15 April 1999

OPTICS COMMUNICATIONS

ELSEVIER

Optics Communications 162 (1999) 237–240

One-dimensional self-focusing in photorefractive $Bi_{12}SiO_{20}$ crystal: theoretical modeling and experimental demonstration

Jinn Tsai [a,*], Arthur Chiou [b], Tai-Chiung Hsieh [c], Ken Hsu [a]

[a] *Institute of Electro-Optical Engineering, National Chiao Tung University, Hsinchu, Taiwan*
[b] *Institute of Electrical Engineering, National Dong Hwa University, Hualien, Taiwan*
[c] *Department of Electro-Physics, National Chiao Tung University, Hsinchu, Taiwan*

Received 19 June 1998; received in revised form 16 February 1999; accepted 24 February 1999

Abstract

We have experimentally demonstrated one-dimensional self-focusing of a laser beam in a photorefractive $Bi_{12}SiO_{20}$ (BSO) crystal with an externally applied DC field. Fractional change in focal length larger than 50% was measured when the biased field was varied from 2 kV/cm to 8 kV/cm. Based on the band transport model, thin crystal approximation, and gradient index (GRIN) lens approximation, we derive a mathematical expression that describes the effect quantitatively.

PACS: 42.65.Jx; 42.65.Hw; 42.81.Ht
Keywords: Self-focusing; Photorefractive effect; GRIN lens

1. Introduction

The self-focusing of a laser beam (457 nm) in a photorefractive strontium barium niobate (SBN) crystal was first observed in 1993 [1], and an analytical theory was proposed [2]. In the case of two-dimensional self-focusing, the theoretical results agree fairly well with the experimental data [3–5], despite the fact that the experimental determination of the Gaussian beam profiles and their changes were measured at no more than a few (usually one or two) characteristic points. In this paper, we report experimental results on one-dimensional self-focusing of a laser beam (632.8 nm) in a photorefractive BSO crystal, and present a GRIN lens theory to explain the observed phenomenon. In our experiment, we measured the Gaussian beam profiles and their changes by repeatedly scanning the whole Gaussian profiles for many cycles. The experimental data were processed with statistical and numerical methods for further rejection of noise. Our theory is derived from the band transport model of photorefractive effect. The theory is simpler than that of the previous works, because the anisotropic property which is important in the two-dimensional case can be ignored in the one-dimensional case.

2. Self-focusing model

Self-focusing of a laser beam in a photorefractive crystal can be explained by the band transport model. In this model, electrons from the donors in the crystal illuminated by light are excited from the donor level in the band gap to conduction band where they can migrate as free carriers. The ionized donors serve as traps to re-capture the free carriers. The free carriers migrate via diffusion or drift effect and are re-captured by the traps where they are transported from the conduction band back to the donor level. When an external electric field is applied across the crystal, spatial charges from the electrons and the ionized donors generate an electric field to screen the externally applied electric field. The resulting field in the media will thus be stronger in the dark region (than in the bright region) and will induce a larger index change there. The

* Corresponding author. E-mail: u8324805@cc.nctu.edu.tw

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PII: S0030-4018(99)00114-5

range of screening can be controlled by illuminating the crystal with an additional uniform bias beam.

In this paper, we consider a transverse electro-optic configuration with the crystallographic orientation of the BSO crystal shown in Fig. 1. By assuming that the crystal is sufficiently thin, we can ignore the variation of the probe beam intensity in the direction of propagation. We also assume that the electric field and the beam intensity are independent of position along the $\langle -110\rangle$ direction. Thus, the current is mainly in the z direction, and the rate equation of the steady state for electrons in the band transport model can be simplified to

$$\frac{\partial J_z(z)}{\partial z} = 0, \tag{1}$$

where J_z is the current density along the z-axis. If the electric field is large enough, then the current is dominated by the drift mechanism and the current density equation in the band transport model can be written as

$$J_z(z) = e\mu n_e(z) E_z(z), \tag{2}$$

where e is the electronic charge, μ is the electronic mobility, n_e is the electron density, and E_z is the electric field along the z-axis. From Eqs. (1) and (2), the electron density and the current density can be related by

