different interpolation algorithms are shown in Figs. 6–8 and are summarized in Table 1. The curve speed fluctuation ratio for each intermediate point is calculated by $\eta_i = (F - V_i)/F$ where V_i is the curve speed from the interpolated point $C(u_i)$ to $C(u_{i+1})$ and F is the given feedrate command. According to the simulation results, most interpolation algorithms except the uniform interpolation result in high contouring accuracy with relatively small chord errors. As shown in Figs. 6–8,

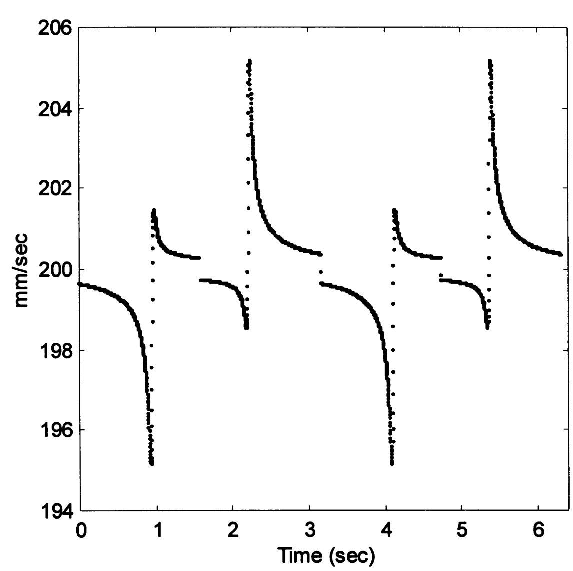

Fig. 6. Simulation results of the first-order approximation.

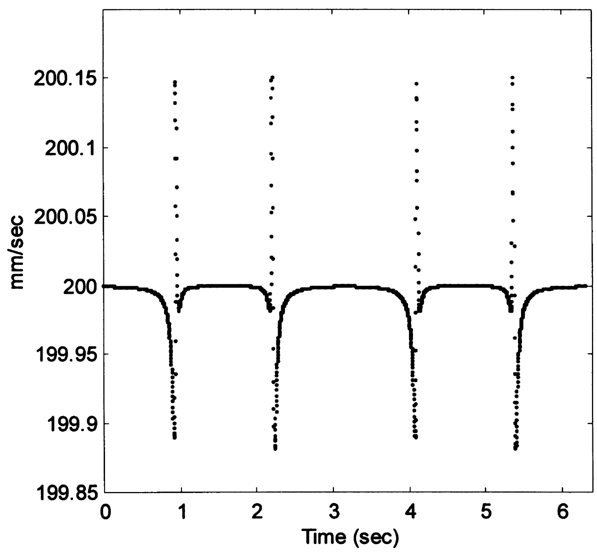

Fig. 7. Simulation results of the second-order approximation.

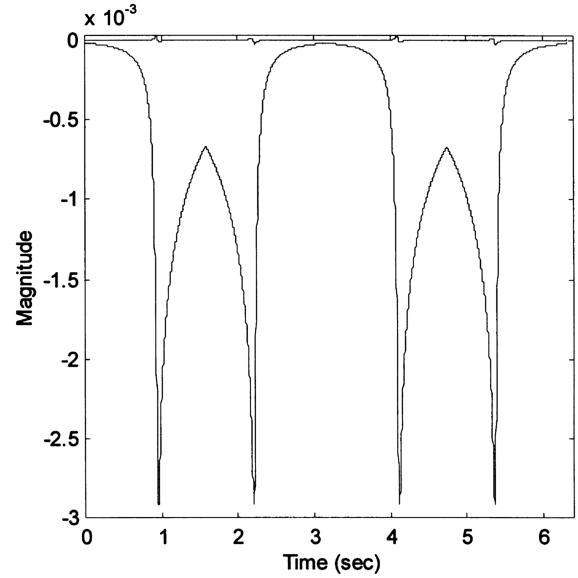

Fig. 9. Roots of quadratic equation $\epsilon_{1,2}(u_i)$.

all the maximum speed deviation occurs at the four sharp corners. Moreover, the speed-controlled interpolation algorithm provides the best curve speed accuracy during the interpolating process. Simulation results show that the curve speed deviates within [−0.004, 0.004] mm/s by applying the present speed-controlled interpolation algorithm. However, the second-order approximation achieves larger speed deviation within [−0.2, 0.2] mm/s and the first-order approximation achieves the largest speed deviation within [−5, 5] mm/s.