$$n_e(z) E_z(z) = \text{const.} \tag{3}$$

If the thermal excitation can be neglected and the photoexcitation is small, i.e., $N_D^+(z) \ll N_D$, then the rate equation for ionized donors in the band transport model can be simplified as

$$n_e(z) = \frac{N_D s}{N_D^+(z)\xi} I(z), \tag{4}$$

where N_D is the donor density, $N_D^+(z)$ is the density of ionized donors, s is the photoexcitation cross-section, ξ is the recombination rate, and $I(z)$ is the intensity of illuminating light [6]. If an electric field E_0 is applied in the $\langle 001\rangle$ direction of the crystal and the crystal is illuminated by a uniform light I_0, then $E_z(z) = E_0$. From the Poisson equation and assuming that $n_e(z) \ll N_A$, the density of the ionized donors

$$N_D^+(z) \approx N_A, \tag{5}$$

where N_A is the acceptor density. If the crystal is illuminated by a probe beam $I(z)$ in the $\langle 110\rangle$ direction, then the electric field, from Eqs. (3) and (4), can be written as

$$E_z(z) = \frac{[N_A + \delta N_D^+(z)] I_0}{N_A[I_0 + I(z)]} E_0, \tag{6}$$

where $\delta N_D^+(z)$ is the variation of density of the ionized donors from that of acceptors. If $\delta N_D^+(z) \ll N_A$, then

$$E_z(z) = \frac{I_0}{I_0 + I(z)} E_0. \tag{7}$$

The assumption about $\delta N_D^+(z)$ can be verified by the Poisson equation, i.e.,

$$\delta N_D^+(z) \cong \frac{\varepsilon E_0}{e} \frac{\partial}{\partial z}\left[\frac{I_0}{I_0 + I(z)}\right], \tag{8}$$

where ε is the dielectric constant and e is the charge of the carrier. To obtain a small value of $\delta N_D^+(z)$, the intensity distribution of the illuminating light must be smooth enough. In our experiments, the probe beam has a Gaussian profile, i.e.,

$$I(z) = I_p \exp\left(-\frac{2z^2}{w^2}\right), \tag{9}$$

where I_p is central peak intensity and w is the beam width at the crystal. If the beam width is large enough so that the gradient of light intensity on the right hand side of Eq. (8) is small, then the condition for Eq. (7) can be satisfied. By substituting Eq. (9) into Eq. (7), we obtain the distribution of the electric field in the crystal,

$$E_z(z) = \frac{E_0}{1 + u\exp(-2z^2/w^2)}, \tag{10}$$

where $u = I_p/I_0$ is the ratio of the peak intensity of the Gaussian beam to the intensity of the uniform beam. Since the electric field E_z is in the direction of $\langle 001\rangle$, the variation of the impermeability tensor can be written as

$$\Delta\left(\frac{1}{n^2}\right)_{i,j} = \begin{bmatrix} 0 & E_z r_{41} & 0 \\ E_z r_{41} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{11}$$

where r_{41} is the linear electro-optic coefficient of the BSO crystal. For the Gaussian beam polarized in the $\langle -110\rangle$ direction, the variation of index is

$$\Delta n = -\frac{1}{4} n_0^3 (-1\,1\,0) \Delta\left(\frac{1}{n^2}\right)_{i,j} \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, \tag{12}$$

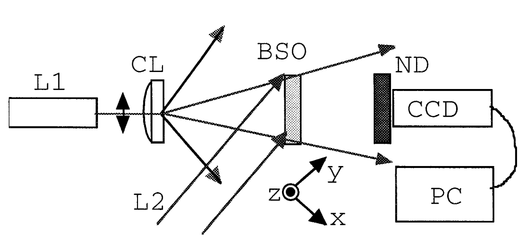

Fig. 1. A schematic diagram of the experimental system and the crystal orientation. L1: He–Ne laser for probe beam, L2: uniform beam from another He–Ne laser, CL: cylindrical lens, ND: neutral density filter, PC: computer.

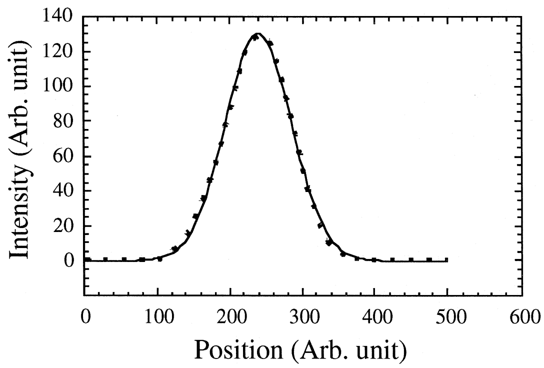

Fig. 2. The experimental curve (dotted line) and the fitted curve of a Gaussian profile with an applied electric field of 8 kV/cm.

where n_0 is the index of the crystal. Thus, from Eqs. (11) and (12), the variation of index can be written as

$$\Delta n = \frac{1}{2} n_0^3 r_{41} E_z(z). \tag{13}$$

With the quadratic GRIN lens approximation of Eq. (10), the index profile in the crystal is given by

$$n(z) = n_0 + \frac{1}{2} n_0^3 r_{41} \left[\frac{1}{1+u} + \frac{2u}{(1+u)^2} \frac{z^2}{w^2} \right] E_0. \tag{14}$$

From the second order term of Eq. (14), it follows that the index profile has a maximum gradient or focusing power when the beam ratio $u = 1$. Thus, we take this condition for the following derivations. Following the conventional notation of the GRIN lens, we can write the distribution of index as

$$n(z) = n_{00} \left(1 - \frac{A}{2} z^2 \right), \tag{15}$$

where

$$A \cong -\frac{n_0^2 r_{41} E_0}{2 w^2}, \tag{16}$$

$$n_{00} = n_0 \left[1 + \frac{n_0^2 E_0 r_{41}}{4} \right] \cong n_0 \tag{17}$$

To obtain a positive lens, the electrical field must be negative, i.e., its direction is opposite to the positive direction of the z-axis. Applying the theory of the GRIN lens, we can write the focal length as [7]

$$f = \frac{1}{n_0 \sqrt{A} \sin\left(d\sqrt{A} \right)}, \tag{18}$$

where d is the thickness of the crystal.

The self-induced GRIN lens will focus the Gaussian beam and change the beam profile. The focal length obtained from the above derivation can be confirmed by measuring the variation in beam width with respect to the electric field at a distance after the lens. From the ABCD law, the beam width at the observing plane in the presence of the electric field can be written as

$$w'^2 = \frac{\lambda \left[\left(l_2 - \frac{a_2}{a_2^2 + b_2^2} \right)^2 + \left(\frac{b_2}{a_2^2 + b_2^2} \right)^2 \right]}{\pi \left[b_2 / \left(a_2^2 + b_2^2 \right) \right]}, \tag{19}$$

where l_2 is the distance from the lens to the observation position.

$$a_2 = \frac{a_1^2 + b_1^2 - a_1 f}{f\left(a_1^2 + b_1^2 \right)}, \quad b_2 = \frac{b_1}{a_1^2 + b_1^2},$$

where

$$a_1 = l_1, \quad b_1 = \frac{\pi w_0^2}{\lambda},$$

where l_1 is the distance between beam waist and the lens, w_0 is the beam width of the input beam, and λ is the wavelength. Thus, the focal length can be derived from the beam-width measurement.

3. Experimental procedure and results

The experimental setup for measuring the variation of the 1D Gaussian beam is shown in Fig. 1. A He–Ne laser beam was expanded in the xy-plane by a cylindrical lens such that the beam profile on the crystal is approximately uniform in the ⟨1–10⟩ direction. The beam profile in the z direction was not modified. By measuring the beam width at two different locations, we obtained the beam waist w_0 (0.52 mm) along the z direction. The beam width w and

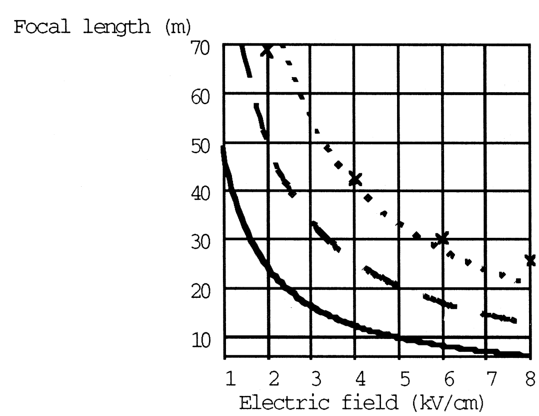

Fig. 3. The dependence of the focal length on the applied electric field. The crosses are the experimental data. The solid curve is the theoretical result without an additional correction factor. The dashed curve is the theoretical result with a correction factor. The dotted curve is the revised theoretical result with a revised value of electro-optical coefficient.