By applying the present algorithm, the roots of the quadratic equation for the compensatory value are also shown in Fig. 9. Fig. 9 indicates that one root is near zero which is adopted here while the other is negative and relatively large. The curve speed accuracy of the interpolator by applying the uniform interpolation algorithm is the worst case because the undefined real map operation of curve and parameters is not uniform. Although the present interpolation algorithm takes a longer period to compute the compensatory value, its processing time can be significantly improved with an updated CPU to achieve desirable performance.

4.3. The ACC/DEC mode

In order to avoid shock or vibration of mechanical systems when starting and slowing down the axial travel, ACC/DEC algorithms are required during the machining process. The conventional ACC/DEC algorithms for parametric curves usually result in larger radial and chord errors.

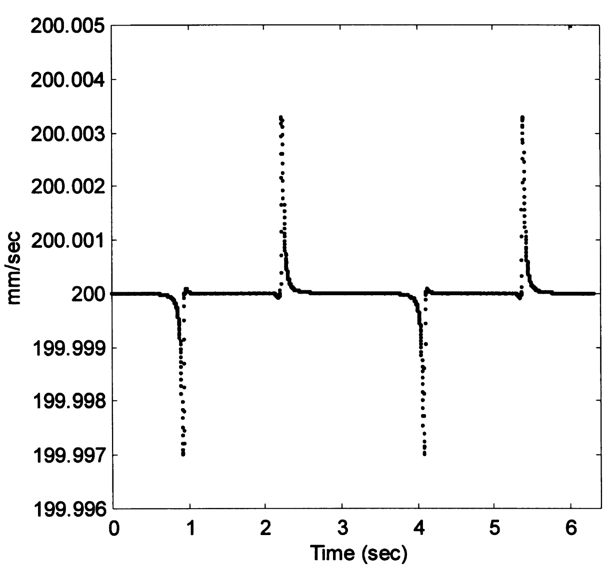

Fig. 8. Simulation results of the present constant-speed interpolation.

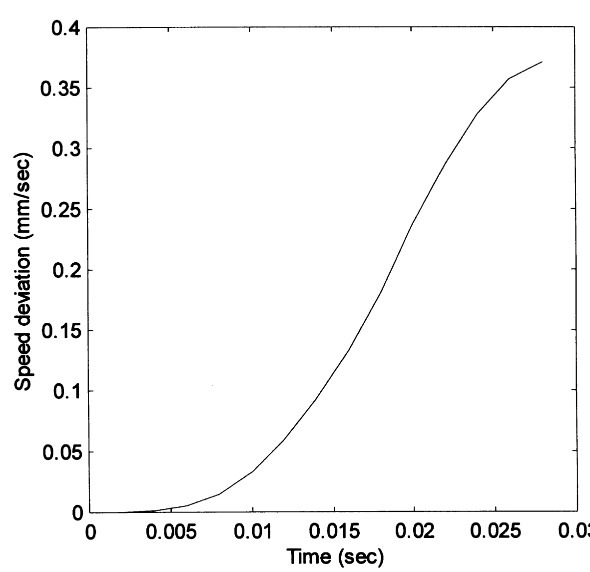

Fig. 10. Speed error of the first-order approximation during acceleration.

Table 1
Simulation results for different interpolation algorithms

Type	Measure				
	Mean-square of speed error $(mm/s)^2$	The maximum speed fluctuation ratio	The maximum chord error (mm)	Computation time with 80 MHz CPU (μs)	Computation time with 200 MHz CPU (μs)
Uniform	9.346×10^{6}	151.5245	0.036053	11	4
First order approximation [1,6,9]	1.3718	0.02583	0.0036181	27	10
Second order approximation [1,6,9]	6.1091×10^{-4}	0.00201	0.003545	60	19
Constant speed	1.679×10^{-7}	1.6398×10^{-5}	0.003540	77	26

Table 2
Measurement of the maximum curve speed error

Interpolation	ACC type		
	Linear (mm/s)	Parabolic (mm/s)	Exponential (mm/s)
First order approximation	3.7364×10^{-1}	3.7152×10^{-1}	4.0231×10^{-1}
Second order approximation	7.0317×10^{-4}	6.9518×10^{-4}	8.1802×10^{-4}
Speed − controlled	1.309×10^{-6}	1.2868×10^{-6}	1.6420×10^{-6}

Kim [8] proposed the parametric ACC/DEC algorithm. As the mapping operation between the curve and the parameter is not uniform in nature, the ACC/DEC in the uniform parameter interpolation algorithm cannot obtain the desirable acceleration and deceleration.

The curve speed errors in different acceleration profiles—linear, parabolic, and exponential profiles with different interpolations as (a) the first-order approximation, (b) the second-order approximation, and (c) the speed-controlled interpolation—are compared. The curve speed errors by applying the parabolic ACC/DEC method are shown in Figs. 10–12 and are also summarized in Table 2. Results indicate that the largest speed error is caused by applying the exponential ACC/DEC method and the smallest curve speed error is obtained by applying the parabolic ACC/DEC method. According to the simulation results, the curve speed is also changed parabolically during the acceleration process. Results also indicate that the present interpolation algorithm achieves the best speed accuracy with error magnitude 3–5 order less than when other methods are used.

In general, ACC/DEC functions are required in real

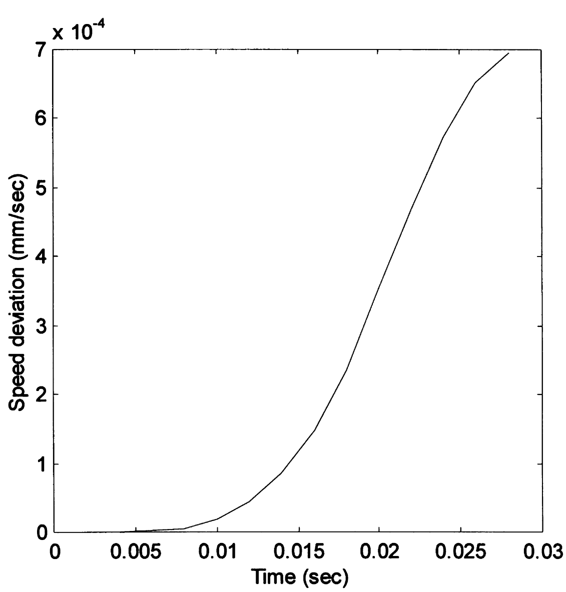

Fig. 11. Speed error of the second-order approximation during acceleration.

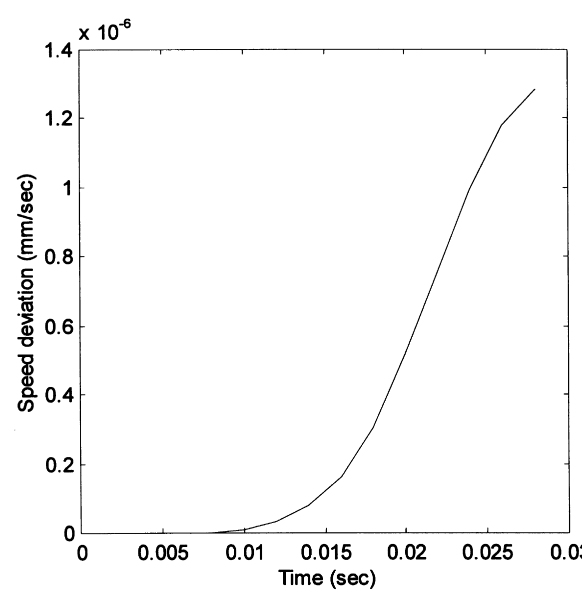

Fig. 12. Speed error of the speed-controlled interpolation during acceleration.

machining with different speed profiles, like linear, parabolic, and exponential types [5,8]. In this paper, the deceleration algorithm is the same as the acceleration algorithm except that the speed is decreasing during the process. Results indicate that the present speed-controlled interpolation algorithm achieves smooth acceleration and deceleration with the specified ACC/DEC speed profiles.

5. Conclusions

The proposed speed-controlled interpolation algorithm has significantly improved curve speed accuracy for parametric curves by including a compensatory value. The constant-speed mode operates the contiguous interpolated points with a constant curve speed and the ACC/DEC mode generates interpolated points successively with smooth speed variation. The proposed interpolation algorithm provides the best accuracy in both position and speed in real machining processes. Although the processing time of the proposed interpolation algorithm is a little longer than the others, it has been successfully implemented with faster computers.

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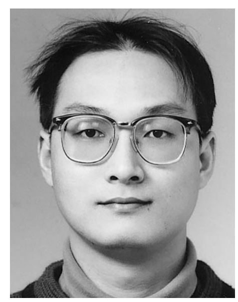

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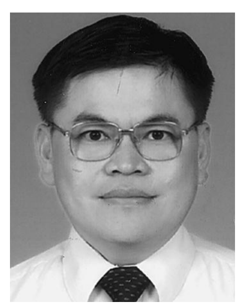

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