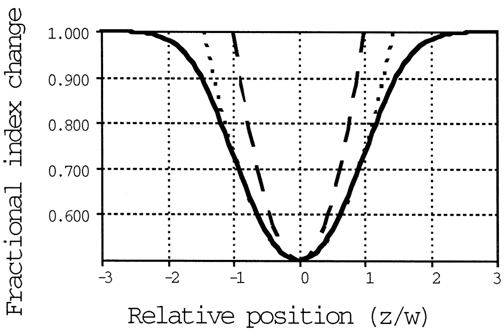

Fig. 4. The fitting results of index change as a function of the normalized position. The vertical axis indicates fractional index change which is relative to the index change caused only by the DC electrical field. The solid line is the original relation. The dashed line is the first order approximation without correction. The dotted line is the approximation with the correction factor of 0.48.

the intensity of the Gaussian peak at the crystal were about 0.76 mm and 140 μW/cm^2, respectively. A second beam from another He–Ne laser was expanded and collimated to form a uniform biased beam which illuminated the entire crystal. The intensity of the uniform beam was chosen to be the same as the peak of the Gaussian beam (140 μW/cm^2). The thickness d, the refractive index n_0, and the electro-optic coefficient r_{41} of the crystal are 4 mm, 2.54, and 3.6×10^{-12} m/V [8], respectively. The electric field applying on the crystal was in the z direction. A neutral density filter was placed in front of the CCD to avoid saturation. The distance between the beam waist and the BSO crystal was 140 cm, and that from the crystal to the observation plane was 60 cm. To reduce the random error in measuring the Gaussian beam width, the intensity profile was averaged over 400 CCD scan lines, and the beam width w was obtained by curve fitting the intensity profile with a biased Gaussian function

$$I(z) = I_p \exp\left[-\frac{2(z-z_0)^2}{w^2} \right] + I_b,$$

where I_p is the peak intensity, z_0 is the position of the peak, and I_b is the bias or background intensity noise. Fig. 2 shows an example of the experimental curve (dotted line) and the fitted curve of a Gaussian profile when the applied electric field is 8 kV/cm. The correlation coefficient is 0.99928. The dependence of the focal length on the applied electric field is shown in Fig. 3 where the crosses are the experimental data and the solid line is the theoretical result from Eq. (15). Apparently, although the theoretical curve shows the correct trend, it deviates significantly from the experimental data. We notice that if Eq. (16) is multiplied by a factor of 0.48 to account for the negligence of the higher order terms, then the revised theoretical curve (the dashed line in Fig. 3) will be closer to the experimental data. The correction factor (0.48) was obtained by curve fitting the fractional refractive index change as approximated by Eq. (14) to that prescribed by Eqs. (10) and (13) in the region of the beam width. The fractional index change is derived from the index change in Eq. (13) divided by that caused only by the DC electrical field E_0. The fitting results of the fractional index change as a function of the normalized position are shown in Fig. 4. The solid line is the original relation prescribed by Eqs. (10) and (13). The dashed line is the approximated relation (14) without correction. The dotted line is the relation with the correction factor (0.48). In addition, if we assume the electro-optical coefficient to be 2.2×10^{-12} (rather than 3.6×10^{-12} m/V), the theoretical result will be much closer to the experimental data. This revised curve is shown as the dotted line in Fig. 3.

4. Conclusions

We have experimentally demonstrated one-dimensional self-focusing effect of a laser beam in a photorefractive crystal. Fractional change in focal length larger than 50% was measured when the biased field was varied from 2 kV/cm to 8 kV/cm. Based on the band transport model and with a gradient index (GRIN) lens approximation, we have presented a mathematical expression that describes the effect quantitatively.

Acknowledgements

This study was supported by the National Science Council, R.O.C. under contract number: NSC-87-2215-E-009-011.

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